Existential quantification and closed vs. open worlds

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March 2019

Inference rule from last week

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   utool(Step,Tool),
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- However, our data usually doesn't specify both the tool and the property. One or the other is mentioned alone.

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- This is a powerful inference, since it lets us reason about one or more individuals that aren't specified.
- But Datalog doesn't give us full existential quantification for reasoning.

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```
needstool(Recipe,something,Property) :-
   requires(Recipe,Ingredient),
   satisfies(Ingredient, Property).
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- We deduce that some tool is needed for dicing, skinning, and trimming.
- But we can't deduce that we need a knife for those actions.
- Under a closed world assumption we would take it as false that a knife is needed for this recipe.

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- But these days many information systems rely on loosely coupled, widely distributed data.
- Systems are designed for uses across institutional boundaries.
- Stakeholders cooperate on maintaining and integrating machine readable data.

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- Understanding how this works requires the concept of an *interpretation.*

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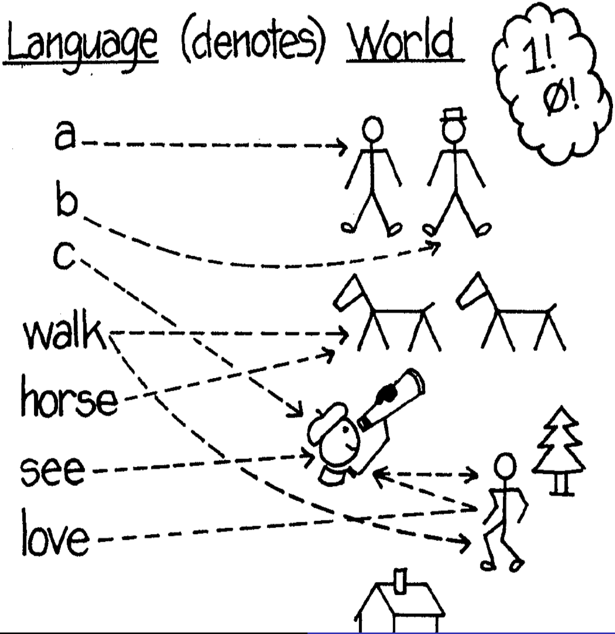
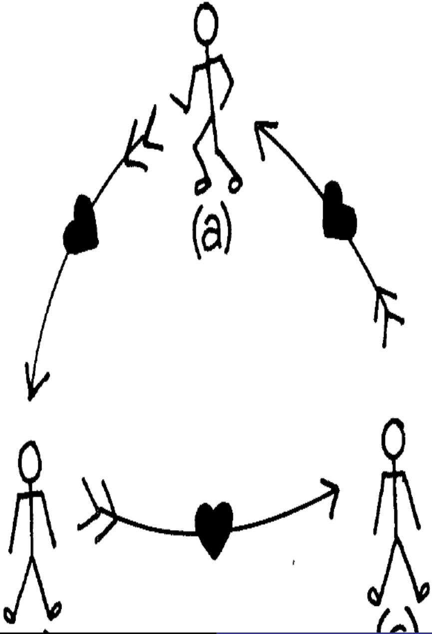
3. Meanings are things that aren't language.


Language (denotes) World
1!
Ø!
a
b
c
walk
horse
see
love

Language and the world



Syntax of Bach's logic subset

$\langle var \rangle ::= w|x|y|z$

$\langle con \rangle ::= a|b|c|d$

$\langle ter \rangle ::= \langle var \rangle | \langle con \rangle$

$\langle 1pp \rangle ::= Run|Walk|Happy|Calm$

$\langle 2pp \rangle ::= Love|Kiss|Like|See$

$\langle wff \rangle ::= \langle 1pp \rangle(\langle ter \rangle)$

$\langle wff \rangle ::= \langle 2pp \rangle(\langle ter \rangle, \langle ter \rangle)$

$\langle wff \rangle ::= -\langle wff \rangle$

$\langle wff \rangle ::= (\langle wff \rangle \vee \langle wff \rangle)$

$\langle wff \rangle ::= (\langle wff \rangle \, \& \, \langle wff \rangle)$

$\langle wff \rangle ::= \forall \langle var \rangle \, \langle wff \rangle$

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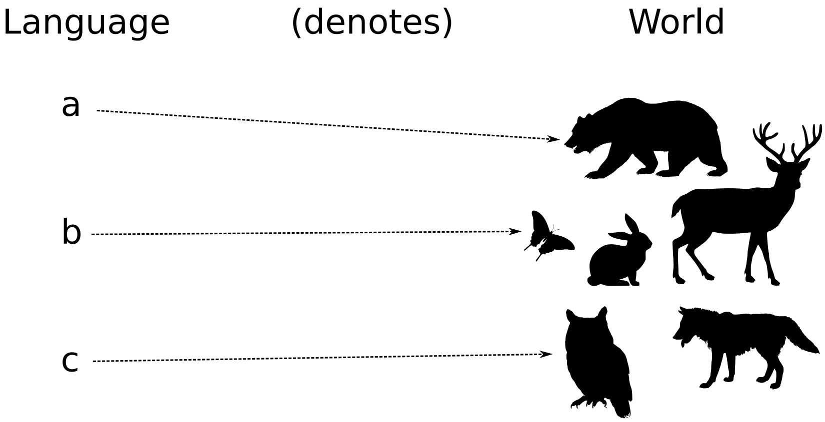
- Bach calls his structure an interpretation, so I'll use I for that.

Language and the world



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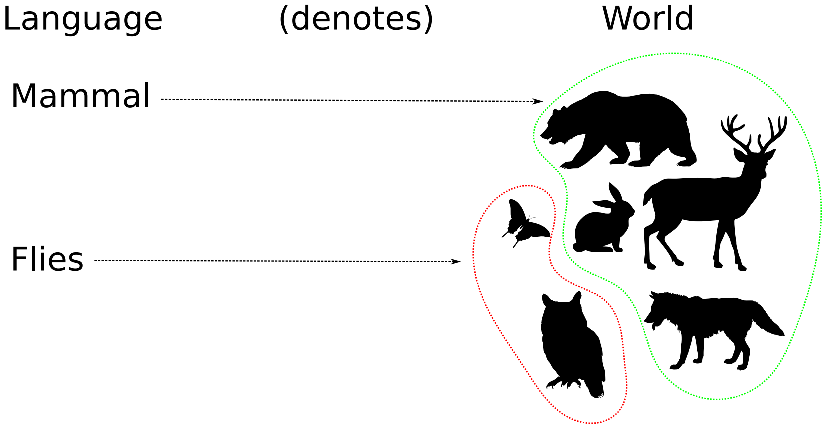
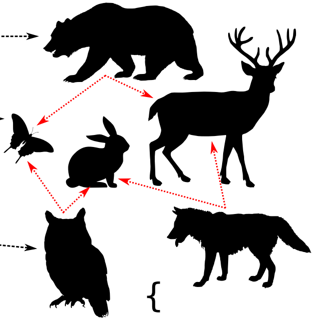


Language (denotes) World

a

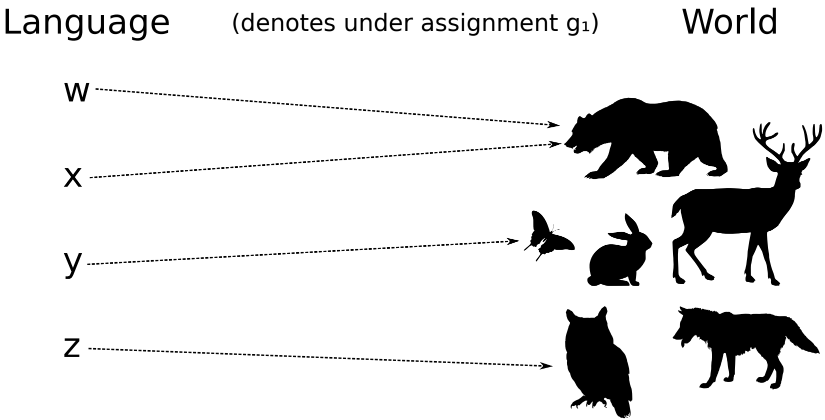
b

c



Eats

$\{\langle a_D, b_D\rangle, \langle c_D, b_D\rangle, \ldots\}$


Language
(denotes under assignment g_1)
World
w
x
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z

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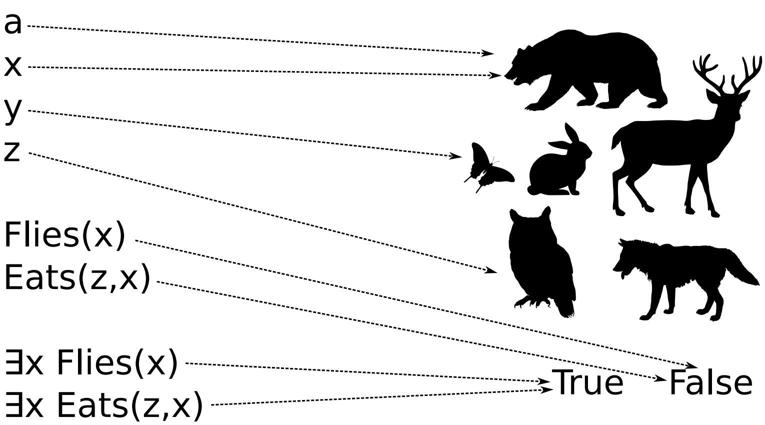

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- Toward the end of the reading he suggests adding a set of times and a set of possible worlds for more expressive languages.

Simple denotations

Individual constants (like a and b) will denote individuals in the domain, so they'll be elements of Bach's set E. The one-place predicates (classes) will denote their extensions (i.e., subsets of the domain). The two-place predicates (binary relations) will denote sets of ordered pairs.

$D(\langle con\rangle) \in E$

$D(\langle 1pp\rangle) \subseteq E$

$D(\langle 2pp\rangle) \subseteq E \times E$

Following a common convention, we adopt the notation φ_D for $D(\varphi)$. So, for example, a_D would be some individual in the Domain set E, and $Love_D$ would be a set of ordered pairs $\langle x, y\rangle$, in each of which x loves y.

Variable assignments

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- Because our functions are cartesian product subsets, two or more different variables can be mapped to the same individual. For example, maybe $g_3 = \{\langle w, a_D\rangle, \langle x, a_D\rangle, \langle y, b_D\rangle, \langle z, a_D\rangle\}$ and $g_4 = \{\langle w, c_D\rangle, \langle x, c_D\rangle, \langle y, c_D\rangle, \langle z, c_D\rangle\}$. But they're functions, so every variable in the domain will be included.

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- For example, $g_1[x{:=}d_D] = \{\langle w, a_D\rangle, \langle x, d_D\rangle, \langle y, c_D\rangle, \langle z, d_D\rangle\}$

Denotation, support, and truth

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- $I \models_g -\varphi$ iff it is not the case that $I \models_g \varphi$

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Denotation, support, and truth

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- $I \models_g \forall v \varphi$ iff for all $e \in E$ it holds that $I \models_{g[v:=e]} \varphi$
- $I \models_g \exists v \varphi$ iff for at least one $e \in E$ it holds that $I \models_{g[v:=e]} \varphi$