These problems are from the G & W

Problem 3.1

Let f denote the original image. First subtract the minimum value of f denoted $f_{\min}$ from f to yield a function whose minimum value is 0:

$$g_1 = f - f_{\min}$$

Next divide g_1 by its maximum value to yield a function in the range $[0, 1]$ and multiply the result by (L-C-1) to yield a function with values in the range [C, L-1]

$$\begin{aligned} g &= \frac{L-1\ -C}{\max(g1)} g1 \quad + C \\ &= \frac{L-1\ -C}{\max(f - f_{\min})}(f - f_{\min}) \quad + C \end{aligned}$$

Keep in mind that $f_{\min}$ is a scalar and f is an image.

Problem 3.5

(a) The number of pixels having different intensity level values would decrease, thus causing the number of components in the histogram to decrease. Because the number of pixels would not change, this would cause the height of some of the remaining histogram peaks to increase in general. Typically, less variability in intensity level values will reduce contrast.

Problem 3.6

All that histogram equalization does is remap histogram components on the intensity scale. To obtain a uniform (flat) histogram would require in general that pixel intensities actually be redistributed so that there are L groups of n/L pixels with the same intensity, where L is the number of allowed discrete intensity levels and $n = MN$ is the total number of pixels in the input image. The histogram equalization method has no provisions for this type of (artificial) intensity redistribution process.

Problem 3.19

(a) Numerically sort the n^2 values. The median is

$$\zeta = [(n^2 + 1)/2]\text{-th largest value.}$$

(b) Once the values have been sorted one time, we simply delete the values in the trailing edge of the neighborhood and insert the values in the leading edge in the appropriate locations in the sorted array.

Problem 3.23

Order of operations can be changed for the linear operators without changing the final results.

Problem 5.1

The solutions are shown in Fig. P5.1, from left to right.

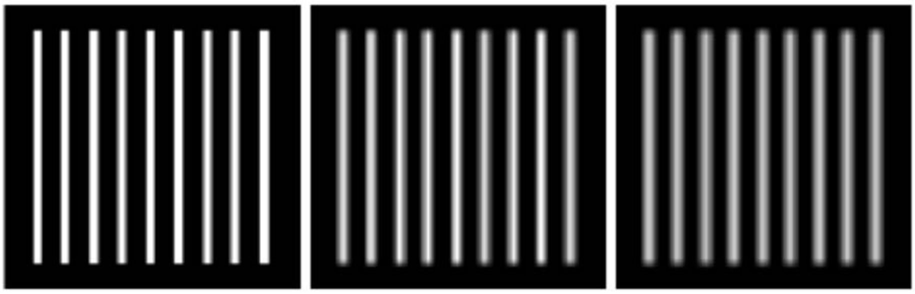

Figure P5.1

Problem 9.7

(a) The dilated image will grow without bound.

(b) A one-element set (i.e., a one-pixel image).

Problem 9.33

(a) From Eq. (9.6-1),

$$
\begin{aligned}
(f \ominus b)^c &= \left[\min_{(s,t)\in b}\{f(x+s,y+t)\}\right]^c \\
&= \left[-\max_{(s,t)\in b}\{-f(x+s,y+t)\}\right]^c \\
&= \max_{(s,t)\in b}\{-f(x+s,y+t)\} \\
&= -f \oplus \hat{b} \\
&= f^c \oplus \hat{b}.
\end{aligned}
$$

The second step follows from the definition of the complement of a gray-scale function; that is, the minimum of a set of numbers is equal to the negative of the maximum of the negative of those numbers. The third step follows from the definition of the complement. The fourth step follows from the definition of gray-scale dilation in Eq. (9.6-2), using the fact that $\hat{b}(x,y) = b(-x-y)$. The last step follows from the definition of the complement, $-f = f^c$. The other duality property is proved in a similar manner.

Eqns 9.6-1 & 2 are given below:

$$[f \ominus b](x,y) = \min_{(s,t) \in b}\{f(x+s, y+t)\}$$

$$[f \oplus b](x,y) = \max_{(s,t) \in b}\{f(x-s, y-t)\}$$

Problem 9.35

(a) The noise spikes are of the general form shown in Fig. P9.35(a), with other possibilities in between. The amplitude is irrelevant in this case; only the shape of the noise spikes is of interest. To remove these spikes we perform an opening with a cylindrical structuring element of radius greater than $R_{\max}$, as shown in Fig. P9.35(b). Note that the shape of the structuring element is matched to the known shape of the noise spikes.

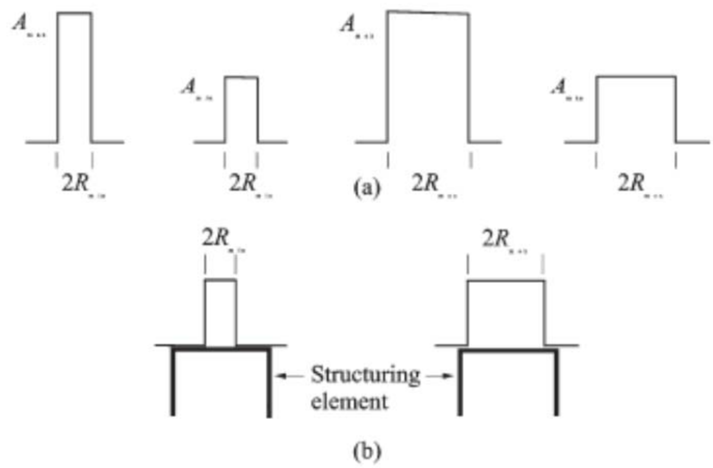

Figure P9.35

Problem 10.2

The masks would have the coefficients shown in Fig. P10.2. Each mask would yield a value of 0 when centered on a pixel of an unbroken 3-pixel segment oriented in the direction favored by that mask. Conversely, the response would be a +2 when a mask is centered on a one-pixel gap in a 3-pixel segment oriented in the direction favored by that mask.

Horizontal			Vertical			$+45^\circ$			-45°		
0	0	0	0	1	0	0	0	1	1	0	0
1	−2	1	0	−2	0	0	−2	0	0	−2	0
0	0	0	0	1	0	1	0	0	0	0	1

Figure P10.2

Problem 10.11

(a) The operators are as follows (negative numbers are shown underlined):

111	110	$10\underline{1}$	$0\underline{1}\underline{1}$	$\underline{1}\underline{1}\underline{1}$	$\underline{1}\underline{1}0$	$\underline{1}01$	011
000	$10\underline{1}$	$10\underline{1}$	$10\underline{1}$	000	$\underline{1}01$	$\underline{1}01$	$\underline{1}01$
$\underline{1}\underline{1}\underline{1}$	$0\underline{1}\underline{1}$	$10\underline{1}$	110	111	011	$\underline{1}01$	$\underline{1}\underline{1}0$

Problem 10.14

(a) We proceeed as follows

$$
\begin{aligned}
\text{Average}\left[\nabla^2 G(x,y)\right] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \nabla^2 G(x,y)\,dx\,dy \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\frac{x^2+y^2-2\sigma^2}{\sigma^4}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}\,dx\,dy \\
&= \frac{1}{\sigma^4}\int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2\sigma^2}}\,dx \int_{-\infty}^{\infty} e^{-\frac{y^2}{2\sigma^2}}\,dy \\
&\qquad + \frac{1}{\sigma^4}\int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2\sigma^2}}\,dy \int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}}\,dx \\
&\qquad - \frac{2}{\sigma^2}\int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2\sigma^2}}\,dx\,dy \\
&= \frac{1}{\sigma^4}\left(\sqrt{2\pi}\sigma \times \sigma^2\right)\left(\sqrt{2\pi}\sigma\right) \\
&\qquad + \frac{1}{\sigma^4}\left(\sqrt{2\pi}\sigma \times \sigma^2\right)\left(\sqrt{2\pi}\sigma\right) \\
&\qquad - \frac{2\left(2\pi\sigma^2\right)}{\sigma^2} \\
&= 4\pi - 4\pi \\
&= 0
\end{aligned}
$$

the fourth line follows from the fact that

$$\text{variance}(z) = \sigma^2 = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2\sigma^2}} dz$$

and

$$\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{-\frac{z^2}{2\sigma^2}} dz = 1.$$

Problem 10.15

(**b**) The answer is yes for functions that meet certain mild conditions, and if the zero crossing method is based on rotational operators like the LoG function and a threshold of 0. Geometrical properties of zero crossings in general are explained in some detail in the paper "On Edge Detection," by V. Torre and T. Poggio, *IEEE Trans. Pattern Analysis and Machine Intell.*, vol. 8, no. 2, 1986, pp. 147-163. Looking up this paper and becoming familiar with the mathematical underpinnings of edge detection is an excellent reading assignment for graduate students.

Problem 10.16

$$\frac{dG}{dr} = \frac{-r}{\sigma^2} e^{\frac{-r^2}{2\sigma^2}}$$

$$\frac{d^2G}{dr^2} = \frac{-1}{\sigma^2} e^{\frac{-r^2}{2\sigma^2}} + \frac{r^2}{\sigma^4} e^{\frac{-r^2}{2\sigma^2}}$$

This is different from the correct answer!

You can derive with *x* and *y* and see the difference. Can you explain the reason for this difference?

Problem 10.22

To transform from Cartesian to Polar,

$$x = \rho cos\theta$$

$$y = \rho sin\theta$$

Problem 10.23

(a) Origin has coordinates $x = 0$ and $y = 0$. Substituting into Eq. (10.2-38) yields $\rho = 0$, which, in a plot of ρ vs. θ, is a straight line.

(b) Only the origin $(0, 0)$ would yield this result.

Eq 10.2-38: $x cos\theta + y sin\theta = \rho$

When we map a particular point (x, y) to Hough space, x and y are constants:

$$\rho = x cos\theta + y sin\theta = A sin(\theta + \beta)$$

Hence, ρ and θ are sinusoidally related.

where $A = \sqrt{x^2 + y^2}$; $x = A sin(\beta)$; $y = A cos(\beta)$

Problem 11.1

(a) The key to this problem is to recognize that the value of every element in a chain code is relative to the value of its predecessor. The code for a boundary that is traced in a consistent manner (e.g., clockwise) is a unique circular set of numbers. Starting at different locations in this set does not change the structure of the circular sequence. Selecting the smallest integer as the starting point simply identifies the same point in the sequence. Even if the starting point is not unique, this method would still give a unique sequence. For example, the sequence 101010 has three possible starting points, but they all yield the same smallest integer 010101.

Problem 11.16

This problem can be solved by using two descriptors: holes and the convex deficiency (see Section 9.5.4 regarding the convex hull and convex deficiency of a set). The decision making process can be summarized in the form of a simple decision, as follows: If the character has two holes, it is an 8. If it has one hole it is a 0 or a 9. Otherwise, it is a 1 or an X. To differentiate between 0 and 9 we compute the convex deficiency. The presence of a "significant" deficiency (say, having an area greater than 20% of the area of a rectangle that encloses the character) signifies a 9; otherwise we classify the character as a 0. We follow a similar procedure to separate a 1 from an X. The presence of a convex deficiency with four components whose centroids are located approximately in the North, East, West, and East quadrants of the character indicates that the character is an X. Otherwise we say that the character is a 1. This is the basic approach. Implementation of this technique in a real character recognition environment has to take into account other factors such as multiple "small" components in the convex deficiency due to noise, differences in orientation, open loops, and the like. However, the material in Chapters 3, 9 and 11 provide a solid base from which to formulate solutions.

Problem 12.2

From the definition of the Euclidean distance,

$$D_j(\mathbf{x}) = \left\|\mathbf{x} - \mathbf{m}_j\right\| = \left[(\mathbf{x} - \mathbf{m}_j)^T(\mathbf{x} - \mathbf{m}_j)\right]^{1/2}$$

Because $D_j(\mathbf{x})$ is non-negative, choosing the smallest $D_j(\mathbf{x})$ is the same as choosing the smallest $D_j^2(\mathbf{x})$, where

$$\begin{aligned} D_j^2(\mathbf{x}) &= \left\|\mathbf{x} - \mathbf{m}_j\right\|^2 = (\mathbf{x} - \mathbf{m}_j)^T(\mathbf{x} - \mathbf{m}_j) \\ &= \mathbf{x}^T\mathbf{x} - 2\mathbf{x}^T\mathbf{m}_j + \mathbf{m}_j^T\mathbf{m}_j \\ &= \mathbf{x}^T\mathbf{x} - 2\left(\mathbf{x}^T\mathbf{m}_j - \frac{1}{2}\mathbf{m}_j^T\mathbf{m}_j\right). \end{aligned}$$

We note that the term $\mathbf{x}^T\mathbf{x}$ is independent of j (that is, it is a constant with respect to j in $D_j^2(\mathbf{x})$, $j = 1, 2, \ldots$). Thus, choosing the minimum of $D_j^2(\mathbf{x})$ is equivalent to choosing the maximum of $\left(\mathbf{x}^T\mathbf{m}_j - \frac{1}{2}\mathbf{m}_j^T\mathbf{m}_j\right)$.

Problem 12.8

(a)

$$\mathbf{m}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{m}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{m}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{C}_1 = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{C}_1^{-1} = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad |\mathbf{C}_1| = 0.25$$

and

$$\mathbf{C}_2 = 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad \mathbf{C}_2^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad |\mathbf{C}_2| = 4.00.$$

Eqn: 2.2-26: $d_j^{'}(\mathbf{x}) = -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln\left|\mathbf{C}_j\right| - \frac{1}{2}\left[(\mathbf{x}-\mathbf{m}_j)^T \mathbf{C}_j^{-1}(\mathbf{x}-\mathbf{m}_j)\right] + \ln p(\omega_j)$

Because the covariance matrices are not equal, it follows from Eq. (12.2-26) that

$$\begin{aligned} d_1(\mathbf{x}) &= -\frac{1}{2}\ln(0.25) - \frac{1}{2}\left\{\mathbf{x}^T \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \mathbf{x}\right\} \\ &= -\frac{1}{2}\ln(0.25) - (x_1^2 + x_2^2) \end{aligned}$$

and

$$\begin{aligned} d_2(\mathbf{x}) &= -\frac{1}{2}\ln(4.00) - \frac{1}{2}\left\{\mathbf{x}^T \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \mathbf{x}\right\} \\ &= -\frac{1}{2}\ln(4.00) - \frac{1}{4}(x_1^2 + x_2^2) \end{aligned}$$

where the term $\ln P(\omega_j)$ was not included because it is the same for both decision functions in this case. The equation of the Bayes decision boundary is

$$d(\mathbf{x}) = d_1(\mathbf{x}) - d_2(\mathbf{x}) = 1.39 - \frac{3}{4}(x_1^2 + x_2^2) = 0.$$

(**b**) Figure P12.8 shows a plot of the boundary.

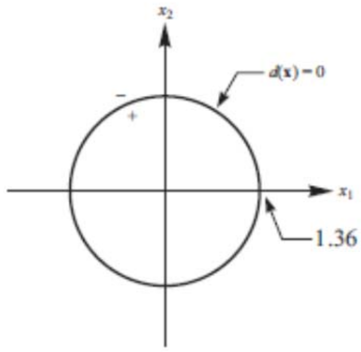

Figure P12.8