



北京航空航天大学  
BEIHANG UNIVERSITY

# Research Summary

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# Outline

➤ Higher-order Topology in Graphyne Families

**Binbin Liu** et al., Phys. Rev. B 106, 035153 (2022).

Xu-Tao Zeng, **Binbin Liu**, et al., arXiv: 2302.13090. (Submitted to PRB.)

➤ Non-centered Inversion Symmetry in Momentum Space

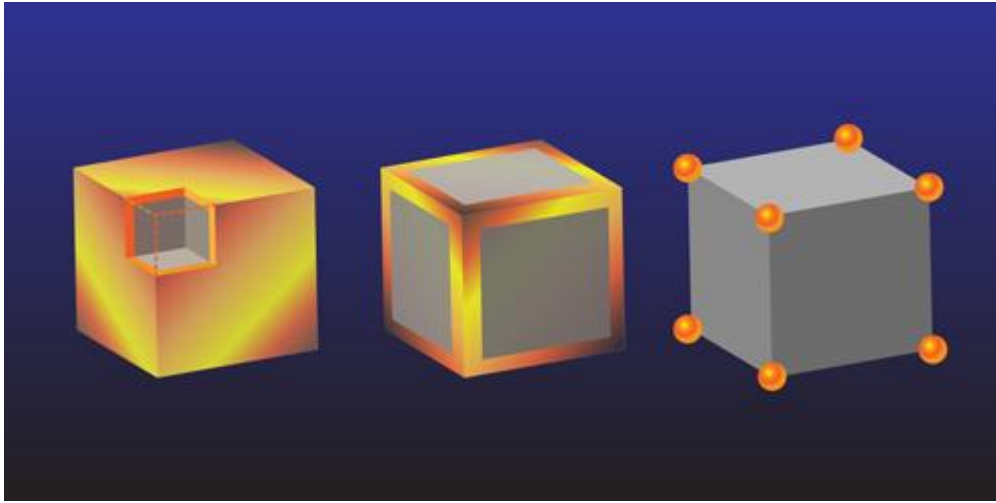
**Binbin Liu**<sup>†</sup>, et al. (To be submitted to PRL.)

➤ Large Bilinear Magnetoresistance (BMR) from Rashba Spin-Splitting on the Surface of a Topological Insulator

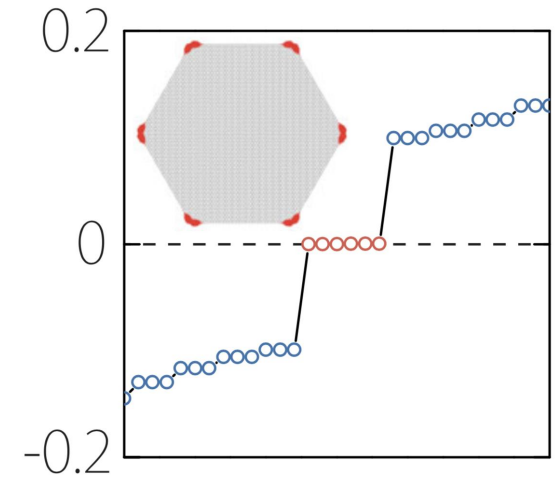
Wang Yang\*, **Binbin Liu**\*, et al., Phys. Rev. B 106, L241401 (2022).

# Higher-order Topology in Graphyne Families

# Introduction



3D Higher-order TI



2D Second-order TI

Real Chern number (PT)

$$(-1)^{\nu_R} = \prod_{i=1}^4 (-1)^{\lfloor (n_{-}^{\Gamma_i} / 2) \rfloor}$$

Fractional Charge (Crystalline)

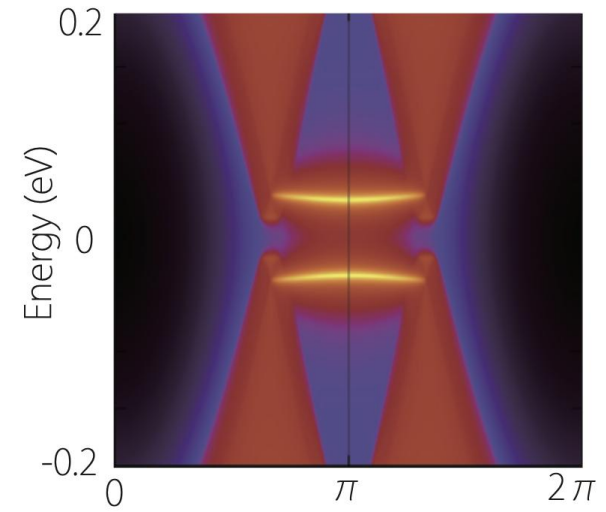
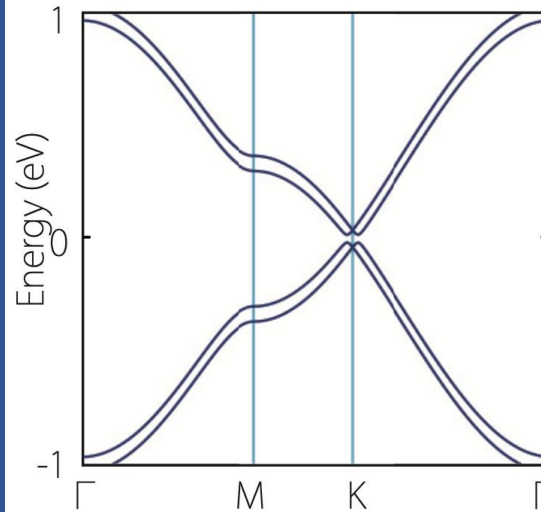
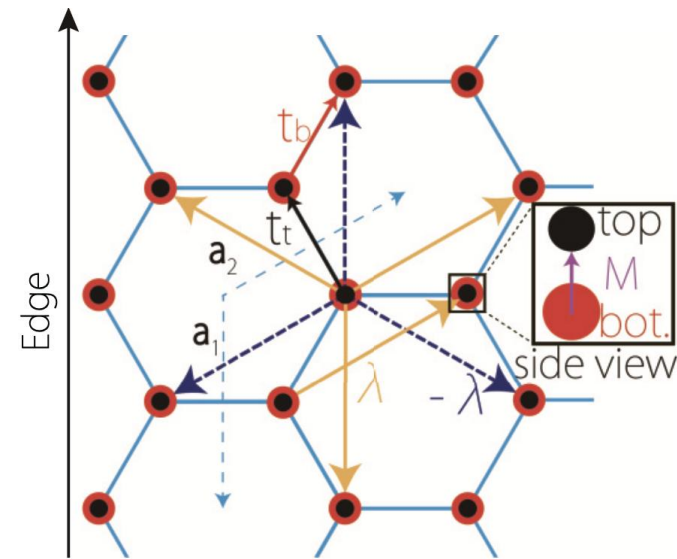
$$Q_{corner}^{(2)} = \frac{e}{4} [-n_{C_2}^X - n_{C_2}^Y + (n_{C_2}^M + n_{C_2}^{\Gamma})] \mod e$$

Real Chern insulator / Stiefel - Whitney insulator

Quadrupole insulator / TCI

$$c_{6z} Q_{corner}^{(2)} = e \frac{\nu_R}{2}.$$

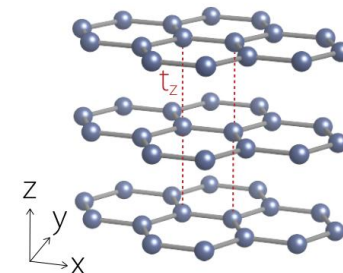
# SOTI models



$$C_{2z}\mathcal{T} \quad C_{6z} \text{ and } C_{2y}$$

$$\nu_R = 1.$$

$$H^{4\text{BTB}}(\mathbf{k}) = \begin{bmatrix} G_t & M\sigma_0 + iS \\ M\sigma_0 - iS & G_b \end{bmatrix}$$

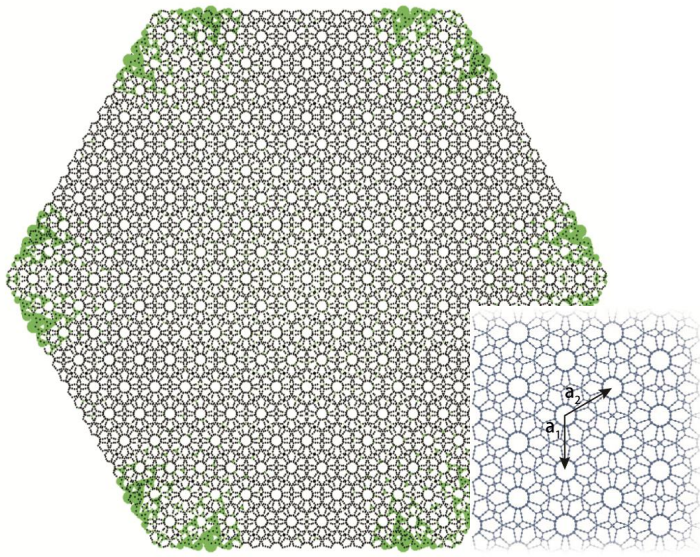


$$\mathcal{H}_{3D} = \mathcal{H}_{2D} + 2t_z \cos k_z \sigma_z \tau_0$$

Can be readily realized in **Graphyne families!**

# SOTI in Graphyne Families

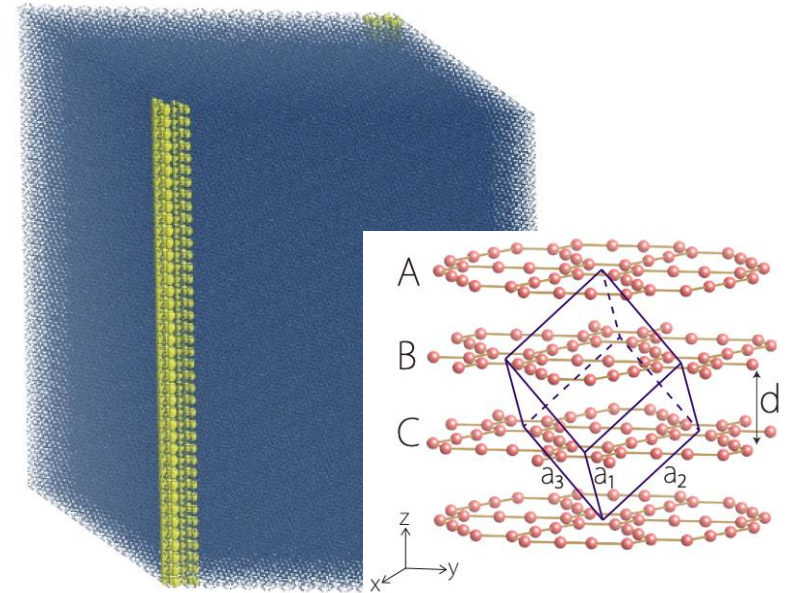
Moiré  $\alpha$ -Graphyne (tbGPY)



**General way to SOTI phase in moiré materials**

**B. Liu et al.**, Phys. Rev. B 106, 035153 (2022).

Bulk  $\gamma$ -Graphyne Nat. Synth. 1, 449 (2022).



**The first 3D RCI material**

X.-T. Zeng, **B. Liu**, et al., arXiv: 2302.13090.

2D SOTI & RCI: Graphdiyne(PRL 2019), **tbGPY (PRB 2022)**  
SO real nodal-line semimetal: 3D graphdiyne (PRL 2022)  
3D SOTI & RCI: **bulk  $\gamma$ -graphyne (arXiv 2023)**

# **Non-centered Inversion Symmetry in Momentum Space**

# Projective Symmetry

Projective symmetry algebra.

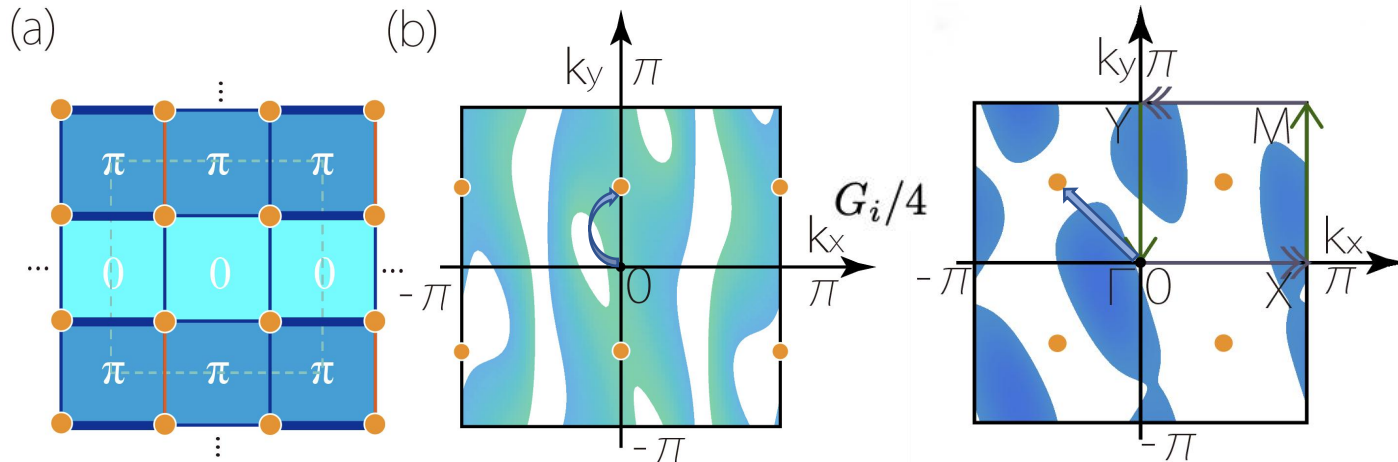
$$(PL_i)^2 = \theta \quad \theta = \pm 1 \quad \text{Cohomology invariant}$$

$$PL_i P = -L_i^{-1} = e^{i(k_i + G_i/2)a_i}$$

$$\mathcal{P}_i = P\mathcal{L}_{G_i/2}$$

**Half** reciprocal lattice translation

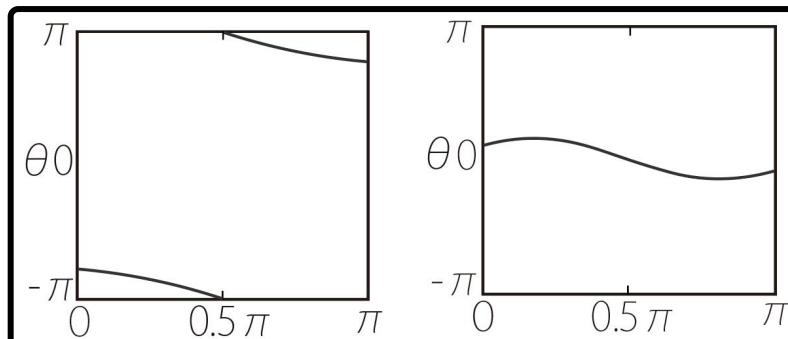
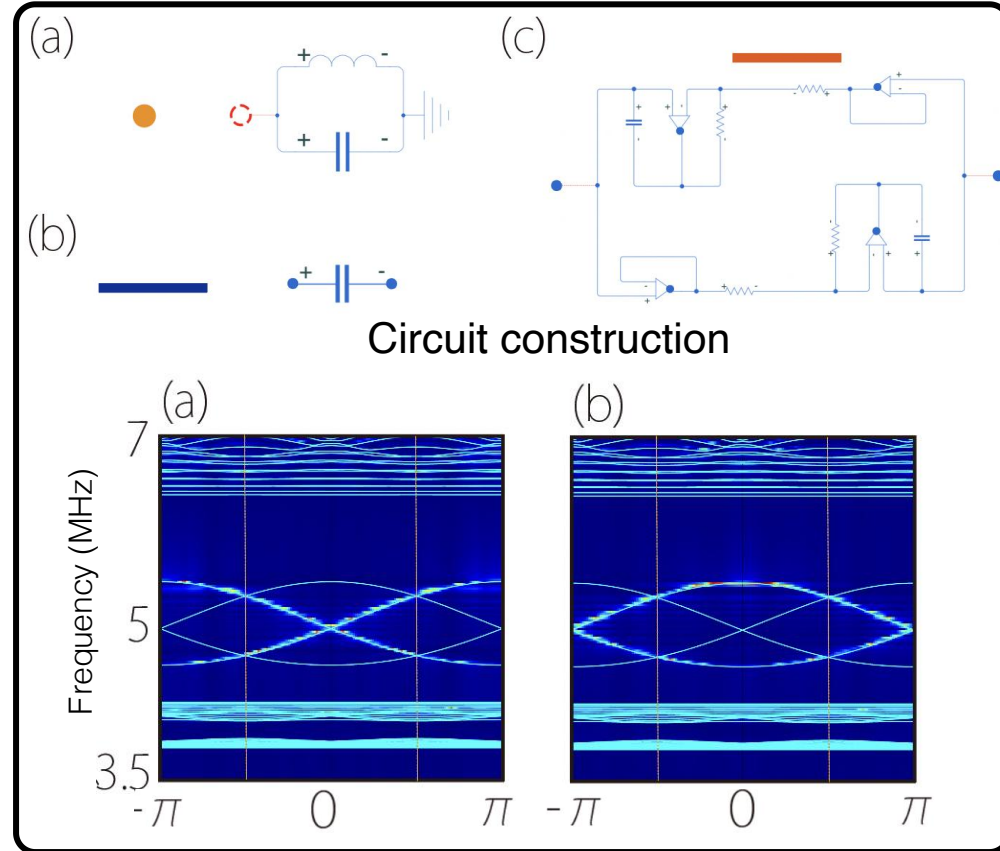
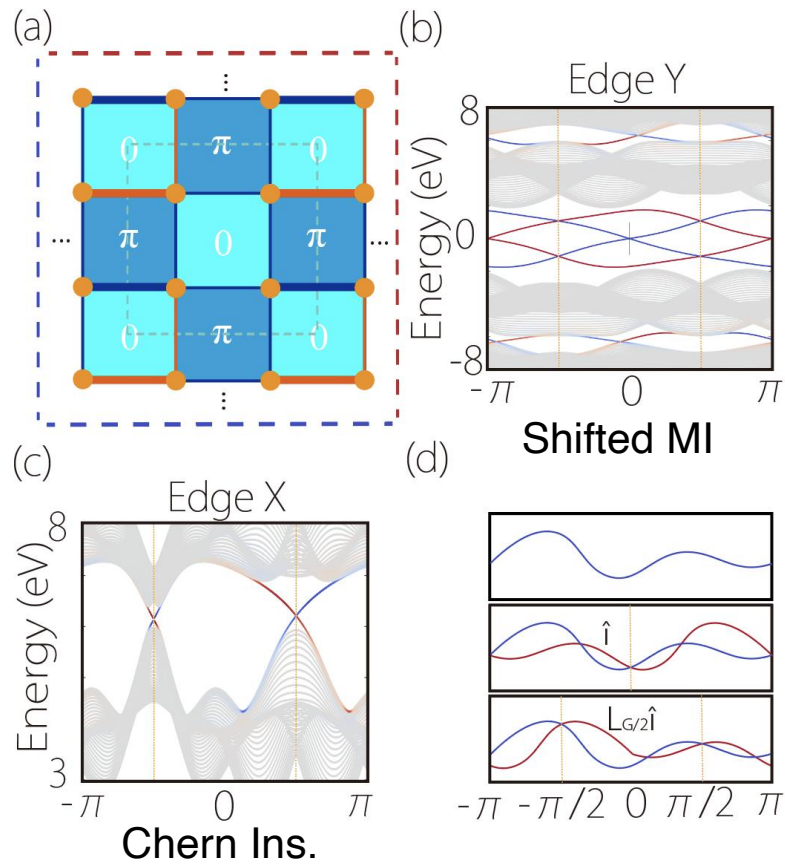
$$UH_i(-\mathbf{k}_{rest}, -k_i + G_i/2)U^\dagger = H_i(\mathbf{k}_{rest}, k_i)$$



Inversion centers: **Quarter** reciprocal lattice translation



# Topological States & Simulations



$$\nu = \frac{1}{2\pi} [\gamma(0) + \gamma(\pi)] \mod 2$$

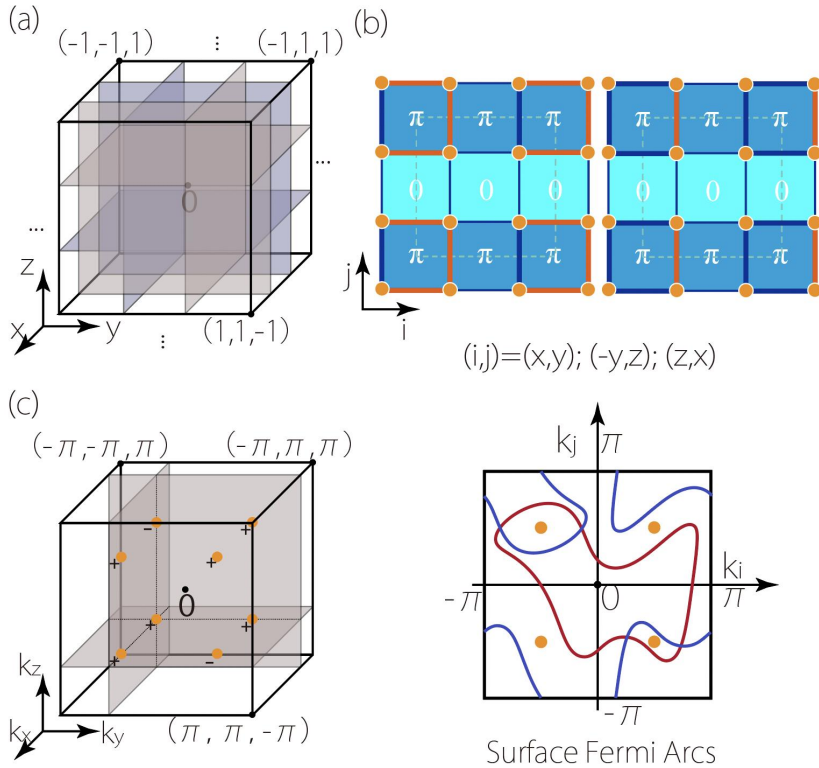
$$= \zeta \mod 2$$

$$(-1)^\nu = \prod_{j=a,b} \prod_i \lambda_i(\Gamma_j)$$

Topological invariant

# Extending to 3D

# POSTER



$$\mathcal{P}_{xyz}^{3D} = U_{3D} \mathcal{L}_{G_x/2} \mathcal{L}_{G_y/2} \mathcal{L}_{G_z/2} \hat{I}_{xyz}$$

To be submitted to PRL.

## Non-centered inversion symmetry in momentum space

Bin-Bin Liu†, Xian-Lei Sheng†, Y. X. Zhao†, and Shengyuan A. Yang  
School of Physics, Beihang University

### Introduction

Symmetry is of paramount import in physics. Spatial inversion symmetry is one of the most important symmetries which defines the parity. It inverts the real space position as well as the momentum of a physical system relative to the original point in the real and momentum spaces. In this letter, we discover a new kind of inversion symmetry which inverts the real space position relative to the origin but is non-centered in the momentum space. We derive this non-centered momentum with projectively enriched symmetry algebras. We demonstrate the idea in Hamiltonians on flux lattices with  $Z_2$  gauge field, which naturally realize the non-centered momentum inversion symmetries. Intriguingly, we discover unconventional topological indicators emergent from the gauge fields, which are protected by the momentum-non-centered symmetries. The topological numbers can be read off from Wilson loops evolving along both reciprocal lattice vectors. We find that non-trivial topological indicators defined along both vectors signal a novel topological insulator, with both of its  $x$  and  $y$  edges (four edges) hosting edge states linked by the non-centered inversion symmetry in each pairs, drastically distinct from previous topological insulators with normal inversion symmetry. We provide circuit simulations which agree with the theoretical prediction. Furthermore, we construct a 3D topological insulator with surface states satisfying the momentum-non-centered inversion symmetry. Our theory opens new routes for exploring physical and topological consequences associated with the momentum-non-centered inversion symmetries which can be realized and tested in circuits and other artificial systems.

### Projective Symmetry

• Normal Inversion symmetry (NIS):  $\mathcal{P} = U\hat{I}$   
 $UH(-k_x, -k_y)U^\dagger = H(k_x, k_y)$ .  
• Momentum-non-centered inversion symmetry (MNCIS) from projective symmetry  $\mathcal{P}\mathcal{L}_y\mathcal{P}^\dagger = -\mathcal{L}_y^{-1} = e^{i(k_y + G_y/2)b}$   
 $\mathcal{P} = U\mathcal{L}_{G_y/2}\hat{I}$  or  $\mathcal{P} = U\mathcal{L}_{G_x/2}\mathcal{L}_{G_y/2}\hat{I}$   
 $UH(-k_x + \pi, -k_y + \pi)U^\dagger = H(k_x, k_y)$

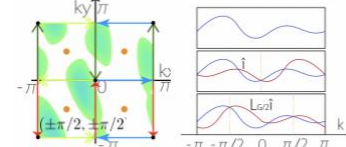


Fig. 1 Bulk states with MNCIS. Fig. 2 Edge states with NIS and MNCIS.

### Topological invariants

$u_i = \frac{1}{2\pi} \int_{\mathcal{R}} \mathcal{F}(\mathbf{k}) d^2\mathbf{k} - \frac{1}{\pi} \gamma(D+R) \bmod 2$   $\gamma$ : Calculated along the rightward (R), downward (D) boundary of each quadrant.  
(a)  $u_i = 0$  (b)  $u_i = 1$  (c)  $u_i = 0$  (d)  $u_i = 1$   
Fig. 3 Topological indicators. The distribution of  $u_i$  for (a) the  $(\pm\pi/2, \pm\pi/2)$  and (b) the  $(\pi, \pm\pi/2)$  cases. The Wilson loop for (c) nontrivial and (d) trivial  $v$  ( $v'$ ).  
 $\nu \ (\nu') = \zeta \bmod 2$   
 $\zeta$  counts the times of crossings of the Wilson loop spectrum with the  $\theta = \pi$  line.

### Topological insulators with MNCIS

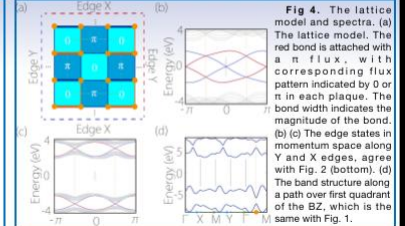


Fig. 4. The lattice model and spectra. (a) The lattice model. The red bond is attached with a  $\pi$  flux, with corresponding flux pattern indicated by 0 or  $\pi$  in each plaquette. The bond width indicates the magnitude of the bond. (b) (c) The edge states in momentum space along Y and X edges, agree with Fig. 2 (bottom). (d) The band structure along a path over first quadrant of the  $BZ$ , which is the same with Fig. 1.

### Circuit simulations

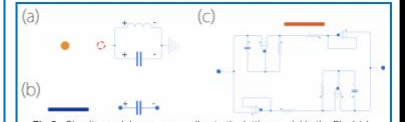


Fig. 5. Circuits corresponding to the lattice model in Fig. 4 (a). Kirchhoff's current law:

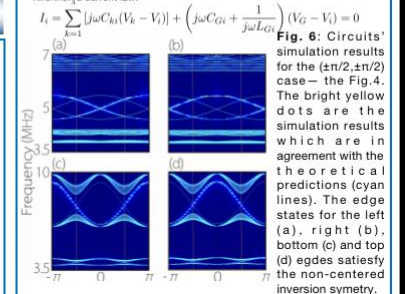


Fig. 6. Circuits' simulation results for the  $(\pm\pi/2, \pm\pi/2)$  case - the Fig. 4. The bright yellow dots are the simulation results which are in agreement with the theoretical predictions (cyan lines). The edge states for the left (a), right (b), bottom (c) and top (d) edges satisfy the non-centered inversion symmetry.

### Conclusions

- Discovered a novel non-centered inversion symmetry momentum space from projective symmetry algebras.
- Discovered novel topological insulator states with the MNCIS.
- Formulated a topological indicator for the states and derived several equivalent ways to calculate it: including the parity counting from the topological quantum chemistry, the integration from K-theory, and the Wilson loop method.
- Proposed circuits setup whose circuits' Laplacian from Kirchhoff's current law to the Hamiltonian. Simulated the circuits with "hspice".
- Detected the oddity of Chern number using the topological invariant.
- Constructed a 3d topological indicator protected by the non-centered inversion symmetry.

### References

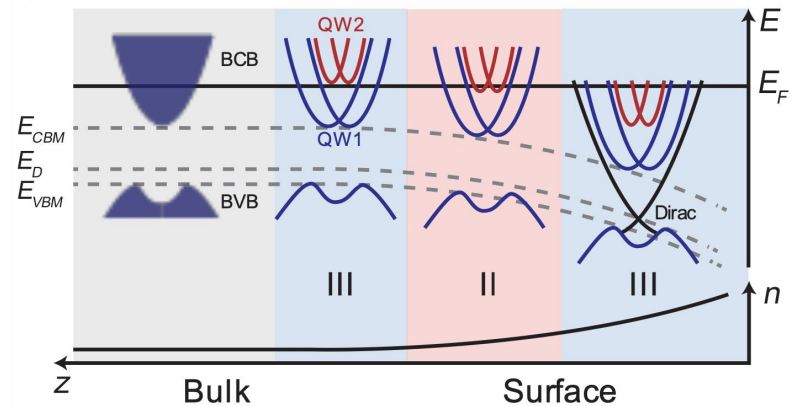
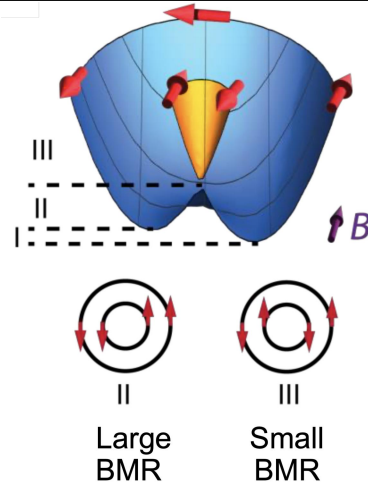
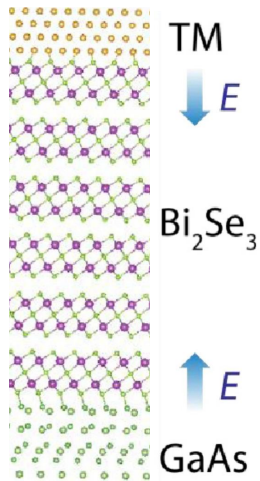
- [1] Z. Y. Chen, S. A. Yang, and Y. X. Zhao, Nature Communications 13, 2215 (2022).
- [2] Y. X. Zhao, Y.-X. Huang, and S. A. Yang, Phys. Rev. B 102, 161117 (2020).

# **Large BMR in Rashba States on the Surface of a TI**

# Explaining the Large BMR

$$R(I, B) = R_0[1 + \beta B^2 + \gamma \mathbf{I} \cdot (\mathbf{P} \times \mathbf{B})]$$

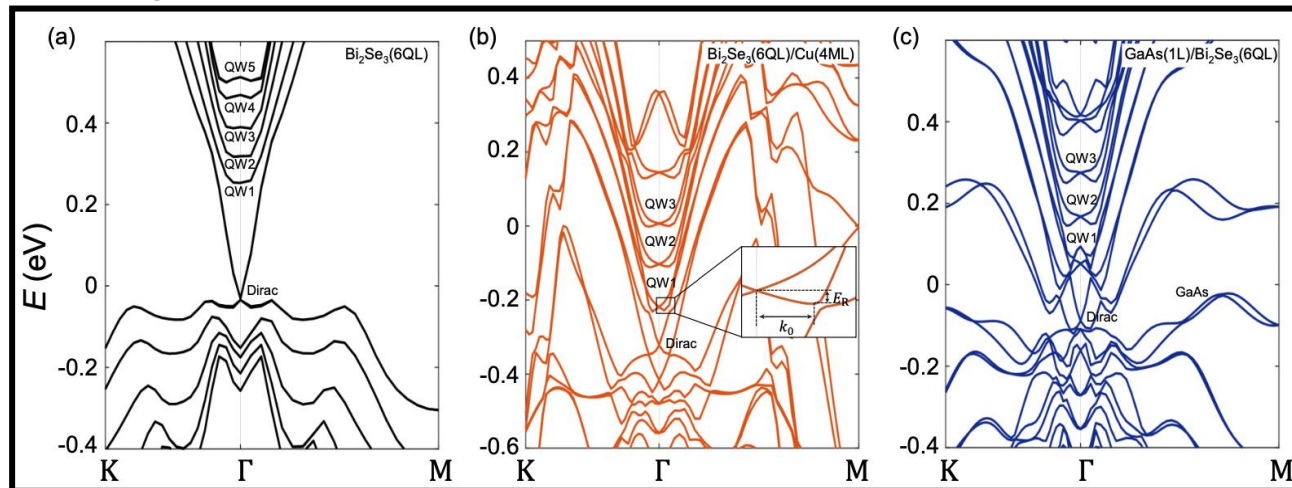
$$\gamma \sim \text{BMR}$$



Surface doping

Rashba spin-splitting

Band bending



DFT results

PRB, Letter (2022).



# Outlooks

nonHermitian  
Tensor networks  
Complex systems Application  
Circuit Hyperbolic fluid  
metamaterial  
Interests  
Topological  
Correlated electrons  
Topological everything  
material

# Outlooks

A word cloud centered around the word 'Specialties'. The word 'Specialties' is the largest and most prominent, rendered in a bold, grey, sans-serif font. Surrounding it are various other words in different sizes and shades of grey and black. The words include: 'Coding' (large, bold, black), 'Teaching' (medium, grey), 'Scientific writing' (small, black), 'Advising' (small, grey), 'Numeric.' (medium, grey), 'Team playing' (medium, grey), 'Analytic' (medium, grey), 'Idea contributing' (small, grey), and 'Fast learning' (small, grey). The words are arranged in a way that they appear to be floating or falling around the central 'Specialties'.

Scientific writing  
Advising  
Teaching  
**Coding**  
Numeric.  
**Specialties**  
Team playing  
Analytic  
Idea contributing Fast learning



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# Thank you!

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