Artificial Languages, Part 1 (Syntax)

Dave Dubin

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5. The cost of processing language is measured in processing time and memory.
6. The more expressive our language, the more expensive it will be to run our software over it.
7. Classifications of languages (like the Chomsky hierarchy) help us recognize what kind of software we need to accomplish our processing goals.

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Bottom-up parse

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- A conforming expression should have *some* path to our start symbol, but how do we program software to make the right choices?

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- It turns out that some languages are easy to parse: consider, for example, the set of strings consisting of the letter 'b'.

An easy URL subset grammar

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- with one of those slashes being the rightmost character.
- We can easily parse this from left to right, and quit right away if one of the rules is broken.

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- `http://www.whatever.something.com/abc/cba/wxy/qrs/`

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- **http://www.whatever.something.**`com/abc/cba/wxy/qrs/`
- **http://www.whatever.something.com/**`abc/cba/wxy/qrs/`
- **http://www.whatever.something.com/abc/**`cba/wxy/qrs/`
- **http://www.whatever.something.com/abc/cba/**`wxy/qrs/`

Parsing a URL

- http://www.whatever.something.com/abc/cba/wxy/qrs/
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- **http://www.whatever.something.com/abc/cba/wxy/**`qrs/`
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- **Success!**

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- Our first diagram is almost deterministic, but may still require some backtracking.
- The second diagram adds some additional states and arcs, but is completely deterministic.

Nondeterministic Finite State Automaton

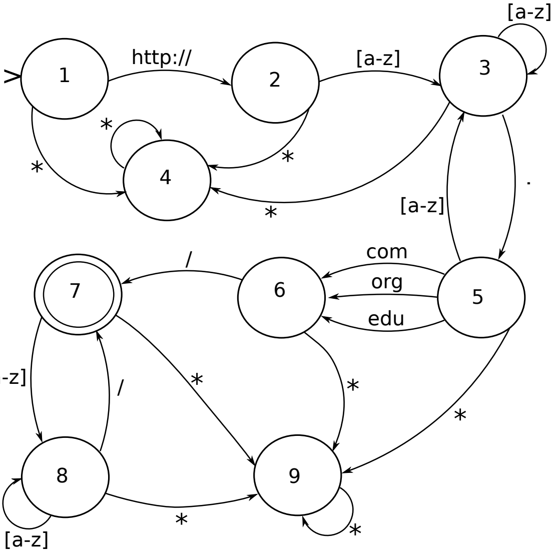

Figure 1: NFA Parser for URL grammar

Deterministic Finite State Automaton

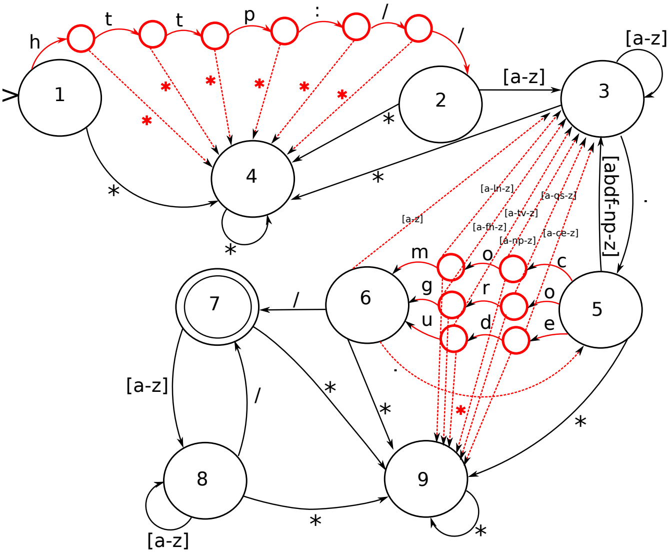

Figure 2: DFA Parser for URL grammar

Top-down derivation

Derivation (a term you read in Rosen) is parsing in reverse.

- φ

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- $\forall\mathbf{v}\varphi$
- $\forall\mathbf{v}(\varphi \leftrightarrow \varphi)$

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- $\forall \mathbf{v} \varphi$
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- $\forall \mathbf{v}(\textbf{Atom} \leftrightarrow \exists \mathbf{v}\textbf{Atom})$
- $\forall \mathbf{v}(\textbf{Pt} \leftrightarrow \exists \mathbf{v}\textbf{Atom})$

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- $\forall \mathbf{v}(\mathbf{Pt} \leftrightarrow \exists \mathbf{vPtt})$

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- $\forall \mathbf{v}(\mathbf{Pt} \leftrightarrow \exists \mathbf{vAtom})$
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- $\forall \mathbf{v}(\mathbf{Pv} \leftrightarrow \exists \mathbf{v} L \mathbf{vv})$
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- $\forall \mathbf{v}(V \mathbf{v} \leftrightarrow \exists z L \mathbf{v} z)$

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- $\forall \mathbf{v}(V \mathbf{v} \leftrightarrow \exists \mathbf{v} L \mathbf{vv})$
- $\forall \mathbf{v}(V \mathbf{v} \leftrightarrow \exists z L \mathbf{v} z)$
- $\forall x(Vx \leftrightarrow \exists z Lxz)$

Formalization: Rosen on vocabularies

A vocabulary (or alphabet) V is a finite nonempty set of elements, called symbols. A word (or sentence) over V is a string of finite length of elements of V. The empty string or null string, denoted by λ, is the string containing no symbols. The set of all words over V is denoted by V^*. A language over V is a subset of V^*.

Rosen on grammars

A phrase-structure grammar $G = \langle V, T, S, P \rangle$ *consists of a vocabulary* V*, a subset* T *of* V *consisting of terminal elements, a start symbol* S *from* V*, and a set of productions* P*. The set* $V - T$ *is denoted by* N*. Elements of* N *are called nonterminal symbols. Every production in* P *must contain at least one nonterminal on its left side.*

Phrase-structure grammar

```
<protocol> ::= http://
<letter>   ::= a|b|c|d|e|f|g|h|i|j|k|l|m
<letter>   ::= n|o|p|q|r|s|t|u|v|w|x|y|z
<slash>    ::= /
<dot>      ::= .
<string>   ::= <letter><string>|<letter>
<host>     ::= <string><dot><host>|<string><dot>
<domain>   ::= com|org|edu
<site>     ::= <host><domain><slash>
<dir>      ::= <string><slash>
<body>     ::= <dir><body>|<dir>
<url>      ::= <protocol><site><body>|<protocol><site>
```

Rosen on languages and derivability

Let $G = \langle V, T, S, P\rangle$ be a phrase-structure grammar. Let $w_0 = lz_0r$ (that is, the concatenation of l, z_0, and r) and $w_1 = lz_1r$ be strings over V. If $z_0 \rightarrow z_1$ is a production of G, we say that w_1 is directly derivable from w_0 and we write $w_0 \Rightarrow w_1$. If $w_0, w_1, \ldots w_n, n \geq 0$, are strings over V such that $w_0 \Rightarrow w_1, w_1 \Rightarrow w_2, \ldots w_{n-1} \Rightarrow w_n$, then we say that w_n is derivable from w_0, and we write $w_0 \stackrel{*}{\Rightarrow} w_n$. The sequence of steps used to obtain w_n from w_0 is called a derivation.

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Let $G = \langle V, T, S, P \rangle$ be a phrase-structure grammar. The language generated by G (or the language of G), denoted by $L(G)$, is the set of all strings of terminals that are derivable from the starting state S. In other words, $L(G) = \{w \in T^* | S \stackrel{*}{\Rightarrow} w\}$.

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Per Rosen, section 10.1:

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- A *type 3* (regular) grammar can have productions only of the form $w_1 \rightarrow w_2$ with $w_1 = A$, and either $w_2 = aB$ or $w_2 = a$, where A and B are nonterminal symbols and a is a terminal symbol, or with $w_1 = S$ and $w_2 = \lambda$.

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- Parsing non-regular, context free languages like our logic grammar always requires a stack memory, and often requires backtracking.
- We can usually tell that a language is not regular by seeing whether its productions meet Rosen's constraints.

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- $\mathbf{t} ::= \mathbf{v}|\mathbf{c}$
- $\mathbf{P} ::= P|Q|R|\ldots$

Context free, non-regular grammars

- $\mathbf{v} ::= x|y|z|\ldots$
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- $\mathbf{Atom} ::= \mathbf{Pt}_1 \dots \mathbf{t}_n$ where n is the arity of $\mathbf{P}$
- $\varphi ::= \mathbf{Atom}|\neg\varphi|(\varphi \wedge \varphi)|(\varphi \vee \varphi)|(\varphi \rightarrow \varphi)|(\varphi \leftrightarrow \varphi)|\forall\mathbf{v}\varphi|\exists\mathbf{v}\varphi$

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- Some grammars that don't satisfy Rosen's type 3 definition still summarize regular languages.
- It can be challenging to verify whether a language is regular just by looking at the productions.

```
<protocol> ::= http://
<letter>   ::= a|b|c|d|e|f|g|h|i|j|k|l|m
<letter>   ::= n|o|p|q|r|s|t|u|v|w|x|y|z
<slash>    ::= /
<dot>      ::= .
<string>   ::= <letter><string>|<letter>
<host>     ::= <string><dot><host>|<string><dot>
<domain>   ::= com|org|edu
<site>     ::= <host><domain><slash>
<dir>      ::= <string><slash>
<body>     ::= <dir><body>|<dir>
<url>      ::= <protocol><site><body>|<protocol><site>
```

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- $\langle var \rangle$ ::= $q|r|s|t|u|v|w|x|y|z$
- $\langle pred \rangle$ ::= $A|B|C|D|E|F|G|H|I|J|K|L|M$

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- $\langle pred \rangle ::= N|O|P|Q|R|S|T|U|V|W|X|Y|Z$

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- $\langle pred \rangle ::= A|B|C|D|E|F|G|H|I|J|K|L|M$
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- $\langle rp \rangle ::=)$

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- $\langle term \rangle$ ::= $\langle const \rangle | \langle var \rangle$

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- $\langle term \rangle$ ::= $\langle const \rangle | \langle var \rangle$
- $\langle atom \rangle$ ::= $\langle pred \rangle \langle term \rangle | \langle atom \rangle \langle term \rangle$
- $\langle phi \rangle$ ::= $\langle atom \rangle | \langle not \rangle \langle phi \rangle | \langle quant \rangle \langle var \rangle \langle phi \rangle$
- $\langle phi \rangle$ ::= $\langle lp \rangle \langle phi \rangle \langle binop \rangle \langle phi \rangle \langle rp \rangle$

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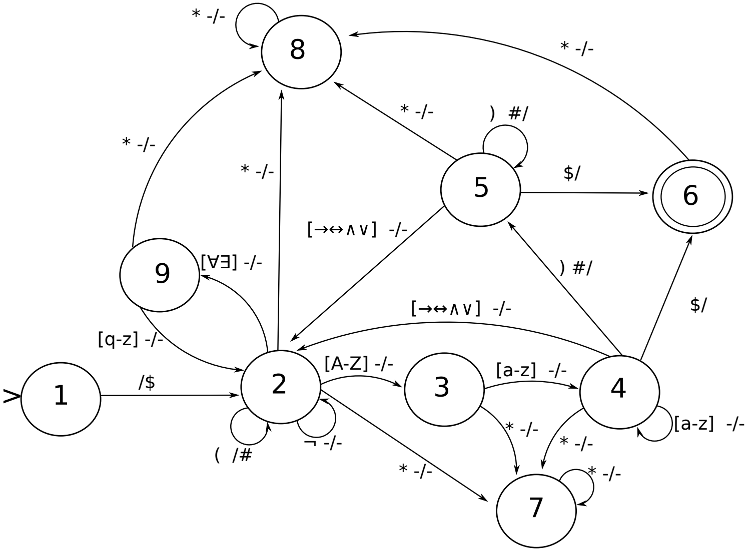

Figure 3: PDA Parser for the predicate logic grammar