Artificial Languages, Part 2 (Semantics)

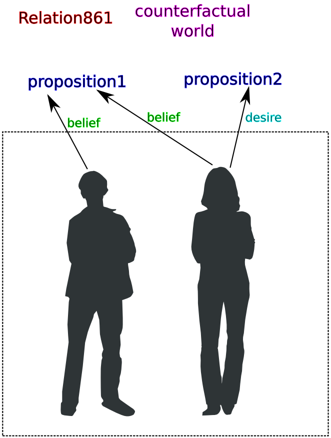
Dave Dubin

April 3, 2017

Language and the mind

We're used to thinking about language content and meaning as things in our minds: mental representations.

Language and the world



The Platonistic view we read in Jubien treats language meaning as abstract objects outside our minds and relationships with those objects.

Bach on Semantics

1. The structure of language can each be described using formal systems.

Bach on Semantics

1. The structure of language can each be described using formal systems.
2. Language has meaning.

Bach on Semantics

1. The structure of language can each be described using formal systems.
2. Language has meaning.
3. Meanings are things that aren't language.

Language and the world

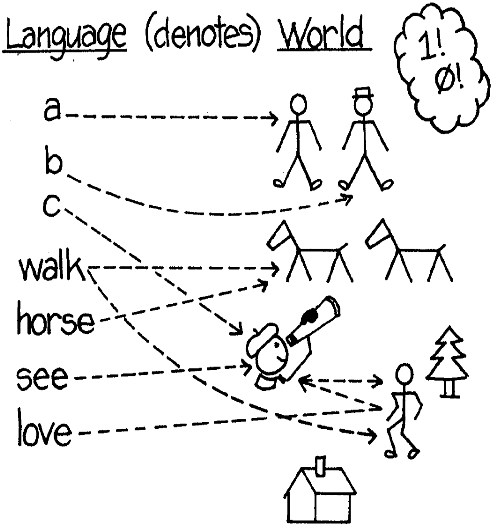


Figure 1: page 10 figure from Bach (1989)

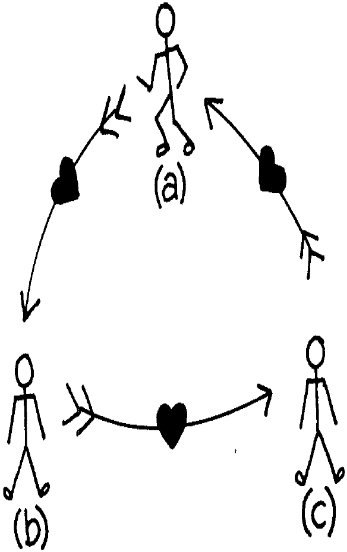


Figure 2: page 12 figure from Bach (1989)

Syntax of Bach's logic subset

$$
\begin{aligned}
&\langle var\rangle ::= w|x|y|z\\
&\langle con\rangle ::= a|b|c|d\\
&\langle ter\rangle ::= \langle var\rangle|\langle con\rangle\\
&\langle 1pp\rangle ::= Run|Walk|Happy|Calm\\
&\langle 2pp\rangle ::= Love|Kiss|Like|See\\
&\langle wff\rangle ::= \langle 1pp\rangle(\langle ter\rangle)\\
&\langle wff\rangle ::= \langle 2pp\rangle(\langle ter\rangle, \langle ter\rangle)\\
&\langle wff\rangle ::= -\langle wff\rangle\\
&\langle wff\rangle ::= (\langle wff\rangle \vee \langle wff\rangle)\\
&\langle wff\rangle ::= (\langle wff\rangle \;\&\; \langle wff\rangle)\\
&\langle wff\rangle ::= \forall\langle var\rangle\ \langle wff\rangle\\
&\langle wff\rangle ::= \exists\langle var\rangle\ \langle wff\rangle
\end{aligned}
$$

Notational differences between the readings

- The & and $\wedge$ symbols both mean conjunction ("and").

Notational differences between the readings

- The & and $\wedge$ symbols both mean conjunction ("and").
- The $\neg$, $\sim$, and $-$ all mean negation ("not").

Notational differences between the readings

- The & and $\wedge$ symbols both mean conjunction ("and").
- The $\neg$, $\sim$, and $-$ all mean negation ("not").
- Sometimes predicates are written with parentheses ($P(x, y)$), sometimes not (Pxy).

Notational differences between the readings

- The & and $\wedge$ symbols both mean conjunction ("and").
- The $\neg$, $\sim$, and $-$ all mean negation ("not").
- Sometimes predicates are written with parentheses ($P(x, y)$), sometimes not (Pxy).
- In other words, $(Pab \wedge \neg Qbc)$ is the same as $(P(a, b)$ & $-Q(b, c))$.

Notational differences between the readings

- The & and $\wedge$ symbols both mean conjunction ("and").
- The $\neg$, $\sim$, and $-$ all mean negation ("not").
- Sometimes predicates are written with parentheses ($P(x, y)$), sometimes not (Pxy).
- In other words, $(Pab \wedge \neg Qbc)$ is the same as $(P(a, b)\ \&\ -Q(b, c))$.
- van Benthem, et al. use D for the domain set and I for the interpretation function.

Notational differences between the readings

- The & and $\wedge$ symbols both mean conjunction ("and").
- The $\neg$, $\sim$, and $-$ all mean negation ("not").
- Sometimes predicates are written with parentheses ($P(x, y)$), sometimes not (Pxy).
- In other words, $(Pab \wedge \neg Qbc)$ is the same as $(P(a, b)\ \&\ -Q(b, c))$.
- van Benthem, et al. use D for the domain set and I for the interpretation function.
- But Bach calls the domain set E (entities) and the interpretation function D (denotes).

Notational differences between the readings

- The & and $\wedge$ symbols both mean conjunction ("and").
- The $\neg$, $\sim$, and $-$ all mean negation ("not").
- Sometimes predicates are written with parentheses ($P(x,y)$), sometimes not (Pxy).
- In other words, $(Pab \wedge \neg Qbc)$ is the same as $(P(a,b)\ \&\ -Q(b,c))$.
- van Benthem, et al. use D for the domain set and I for the interpretation function.
- But Bach calls the domain set E (entities) and the interpretation function D (denotes).
- I will bring in some details from the van Benthem readings, but use Bach's notation in this presentation.

Notational differences between the readings

- The & and $\wedge$ symbols both mean conjunction ("and").
- The $\neg$, $\sim$, and $-$ all mean negation ("not").
- Sometimes predicates are written with parentheses ($P(x, y)$), sometimes not (Pxy).
- In other words, $(Pab \wedge \neg Qbc)$ is the same as $(P(a, b)\ \&\ -Q(b, c))$.
- van Benthem, et al. use D for the domain set and I for the interpretation function.
- But Bach calls the domain set E (entities) and the interpretation function D (denotes).
- I will bring in some details from the van Benthem readings, but use Bach's notation in this presentation.
- Bach calls his structure an interpretation, so I'll use I for that.

Summary of notation

Meaning	van Benthem	Bach	Dubin
and	$\wedge$	&	&
not	$\neg$	$-$	$-$
or	$\vee$	$\vee$	$\vee$
predicate	Pxy	Word(x,y)	Word(x,y)
domain	D	E	E
denotation	I	D	D
referent	φ_I^g	$D(\varphi)$	$D(\varphi)$ or φ_D^g
var asgts	$g \in G$	$g \in G$	$g \in G$

The semantics of propositional logic vs. predicate logic

van Benthem, et al. write:

> *In propositional logic, the link was the valuation mapping proposition letters to truth values. But this will no longer do. For checking whether a statement saying that a certain object has a certain property, or that certain objects are in a certain relation is true we need something more refined. Instead of just saying that "John is boy" is assigned the value true, we now need an interpretation for "John" and an interpretation for "being a boy".*

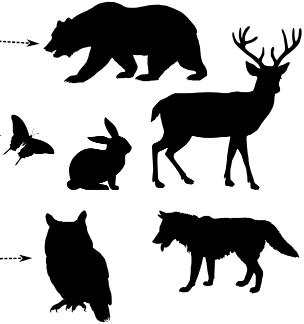
Recall that the only non-language view of the world or domain that we needed consisted of truth values and propositions that concerned domain entities. Now we're going to include domain elements themselves in our model, not just propositions concerning them.

Language (denotes) World

a

b

c

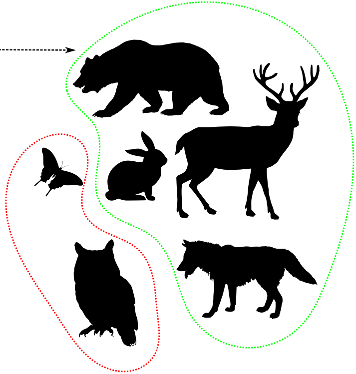


Language and the world

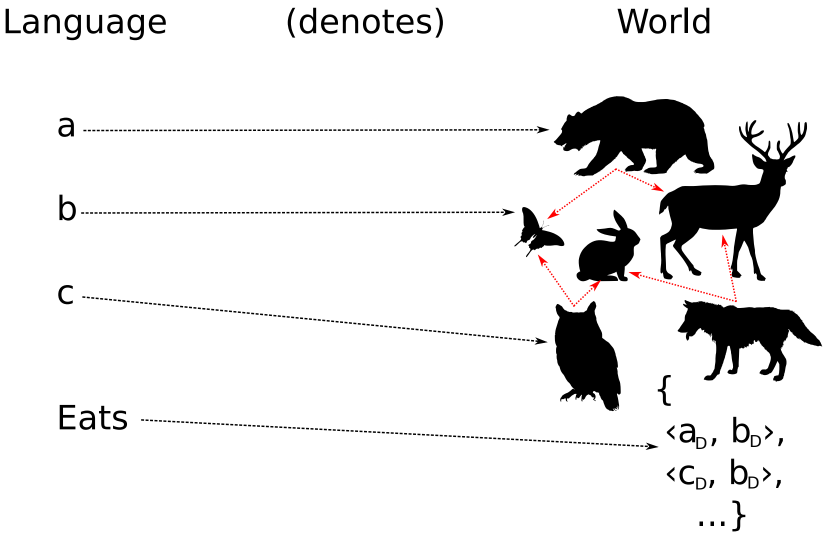
Language (denotes) World

Mammal

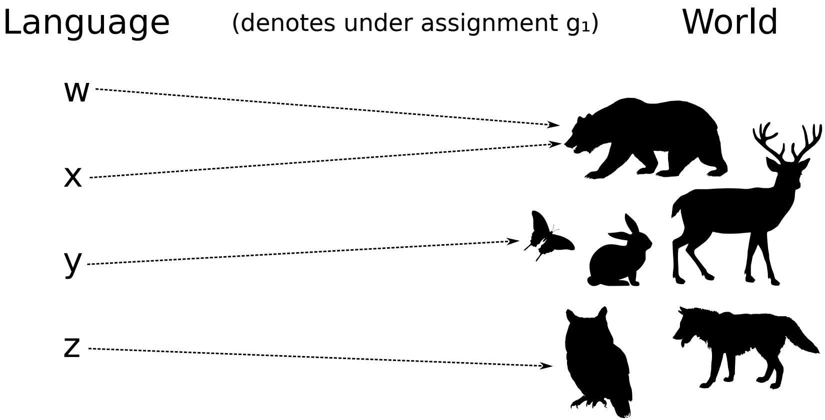
Flies



Language and the world



Language and the world



Language and the world

Language (denotes under assignment g_1) World

a

x

y

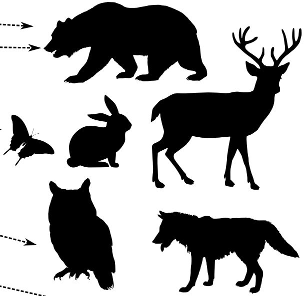
z

Flies(x)

Eats(z,x)

∃x Flies(x)

∃x Eats(z,x)



True False

Models and interpretations

Bach writes:

> *An interpretation is a way of assigning denotations in a certain model structure to expressions in a language.*

- While van Benthem's formalization is simply $M = \langle D, I \rangle$, Bach's approach gives us: $I = \langle M1, D, G \rangle$ where:

Models and interpretations

Bach writes:

> *An interpretation is a way of assigning denotations in a certain model structure to expressions in a language.*

- While van Benthem's formalization is simply $M = \langle D, I \rangle$, Bach's approach gives us: $I = \langle M1, D, G \rangle$ where:
    - $M1$ is the first model structure he proposes;

Models and interpretations

Bach writes:

> *An interpretation is a way of assigning denotations in a certain model structure to expressions in a language.*

- While van Benthem's formalization is simply $M = \langle D, I \rangle$, Bach's approach gives us: $I = \langle M1, D, G \rangle$ where:
  - $M1$ is the first model structure he proposes;
  - G is a set of assignments of values to variables (same as van Benthem, et al.)

Models and interpretations

Bach writes:

> *An interpretation is a way of assigning denotations in a certain model structure to expressions in a language.*

- While van Benthem's formalization is simply $M = \langle D, I \rangle$, Bach's approach gives us: $I = \langle M1, D, G \rangle$ where:
  - $M1$ is the first model structure he proposes;
  - G is a set of assignments of values to variables (same as van Benthem, et al.)
  - D is the denoting evaluation function (van Benthem's I).

Models and interpretations

Bach writes:

> *An interpretation is a way of assigning denotations in a certain model structure to expressions in a language.*

- While van Benthem's formalization is simply $M = \langle D, I \rangle$, Bach's approach gives us: $I = \langle M1, D, G \rangle$ where:
  - $M1$ is the first model structure he proposes;
  - G is a set of assignments of values to variables (same as van Benthem, et al.)
  - D is the denoting evaluation function (van Benthem's I).
- Bach's $M1$ is itself a two-place tuple consisting of:

Models and interpretations

Bach writes:

> *An interpretation is a way of assigning denotations in a certain model structure to expressions in a language.*

- While van Benthem's formalization is simply $M = \langle D, I \rangle$, Bach's approach gives us: $I = \langle M1, D, G \rangle$ where:
  - $M1$ is the first model structure he proposes;
  - G is a set of assignments of values to variables (same as van Benthem, et al.)
  - D is the denoting evaluation function (van Benthem's I).
- Bach's $M1$ is itself a two-place tuple consisting of:
  - Set E of individuals (van Benthem's D);

Models and interpretations

Bach writes:

> *An interpretation is a way of assigning denotations in a certain model structure to expressions in a language.*

- While van Benthem's formalization is simply $M = \langle D, I \rangle$, Bach's approach gives us: $I = \langle M1, D, G \rangle$ where:
  - $M1$ is the first model structure he proposes;
  - G is a set of assignments of values to variables (same as van Benthem, et al.)
  - D is the denoting evaluation function (van Benthem's I).
- Bach's $M1$ is itself a two-place tuple consisting of:
  - Set E of individuals (van Benthem's D);
  - The set of truth values $\{1, 0\}$.

Models and interpretations

Bach writes:

> *An interpretation is a way of assigning denotations in a certain model structure to expressions in a language.*

- While van Benthem's formalization is simply $M = \langle D, I \rangle$, Bach's approach gives us: $I = \langle M1, D, G \rangle$ where:
  - $M1$ is the first model structure he proposes;
  - G is a set of assignments of values to variables (same as van Benthem, et al.)
  - D is the denoting evaluation function (van Benthem's I).
- Bach's $M1$ is itself a two-place tuple consisting of:
  - Set E of individuals (van Benthem's D);
  - The set of truth values $\{1, 0\}$.
- Toward the end of the reading he suggests adding a set of times and a set of possible worlds for more expressive languages.

Simple denotations

Individual constants (like a and b) will denote individuals in the domain, so they'll be elements of (Bach's) set E. The one-place predicates (classes) will denote their extensions (i.e., subsets of the domain). The two-place predicates (binary relations) will denote sets of ordered pairs.

$D(\langle con \rangle) \in E$
$D(\langle 1pp \rangle) \subseteq E$
$D(\langle 2pp \rangle) \subseteq E \times E$

Following the convention in your logic readings, we adopt the notation φ_D for $D(\varphi)$. So, for example, a_D would be some individual in the Domain set E, and $Love_D$ would be a set of ordered pairs $\langle x, y \rangle$, in each of which x loves y.

Variable assignments

- Let V be the set of variables in the language. Bach's G is a set of variable assignments, and each $g \in G$ is a function from variables ($v \in V$) to individuals in E, i.e.: $g \subseteq V \times E$.

Variable assignments

- Let V be the set of variables in the language. Bach's G is a set of variable assignments, and each $g \in G$ is a function from variables ($v \in V$) to individuals in E, i.e.: $g \subseteq V \times E$.
- For example, suppose $g_1 = \{\langle w, a_D \rangle, \langle x, b_D \rangle, \langle y, c_D \rangle, \langle z, d_D \rangle\}$, but $g_2 = \{\langle w, b_D \rangle, \langle x, c_D \rangle, \langle y, d_D \rangle, \langle z, a_D \rangle\}$.

Variable assignments

- Let V be the set of variables in the language. Bach's G is a set of variable assignments, and each $g \in G$ is a function from variables ($v \in V$) to individuals in E, i.e.: $g \subseteq V \times E$.
- For example, suppose $g_1 = \{\langle w, a_D \rangle, \langle x, b_D \rangle, \langle y, c_D \rangle, \langle z, d_D \rangle\}$, but $g_2 = \{\langle w, b_D \rangle, \langle x, c_D \rangle, \langle y, d_D \rangle, \langle z, a_D \rangle\}$.
- Because our functions are cartesian product subsets, two or more different variables can be mapped to the same individual. For example, maybe $g_3 = \{\langle w, a_D \rangle, \langle x, a_D \rangle, \langle y, b_D \rangle, \langle z, a_D \rangle\}$ and $g_4 = \{\langle w, c_D \rangle, \langle x, c_D \rangle, \langle y, c_D \rangle, \langle z, c_D \rangle\}$. But they're functions, so every variable in the domain will be included.

Variable assignments

- Let V be the set of variables in the language. Bach's G is a set of variable assignments, and each $g \in G$ is a function from variables ($v \in V$) to individuals in E, i.e.: $g \subseteq V \times E$.
- For example, suppose $g_1 = \{\langle w, a_D\rangle, \langle x, b_D\rangle, \langle y, c_D\rangle, \langle z, d_D\rangle\}$, but $g_2 = \{\langle w, b_D\rangle, \langle x, c_D\rangle, \langle y, d_D\rangle, \langle z, a_D\rangle\}$.
- Because our functions are cartesian product subsets, two or more different variables can be mapped to the same individual. For example, maybe $g_3 = \{\langle w, a_D\rangle, \langle x, a_D\rangle, \langle y, b_D\rangle, \langle z, a_D\rangle\}$ and $g_4 = \{\langle w, c_D\rangle, \langle x, c_D\rangle, \langle y, c_D\rangle, \langle z, c_D\rangle\}$. But they're functions, so every variable in the domain will be included.
- Following van Benthem, et al., the expression $g[v{:=}e]$ is understood to mean "the variable assignment that is completely like g except that v now gets the value e (where g might have assigned a different value)."

Variable assignments

- Let V be the set of variables in the language. Bach's G is a set of variable assignments, and each $g \in G$ is a function from variables ($v \in V$) to individuals in E, i.e.: $g \subseteq V \times E$.
- For example, suppose $g_1 = \{\langle w, a_D \rangle, \langle x, b_D \rangle, \langle y, c_D \rangle, \langle z, d_D \rangle\}$, but $g_2 = \{\langle w, b_D \rangle, \langle x, c_D \rangle, \langle y, d_D \rangle, \langle z, a_D \rangle\}$.
- Because our functions are cartesian product subsets, two or more different variables can be mapped to the same individual. For example, maybe $g_3 = \{\langle w, a_D \rangle, \langle x, a_D \rangle, \langle y, b_D \rangle, \langle z, a_D \rangle\}$ and $g_4 = \{\langle w, c_D \rangle, \langle x, c_D \rangle, \langle y, c_D \rangle, \langle z, c_D \rangle\}$. But they're functions, so every variable in the domain will be included.
- Following van Benthem, et al., the expression $g[v{:=}e]$ is understood to mean "the variable assignment that is completely like g except that v now gets the value e (where g might have assigned a different value)."
- For example, $g_1[x{:=}d_D] = \{\langle w, a_D \rangle, \langle x, d_D \rangle, \langle y, c_D \rangle, \langle z, d_D \rangle\}$

Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

- $[\![v]\!]_D^g = g(v)$

Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

- $[\![v]\!]_D^g = g(v)$
- $[\![c]\!]_D^g = c_D$

Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

- $[\![v]\!]_D^g = g(v)$
- $[\![c]\!]_D^g = c_D$
- $I \models_g P(t)$ iff $[\![t]\!]_D^g \in P_D$

Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

- $[\![v]\!]^g_D = g(v)$
- $[\![c]\!]^g_D = c_D$
- $I \models_g P(t)$ iff $[\![t]\!]^g_D \in P_D$
- $I \models_g R(s,t)$ iff $\langle [\![s]\!]^g_D, [\![t]\!]^g_D \rangle \in R_D$

Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

- $[\![v]\!]^g_D = g(v)$
- $[\![c]\!]^g_D = c_D$
- $I \models_g P(t)$ iff $[\![t]\!]^g_D \in P_D$
- $I \models_g R(s,t)$ iff $\langle [\![s]\!]^g_D, [\![t]\!]^g_D \rangle \in R_D$
- $I \models_g -\varphi$ iff it is not the case that $I \models_g \varphi$

Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

- $[\![v]\!]^g_D = g(v)$
- $[\![c]\!]^g_D = c_D$
- $I \models_g P(t)$ iff $[\![t]\!]^g_D \in P_D$
- $I \models_g R(s,t)$ iff $\langle [\![s]\!]^g_D, [\![t]\!]^g_D \rangle \in R_D$
- $I \models_g -\varphi$ iff it is not the case that $I \models_g \varphi$
- $I \models_g (\varphi_1 \ \&\ \varphi_2)$ iff $I \models_g \varphi_1$ and $I \models_g \varphi_2$

Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

- $[\![v]\!]^g_D = g(v)$
- $[\![c]\!]^g_D = c_D$
- $I \models_g P(t)$ iff $[\![t]\!]^g_D \in P_D$
- $I \models_g R(s,t)$ iff $\langle [\![s]\!]^g_D, [\![t]\!]^g_D \rangle \in R_D$
- $I \models_g -\varphi$ iff it is not the case that $I \models_g \varphi$
- $I \models_g (\varphi_1 \mathbin{\&} \varphi_2)$ iff $I \models_g \varphi_1$ and $I \models_g \varphi_2$
- $I \models_g (\varphi_1 \vee \varphi_2)$ iff $I \models_g \varphi_1$ or $I \models_g \varphi_2$

Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

- $[\![v]\!]^g_D = g(v)$
- $[\![c]\!]^g_D = c_D$
- $I \models_g P(t)$ iff $[\![t]\!]^g_D \in P_D$
- $I \models_g R(s,t)$ iff $\langle [\![s]\!]^g_D, [\![t]\!]^g_D \rangle \in R_D$
- $I \models_g -\varphi$ iff it is not the case that $I \models_g \varphi$
- $I \models_g (\varphi_1 \ \& \ \varphi_2)$ iff $I \models_g \varphi_1$ and $I \models_g \varphi_2$
- $I \models_g (\varphi_1 \vee \varphi_2)$ iff $I \models_g \varphi_1$ or $I \models_g \varphi_2$
- $I \models_g \forall v \varphi$ iff for all $e \in E$ it holds that $I \models_{g[v:=e]} \varphi$

Denotation, support, and truth

An expression in our language will be true or false in an interpretation, relative to a particular assignment of individuals to variables. So, carrying forward our notion of support from the propositional logic semantics, we write $I \models_g \varphi$ to mean that assignment g *satisfies* expression φ in interpretation I (or, in other words, φ is true in interpretation I under assignment g).

- $[\![v]\!]_D^g = g(v)$
- $[\![c]\!]_D^g = c_D$
- $I \models_g P(t)$ iff $[\![t]\!]_D^g \in P_D$
- $I \models_g R(s,t)$ iff $\langle [\![s]\!]_D^g, [\![t]\!]_D^g \rangle \in R_D$
- $I \models_g -\varphi$ iff it is not the case that $I \models_g \varphi$
- $I \models_g (\varphi_1 \mathbin{\&} \varphi_2)$ iff $I \models_g \varphi_1$ and $I \models_g \varphi_2$
- $I \models_g (\varphi_1 \vee \varphi_2)$ iff $I \models_g \varphi_1$ or $I \models_g \varphi_2$
- $I \models_g \forall v \varphi$ iff for all $e \in E$ it holds that $I \models_{g[v:=e]} \varphi$
- $I \models_g \exists v \varphi$ iff for at least one $e \in E$ it holds that $I \models_{g[v:=e]} \varphi$

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.
- Possible worlds are part of the semantics of modal logics such as:

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.
- Possible worlds are part of the semantics of modal logics such as:
  - Alethic logics, in which necessary truth and falsehood are distinguished from contingent truth and falsehood.

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.
- Possible worlds are part of the semantics of modal logics such as:
  - Alethic logics, in which necessary truth and falsehood are distinguished from contingent truth and falsehood.
  - Deontic logics, which concern things that permissible and obligatory.

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.
- Possible worlds are part of the semantics of modal logics such as:
    - Alethic logics, in which necessary truth and falsehood are distinguished from contingent truth and falsehood.
    - Deontic logics, which concern things that permissible and obligatory.
    - Epistemic logic, which concern what must or might be true, based on what we know.

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.
- Possible worlds are part of the semantics of modal logics such as:
  - Alethic logics, in which necessary truth and falsehood are distinguished from contingent truth and falsehood.
  - Deontic logics, which concern things that permissible and obligatory.
  - Epistemic logic, which concern what must or might be true, based on what we know.
- Modal logics use predicate logic notation, along with two new symbols:

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.
- Possible worlds are part of the semantics of modal logics such as:
  - Alethic logics, in which necessary truth and falsehood are distinguished from contingent truth and falsehood.
  - Deontic logics, which concern things that permissible and obligatory.
  - Epistemic logic, which concern what must or might be true, based on what we know.
- Modal logics use predicate logic notation, along with two new symbols:
  - The $\Box$ operator is read "necessarily."

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.
- Possible worlds are part of the semantics of modal logics such as:
  - Alethic logics, in which necessary truth and falsehood are distinguished from contingent truth and falsehood.
  - Deontic logics, which concern things that permissible and obligatory.
  - Epistemic logic, which concern what must or might be true, based on what we know.
- Modal logics use predicate logic notation, along with two new symbols:
  - The $\Box$ operator is read "necessarily."
  - The $\Diamond$ operator is read "possibly."

Possible worlds and modal logics

- Bach's M2 model structure includes a set of times and a set of possible worlds.
- Possible worlds are part of the semantics of modal logics such as:
  - Alethic logics, in which necessary truth and falsehood are distinguished from contingent truth and falsehood.
  - Deontic logics, which concern things that permissible and obligatory.
  - Epistemic logic, which concern what must or might be true, based on what we know.
- Modal logics use predicate logic notation, along with two new symbols:
  - The $\Box$ operator is read "necessarily."
  - The $\Diamond$ operator is read "possibly."
  - For example, $\forall x \Diamond \exists y (Px \vee \neg \Box Ryx)$

Modal logics and information modeling

- We'll revisit alethic modal logic when we discuss best practices for designing ontologies and other RDF vocabularies.

Modal logics and information modeling

- We'll revisit alethic modal logic when we discuss best practices for designing ontologies and other RDF vocabularies.
- Deontic logic is beyond the scope of this course, but can be useful for modeling constraints like those listed in these quotes from the Harvard Personnel Manual:

Modal logics and information modeling

- We'll revisit alethic modal logic when we discuss best practices for designing ontologies and other RDF vocabularies.
- Deontic logic is beyond the scope of this course, but can be useful for modeling constraints like those listed in these quotes from the Harvard Personnel Manual:
    1. "The Commonwealth of Massachusetts requires all employees under the age of 18 to secure and present an educational certificate."

Modal logics and information modeling

- We'll revisit alethic modal logic when we discuss best practices for designing ontologies and other RDF vocabularies.
- Deontic logic is beyond the scope of this course, but can be useful for modeling constraints like those listed in these quotes from the Harvard Personnel Manual:
  1. "The Commonwealth of Massachusetts requires all employees under the age of 18 to secure and present an educational certificate."
  2. "Applicable state and federal laws provide that employees under 18 years of age may not: [...] work more than 9 hours per day."

Modal logics and information modeling

- We'll revisit alethic modal logic when we discuss best practices for designing ontologies and other RDF vocabularies.
- Deontic logic is beyond the scope of this course, but can be useful for modeling constraints like those listed in these quotes from the Harvard Personnel Manual:
    1. "The Commonwealth of Massachusetts requires all employees under the age of 18 to secure and present an educational certificate."
    2. "Applicable state and federal laws provide that employees under 18 years of age may not: [...] work more than 9 hours per day."
    3. "New employees must present a Social Security number at the time of hire or immediately thereafter."