WMMSE Beamforming for User-Centric Cell-Free Networks with Non-Coherent Joint Transmission

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***Abstract*—We consider downlink beamforming design to maximize the weighted sum rate (WSR) in a user-centric cell-free network, where distributed access points (APs) are organized in a cluster to jointly serve each user equipment (UE). This architecture ensures uniform service quality even at the cell edge, but synchronization between APs can be problematic, necessitating the utilization of non-coherent joint transmission (NCJT) that eliminates the need for strict synchronization. Most existing works on beamforming design for NCJT assume that the classic weighted minimum mean square error (WMMSE) approach is not applicable, and design their beamforming algorithms based on the successive convex approximation (SCA) method with high computational complexity. In this work, we for the first time demonstrate the applicability of the WMMSE approach for NCJT in cell-free networks. Based on the unique observations on the structures of the WSR maximization problem for NCJT, we propose an efficient WMMSE based beamforming algorithm. Our proposed algorithm is guaranteed to converge to a stationary point of the WSR maximization problem. Furthermore, our beamforming updates are in closed form with low computational complexity. Simulation results demonstrate substantial performance gain of our proposed algorithm over the current best SCA based alternatives in both computational complexity and convergence time.**

I. Introduction

In traditional cellular networks, edge user equipments (UEs) suffer from inherent inter-cell interference and rate degradation due to co-located antenna layouts, which limit the improvement of quality of service (QoS) for wireless communication [1]. To overcome these limitations, [2] proposed the concept of the cell-free network, where distributed access points (APs) cooperate with each other to jointly serve the UEs, such that each UE is served by multiple APs and cell boundary no longer exists. This approach can mitigate inter-cell interference and reduce the rate variation among UEs. However, in practical cell-free networks, the UE received signals from the remote APs may experience a considerable amount of attenuation due to pathloss. In other words, these remote APs contribute little signal-to-noise ratio (SNR) to their serving UEs, while consuming substantial transmit power and causing additional interference to the other UEs [3]. Therefore, it is desirable that the UEs are served only by nearby APs, generally referred to as user-centric cell-free networks. It has been shown that the user-centric cell-free networks can outperform the standard cell-free networks in terms of both system performance and network scalability [4].

In user-centric cell-free networks, multiple APs perform joint transmission (JT) to provide high QoS to their serving UEs with the assistance of a central unit (CU) [5]. A common JT strategy is coherent joint transmission (CJT), where multiple APs act as a virtual multiple-input multiple-output (MIMO) system and transmit the same signals to their serving UEs [6]. However, CJT requires strict synchronization among APs and the synchronization cost is high for dense cell-free networks. In non-coherent joint transmission (NCJT), the APs send different signals to their serving UEs. Since each UE decodes the received signals independently, no strict synchronization is required for NCJT [6]. Similar to the CJT strategy, NCJT requires a cooperative beamforming design for the APs to enhance constructive signals and suppress destructive signals at the UEs [7].

Heuristic maximum ratio transmission (MRT) and zero forcing (ZF) precoding for NCJT were adopted in a cell-free MIMO network [8]. However, the MRT precoding cannot resist any interference, and neither the ZF precoding nor the MRT precoding aims to maximize the weighted sum rate (WSR) of the UEs. For dense small-cell networks, a downlink precoding algorithm based on semi-definite relaxation (SDR) was proposed to minimize the transmit power [9]. Although the SDR based algorithm in [9] can provide a feasible solution in polynomial time, it is not applicable to the WSR maximization problem. In [10], the authors developed a branch reduce and bound framework to find the global optimum of the WSR problem but at the expense of exponential complexity. To reduce the computational complexity, they utilized the inner approximation method to relax the original non-convex WSR maximization problem to a convex approximation subproblem, which is then solved by the standard alternating direction method of multipliers method. WSR maximization with fronthaul capacity constraints was investigated for NCJT in cell-free networks [6]. However, the algorithm proposed in [6] requires CVX [11] to solve a second order cone programming (SOCP) problem, which causes high computational costs.

All of the above works on NCJT for cell-free networks assume single-antenna receivers, which greatly simplifies the beamforming design. But it is challenging to consider the more commonly encountered scenarios with multi-antenna receivers. Furthermore, most existing precoding designs for NCJT in cell-free networks are based on successive convex approximation (SCA), which is known to be of high computational complexity. For WSR maximization in CJT, the classic weighted minimum mean square error (WMMSE) approach yields closed form beamforming updates with low computational complexity [12], [13]. However, for NCJT, whether the WMMSE approach is applicable remains an open problem.

In this work, we aim at developing a low-complexity beamforming algorithm to maximize the WSR for user-centric cell-free MIMO networks with NCJT. The main contributions of this paper are summarized below:

1) We for the first time show the applicability of the classic WMMSE approach to NCJT in cell-free networks, based on unique observations on the structures of the WSR expression to equivalently transform the non-convex WSR maximization problem into a convex weighted mean square error (MSE) minimization problem.
2) We propose an efficient beamforming algorithm under the WMMSE framework, to maximize the WSR subject to per-AP power constraints. We show that our proposed algorithm converges to a stationary point of the original WSR maximization problem. Furthermore, our beamforming updates are in closed form with low computational complexity.
3) Our simulation results demonstrate that the proposed WMMSE based algorithm substantially outperforms the current best SCA based algorithms, in terms of both computational complexity and convergence speed.

Notations: We use bold lower and upper case letters to represent vectors and matrices, i.e., $\mathbf{a}$, $\mathbf{A}$. The Hermitian transpose, inverse, and trace of a matrix $\mathbf{A}$ are denoted by $\mathbf{A}^H$, $\mathbf{A}^{-1}$, and $\mathrm{Tr}(\mathbf{A})$, respectively. The notation $\mathbf{A} \succ \mathbf{0}$ indicates positive definite. The notation $\mathrm{blkdiag}(\mathbf{A}_1, \ldots, \mathbf{A}_n)$ denotes a block diagonal matrix with matrices $\mathbf{A}_1, \ldots, \mathbf{A}_n$. Notation $\mathbf{I}$ denotes an identity matrix. The real part of a complex number a is denoted as $\mathrm{Re}(a)$. For $\mathbf{x}$ being a vector, $\mathbf{x} \sim \mathcal{CN}(\mathbf{m}, \mathbf{R})$ means that $\mathbf{x}$ is a circular complex Gaussian random variable with mean $\mathbf{m}$ and covariance $\mathbf{R}$. The binary and complex space are denoted as $\mathbb{B}$ and $\mathbb{C}$.

II. System Model

We consider a user-centric cell-free MIMO network that consists of I APs and K UEs, denoted by indices $\mathcal{I} = \{1, \ldots, I\}$ and $\mathcal{U} = \{1, \ldots, K\}$, respectively. As shown in Fig. 1, the APs are distributed geographically in the network to get close access to the UEs. All the APs cooperate with each other via a CU, where baseband signal processing tasks are performed. In the user-centric network, each AP i is equipped with M_i transmit antennas, serving a set of UEs $\mathcal{U}_i \subseteq \mathcal{U}$. Each UE k is equipped with N_k receive antennas and is served by APs $\mathcal{I}_k \subseteq \mathcal{I}$ that are close to it.

A. Non-Coherent Joint Transmission

In NCJT, each UE k receives different signals $\mathbf{s}_{i,k} \in \mathbb{C}^{D_{i,k} \times 1}$ from all the AP i, where $D_{i,k}$ is the number of data streams. Note that only $\mathbf{s}_{i,k}$ transmitted from AP $i \in \mathcal{I}_k$

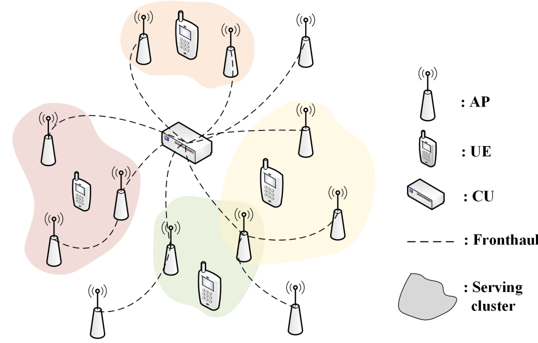

Fig. 1. An illustration of the user-centric cell-free network.

contains useful signals for UE k. Let $\mathbf{H}_{i,k} \in \mathbb{C}^{N_k \times M_i}$ and $\mathbf{P}_{i,k} \in \mathbb{C}^{M_i \times D_{i,k}}$ denote the channel matrix and the beamforming matrix between AP i and UE k. The received signals at each UE k can be written as

$$\mathbf{y}_k = \underbrace{\sum_{i \in \mathcal{I}_k} \mathbf{H}_{i,k}\mathbf{P}_{i,k}\mathbf{s}_{i,k}}_{\text{useful signals}} + \underbrace{\sum_{l \in \mathcal{U}_{-k}} \sum_{j \in \mathcal{I}_l} \mathbf{H}_{j,k}\mathbf{P}_{j,l}\mathbf{s}_{j,l}}_{\text{inter-user interference}} + \mathbf{z}_k \quad (1)$$

where $\mathcal{U}_{-k} \triangleq \{\mathcal{U} \backslash k\}$ and $\mathbf{z}_k \sim \mathcal{CN}\left(\mathbf{0}, {\sigma_k}^2\mathbf{I}\right)$ is the additive Gaussian noise at UE k.

For NCJT, successive interference cancellation (SIC) is usually adopted at the UEs to detect the received signals from their serving APs [6], [10]. With SIC, the achievable data rate of UE k is

$$R_k = \log\det\left(\mathbf{I} + \left(\sum_{i \in \mathcal{I}_k} \mathbf{H}_{i,k}\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\mathbf{H}_{i,k}^H\right)\mathbf{N}_k^{-1}\right) \quad (2)$$

where $\mathbf{N}_k \in \mathbb{C}^{N_k \times N_k}$ is the interference-plus-noise term

$$\mathbf{N}_k \triangleq \mathbf{J}_k + \sigma_k^2\mathbf{I} = \sum_{l \in \mathcal{U}_{-k}} \sum_{j \in \mathcal{I}_l} \mathbf{H}_{j,k}\mathbf{P}_{j,l}\mathbf{P}_{j,l}^H\mathbf{H}_{j,k}^H + \sigma_k^2\mathbf{I}. \quad (3)$$

B. Problem Formulation

We aim at designing the beamforming matrices $\{\mathbf{P}_{i,k}\}$ to maximize the WSR of the user-centric cell-free MIMO network, subject to individual power budget constraints at the APs. Specifically, the power constrained WSR maximization problem is formulated as

$$\begin{aligned} \max_{\{\mathbf{P}_{i,k}\}} \quad & \sum_{k=1}^{K} \alpha_k R_k \\ \text{s. t.} \quad & \sum_{k \in \mathcal{U}_i} \mathrm{Tr}\left(\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\right) \leq P_{\max,i}, \ \forall i \in \mathcal{I}. \end{aligned} \quad (4)$$

where $\alpha_k > 0$ is the priority of UE k and $P_{\max,i}$ is the power budget of AP i.

Remark 1. For NP-hard problems such as problem (4), it is assumed by prior works [6] and [10] that the low-complexity WMMSE algorithm that has been widely adopted for CJT is not applicable to NCJT in cell-free MIMO networks. In [6], a SCA based algorithm was proposed to solve the WSR maximization problem for NCJT. However, the SCA approach is of high computational complexity in general. Furthermore, the beamforming algorithm in [6] requires CVX to solve a SOCP problem, which causes high computational costs and is thus impractical for real-world communication systems. We will show later in Section III that, based on the unique observations of the WSR maximization problem structure, the classic WMMSE approach is still applicable for NCJT.

III. Efficient WMMSE Based Algorithm for Weight Sum Rate Maximization in NCJT

In this section, from our unique observations on the structures of the WSR maximization problem (4) for NCJT in user-centric cell-free MIMO networks, we propose an efficient WMMSE based beamforming algorithm. Our proposed algorithm is guaranteed to converge to the stationary point of the original WSR maximization problem. Furthermore, our beamforming updates are in closed form with low computational complexity.

A. The WMMSE Approach

We reveal the following lemma in [13] to clarify the equivalent transformation in the WMMSE approach.

Lemma 1. [13, Lemma 4.1] For any $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{N} \succ \mathbf{0}$, the following transformation holds:

$$\begin{aligned}&\log\det(\mathbf{I}+\mathbf{A}\mathbf{B}\mathbf{B}^H\mathbf{A}^H\mathbf{N}^{-1})\\&=\max_{\mathbf{W}\succ\mathbf{0},\mathbf{U}}\log\det(\mathbf{W})-\mathrm{Tr}(\mathbf{W}\mathbf{E}(\mathbf{U},\mathbf{B}))+m,\end{aligned}\tag{5}$$

where $\mathbf{W}$ is an auxiliary weight matrix of size $m\times m$, the MSE matrix $\mathbf{E}\in\mathbb{C}^{m\times m}$ is defined as

$$\mathbf{E}(\mathbf{U},\mathbf{B})\triangleq\left(\mathbf{I}-\mathbf{U}^H\mathbf{A}\mathbf{B}\right)\left(\mathbf{I}-\mathbf{U}^H\mathbf{A}\mathbf{B}\right)^H+\mathbf{U}^H\mathbf{N}\mathbf{U}.\tag{6}$$

Besides, we give the closed-form expression of the optimal $\mathbf{U}^*$ as

$$\begin{aligned}\mathbf{U}^*&\triangleq\arg\min_{\mathbf{U}}\mathrm{Tr}(\mathbf{W}\mathbf{E}(\mathbf{U},\mathbf{B}))\\&=\left(\mathbf{N}+\mathbf{A}\mathbf{B}\mathbf{B}^H\mathbf{A}^H\right)^{-1}\mathbf{A}\mathbf{B},\end{aligned}\tag{7}$$

and the optimal $\mathbf{W}^*$ as

$$\mathbf{W}^*\triangleq\arg\max_{\mathbf{W}\succ\mathbf{0}}\log\det(\mathbf{W})-\mathrm{Tr}(\mathbf{W}\mathbf{E})=\mathbf{E}^{-1}.\tag{8}$$

Consequently, we have

$$-\log\det(\mathbf{E})=\max_{\mathbf{W}\succ\mathbf{0}}\log\det(\mathbf{W})-\mathrm{Tr}(\mathbf{W}\mathbf{E})+m.\tag{9}$$

Furthermore, substituting (7) into (6), the optimal MSE matrix $\mathbf{E}^*$ is given by

$$\begin{aligned}\mathbf{E}^*(\mathbf{U}^*,\mathbf{B})&=\mathbf{I}-\mathbf{U}^H\mathbf{A}\mathbf{B}\\&=\left(\mathbf{I}+\mathbf{B}^H\mathbf{A}^H\mathbf{N}^{-1}\mathbf{A}\mathbf{B}\right)^{-1}.\end{aligned}\tag{10}$$

Remark 2. We remark here that any problem that follows the structure of $\log\det(\mathbf{I}+\mathbf{A}\mathbf{B}\mathbf{B}^H\mathbf{A}^H\mathbf{N}^{-1})$ can be equivalently transformed to a weighted MSE form as shown in (5).

B. Transforming WSR Maximization to MSE Minimization

Previous studies assume that WMMSE cannot be used in the problem with the structure of $\log\left(1+\mathbf{b}^H\mathbf{A}\mathbf{b}n^{-1}\right)$, since the rank of $\mathbf{A}=\bar{\mathbf{A}}^H\bar{\mathbf{A}}$ is higher than one [6], [10]. However, following the fact that $1+\mathbf{v}^H\mathbf{u}=\det\left(\mathbf{I}+\mathbf{u}\mathbf{v}^H\right)$, the seemingly unsolvable problem in [6] can be subtly rewritten as $\log\det\left(\mathbf{I}+\bar{\mathbf{A}}\mathbf{b}\mathbf{b}^H\bar{\mathbf{A}}^Hn^{-1}\right)$ to yield the structure of WMMSE transformation.

Similarly, we show that the WSR objective of problem (4) is in the same structure as the WMMSE transformation (5) in Lemma 1, by reformulating problem (4) as

$$\begin{aligned}\max_{\{\mathbf{P}_{i,k}\}}\quad&\sum_{k=1}^{K}\alpha_k\log\det\left(\mathbf{I}+\mathbf{H}_k\mathbf{P}_k{\mathbf{P}_k}^H{\mathbf{H}_k}^H{\mathbf{N}_k}^{-1}\right)\\\mathrm{s.t.}\quad&\sum_{k\in\mathcal{U}_i}\mathrm{Tr}\left(\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\right)\le P_{\max},\ \forall i\in\mathcal{I}\end{aligned}\tag{11}$$

where $\mathbf{H}_k\in\mathbb{C}^{N_k\times\sum_{i\in\mathcal{I}_k}M_i}$ is the channel matrix between UE k and its serving APs

$$\mathbf{H}_k\triangleq\left[\mathbf{H}_{i_1,k},\mathbf{H}_{i_2,k},\ldots,\mathbf{H}_{i_{|\mathcal{I}_k|},k}\right],\tag{12}$$

and $\mathbf{P}_k\in\mathbb{C}^{\sum_{i\in\mathcal{I}_k}M_i\times\sum_{i\in\mathcal{I}_k}D_{i,k}}$ is the beamforming matrix for UE k,

$$\mathbf{P}_k\triangleq\mathrm{blkdiag}\left(\mathbf{P}_{i_1,k},\mathbf{P}_{i_2,k},\ldots,\mathbf{P}_{i_{|\mathcal{I}_k|},k}\right)\tag{13}$$

with $|\mathcal{I}_k|$ being the number of serving APs $\mathcal{I}_k$ of UE k.

We can see that the objective of the transformed problem (11) is in the same structure as (5) in Lemma 1. Therefore, following (9), the WSR maximization problem (11) can be equivalently transformed to the MSE minimization problem given by

$$\begin{aligned}\max_{\substack{\{\mathbf{P}_{i,k}\},\{\mathbf{U}_k\}\\\{\mathbf{W}_k\succ\mathbf{0}\}}}\quad&\sum_{k=1}^{K}\alpha_k(\log\det(\mathbf{W}_k)-\mathrm{Tr}\left(\mathbf{W}_k\mathbf{E}_k(\mathbf{U}_k,\mathbf{P})\right))\\\mathrm{s.t.}\quad&\sum_{k\in\mathcal{U}_i}\mathrm{Tr}\left(\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\right)\le P_{\max,i},\ \forall i\in\mathcal{I}.\end{aligned}\tag{14}$$

where $\mathbf{P}\triangleq\{\mathbf{P}_{i,k}\}$, $\mathbf{U}_k\in\mathbb{C}^{N_k\times\sum_{i\in\mathcal{I}_k}D_{i,k}}$ and the weighted matrix $\mathbf{W}_k\in\mathbb{C}^{\sum_{i\in\mathcal{I}_k}D_{i,k}\times\sum_{i\in\mathcal{I}_k}D_{i,k}}$ are some auxiliary variables, and $\mathbf{E}_k\in\mathbb{C}^{\sum_{i\in\mathcal{I}_k}D_{i,k}\times\sum_{i\in\mathcal{I}_k}D_{i,k}}$ is the MSE matrix defined as

$$\begin{aligned}\mathbf{E}_k\left(\mathbf{U}_k,\mathbf{P}\right)\triangleq&\left(\mathbf{I}-\mathbf{U}_k^H\mathbf{H}_k\mathbf{P}_k\right)\left(\mathbf{I}-\mathbf{U}_k^H\mathbf{H}_k\mathbf{P}_k\right)^H\\&+\mathbf{U}_k^H\mathbf{N}_k\mathbf{U}_k.\end{aligned}\tag{15}$$

C. WMMSE Based Beamforming Solution

Note that the transformed problem (14) is convex with respect to (w.r.t.) $\{\mathbf{U}_k\}$, $\{\mathbf{W}_k\}$, and $\{\mathbf{P}_k\}$, respectively. We now provide closed-form solutions to problem (14) using the block coordinate descent (BCD) method [14]. Using (7) in Lemma 1, we fix $\{\mathbf{W}_k\}$ and $\{\mathbf{P}_k\}$, and update the optimal $\{\mathbf{U}_k^*\}$ as

$$\mathbf{U}_k^*=\left(\mathbf{N}_k+\mathbf{H}_k\mathbf{P}_k\mathbf{P}_k^H\mathbf{H}_k^H\right)^{-1}\mathbf{H}_k\mathbf{P}_k,\ \forall k.\tag{16}$$

Following (8) and (10) in Lemma 1, the optimal $\{\mathbf{W}_k^*\}$ is given by

$$\mathbf{W}_k^* = (\mathbf{E}_k^*)^{-1} = \left(\mathbf{I} - (\mathbf{U}_k^*)^H \mathbf{H}_k \mathbf{P}_k\right)^{-1}, \ \forall k. \tag{17}$$

Fixing $\{\mathbf{U}_k\}$ and $\{\mathbf{W}_k\}$, the optimal beamformer $\{\mathbf{P}_{i,k}^*\}$ can be obtained by solving the following optimization problem:

$$\begin{aligned} \min_{\{\mathbf{P}_{i,k}\}} \ & \sum_{k=1}^{K} \alpha_k \mathrm{Tr}\left(\mathbf{W}_k \mathbf{E}_k\left(\mathbf{U}_k, \mathbf{P}\right)\right) \\ \text{s. t.} \ & \sum_{k \in \mathcal{U}_i} \mathrm{Tr}\left(\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\right) \le P_{\max,i}, \ \forall i \in \mathcal{I}. \end{aligned} \tag{18}$$

The optimization problem of $\{\mathbf{P}_{i,k}\}$ can then be equivalently reformulated as (19) (in the top of next page). Particularly, in (19b), (a) follows directly from the definition of $\mathbf{E}_k\left(\mathbf{U}_k, \mathbf{P}\right)$; (b) is due to $\mathrm{Tr}\left(\mathbf{AB}\right) = \mathrm{Tr}\left(\mathbf{BA}\right)$; (c) is because of replacing $\mathbf{P}_k$ in (19c) with its definition in (13) and from the fact that $\mathrm{Tr}\left(\mathbf{A}\right)$ equals the sum of the diagonal elements of $\mathbf{A}$, $\mathbf{\Xi}_{i,k} \in \mathbb{B}^{D_{i,k} \times \sum_{i \in \mathcal{I}_k} D_{i,k}}$ being introduced to extract $\mathbf{P}_{i,k}$ from $\mathbf{W}_k\mathbf{U}_k^H\mathbf{H}_k\mathbf{P}_k$ by selecting the corresponding $D_{i,k}$ rows, and hence, for $i \in \mathcal{I}_k$, the $(i-1)\times D_{i,k}+1$ to $i \times D_{i,k}$ columns of $\mathbf{\Xi}_{i,k}$ are the identity matrix, the remaining elements are zero. Explicitly, (19) can be rewritten as

$$\begin{aligned} \min_{\{\mathbf{P}_{i,k}\}} & \sum_{k=1}^{K} \alpha_k \sum_{i \in \mathcal{I}_k} \mathrm{Tr}\left(\mathbf{P}_{i,k}^H \mathbf{H}_{i,k}^H \mathbf{A}_k \mathbf{H}_{i,k} \mathbf{P}_{i,k}\right) \\ & - 2\sum_{k=1}^{K} \alpha_k \sum_{i \in \mathcal{I}_k} \mathrm{Re}\left\{\mathrm{Tr}\left(\mathbf{\Xi}_{i,k}\mathbf{W}_k\mathbf{U}_k^H\mathbf{H}_{i,k}\mathbf{P}_{i,k}\right)\right\} \\ & + \sum_{k=1}^{K} \alpha_k \mathrm{Tr}\left(\mathbf{A}_k \sum_{l \in \mathcal{U}_{-k}} \sum_{j \in \mathcal{I}_l} \mathbf{H}_{j,k}\mathbf{P}_{j,l}\mathbf{P}_{j,l}^H\mathbf{H}_{j,k}^H\right) \\ \text{s. t.} & \sum_{k \in \mathcal{U}_i} \mathrm{Tr}\left(\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\right) \le P_{\max,i}, \ \forall i \in \mathcal{I}, \end{aligned} \tag{20}$$

Note that the update of beamformers can be decoupled between APs by extracting the items related to $\{\mathbf{P}_{i,k}, k \in \mathcal{U}_i\}$ from (20), the beamforming optimization problem of each AP i can be rewritten as

$$\begin{aligned} \min_{\{\mathbf{P}_{i,k}, k \in \mathcal{U}_i\}} \ & \sum_{l \in \mathcal{U}} \sum_{k \in \mathcal{U}_i} \alpha_l \mathrm{Tr}\left(\mathbf{A}_l \mathbf{H}_{i,l}\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\mathbf{H}_{i,l}^H\right) \\ & -2\sum_{k \in \mathcal{U}_i} \alpha_k \mathrm{Re}\left\{\mathrm{Tr}\left(\mathbf{\Xi}_{i,k}\mathbf{W}_k\mathbf{U}_k^H\mathbf{H}_{i,k}\mathbf{P}_{i,k}\right)\right\} \\ \text{s. t.} \ & \sum_{k \in \mathcal{U}_i} \mathrm{Tr}\left(\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\right) \le P_{\max,i}. \end{aligned} \tag{21}$$

where the first item of (21) is derived by merging the first and the third item of (20).

Remark 3. We observe that the per-AP optimization problem (21) does not involve the beamforming matrices of any other APs. This reveals that the cooperative beamforming design is separable among APs, allowing for parallel computation of the beamformers across all APs.

Algorithm 1 WMMSE Based Beamforming Algorithm

1: **Initialize** $\{\mathbf{P}_{i,k}\}$ such that $\sum_{k \in \mathcal{U}_i} \mathrm{Tr}\left(\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\right) \le P_{\max}, \forall i \in \mathcal{I}$, do the following:
2: **Repeat**
3: Update $\{\mathbf{U}_k\}$ via (16);
4: Update $\{\mathbf{W}_k\}$ via (17);
5: Update $\{\mathbf{P}_{i,k}\}$ via (22);
6: **Until** Converge.

The objective function of (21) is a quadratic function that is convex w.r.t. $\mathbf{P}_{i,k}$. Hence, we use the Lagrangian multiplier method to solve for $\mathbf{P}_{i,k}^*$, which is given by

$$\mathbf{P}_{i,k}^* = \left(\sum_{l \in \mathcal{U}} \alpha_l \mathbf{H}_{i,l}^H \mathbf{A}_l \mathbf{H}_{i,l} + \mu_i \mathbf{I}\right)^{-1} \alpha_k \mathbf{H}_{i,k}^H \mathbf{U}_k \mathbf{W}_k \mathbf{\Xi}_{i,k}^H, \tag{22}$$

where $\mu_i \ge 0$ is a Lagrangian multiplier that can be found by one dimensional search techniques such as bisection method [12]. An outline of the proposed WMMSE based algorithm is given in Algorithm 1.

Remark 4. The complexity of our proposed algorithm is $\mathcal{O}\left(\sum_{i \in \mathcal{I}} M_i^3\right)$. In contrast, the updates of matrices $\{\mathbf{P}_{i,k}\}$ in [6] require the utilization of the CVX package, which utilizes the interior point method with a complexity of $\mathcal{O}\left(\left(\sum_{i \in \mathcal{I}} M_i \sum_{k \in \mathcal{U}_i} D_{i,k}\right)^3\right)$ [15]. Furthermore, the beamforming matrices can be updated in parallel among the APs in our proposed algorithm.

Remark 5. The convergence of the proposed algorithm is guaranteed by the classic convergence theory of BCD method [14], since the objective function in (14) is continuously differentiable and convex in each block, i.e., $\{\mathbf{P}_{i,k}\}$, $\{\mathbf{W}_k \succ \mathbf{0}\}$ and $\{\mathbf{U}_k\}$. Therefore, any limit point $\left(\{\mathbf{U}_k^*\}, \{\mathbf{W}_k^*\}, \{\mathbf{P}_{i,k}^*\}\right)$ of the WMMSE iterative sequence is a stationary point of problem (4). We refer the interested readers to [12], [16].

IV. Simulation Results

In this section, we study the performance of the proposed algorithm. We model the channel as the Rayleigh channel with circularly symmetric standard complex normal distribution. We model the pathloss as $128.1 + 37.6\log_{10}(d)[\mathrm{dB}]$ [7], where $d \in [0.1, 0.3]$ in kilometers is the distance between the AP and the UE. Without loss of generality, we assume that the APs have the same number of transmit antennas, the UEs have the same number of receive antennas, and number of transmit streams for the UEs are the same, i.e., $M_i = M$, $N_k = N$, and $D_{i,k} = D$. In addition, the size of serving APs for each UE is assumed to be the same, i.e., $|\mathcal{I}_k| = L$. We set the maximum transmit power to be $P_{\max}$ for all APs. Finally, the priority α_k and noise power $\sigma_k^2 = 10^{\frac{1}{K}\sum_k \log_{10}\frac{1}{N_k}\|\mathbf{H}_k\|_F^2} \times 10^{-\frac{\mathrm{SNR}}{10}}$ are set equally for all UEs, where SNR is the average received SNR for all UEs without beamforming.

Unless otherwise stated, the system parameters are set as $I = 2$, $K = 4$, $P_{\max} = 1[\mathrm{W}]$, $L = 2$, and $\alpha_k = 1$. Monte Carlo simulations are performed over 100 randomly

$$
\begin{aligned}
&\min_{\{\mathbf{P}_{i,k}\}} \sum_{k=1}^{K} \alpha_k \mathrm{Tr}\left(\mathbf{W}_k \mathbf{E}_k\left(\mathbf{U}_k, \mathbf{P}\right)\right) && \text{(19a)}\\
\overset{(a)}{\Longleftrightarrow}\; &\min_{\{\mathbf{P}_{i,k}\}} \sum_{k=1}^{K} \alpha_k \mathrm{Tr}\left(\mathbf{W}_k\left(\mathbf{I}-\mathbf{U}_k^H \mathbf{H}_k \mathbf{P}_k\right)\left(\mathbf{I}-\mathbf{U}_k^H \mathbf{H}_k \mathbf{P}_k\right)^H + \mathbf{W}_k \mathbf{U}_k^H \mathbf{J}_k \mathbf{U}_k\right) && \text{(19b)}\\
\overset{(b)}{\Longleftrightarrow}\; &\min_{\{\mathbf{P}_{i,k}\}} \sum_{k=1}^{K} \alpha_k \left\{\mathrm{Tr}\left(\mathbf{P}_k^H \mathbf{H}_k^H \underbrace{\mathbf{U}_k \mathbf{W}_k \mathbf{U}_k^H}_{\mathbf{A}_k} \mathbf{H}_k \mathbf{P}_k\right) - 2\,\mathrm{Re}\left\{\mathrm{Tr}\left(\mathbf{W}_k \mathbf{U}_k^H \mathbf{H}_k \mathbf{P}_k\right)\right\} + \mathrm{Tr}\left(\underbrace{\mathbf{U}_k \mathbf{W}_k \mathbf{U}_k^H}_{\mathbf{A}_k} \mathbf{J}_k\right)\right\} && \text{(19c)}\\
\overset{(c)}{\Longleftrightarrow}\; &\min_{\{\mathbf{P}_{i,k}\}} \sum_{k=1}^{K} \alpha_k \left\{\mathrm{Tr}\begin{bmatrix} \mathbf{P}_{i_1,k}^H \mathbf{H}_{i_1,k}^H \mathbf{A}_k \mathbf{H}_{i_1,k} \mathbf{P}_{i_1,k} & & \\ & \ddots & \\ & & \mathbf{P}_{i_{|\mathcal{I}_k|},k}^H \mathbf{H}_{i_{|\mathcal{I}_k|},k}^H \mathbf{A}_k \mathbf{H}_{i_{|\mathcal{I}_k|},k} \mathbf{P}_{i_{|\mathcal{I}_k|},k} \end{bmatrix}\right\} && \text{(19d)}\\
&\quad - \sum_{k=1}^{K} 2\alpha_k \,\mathrm{Re}\left\{\mathrm{Tr}\begin{bmatrix} \underbrace{\left[\mathbf{I}_{D_{i_1,k}}, \mathbf{0}\right]}_{\boldsymbol{\Xi}_{i_1,k}} \mathbf{W}_k \mathbf{U}_k \mathbf{H}_{i_1,k} \mathbf{P}_{i_1,k} & & \\ & \ddots & \\ & & \underbrace{\left[\mathbf{0}, \mathbf{I}_{D_{i_{|\mathcal{I}_k|},k}}\right]}_{\boldsymbol{\Xi}_{i_{|\mathcal{I}_k|},k}} \mathbf{W}_k \mathbf{U}_k \mathbf{H}_{i_{|\mathcal{I}_k|},k} \mathbf{P}_{i_{|\mathcal{I}_k|},k} \end{bmatrix}\right\}\\
&\quad + \sum_{k=1}^{K} \alpha_k \mathrm{Tr}\left(\mathbf{A}_k \sum_{l \in \mathcal{U}_{-k}} \sum_{j \in \mathcal{I}_l} \mathbf{H}_{j,k} \mathbf{P}_{j,l} \mathbf{P}_{j,l}^H \mathbf{H}_{j,k}^H\right).
\end{aligned}
$$

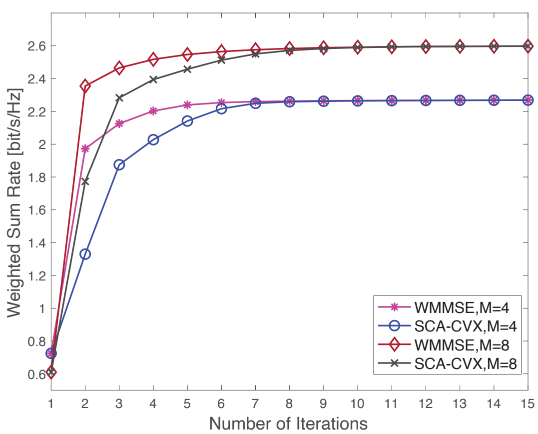

Fig. 2. Comparison of the WSR between the proposed WMMSE algorithm and the SCA-CVX benchmark under different values of transmit antennas M.

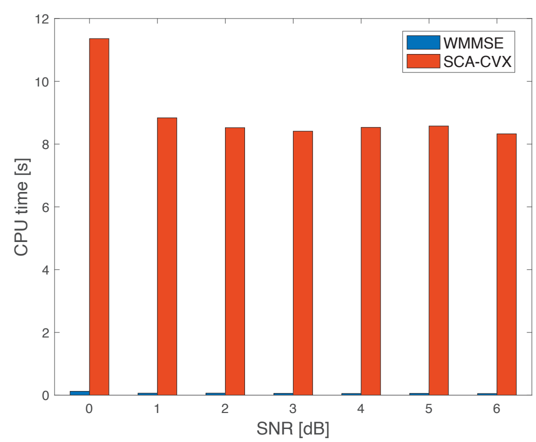

Fig. 3. Comparison of the CPU time between the proposed WMMSE algorithm and the SCA-CVX benchmark under different values of the SNR.

generated channel realizations. We take the SCA-CVX based algorithm proposed in [6] as a benchmark, which is the only existing beamforming method designed for NCJT cell-free networks as far as we know. Since the SCA-CVX method is designed for UE with single antenna, we introduce the classic MRT as a benchmark for the multi-antenna UE scenarios $\mathbf{P}_{i,k}^{\mathrm{MRT}} = \frac{\sqrt{P_{\max}}}{|\mathcal{U}_i|} \frac{\tilde{\mathbf{V}}_{i,k}^H}{\left\|\tilde{\mathbf{V}}_{i,k}\right\|}$, where $\mathbf{H}_{i,k} = \mathbf{U}_{i,k} \boldsymbol{\Sigma}_{i,k} \mathbf{V}_{i,k}^H$ is the singular value decomposition of $\mathbf{H}_{i,k}$, $\tilde{\mathbf{V}}_{i,k}^H$ consists of the right singular vectors corresponding to the $D_{i,k}$ largest singular values.

We first examine the convergence performance of the proposed WMMSE based algorithm for different number of transmit antennas with SNR $= 2$ [dB]. Note that the SCA-CVX based low-complexity algorithm in [6] is designed for UEs with a single receive antenna, we set $N = 1$ and $D = 1$ in Fig. 2. We observe that the proposed WMMSE based algorithm converges faster than the SCA-CVX benchmark. In addition, the WSR increases as the number of transmit antennas increasing, since the APs have more degrees of freedom for their beamforming designs to mitigate the inter-UE interference.

In Fig. 3, we compare the CPU time of our proposed WMMSE algorithm with that of the SCA-CVX benchmark. We observe that the run time of WMMSE is negligible compared with the one of the SCA-CVX benchmark, showing the efficiency of our method. Upon further analysis, this is due

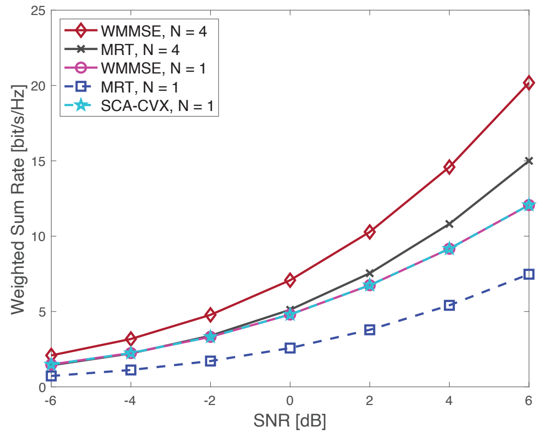

Fig. 4. Comparison of the WSR among the proposed WMMSE algorithm, the MRT benchmark, and the SCA-CVX benchmark in large-scale networks.

to the facts that: 1) the beamformers are computed in parallel among APs in our WMMSE algorithm; 2) the SCA-CVX benchmark has substantially higher computational complexity, as stated in Section III. Consequently, our WMMSE algorithm offers a more effective and practical engineering solution.

Fig. 4 illustrates the WSR performance in NCJT cell-free networks with $N = 1$ and $N = 4$. In order to evaluate the performance of the proposed algorithm on a larger-scale network, we set $I = 20$, $K = 10$, $M = 64$ and $D = 2$, and select the two APs closest to the UE k to form $\mathcal{I}_k, \forall k$. Fig. 4 shows that the proposed algorithm achieves much higher WSR than the MRT method, while achieving the same WSR as SCA-CVX when each UE has a single antenna.

V. Conclusions

In this work, we have investigated the beamforming design for maximizing the WSR in NCJT user-centric cell-free MIMO networks. We for the first time show the applicability of a classic WMMSE algorithm in NCJT, which is assumed to be inapplicable for NCJT in previous studies. Based on the unique observations of the WSR maximization problem, we propose a low-complexity WMMSE-based algorithm structures, which is guaranteed to converge to a stationary point of the original WSR maximization problem with closed-form beamforming updates. Simulation results demonstrate that our proposed algorithm substantially outperforms the current best SCA-CVX method in both convergence speed and computational complexity. Additionally, we acknowledge the centralized nature of our algorithm and suggest distributed beamforming as a direction for future research.

Appendix

The Lagrangian function of (21) can be written as

$$\begin{aligned}\mathcal{L}(\{\mathbf{P}_{i,k}\}) = &\sum_{l\in\mathcal{U}}\sum_{k\in\mathcal{U}_i}\alpha_l \mathrm{Tr}\left(\mathbf{A}_l\mathbf{H}_{i,l}\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\mathbf{H}_{i,l}^H\right)\\ &-2\sum_{k\in\mathcal{U}_i}\alpha_k \mathrm{Re}\left\{\mathrm{Tr}\left(\mathbf{\Xi}_{i,k}\mathbf{W}_k\mathbf{U}_k^H\mathbf{H}_{i,k}\mathbf{P}_{i,k}\right)\right\}\\ &+\mu_i\left(\sum_{k\in\mathcal{U}_i}\mathrm{Tr}\left(\mathbf{P}_{i,k}\mathbf{P}_{i,k}^H\right)-P_{\max,i}\right).\end{aligned} \tag{23}$$

Setting the derivative of function $\mathcal{L}(\{\mathbf{P}_{i,k}\})$ w.r.t. $\mathbf{P}_{i,k}$ to be $\mathbf{0}$, we have

$$\begin{aligned}\nabla_{\mathbf{P}_{i,k}}\mathcal{L}(\{\mathbf{P}_{i,k}\}) = &\left(\sum_{l\in\mathcal{U}}\alpha_l\mathbf{H}_{i,l}^H\mathbf{A}_l\mathbf{H}_{i,l}+\mu_i\mathbf{I}\right)\mathbf{P}_{i,k}\\ &-\alpha_k\mathbf{H}_{i,k}^H\mathbf{U}_k\mathbf{W}_k\mathbf{\Xi}_{i,k}^H=\mathbf{0},\end{aligned} \tag{24}$$

yields the closed form expression of $\mathbf{P}_{i,k}^*$ in (22).

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