# Propositional Logic

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(2.4)

If you take my medication, you will get better
But you are not taking my medication.

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So, you will not get better. $\therefore$

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- van Benthem, et al. contrast arguments like this invalid one:

$$\text{(2.4)}\quad \frac{\begin{array}{l}\text{If you take my medication, you will get better}\\ \text{But you are not taking my medication.}\end{array}}{\text{So, you will not get better.} \qquad \therefore}$$

- ...with this valid argument:

$$\text{(2.6)}\quad \frac{\begin{array}{l}\text{If you take my medication, you will get better}\\ \text{But you are not getting better.}\end{array}}{\text{So, you have not taken my medication.} \qquad \therefore}$$

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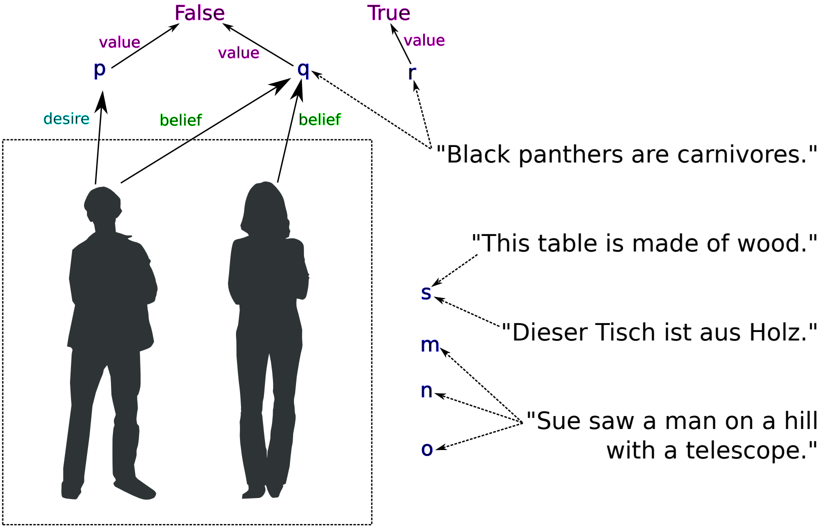
# Reasoning vs. representation

- In this class we'll give some attention to logical reasoning and inference.
- The same rules that govern valid argumentation also enable information systems to answer interesting questions.
- But it takes an entire semester course in logic before you start to get good at logical inference.
- This semester we'll focus mainly on logic as a means to describe a *domain* (some part of the world we're interested in).

# Propositions, statements, and sentences



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- States of affairs are the parts of reality responsible for making propositions true or false.

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- A set of proposition letters generates a set of states of affairs: ways the world might be.
- $n$ proposition letters generate $2^n$ states of affairs.
- $\{pqr, pq\overline{r}, p\overline{q}r, p\overline{qr}, \overline{p}qr, \overline{p}q\overline{r}, \overline{pq}r, \overline{pqr}\}$

# Logical Operators

Table 1: 2.15 from van Benthem, et al.

| Symbol | In natural language | Technical name |
|---|---|---|
| $\neg$ | not | negation |
| $\wedge$ | and | conjunction |
| $\vee$ | or | disjunction |
| $\rightarrow$ | if . . . then | implication |
| $\leftrightarrow$ | if and only if | equivalence |

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- The sentence is mapped to a truth value via the following tables

# Semantics of the operators

| $\varphi$ | $\neg\varphi$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

Table 3: 2.18 from van Benthem, et al.

| $\varphi$ | $\psi$ | $\varphi \wedge \psi$ | $\varphi \vee \psi$ | $\varphi \to \psi$ | $\varphi \leftrightarrow \psi$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |

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- $e =$ "A foreign national has legal employment."
- $l =$ "He has had legal employment less than three years ago."
- $a =$ "He is currently employed abroad."
- $s =$ "He is entitled to social security."

# Drawing truth tables for expressions

| p | q | r | (( | $\neg$ | p | $\vee$ | q | ) | $\rightarrow$ | r | ) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | 0 | 1 | 1 | 1 | | 1 | 1 | |
| 1 | 1 | 0 | | 0 | 1 | 1 | 1 | | 0 | 0 | |
| 1 | 0 | 1 | | 0 | 1 | 0 | 0 | | 1 | 1 | |
| 1 | 0 | 0 | | 0 | 1 | 0 | 0 | | 1 | 0 | |
| 0 | 1 | 1 | | 1 | 0 | 1 | 1 | | 1 | 1 | |
| 0 | 1 | 0 | | 1 | 0 | 1 | 1 | | 0 | 0 | |
| 0 | 0 | 1 | | 1 | 0 | 1 | 0 | | 1 | 1 | |
| 0 | 0 | 0 | | 1 | 0 | 1 | 0 | | 0 | 0 | |

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| p | q | r | (( | ¬ | p | ∨ | q | ) → | r | ) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | 0 | 1 | 1 | 1 | 1 | 1 | |
| 1 | 1 | 0 | | 0 | 1 | 1 | 1 | 0 | 0 | |
| 1 | 0 | 1 | | 0 | 1 | 0 | 0 | 1 | 1 | |
| 1 | 0 | 0 | | 0 | 1 | 0 | 0 | 1 | 0 | |
| 0 | 1 | 1 | | 1 | 0 | 1 | 1 | 1 | 1 | |
| 0 | 1 | 0 | | 1 | 0 | 1 | 1 | 0 | 0 | |
| 0 | 0 | 1 | | 1 | 0 | 1 | 0 | 1 | 1 | |
| 0 | 0 | 0 | | 1 | 0 | 1 | 0 | 0 | 0 | |

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|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | 0 | 1 | 1 | 1 | | 1 | 1 | |
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| 1 | 0 | 0 | | 0 | 1 | 0 | 0 | | 1 | 0 | |
| 0 | 1 | 1 | | 1 | 0 | 1 | 1 | | 1 | 1 | |
| 0 | 1 | 0 | | 1 | 0 | 1 | 1 | | 0 | 0 | |
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| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |

# Exercise 2.11

| p | q | (( p | → | q ) | ∨ | ( q | → | p )) |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |

# Exercise 2.11

| $p$ | $q$ | $r$ | $(((p$ | $\lor$ | $\neg$ | $q)$ | $\land$ | $r)$ | $\leftrightarrow$ | $(\neg$ | $(p$ | $\land$ | $r)$ | $\lor$ | $q))$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |

# Grammar of propositional logic

Let $P$ be a set of proposition letters and let $p \in P$.
The following expression defines the recursive grammar for a logical expression $\varphi$ in Backus–Naur Form:
$\varphi ::= p|\neg\varphi|(\varphi \wedge \varphi)|(\varphi \vee \varphi)|(\varphi \rightarrow \varphi)|(\varphi \leftrightarrow \varphi)$

# Syntactically conforming expressions

Let $P = \{o, q, r, s\}$
Examples of grammatically conforming expressions include:

- $r$

Grammatically *incorrect* expressions would include:

How many correct expressions are consistent with the last one?

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- $(\neg(s \leftrightarrow \neg\neg\neg o) \rightarrow (q \wedge q))$

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- $\vee)p\neg$
- $\neg p \vee q \rightarrow r$

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# Syntactic ambiguity

These conforming expressions are all consistent with $\neg p \vee q \rightarrow r$

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- $(\neg(p \vee q) \to r)$
- $\neg((p \vee q) \to r)$
- $(\neg p \vee (q \to r))$

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- $\neg((p \vee q) \rightarrow r)$
- $(\neg p \vee (q \rightarrow r))$
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- $p\neg q$

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- "$V(\varphi) = 1$" means the formula (or sentence) $\varphi$ is true in the state of affairs represented by the function $V$. "$V(\varphi) = 0$" means that $\varphi$ is false in the state of affairs represented by the function $V$.

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- For "$V(\varphi) = 1$" we also write "$V \models \varphi$" read as "V is a model of $\varphi$" or "V satisfies $\varphi$."
- If V doesn't satisfy $\varphi$ we write "$V \not\models \varphi$". In other words $V(\varphi) = 0$.

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# Consistency

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- A set of propositional logic statements is consistent if at least one state of affairs satisfies every statement in the set.
- A set of propositional logic statements is inconsistent if no state of affairs satisfies every statement in the set.

# Inference and validity

- A conclusion is *valid* with respect to a set of premises if the conclusion is true in every sitation where the premises are true (van Benthem, et al, page 2-4).

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- A conclusion is *valid* with respect to a set of premises if the conclusion is true in every sitation where the premises are true (van Benthem, et al, page 2-4).
- One can validly infer a conclusion $\varphi$ from a set of premises $P$ if the negation of $\varphi$ is inconsistent with the set of statements $P$.

# Computation and expressive power

(From van Benthem, et al., chapter 2)

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1. Computing a truth value for a formula takes linear time.
2. Computing a truth table for validity takes exponential time.
3. The problem of testing for validity in propositional logic is decidable: there exists a mechanical method that computes the answer, at least in principle.