Artificial Languages, Part 2 (Semantics)

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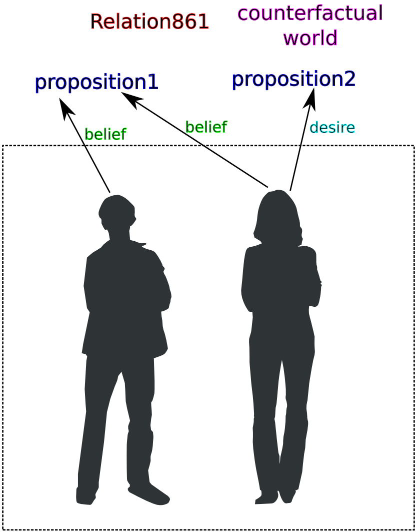
April 3, 2017

We're used to thinking about language content and meaning as things in our minds: mental representations.



The Platonistic view we read in Jubien treats language meaning as abstract objects outside our minds and relationships with those objects.

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Bach on Semantics

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3. Meanings are things that aren't language.

Language and the world

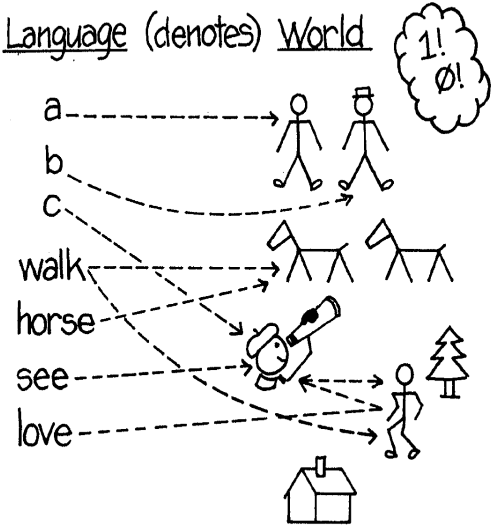

Figure 1: page 10 figure from Bach (1989)

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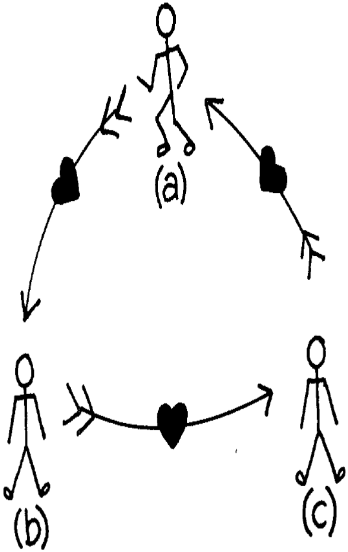

Figure 2: page 12 figure from Bach (1989)

Syntax of Bach's logic subset

$\langle var \rangle ::= w|x|y|z$
$\langle con \rangle ::= a|b|c|d$
$\langle ter \rangle ::= \langle var \rangle | \langle con \rangle$
$\langle 1pp \rangle ::= Run|Walk|Happy|Calm$
$\langle 2pp \rangle ::= Love|Kiss|Like|See$
$\langle wff \rangle ::= \langle 1pp \rangle (\langle ter \rangle)$
$\langle wff \rangle ::= \langle 2pp \rangle (\langle ter \rangle, \langle ter \rangle)$
$\langle wff \rangle ::= -\langle wff \rangle$
$\langle wff \rangle ::= (\langle wff \rangle \vee \langle wff \rangle)$
$\langle wff \rangle ::= (\langle wff \rangle \ \& \ \langle wff \rangle)$
$\langle wff \rangle ::= \forall \langle var \rangle \ \langle wff \rangle$
$\langle wff \rangle ::= \exists \langle var \rangle \ \langle wff \rangle$

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- Bach calls his structure an interpretation, so I'll use I for that.

Summary of notation

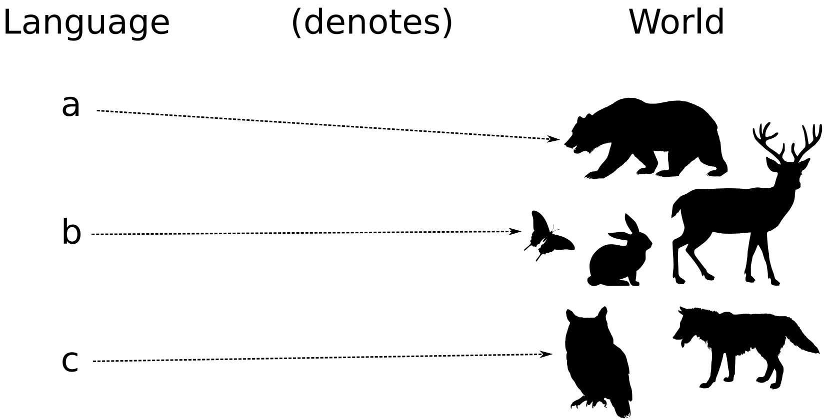
Meaning	van Benthem	Bach	Dubin
and	$\wedge$	&	&
not	$\neg$	—	—
or	$\vee$	$\vee$	$\vee$
predicate	Pxy	Word(x,y)	Word(x,y)
domain	D	E	E
denotation	I	D	D
referent	φ_I^g	$D(\varphi)$	$D(\varphi)$ or φ_D^g
var asgts	$g \in G$	$g \in G$	$g \in G$

The semantics of propositional logic vs. predicate logic

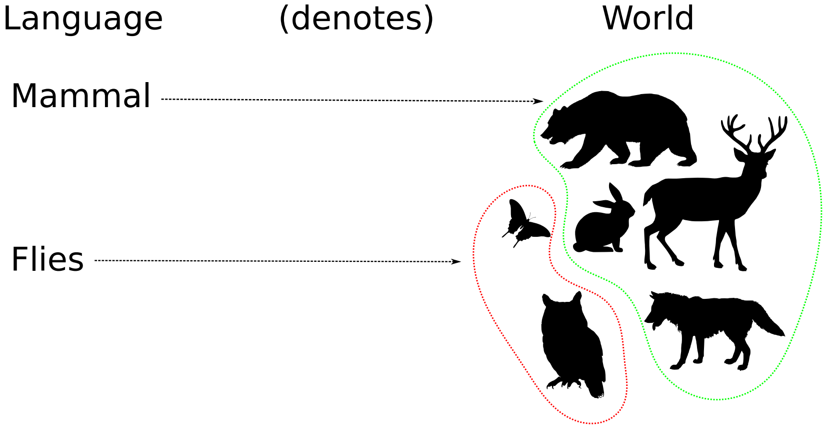
van Benthem, et al. write:

> *In propositional logic, the link was the valuation mapping proposition letters to truth values. But this will no longer do. For checking whether a statement saying that a certain object has a certain property, or that certain objects are in a certain relation is true we need something more refined. Instead of just saying that "John is boy" is assigned the value true, we now need an interpretation for "John" and an interpretation for "being a boy".*

Recall that the only non-language view of the world or domain that we needed consisted of truth values and propositions that concerned domain entities. Now we're going to include domain elements themselves in our model, not just propositions concerning them.


Language
(denotes)
World
a
b
c

Language and the world



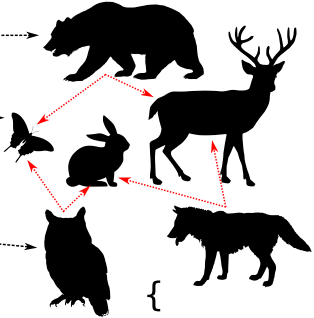
Language and the world

Language (denotes) World

a

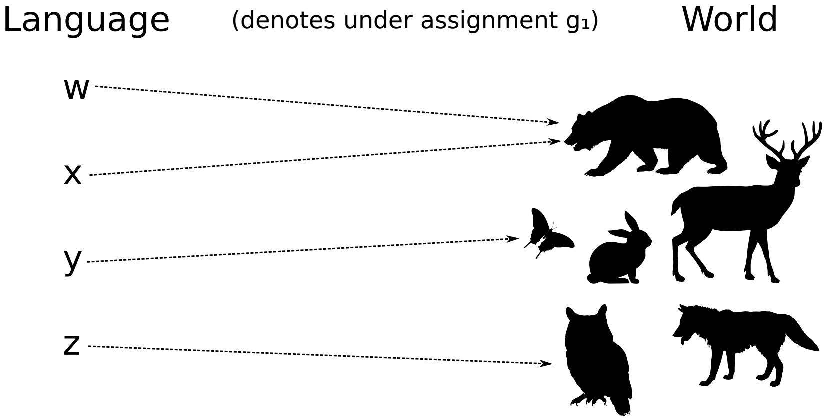
b

c

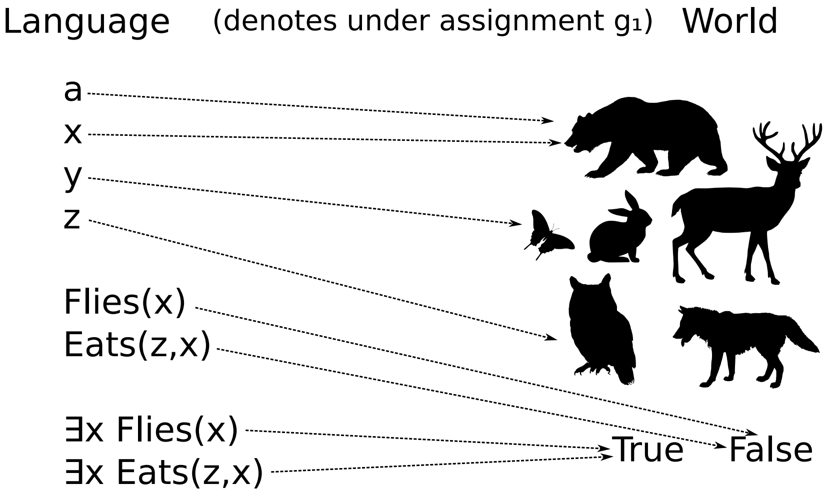


Eats

$$\{ \langle a_D, b_D \rangle, \langle c_D, b_D \rangle, \ldots \}$$


Language
(denotes under assignment g₁)
World
w
x
y
z

Language and the world



Models and interpretations

Bach writes:

> *An interpretation is a way of assigning denotations in a certain model structure to expressions in a language.*

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  - Set E of individuals (van Benthem's D);
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- Toward the end of the reading he suggests adding a set of times and a set of possible worlds for more expressive languages.

Simple denotations

Individual constants (like a and b) will denote individuals in the domain, so they'll be elements of (Bach's) set E. The one-place predicates (classes) will denote their extensions (i.e., subsets of the domain). The two-place predicates (binary relations) will denote sets of ordered pairs.

$D(\langle con\rangle) \in E$

$D(\langle 1pp\rangle) \subseteq E$

$D(\langle 2pp\rangle) \subseteq E \times E$

Following the convention in your logic readings, we adopt the notation φ_D for $D(\varphi)$. So, for example, a_D would be some individual in the Domain set E, and $Love_D$ would be a set of ordered pairs $\langle x, y\rangle$, in each of which x loves y.

Variable assignments

- Let V be the set of variables in the language. Bach's G is a set of variable assignments, and each $g \in G$ is a function from variables ($v \in V$) to individuals in E, i.e.: $g \subseteq V \times E$.

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- For example, suppose $g_1 = \{\langle w, a_D \rangle, \langle x, b_D \rangle, \langle y, c_D \rangle, \langle z, d_D \rangle\}$, but $g_2 = \{\langle w, b_D \rangle, \langle x, c_D \rangle, \langle y, d_D \rangle, \langle z, a_D \rangle\}$.

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- Because our functions are cartesian product subsets, two or more different variables can be mapped to the same individual. For example, maybe $g_3 = \{\langle w, a_D\rangle, \langle x, a_D\rangle, \langle y, b_D\rangle, \langle z, a_D\rangle\}$ and $g_4 = \{\langle w, c_D\rangle, \langle x, c_D\rangle, \langle y, c_D\rangle, \langle z, c_D\rangle\}$. But they're functions, so every variable in the domain will be included.

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Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

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- $I \models_g \exists v \varphi$ iff for at least one $e \in E$ it holds that $I \models_{g[v:=e]} \varphi$

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  - Epistemic logic, which concern what must or might be true, based on what we know.
- Modal logics use predicate logic notation, along with two new symbols:
  - The $\Box$ operator is read "necessarily."
  - The $\Diamond$ operator is read "possibly."

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.
- Possible worlds are part of the semantics of modal logics such as:
  - Alethic logics, in which necessary truth and falsehood are distinguished from contingent truth and falsehood.
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- Modal logics use predicate logic notation, along with two new symbols:
  - The $\Box$ operator is read "necessarily."
  - The $\Diamond$ operator is read "possibly."
  - For example, $\forall x \Diamond \exists y (Px \vee \neg \Box Ryx)$

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    3. "New employees must present a Social Security number at the time of hire or immediately thereafter."