From graphs to predicates

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- Propositions are the objects of *propositional attitudes*. They are the kinds of things that can be believed, desired, doubted, expected, or feared.

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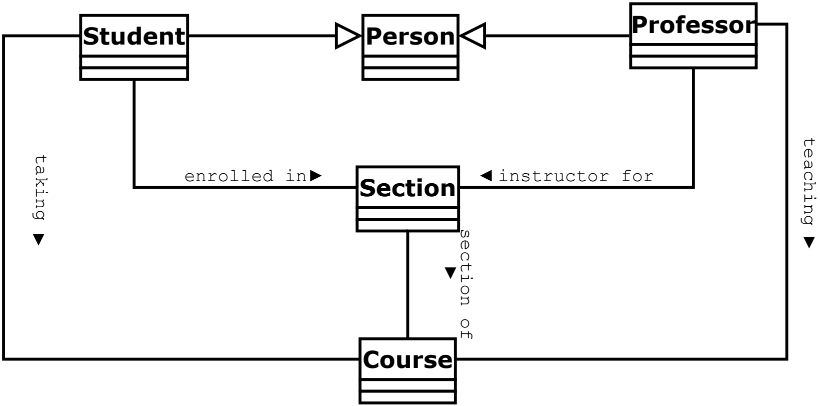
- Lower case letters from the end of the alphabet (like x and y) are variables that can denote different individuals under different assignments—just like variables in algebraic expressions.
- Capital letters represent properties that an individual might have, classes they might belong to, or relations they might stand in. Think of them like relations in a relational database.

UML Class Diagram



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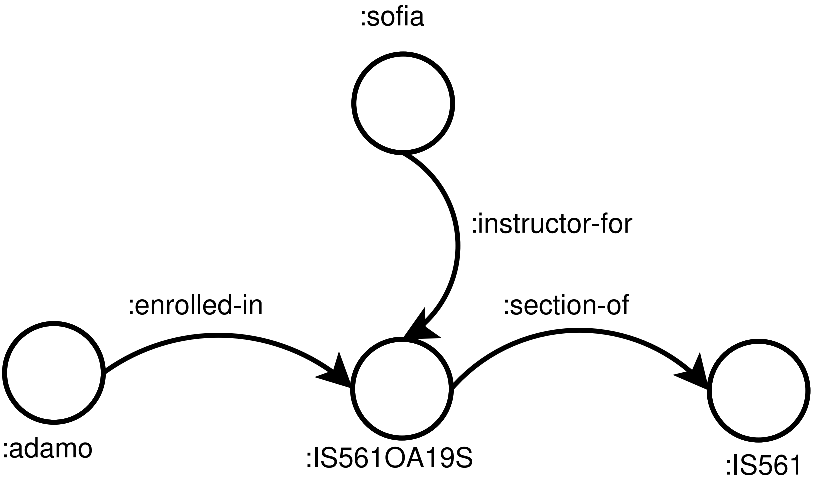
- o means "IS561-OA Spring 2019."

Instance diagram



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- IS561-OA is a section of IS561: *Oom*

Logical Operators

Symbol	In natural language	Technical name
$\neg$	not	negation
$\wedge$	and	conjunction
$\vee$	or	disjunction
$\rightarrow$	if ... then	implication
$\leftrightarrow$	if and only if	equivalence

Examples of predicate logic expressions

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- $\forall x (Vx \leftrightarrow \exists z Lxz)$ means "a lover is someone who loves."

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- We can express the second as $\forall y(Sy \rightarrow \exists x(Cx \land Tyx))$. On this interpretation, every student was taking some course, but no particular course was necessarily taken by every student.

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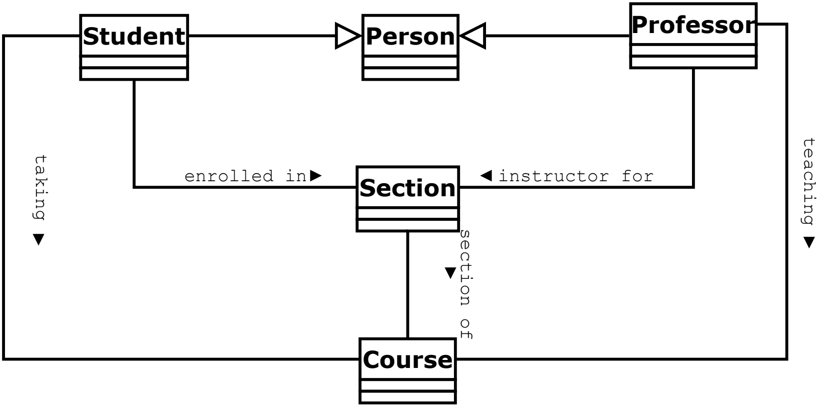
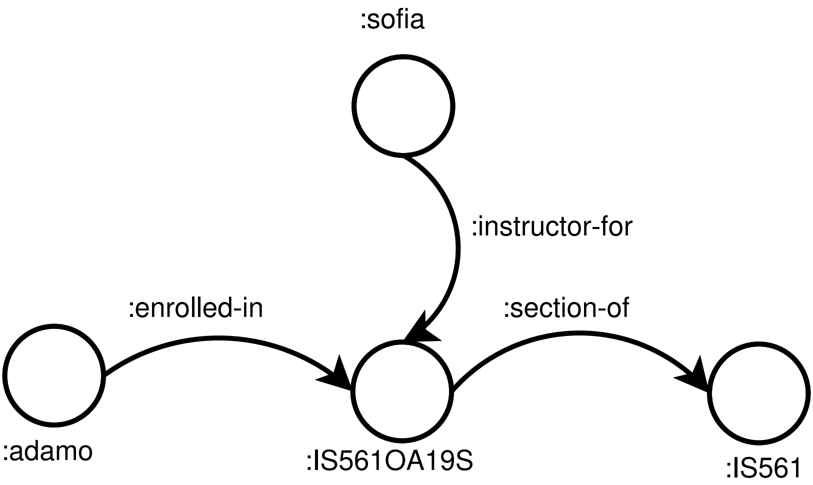


Back to the Instance diagram



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