Online MIMO Wireless Network Virtualization over Time-Varying Channels with Periodic Updates

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***Abstract*—We consider online downlink precoding design for multiple-input multiple-output (MIMO) wireless network virtualization (WNV) over time-varying channels. In our WNV framework, each service provider (SP) locally designs virtual precoding to serve its own users, while the infrastructure provider (InP) designs the actual global precoding to meet the service demands from the SPs. We consider a periodic update model where partial channel state information (CSI) feedbacks may be received out of order due to delay and the precoding is updated per period. Our goal is to minimize the accumulated deviation of the actual precoding designed by the InP from the virtualization demands made by the SPs, subject to both long-term and short-term transmit power constraints. We propose an online MIMO WNV algorithm, which achieves $\mathcal{O}(\sqrt{T})$ growth of regret on the precoding deviation from the virtualization demand over time horizon T, and maintains $\mathcal{O}(1)$ violation of the long-term transmit power constraint. Our proposed online precoding solution is in closed-form. Furthermore, it does not require the knowledge of the channel distribution and only depends on the past CSI. Finally, its effectiveness is shown in simulation and comparison with existing alternatives.**

I. Introduction

Wireless network virtualization (WNV) aims at sharing a common network infrastructure among multiple virtual networks to reduce the capital and operational expenses of wireless networks. In WNV, the infrastructure provider (InP) virtualizes the physical infrastructure and radio resources into virtual slices; the service providers (SPs) lease these virtualized resources to serve their users. Different from wired network virtualization, simultaneous sharing of both the wireless hardware and the radio spectrum brings new challenges to guarantee the isolation among virtual networks [1].

In this work, we consider online downlink WNV in a multiple-input multiple-output (MIMO) system, where the InP virtualizes the base station (BS) equipped with multiple antennas to serve multiple SPs. Existing works on MIMO WNV can be categorized into two streams: physical isolation [2]-[5] and spatial isolation [6]-[8]. Inherited from wired network virtualization, the idea of physical isolation is to allocate exclusive subsets of antennas or orthogonal subchannels among the SPs. In contrast, in spatial isolation, the SPs share all antennas and wireless spectrum resources simultaneously through spatial spectrum sharing enabled by MIMO beamforming, which substantially outperforms the physical isolation approach. In this work, we adopt the spatial isolation approach first proposed in [6].

Online MIMO WNV with instantaneous perfect and imperfect channel state information (CSI) was respectively studied in [7] and [8]. Both works assume the InP can design a precoder at each time slot. However, in practical Long-Term Evolution (LTE) networks [9], MIMO precoding relies on CSI feedback and is fixed for a period, *i.e.,* the duration of one resource block, while the underlying channel state keeps varying over time. Motivated by this discrepancy, in this work, we consider a scenario where the InP designs a fixed precoder per update period of multiple time slots, while the channel state can vary arbitrarily over time, and the CSI feedbacks can be delayed and imperfectly received.

Thus, we consider a unique constrained online convex optimization (OCO) framework to handle periodic updating by the InP. This is in contrast to existing works on constrained OCO [10]-[13], all of which assume the standard per-time-slot setting. Our goal is to minimize the accumulated deviation of received signals due to the InP's global precoding from those locally demanded by the SPs, subject to both long-term and short-term transmit power constraints. Note that the long-term transmit power constraint introduces correlation among the precoding solutions over time, and the online problem is particularly challenging when the precoding is fixed per period while the channel state varies arbitrarily over time.

The main contributions of this paper are summarized below:
1) We observe that the above downlink MIMO WNV precoding design problem is a novel constrained OCO problem. At the beginning of each update period, which can last multiple time slots, the InP designs a fixed precoder to meet the virtualization demand from SPs, while the channel state varies arbitrarily over time. The CSI feedbacks can be delayed for multiple time slots, received out of order, and partly missing.
2) We propose an online MIMO WNV algorithm to minimize the accumulated precoding deviation from virtualization demand under both long-term and short-term transmit power constraints. We show that it achieves $\mathcal{O}(\sqrt{T})$ regret and maintains $\mathcal{O}(1)$ violation of the long-term transmit power constraint. Our proposed online precoding solution is in closed-form, and it does not require the knowledge of the channel distribution.
3) Our simulation results show that the proposed algorithm converges fast. Further performance studies under typical urban micro-cell LTE settings demonstrate its advantage over the best-known alternatives.

This work has been funded in part by Ericsson Canada and by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

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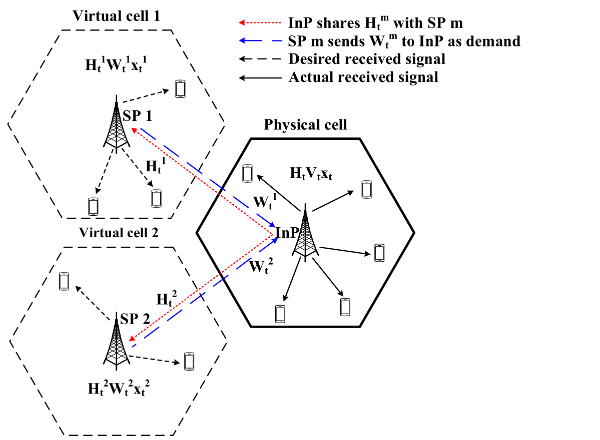

Fig. 1. An illustration of idealized MIMO virtualization in a cell with one InP and two SPs serving users in their respective virtual cells.

II. System Model and Problem Formulation

A. System Model

Consider an InP performing WNV in a MIMO cellular network. In each cell, the InP owns a BS equipped with N antennas, serving M SPs. Each SP m has K_m users. Let the total number of users in the cell be K. We consider a time-slotted system with time indexed by t. Denote by $\mathbf{H}_t^m \in \mathbb{C}^{K_m \times N}$ the local CSI between the BS and the K_m users of SP m at time t. Let $\mathcal{M} = \{1, \dots, M\}$ and $\mathcal{K} = \{1, \dots, K\}$.

For ease of exposition, we first consider an idealized WNV framework, where CSI is feedback per time slot without delay, as shown in Fig. 1. At each time slot t, the InP shares the corresponding local CSI $\mathbf{H}_t^m$ with SP m, and it allocates transmit power P_m to the SP. The power allocation is limited by the total transmit power budget $P_{\max}$ as $\sum_{m\in\mathcal{M}} P_m \le P_{\max}$. Using $\mathbf{H}_t^m$, each SP m designs its own precoding matrix $\mathbf{W}_t^m \in \mathbb{C}^{N \times K_m}$ based on the service needs of its users, while ensuring $\|\mathbf{W}_t^m\|_F^2 \le P_m$. The SP then sends $\mathbf{W}_t^m$ to the InP as its virtual precoding matrix. Note that each SP m designs $\mathbf{W}_t^m$ based only on its local CSI, not knowing the users of other SPs. For SP m, the *desired* received signal vector $\tilde{\mathbf{y}}_t^m$ (noiseless) at its K_m users is given by

$$\tilde{\mathbf{y}}_t^m = \mathbf{H}_t^m \mathbf{W}_t^m \mathbf{x}_t^m, \quad \forall m \in \mathcal{M}$$

where $\mathbf{x}_t^m$ is the transmitted signal vector from SP m to its K_m users. Let $\tilde{\mathbf{y}}_t \triangleq [\tilde{\mathbf{y}}_t^{1H}, \dots, \tilde{\mathbf{y}}_t^{MH}]^H$ be the desired received signal vector at all K users, and $\mathbf{D}_t \triangleq \mathrm{blkdiag}\{\mathbf{H}_t^1\mathbf{W}_t^1, \dots, \mathbf{H}_t^M\mathbf{W}_t^M\}$ be the virtualization demand made by the SPs. Let $\mathbf{x}_t \triangleq [\mathbf{x}_t^{1H}, \dots, \mathbf{x}_t^{MH}]^H$. Then we have $\tilde{\mathbf{y}}_t = \mathbf{D}_t \mathbf{x}_t$. The transmitted signals to all K users are assumed independent to each other, with $\mathbb{E}\{\mathbf{x}_t\mathbf{x}_t^H\} = \mathbf{I}, \forall t$.

At each time slot t, the InP has the global CSI $\mathbf{H}_t = [\mathbf{H}_t^{1H}, \dots, \mathbf{H}_t^{MH}]^H \in \mathbb{C}^{K\times N}$ and designs the actual global downlink precoding matrix $\mathbf{V}_t \triangleq [\mathbf{V}_t^1, \dots, \mathbf{V}_t^M] \in \mathbb{C}^{N\times K}$ to serve all K users, where $\mathbf{V}_t^m \in \mathbb{C}^{N\times K_m}$ is the actual downlink precoding matrix for SP m, and $\mathbf{V}_t$ is constrained by the compact convex set

$$\mathcal{V}_0 \triangleq \{\mathbf{V}_t : \|\mathbf{V}_t\|_F^2 \le P_{\max}\} \tag{1}$$

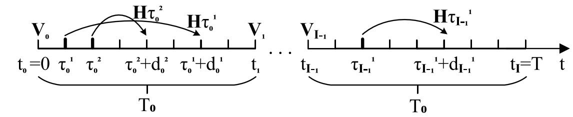

Fig. 2. A timeline illustrating online MIMO WNV with periodic updates.

to meet the short-term transmit power constraint, where $\|\cdot\|_F$ denotes the Frobenius norm. We also consider a long-term transmit power constraint, with

$$g(\mathbf{V}_t) \triangleq \|\mathbf{V}_t\|_F^2 - \bar{P} \tag{2}$$

being the long-term transmit power constraint function, where $\bar{P} \le P_{\max}$ is the average transmit power budget.

The *actual* received signal vector $\mathbf{y}_t^m$ (excluding noise) at the users of SP m is given by

$$\mathbf{y}_t^m = \mathbf{H}_t^m \mathbf{V}_t^m \mathbf{x}_t^m + \sum_{l\in\mathcal{M}, l\neq m} \mathbf{H}_t^m \mathbf{V}_t^l \mathbf{x}_t^l, \quad \forall m \in \mathcal{M}$$

where the second term is the inter-SP interference from the other SPs to the users of SP m. The actual received signal vector $\mathbf{y}_t \triangleq [\mathbf{y}_t^{1H}, \dots, \mathbf{y}_t^{MH}]^H$ at all K users is given by $\mathbf{y}_t = \mathbf{H}_t\mathbf{V}_t\mathbf{x}_t$.

Under the transmission structure of a typical cellular network, such as LTE, we consider a periodic demand-response mechanism for online virtualization. As shown in Fig. 2, the total time horizon T is segmented into I update periods, each having a duration of T_0 time slots, so that $T = T_0 I$. Let $\mathcal{T} = \{0, \dots, T-1\}$ and $\mathcal{I} = \{0, \dots, I-1\}$. In LTE, an update period may correspond to the duration of one or multiple resource blocks. Within each update period, neither the virtual precoding matrices $\mathbf{W}_t^m$ nor the actual precoding matrix $\mathbf{V}_t$ can change. With slight abuse of notation, we use $\mathbf{V}_i$ to denote the precoding matrix chosen by the InP at the beginning of update period $i \in \mathcal{I}$, *i.e.,* $\mathbf{V}_t = \mathbf{V}_i, \forall t \in [iT_0, (i+1)T_0)$.

Let t_i denote the beginning time slot of update period i. Within each update period i, the InP receives multiple delayed CSI feedbacks $\mathbf{H}_t$ for $t_i \le t < t_{i+1}$. Assume there are S_i CSI feedbacks in period i, where $S_i \ge 1$, and let $\mathcal{S}_i = \{1, \dots, S_i\}$. Let τ_i^s denote the time slot at which the s-th CSI feedback in period i is sent, $s \in \mathcal{S}_i$, and let d_i^s be the corresponding delay to arrive at the InP. Then, $\mathbf{H}_{\tau_i^s}$ is received by the InP at the end of time slot $\tau_i^s + d_i^s - 1$. We assume that the delay may be multiple time slots, but is limited such that the CSI feedback is received by the InP in the same period i before it designs the next precoding $\mathbf{V}_{i+1}$, *i.e.,* $1 \le d_i^s \le t_{i+1} - \tau_i^s, \forall s \in \mathcal{S}_i, \forall i \in \mathcal{I}$. Due to unequal delays, the CSI feedbacks may not be received in the order as they are sent.

B. Problem Formulation

The InP designs the actual precoding matrix to mitigate the inter-SP interference in order to meet the virtualization demand received from the SPs. The expected deviation of the received signals by the InP's actual precoding from that of the SPs' virtualization demand is given by $\mathbb{E}_{\mathbf{x}_t}\{\|\mathbf{y}_t - \tilde{\mathbf{y}}_t\|_F^2\} = \|\mathbf{H}_t\mathbf{V}_t - \mathbf{D}_t\|_F^2$. When $\mathbf{H}_t\mathbf{V}_t - \mathbf{D}_t = \mathbf{0}$, the InP-designed precoding matrix $\mathbf{V}_t$ nulls the inter-SP interference and meets the virtual demands of all SPs. We define the deviation of InP's precoding from the virtualization demand as

$$f_t(\mathbf{V}) \triangleq \|\mathbf{H}_t\mathbf{V} - \mathbf{D}_t\|_F^2, \quad \forall t \in \mathcal{T} \tag{3}$$

which is a convex loss function we use as design metric for MIMO WNV.

For the online precoding design of the MIMO WNV, we consider the following two goals typically considered for the constrained OCO, with a slight modification tailored to our problem. The first goal is to design $\mathbf{V}_i$, such that the accumulated precoding deviation in (3) is competitive with the optimal offline fixed precoding matrix with the same set of CSI feedbacks. The latter is defined by

$$\mathbf{V}_p^\circ \triangleq \arg\min_{\mathbf{V}\in\mathcal{V}} \sum_{i\in\mathcal{I}} \frac{T_0}{S_i} \sum_{s\in\mathcal{S}_i} f_{\tau_i^s}(\mathbf{V}) \tag{4}$$

where $\mathcal{V} \triangleq \{\mathbf{V} \in \mathcal{V}_0 : g(\mathbf{V}) \leq 0\}$ is the set of precoding matrices satisfying both long-term and short-term transmit power constraints. For the above goal, we define the regret under partial CSI feedbacks as follows:

$$\text{RE}_p(T) \triangleq \sum_{i\in\mathcal{I}} \frac{T_0}{S_i} \sum_{s\in\mathcal{S}_i} f_{\tau_i^s}(\mathbf{V}_i) - f_{\tau_i^s}(\mathbf{V}_p^\circ). \tag{5}$$

The second goal of our online design is to ensure a sub-linear violation of the long-term transmit power constraint within total T time slots, defined as

$$\text{VO}(T) \triangleq \sum_{i\in\mathcal{I}} g(\mathbf{V}_i)T_0. \tag{6}$$

Remark: For partial CSI feedbacks, *i.e.,* $\sum_{i\in\mathcal{I}} S_i < T$, it is unfair to compare our online algorithm with the standard offline performance benchmark assuming all CSIs within T time slots are known apriori, *i.e.,* $\mathbf{V}^\circ \triangleq \arg\min_{\mathbf{V}\in\mathcal{V}} \sum_{t\in\mathcal{T}} f_t(\mathbf{V})$. For each update period i, let $\mathcal{T}_i = \{\tau_i^1, \dots, \tau_i^{S_i}\}$ be the set of time slots with CSI feedbacks and $\mathcal{F}_i = \{t_i, \dots, t_{i+1} - 1\}$ be the set of time slots where precoding matrix $\mathbf{V}_i$ is applied. For our offline performance benchmark $\mathbf{V}_p^\circ$, we assume the precoding deviation function at each time slot $t \in \mathcal{F}_i \backslash \mathcal{T}_i$ is the average of those where CSI feedbacks are provided, *i.e.,* $\frac{1}{S_i}\sum_{s\in\mathcal{S}_i} f_{\tau_i^s}(\mathbf{V}), \forall i \in \mathcal{I}$. As such, the offline performance benchmark $\mathbf{V}_p^\circ$ has the same amount of CSI feedbacks as our online algorithm, except being apriori.

Different from the standard constrained OCO problem considered in [10]-[13], in this work, we aim to find a precoding solution $\{\mathbf{V}_i\}$ to provide sub-linear regret in (5) and long-term constraint violation in (6) based on $\mathbf{H}_{\tau_i^s}$ and $\mathbf{D}_{\tau_i^s}, s \in \mathcal{S}_i$, which is a new constrained OCO problem.

III. Online MIMO WNV Algorithm

We leverage the OCO technique to design a new online MIMO WNV algorithm with periodic precoder updates. The existing constrained OCO algorithms are under a strict per-time-slot setting with $T_0 = 1$ [10]-[13], *i.e.,* per-time-slot feedback and decision update. Different from these algorithms, we develop new techniques to accommodate imperfect CSI feedbacks and periodic precoding updates.

Algorithm 1 Online MIMO WNV with Periodic Updates

1: Let $\gamma, \alpha > 0$ be constants. Initialize T_0, $Q_0 = 0$, and $\mathbf{V}_0 \in \mathcal{V}_0$.
2: At the beginning of each update period $(i+1)$, *i.e.,* time slot t_{i+1}, use $\{\mathbf{H}_{\tau_i^s}, \mathbf{D}_{\tau_i^s}, s \in \mathcal{S}_i\}$ to compute the following:
3: Update the virtual queue as

$$Q_{i+1} = \max\{-\tilde{g}(\mathbf{V}_i)T_0, Q_i + \tilde{g}(\mathbf{V}_i)T_0\}$$

where $\tilde{g}(\mathbf{V}) \triangleq \gamma g(\mathbf{V})$.
4: Solve the per-period problem $\mathbf{P1}$ for $\mathbf{V}_{i+1}$:

$$\mathbf{P1}: \min_{\mathbf{V}\in\mathcal{V}_0} \frac{T_0}{S_i} \sum_{s\in\mathcal{S}_i} 2\Re\{\text{tr}\{\nabla_{\mathbf{V}_i^*} f_{\tau_i^s}^H(\mathbf{V}_i)(\mathbf{V} - \mathbf{V}_i)\}\} + \alpha\|\mathbf{V} - \mathbf{V}_i\|_F^2 + [Q_{i+1} + \tilde{g}(\mathbf{V}_i)T_0]\tilde{g}(\mathbf{V})T_0.$$

A. Online Algorithm

We assume that the channel gain is bounded by a constant $B \geq 0$ at any time t, given by

$$\|\mathbf{H}_t\|_F \leq B, \quad \forall t \in \mathcal{T}. \tag{7}$$

We show in the following lemma that our online MIMO WNV problem satisfies several general assumptions of OCO: 1) The impact of the compact convex set $\mathcal{V}_0$ is bounded; 2) The gradient of the convex loss function $f_t(\mathbf{V})$ is bounded $\forall t$; 3) The convex long-term constraint function $g(\mathbf{V})$ is Lipschitz continuous; 4) The impact of $g(\mathbf{V})$ is bounded; 5) There exists an interior point $\mathbf{V}' \in \mathcal{V}_0$ for $g(\mathbf{V})$. The proof is omitted due to space limitation.

Lemma 1. Assume the bounded channel gain in (7). Then, the following statements hold:

$$\|\mathbf{V} - \mathbf{V}'\|_F \leq R, \quad \forall \mathbf{V}, \mathbf{V}' \in \mathcal{V}_0, \tag{8}$$

$$\|\nabla f_t(\mathbf{V})\|_F \leq D, \quad \forall \mathbf{V} \in \mathcal{V}_0, \quad \forall t \in \mathcal{T}, \tag{9}$$

$$|g(\mathbf{V}) - g(\mathbf{V}')| \leq R\|\mathbf{V} - \mathbf{V}'\|_F, \quad \forall \mathbf{V}, \mathbf{V}' \in \mathcal{V}_0, \tag{10}$$

$$|g(\mathbf{V})| \leq G, \quad \forall \mathbf{V} \in \mathcal{V}_0, \tag{11}$$

$$\exists \mathbf{V}' \in \mathcal{V}_0, \quad g(\mathbf{V}') \leq -\bar{P} \tag{12}$$

where $R = 2\sqrt{P_{\max}}$, $D = 4B^2\sqrt{P_{\max}}$, and $G = \sqrt{\max\{\bar{P}^2, (P_{\max} - \bar{P})^2\}}$.

Our proposed OCO algorithm with periodic updates is shown in Algorithm 1, where $\tilde{g}(\mathbf{V}) \triangleq \gamma g(\mathbf{V})$, $\nabla_{\mathbf{V}_i^*} f_{\tau_i^s}(\mathbf{V}_i)$ is the partial derivative of $f_{\tau_i^s}(\mathbf{V}_i)$ with respect to $\mathbf{V}_i^*$. At each update period, the algorithm selects a precoding matrix $\mathbf{V}_i$ and a virtual queue length Q_i, which can be viewed as the primal and dual variables in the saddle-point-typed OCO algorithms [10]-[11]. The main difference is that Algorithm 1 uses the virtual queue as a backlog queue of the long-term constraint violation, and thus an upper bound on the virtual queue can be readily transformed into an upper bound on the long-term constraint violation, which will be illustrated in Section III-C. Note that a virtual-queue-based algorithm has been proposed for the standard constrained OCO problem in [12], which requires per-time-slot feedback and decision update. In contrast, the virtual-queue update rule in Step 3 of Algorithm 1 converts the impact of periodic precoding updates on the long-term transmit power constraint into the virtual queue dynamics. Furthermore, the per-period optimization problem **P1** in Algorithm 1 accommodates periodic precoding design with out-of-order delayed partial CSI feedbacks. Compared with the OCO algorithm in [13] under the standard setting, Algorithm 1 only uses the gradient information, instead of the complete information of the loss function information in (3).

B. Online Precoding Solution

Now we solve **P1** in Algorithm 1 for the downlink precoding matrix $\mathbf{V}_{i+1}$. Since **P1** is a convex optimization problem with strong duality, we solve it by studying the Karush-Kuhn-Tucker (KKT) conditions. The Lagrangian for **P1** is

$$L(\mathbf{V},\lambda)=\frac{T_0}{S_i}\sum_{s\in\mathcal{S}_i}2\Re\{\mathrm{tr}\{\nabla_{\mathbf{V}_i^*}f_{\tau_i^s}^H(\mathbf{V}_i)(\mathbf{V}-\mathbf{V}_i)\}\}$$
$$+\alpha\|\mathbf{V}-\mathbf{V}_i\|_F^2+[Q_{i+1}+\tilde{g}(\mathbf{V}_i)T_0]\tilde{g}(\mathbf{V})T_0+\lambda(\|\mathbf{V}\|_F^2-P_{\max})$$

where λ is the Lagrange multiplier associated with the short-term transmit power constraint (1). The KKT conditions for $(\mathbf{V}^\circ,\lambda^\circ)$ being globally optimal are given by $\|\mathbf{V}^\circ\|_F^2-P_{\max}\leq 0$, $\lambda^\circ\geq 0$, $\lambda^\circ(\|\mathbf{V}^\circ\|_F^2-P_{\max})=0$, and

$$\mathbf{V}^\circ=\frac{\alpha\mathbf{V}_i-\frac{T_0}{S_i}\sum_{s\in\mathcal{S}_i}\mathbf{H}_{\tau_i^s}^H(\mathbf{H}_{\tau_i^s}\mathbf{V}_i-\mathbf{D}_{\tau_i^s})}{\alpha+[Q_{i+1}+\tilde{g}(\mathbf{V}_i)T_0]\gamma T_0+\lambda^\circ}, \tag{13}$$

which follows by setting $\nabla_{\mathbf{V}^*}L(\mathbf{V},\lambda)=\mathbf{0}$ and $\nabla_{\mathbf{V}_i^*}f_{\tau_i^s}(\mathbf{V}_i)=\mathbf{H}_{\tau_i^s}^H(\mathbf{H}_{\tau_i^s}\mathbf{V}_i-\mathbf{D}_{\tau_i^s})$. From the KKT conditions, and noting that λ° serves as a power regularization factor for $\mathbf{V}^\circ$ in (13), we have a closed-form solution for $\mathbf{V}_{i+1}$, given by

$$\mathbf{V}_{i+1}=\begin{cases}\mathbf{X}_i, & \text{if } \|\mathbf{X}_i\|_F^2\leq P_{\max}\\ \sqrt{P_{\max}}\frac{\mathbf{X}_i}{\|\mathbf{X}_i\|_F}, & \text{o.w.}\end{cases} \tag{14}$$

where $\mathbf{X}_i=\frac{\alpha\mathbf{V}_i-\frac{T_0}{S_i}\sum_{s\in\mathcal{S}_i}\mathbf{H}_{\tau_i^s}^H(\mathbf{H}_{\tau_i^s}\mathbf{V}_i-\mathbf{D}_{\tau_i^s})}{\alpha+[Q_{i+1}+\tilde{g}(\mathbf{V}_i)T_0]\gamma T_0}$.

Remark: Note that although not considered in our model, the short-term per-antenna transmit power constraints can be incorporated in the convex set $\mathcal{V}_0$ as well. The per-period problem **P1** can still be equivalently decomposed into N sub-problems, each with a closed-form solution similar to (14).

C. Performance Bounds

We now analyze the impacts of periodic update and partial feedback on the regret and the long-term constraint violation bounds. Our analysis is unique from existing analysis for constrained OCO algorithms, which applies only to the standard per-time-slot setting [10]-[13]. We omit the proofs due to space constraint. The following lemma provide bounds for the virtual queue $\{Q_i\}$ produced by Algorithm 1.

Lemma 2. The following statements hold for the all $i\in\mathcal{I}$:

$$Q_{i+1}+\tilde{g}(\mathbf{V}_i)T_0\geq 0, \tag{15}$$
$$Q_{i+1}\geq|\tilde{g}(\mathbf{V}_i)T_0|, \tag{16}$$
$$Q_{i+1}\leq Q_i+|\tilde{g}(\mathbf{V}_i)T_0|. \tag{17}$$

Define $L_i\triangleq\frac{1}{2}Q_i^2$ as the quadratic Lyapunov function and $\Delta_i\triangleq L_{i+1}-L_i$ as the Lyapunov drift for each update period $i\in\mathcal{I}$. Leveraging the results in Lemma 2, we provide an upper bound on the Lyapunov drift Δ_i in the following lemma.

Lemma 3. The Lyapunov drift is upper bounded as follows:

$$\Delta_i\leq Q_i[\tilde{g}(\mathbf{V}_i)T_0]+[\tilde{g}(\mathbf{V}_i)T_0]^2,\quad\forall i\in\mathcal{I}. \tag{18}$$

The following lemma shows that an upper bound on the virtual queue can be readily transferred to bound the long-term constraint violation.

Lemma 4. For the virtual queue $\{Q_i\}$ produced by Algorithm 1 within I update periods, we have $\mathrm{VO}(T)\leq\frac{1}{\gamma}Q_I$.

Using Lemmas 1-4 and OCO techniques, we provide performance bounds for Algorithm 1 in the following theorem.

Theorem 5. For any $I>0$ with $T=IT_0$, set $\gamma^2=\frac{\sqrt{T}}{T_0}$ and $\alpha=\frac{1}{2}(1+R^2)T_0\sqrt{T}$ in Algorithm 1. Then, the following statements hold for $\{\mathbf{V}_i\}$ produced by Algorithm 1:

$$\mathrm{RE}_p(T)\leq\frac{D^2+(1+R^2)R^2+G^2}{2}T_0\sqrt{T}=\mathcal{O}(\sqrt{T}), \tag{19}$$
$$\mathrm{VO}(T)\leq\frac{2DR+(1+R^2)R^2+4G(\bar{P}+G)}{2\bar{P}}T_0=\mathcal{O}(1) \tag{20}$$

where R, D, G are defined in Lemma 1.

Theorem 5 shows that, for Algorithm 1, the regret over T in (5) grows in the order of $\mathcal{O}(\sqrt{T})$, and the long-term constraint violation over T in (6) is maintained in the order of $\mathcal{O}(1)$.

Remark: The results in Theorem 5 can be extended to a more general aperiodic update scenario. Consider a time-varying update period such that the i-th update period contains T_i time slots. Define the variation of update periods as $V_T\triangleq\sum_{i\in\mathcal{I}}(T_i-T_{i+1})^2$. Assume $T_i\leq T_{\max}$, for some $T_{\max}>0$, and $V_T=\mathcal{O}(1)$, we can show that Algorithm 1 still yields $\mathcal{O}(\sqrt{T})$ regret and $\mathcal{O}(1)$ violation of the long-term constraint.

IV. Simulation Results

We consider a urban hexagon micro-cell of radius 500 m. An InP owns the BS equipped with $N=32$ antennas. The InP performs WNV and serves $M=4$ SPs. Each SP m serves $K_m=2$ users uniformly distributed in the cell, for a total of $K=8$ users in the cell. We set bandwidth $B_W=15$ kHz, $P_{\max}=33$ dBm, and $\bar{P}=32$ dBm as default. We model the fading channel as a first-order Gaussian-Markov process $\mathbf{h}_{t+1}^k=\alpha_{\mathbf{h}}\mathbf{h}_t^k+\mathbf{z}_t^k,\forall k\in\mathcal{K}$, where $\mathbf{h}_t^k\sim\mathcal{CN}(\mathbf{0},\beta_k\mathbf{I})$ with β_k representing path-loss and shadowing, $\alpha_{\mathbf{h}}\in[0,1]$ is the channel correlation coefficient, and $\mathbf{z}_t^k\sim\mathcal{CN}(\mathbf{0},(1-\alpha_{\mathbf{h}}^2)\beta_k\mathbf{I})$ is independent of $\mathbf{h}_t^k$. We set $\alpha_{\mathbf{h}}=0.995$ as default, which corresponds to user speed 3 km/h, under the standard LTE transmission structure [9]. We set the time slot duration to be $\Delta t=\frac{1}{B_W}$, and assume there is only one CSI feedback at the beginning of each update period, *i.e.*, $\mathcal{T}_i=\{iT_0\},\forall i\in\mathcal{I}$. We set the update period $T_0=8$ time slots as default, such that the update period is similar to the duration of one resource block in LTE. We set the upper bound of channel gain in (7) as $B=1.645\sqrt{N\sum_{k\in\mathcal{K}}\beta_k}$; it is chosen such that, based on the Chernoff bound, the probability that the gain of unbounded Rayleigh fading channel exceeds this bound is $\mathbb{P}\{\|\mathbf{H}_t\|_F>B\}\leq 1.1\times 10^{-9}$.

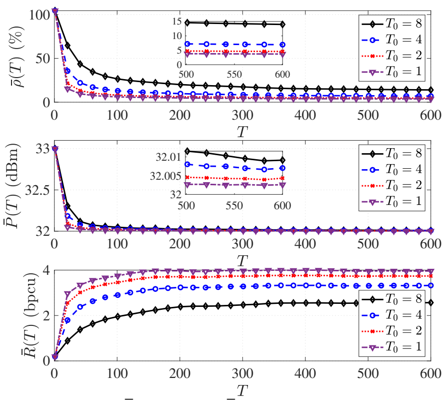

Fig. 3. $\bar{\rho}(T)$, $\bar{P}(T)$, and $\bar{R}(T)$ vs. T with different T_0.

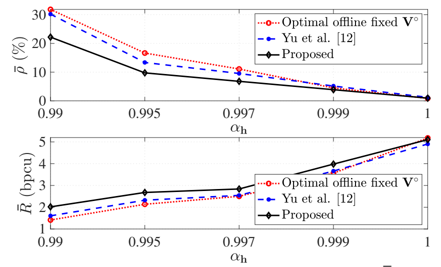

Fig. 4. Performance comparison on $\bar{\rho}$ and $\bar{R}$ vs. $\alpha_{\mathbf{h}}$.

We assume that each SP $m \in \mathcal{M}$ uses the precoding method of maximum ratio transmission (MRT) to design its virtual precoding matrix: $\mathbf{W}_t^m = \sqrt{P_m}\frac{\mathbf{H}_t^m}{\|\mathbf{H}_t^m\|_F}$, where $P_m = \frac{P_{\max}}{M}$. For performance evaluation, we define the time-averaged precoding deviation normalized against the virtualization demand as $\bar{\rho}(T) \triangleq \frac{1}{T}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{F}_i}\frac{\hat{f}_t(\mathbf{V}_i)}{\|\mathbf{D}_t\|_F^2}$, the time-averaged transmit power as $\bar{P}(T) \triangleq \frac{1}{T}\sum_{i\in\mathcal{I}}\|\mathbf{V}_i\|_F^2 T_0$, and the time-averaged per-user rate as $\bar{R}(T) \triangleq \frac{1}{TK}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{F}_i}\sum_{k\in\mathcal{K}}\log_2\left(1+\frac{|\mathbf{h}_t^{kT}\mathbf{v}_i^k|^2}{\sum_{k'\in\mathcal{K},k'\neq k}|\mathbf{h}_t^{kT}\mathbf{v}_i^{k'}|^2+\sigma_n^2}\right)$.

We set the time horizon $T = 600$. To select γ and α, we solve the following geometric programming problem **P2** for γ and α in Algorithm 1, which yields the tightest regret bound subject to the constraint on the long-term constraint violation.

$$\textbf{P2:} \quad \min_{\gamma>0,\alpha\geq\frac{1}{2}(T_0\sqrt{T}+T_0^2R^2\gamma^2)} \quad D^2\sqrt{T}+2\alpha R^2+T_0^2\gamma^2G^2$$
$$\text{s.t.} \quad 2T_0G+\frac{T_0DR+\alpha R^2+2T_0^2\gamma^2G^2}{T_0\gamma^2\bar{P}}\leq(P_{\max}-\bar{P})\sqrt{T}.$$

Fig. 3 shows $\bar{\rho}(T)$, $\bar{P}(T)$, and $\bar{R}(T)$ versus T for different values of the update period T_0. We observe that the proposed algorithm has fast convergence. Furthermore, the performance deteriorates as T_0 increases, *i.e.,* updates are less frequent. This illustrates the impact of channel variation over time, while $\mathbf{V}_i$ is fixed for the update period and only one CSI feedback is received per period.

Next, we compare our proposed algorithm, the standard optimal offline fixed precoding $\mathbf{V}^\circ$, and the online algorithm from [12]. Note that [12] considers only the standard OCO setting with per-time-slot updates. In order to apply it to the periodic update scenario of our work, we treat each update period of T_0 time slots as one super time slot. However, [12] no longer provides any performance guarantee in this scenario. Fig. 4 shows the performance of $\bar{\rho}$ and $\bar{R}$ by these algorithms. We see that our proposed algorithm provides better interference suppression and thus a higher system throughput than that of the other two schemes.

V. Conclusions

In this work, we have considered online downlink precoding design for MIMO WNV over time-varying channels. We study a periodic update model where partial CSI feedbacks may be received out of order due to delay and the precoding is updated per period. Our aim is to minimize the accumulated deviation of the InP's actual precoder from the virtualization demands by SPs, subject to both long-term and short-term transmit power constraints. Due the periodic update model, this is a unique constrained OCO problem different from existing ones. We have proposed an online periodic precoding update algorithm, which achieves $\mathcal{O}(\sqrt{T})$ growth of regret over T on the precoding deviation from the virtualization demand, and maintains $\mathcal{O}(1)$ violation of the long-term transmit power constraint. The proposed online precoding solution is in closed-form. It does not require the knowledge of channel distribution and only depends on the past CSI. Simulation results have demonstrated the effectiveness of the proposed algorithm.

References

[1] M. Richart, J. Baliosian, J. Serrat, and J. Gorricho, "Resource slicing in virtual wireless networks: A survey," *IEEE Trans. Netw. Service Manag.*, vol. 13, pp. 462–476, Sep. 2016.

[2] K. Zhu and E. Hossain, "Virtualization of 5G cellular networks as a hierarchical combinatorial auction," *IEEE Trans. Mobile Comput.*, vol. 15, pp. 2640–2654, Oct. 2016.

[3] Z. Chang, Z. Han, and T. Ristaniemi, "Energy efficient optimization for wireless virtualized small cell networks with large-scale multiple antenna," *IEEE Trans. Commun.*, vol. 65, pp. 1696–1707, Apr. 2017.

[4] S. Parsaeefard, R. Dawadi, M. Derakhshani, T. Le-Ngoc, and M. Baghani, "Dynamic resource allocation for virtualized wireless networks in massive-MIMO-aided and fronthaul-limited C-RAN," *IEEE Trans. Veh. Technol.*, vol. 66, pp. 9512–9520, Oct. 2017.

[5] Y. Liu, M. Derakhshani, S. Parsaeefard, S. Lambotharan, and K. Wong, "Antenna allocation and pricing in virtualized massive MIMO networks via Stackelberg game," *IEEE Trans. Commun.*, vol. 66, pp. 5220–5234, Nov. 2018.

[6] M. Soltanizadeh, B. Liang, G. Boudreau, and S. H. Seyedmehdi, "Power minimization in wireless network virtualization with massive MIMO," in *Proc. Intel. Conf. Commun. Workshops (ICC)*, May 2018.

[7] J. Wang, M. Dong, B. Liang, and G. Boudreau, "Online downlink MIMO wireless network virtualization in fading environments," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, Dec. 2019.

[8] J. Wang, M. Dong, B. Liang, and G. Boudreau, "Online downlink MIMO wireless network virtualization with imperfect CSI," in *Proc. IEEE Conf. Comput. Commun. (INFOCOM)*, Jul. 2020.

[9] H. Holma and A. Toskala, *WCDMA for UMTS - HSPA evolution and LTE*. John Wiely & Sons, 2010.

[10] M. Mahdavi, R. Jin, and T. Yang, "Trading regret for efficiency: Online convex optimization with long term constraints," *J. Mach. Learn. Res.*, vol. 13, pp. 2503–2528, Sep. 2012.

[11] R. Jenatton, J. Huang, and C. Archambeau, "Adaptive algorithms for online convex optimization with long-term constraints," in *Proc. Intel. Conf. Mach. Learn. (ICML)*, Jun. 2016.

[12] H. Yu and M. J. Neely, "A low complexity algorithm with $O(\sqrt{T})$ regret and $O(1)$ constraint violations for online convex optimization with long term constraints," *J. Mach. Learn. Res.*, vol. 21, pp. 1–24, Feb. 2020.

[13] X. Cao, J. Zhang, and H. V. Poor, "A virtual-queue-based algorithm for constrained online convex optimization with applications to data center resource allocation," *IEEE J. Sel. Topics Signal Process.*, vol. 12, pp. 703–716, Aug. 2018.