Mid copies Answer Draft

#linear_algebra
21 Fall Mid.pdf这个对不上
Mid Answer copies.pdf

21 spring midterm

试卷21 Spring Midterm.pdf
答案21 Spring Midterm Answer电子版可用.pdf
这里结合手写的Mid Answer copies.pdf，把答案做成题目+答案的形式
小题直接给答案，答题给原题+过程

Midterm Copy 1
November. 4th, 2021 Dr.Y.Che Fall 2021

Suggested Solutions.
1.DDCBB

2. (1) $\begin{bmatrix} a & b \\ 2-a & 3-b \end{bmatrix}, a, b \in R$
   (2) $t = 5$
   (3) $k = 10$
   (4) $\dim N(A^T A) = 1$
   (5) $\hat{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3.(10 points) Suppose there are three linearly independent solutions to the system

$$\left\{\begin{array}{l} x_1+x_2+x_3+x_4=-1 \\ 4x_1+3x_2+5x_3-x_4=-1 \\ ax_1+x_2+3x_3+bx_4=1 \end{array}\right.$$

(a) Prove that the coefficient matrix of the system has the rank: rank $(A)=2$;

(b) Find the values of a, b,and solve the system of linear equations.

已知线性方程组

$$\left\{\begin{array}{l} x_1+x_2+x_3+x_4=-1 \\ 4x_1+3x_2+5x_3-x_4=-1 \\ ax_1+x_2+3x_3+bx_4=1 \end{array}\right.$$

有三个线性无关的解.

(a) 证明：方程组系数矩阵A的秩 rank$(A)=2$;

(b)求 a, b 的值及方程组的通解.

(1) let $\epsilon_1, \epsilon_2, \epsilon_3,$ be three linearly independant solutions
$\epsilon_1 - \epsilon_2, \epsilon_1 - \epsilon_3$ linearly independent solutions to $Ax = 0$
$\Rightarrow 4 - rank(A) \geq 2 \Rightarrow rank(A) \leq 2$
Also, $\text{rank}(A) \geq 2 \quad \Longrightarrow \quad \text{rank}(A) = 2.$
(the first two rows of A are linearly independent)

$$(2) \begin{bmatrix} 1 & 1 & 1 & 1 & -1 \\ 4 & 3 & 5 & -1 & -1 \\ a & 1 & 3 & b & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 2 & -4 & 2 \\ 0 & +1 & -1 & 5 & -3 \\ 0 & 0 & 4-2a & 4a+b & 4-2a \end{bmatrix}$$

$$rank(A) = 2.$$
$$\Rightarrow \quad 4 - 2a = 0 \quad 6a + b - 5 = 0$$
$$\Rightarrow \quad a = 2, \quad b = -3$$

Complete solution:

$$x = \begin{bmatrix} 2 \\ -3 \\ 0 \\ -0 \end{bmatrix} + k_1 \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 4 \\ -5 \\ 0 \\ 1 \end{bmatrix}, k_1, k_2 \in R.$$

3.

$$(1)A = LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & a & 1 & 0 \\ -0 & a^2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(2)A^{-1} = \begin{bmatrix} 1 & -\frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{a^2} & 0 & 0 \\ 0 & -a & 1 & 0 \\ 0 & -a^2 & 0 & 1 \end{bmatrix}$$

$$(3)x = \begin{bmatrix} 0 \\ \frac{1}{a} \\ 0 \\ 0 \end{bmatrix}$$

5.(10 points) Let

$$A = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 1 & 3 & 0 & 5 \\ 1 & 1 & 6 & 3 \end{bmatrix}.$$

(a) Find a basis for the nullspace of A.

(b) Find a basis for the row space of A.

(c) Find a basis for the column space of A.

(d) For each column vector which is not in the basis that you obtained in part (c), express it as a linear combination of the basis vectors for the column space of A(as obtained in part (c)) .

$$\text{设}A = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 1 & 3 & 0 & 5 \\ 1 & 1 & 6 & 3 \end{bmatrix}.$$

(a)求矩阵 A 的零空间的一组基.

(b)求矩阵 A 的行空间的一组基.

(c)求矩阵 A 的列空间的一组基.

(d)把矩阵A不在(c)中基向量组中的列向量表示成(c)中得到的基向量的线性组合.

$$A=\begin{bmatrix}2&4&6&8\\1&3&0&5\\1&1&6&3\end{bmatrix}\to\begin{bmatrix}1&3&0&5\\0&1&-3&1\\0&0&0&0\end{bmatrix}\to\begin{bmatrix}1&0&9&2\\0&1&-3&1\\0&0&0&0\end{bmatrix}$$

$$(a)\quad N(A)'s\quad basis:\quad \left\{\begin{bmatrix}-9\\3\\1\\0\end{bmatrix},\begin{bmatrix}-2\\4\\0\\1\end{bmatrix}\right\}$$

$$(b)\quad C(A^T)'s\quad basis:\left\{\begin{bmatrix}1\\0\\9\\2\end{bmatrix},\begin{bmatrix}0\\1\\-3\\1\end{bmatrix}\right\}$$

$$(c)\quad C(A)'s\quad basis:\quad \left\{\begin{bmatrix}2\\1\\1\end{bmatrix},\begin{bmatrix}4\\3\\1\end{bmatrix}\right\}$$

$$(d)\quad \begin{bmatrix}6\\0\\6\end{bmatrix}=9\begin{bmatrix}2\\1\\1\end{bmatrix}-3\begin{bmatrix}4\\3\\1\end{bmatrix},\quad \begin{bmatrix}8\\5\\3\end{bmatrix}=2\begin{bmatrix}2\\1\\1\end{bmatrix}+\begin{bmatrix}4\\3\\1\end{bmatrix}$$

$$6.(a)v_1=\begin{bmatrix}1\\1\\0\end{bmatrix},\quad v_2=\begin{bmatrix}-\frac{1}{2}\\\frac{1}{2}\\1\end{bmatrix}\quad (b)\qquad \text{L's basis: }\left\{\begin{bmatrix}1\\-1\\0\end{bmatrix}\right\}$$

$$(c)\text{Projection: }\begin{bmatrix}3/2\\3/2\\0\end{bmatrix}.$$

7.(a)

$$[A \quad b] = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ 0 & -4 & -2 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow x_n = \begin{bmatrix} +\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}, \quad x_p = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} \Rightarrow \xi_2 = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} + c \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}, \quad c \in R.$$

$$A^2 = \begin{bmatrix} 2 & 2 & 0 \\ -2 & -2 & 0 \\ 4 & 4 & 0 \end{bmatrix} \Rightarrow [A^2 \quad b] = \begin{bmatrix} 2 & 2 & 0 & -1 \\ -2 & -2 & 0 & 1 \\ 4 & 4 & 0 & -2 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 0 & -1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow S_3 = \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 0 \end{bmatrix} + k_1 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, k_1, k_2 \in \mathbb{R}.$$

(b)

$$\begin{aligned}
&c_1\xi_1 + c_2\xi_2 + c_3\xi_3 = 0 \\
&A\xi_1 = 0, \quad A^2(c_1\xi_1 + c_2\xi_2 + c_3\xi_3) = A^2 0 \\
&A\xi_2 = \xi_1 \quad \Rightarrow C_3 = 0 \quad A(C_1\xi_1 + C_2\xi_2) = 0 \\
&A\xi_3 = \xi_2, \quad C_1 = 0 \Rightarrow C_1 = C_2 = C_3 = 0 \\
&Y_1, Y_2, Y_3, \text{ are lineady independent.}
\end{aligned}$$

$$8.(a) A^{-1} = I_n + \frac{uv^T}{1 - v^T u}$$

$$(b)(I_n - UV^T)^{-1} = I_n + U(\underbrace{I_m - v^T U}\,)^{-1} V^T$$

Assume $I_m - V^T U$ is invetible.

以下未指出具体的卷子，需要比对