24 Spring Midterm Question

24 Spring Midterm Question 中英试题分离 分页留空版本
线性代数2023-2024学年春季学期期中考试

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1. (共15分，每小题3分)选择题，只有一个选项是正确的.

(1) 假定

$$\alpha_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \alpha_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \alpha_3 = \begin{bmatrix} 7 \\ 3 \\ c \end{bmatrix}.$$

若$\alpha_1, \alpha_2, \alpha_3$线性相关，则$c$的取值为

$(A)5.$
$(B)6.$
$(C)7.$
$(D)8.$

(2) 设 A 为一个 $m \times n$ 实矩阵，b 为一个 m 维实列向量，以下说法一定是**正确**的是？

(A)若$A\mathbf{x} = \mathbf{b}$无解，则$A\mathbf{x} = \mathbf{0}$只有零解.
(B)若$A\mathbf{x} = \mathbf{0}$有无穷多解，则$A\mathbf{x} = \mathbf{b}$有无穷多解。
(C)若$m < n$,则$A\mathbf{x} = \mathbf{b}$和$A\mathbf{x} = \mathbf{0}$都有无穷多解。
(D)若A的秩为n,则$A\mathbf{x} = 0$只有零解.

(3) 如果以下线性方程组有两个自由变量

$$\begin{cases} x_1 + 2x_2 - 4x_3 + 3x_4 = 0, \\ x_1 + 3x_2 - 2x_3 - 2x_4 = 0, \\ x_1 + 5x_2 + (5-k)x_3 - 12x_4 = 0, \end{cases}$$

k的取值为

$(A)5.$
$(B)4.$
$(C)3.$
$(D)2.$

(4) 设 $u, v \in \mathbb{R}^3, \lambda \in \mathbb{R}$. 以下说法**错误**的是?

(A)如果u和v为满足$u^T v = 0$的非零向量，则u和v线性无关.
(B)如果$u+v$和$u-v$正交，则$||u|| = ||v||$.
$(C)u^T v = 0$当且仅当 $u = 0$ or $v = 0$.
$(D)\lambda v = 0$当且仅当 $v = 0$ or $\lambda = 0$.

(5) 设A和B都为n阶矩阵.以下说法**错误**的是?

(A)如果A, B为对称矩阵，则AB也为一个对称矩阵。
(B)如果A, B 为可逆矩阵，则 AB 也为一个可逆矩阵.
(C)如果A, B为置换矩阵，则AB也为一个置换矩阵.
(D)如果A, B为上三角矩阵，则AB也为上三角矩阵。

2. (20 points, 5 points each) 填空，共4题。

(1)$A=\begin{bmatrix}1&0&0\\a&1&0\\b&3&2\end{bmatrix}, a,b\in\mathbb{R}$, 则$A^{-1}=$______.

(2)设A为一个4×3的实矩阵，B为$\begin{bmatrix}1&0&0\\0&2&0\\-1&0&3\end{bmatrix}$.

如果矩阵A的秩为 2,则AB的秩为______.

(3)设 $A=\begin{bmatrix}1&-1&1\\-1&1&-1\\2&-2&2\end{bmatrix}$, 则 $A^{2024}=$______.

(4)考虑以下线性方程组：

$$A\mathbf{x}=\mathbf{b}:\begin{cases}x&=&2\\y&=&3\\x+y&=&6\end{cases}$$

该线性方程组的最小二乘解为______.

3. (10points)设

$$A=\begin{bmatrix}1 & -2 & 3\\ 2 & -5 & 1\\ 1 & -4 & -7\end{bmatrix}.$$

求矩阵A的一个LU分解

4. 考虑以下 4× 5 矩阵 A 以及 4 维列向量 $\mathbf{b}$:

$$A = \begin{bmatrix} 0 & 2 & 4 & 1 & 6 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 4 & 10 & 1 & 2 \\ 0 & -1 & -5 & 1 & 7 \end{bmatrix},\ \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ -5 \\ 10 \end{bmatrix}$$

(a)分别求矩阵 A 的四个基本子空间的一组基向量。

(b)求$Ax = b$的所有解.

5. (20 points)设 Let $A=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}$, T为按照以下方式定义的从$\mathbb{R}^{2\times 2}$到 $\mathbb{R}^{2\times 2}$线性变换:

$$T\left(X\right)=XA+AX, X\in\mathbb{R}^{2\times 2}.$$

其中$\mathbb{R}^{2\times 2}$表示所有2×2实矩阵构成的向量空间.

(a)求T在以下有序基

$$v_1=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, v_2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, v_3=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}, v_4=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$$

下的矩阵表示.

(b)求一个矩阵B使得

$$T(B)=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}.$$

(c)求一个矩阵C使得

$$T\left(C\right)=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}.$$

6. (5 points) 设A, B为满足$A^2 = A$和$B^2 = B$的n阶实矩阵.证明:
如果$(A + B)^2 = A + B$,则$AB = O$.其中O 表示n阶零矩阵。

7. (6 points)设A为 3×2矩阵，B 为 2×3矩阵，并且

$$AB = \begin{bmatrix} 8 & 0 & -4 \\ -\frac{3}{2} & 9 & -6 \\ -2 & 0 & 1 \end{bmatrix}.$$

(a)计算$(AB)^2$.

(b)求BA.

1. (15 points, 3 points each) Multiple Choice. Only one choice is correct.
(共15分，每小题3分)选择题，只有一个选项是正确的.

(1)Let

$$\alpha_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \alpha_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \alpha_3 = \begin{bmatrix} 7 \\ 3 \\ c \end{bmatrix}.$$

If $\alpha_1, \alpha_2, \alpha_3$ are linearly dependent, then c equals
$(A)5.$
$(B)6.$
$(C)7.$
$(D)8.$

(2) let A be an $m \times n$ real matrix and b be an $m \times 1$ real column vector. Which of the following statements is correct?
(A) If $A\mathbf{x} = \mathbf{b}$ does not have any solution, then $A\mathbf{x} = \mathbf{0}$ has only the zero solution.
(B) If $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.
(C) If $m < n$, both $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$ have infinitely many solutions.
(D) If the rank of A is n, then $A\mathbf{x} = \mathbf{0}$ has only the zero solution.

(3) For which value of k does the system

$$\begin{cases} x_1 + 2x_2 - 4x_3 + 3x_4 = 0, \\ x_1 + 3x_2 - 2x_3 - 2x_4 = 0, \\ x_1 + 5x_2 + (5-k)x_3 - 12x_4 = 0, \end{cases}$$

have exactly two free variables?

$(A)5.$

$(B)4.$

$(C)3.$

$(D)2.$

(4) Let $u, v \in \mathbb{R}^3$ and $\lambda \in \mathbb{R}$. Which of the following statements is false?

(A) If u and v are nonzero vectors satisfying $u^T v = 0$, then u and v are linearly independent.

(B)If $u + v$ is orthogonal to $u - v$, then $||u|| = ||v||$.

$(C)u^T v = 0$ if and only if $u = 0$ or $v = 0$.

$(D)\lambda v = 0$ if and only if $v = 0$ or $\lambda = 0$.

(5) Let A and B be two $n \times n$ matrices. Which of the following assertions is **false**?

(A) If A, B are symmetric matrices, then AB is a symmetric matrix.

(B) If A, B are invertible matrices, then AB is an invertible matrix.

(C) If A, B are permutation matrices, then AB is a permutation matrix.

(D) If A, B are upper triangular matrices, then AB is an upper triangular matrix.

2. (20 points, 5 points each) Fill in the blanks.

(1)Let $A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 3 & 2 \end{bmatrix}, a, b \in \mathbb{R}$. Then $A^{-1} =$ ______.

(2) Let A be a 4×3 real matrix with rank 2 and $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$.

Then the rank AB is ______.

(3)Let $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 2 \end{bmatrix}$. Then$A^{2024} =$ ______.

(4) Consider the system of linear equations:

$$Ax = b : \begin{cases} x & = & 2 \\ y & = & 3 \\ x + y & = & 6 \end{cases}$$

The least-squares solution for the system is ______.

3. (10points)Let

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 1 \\ 1 & -4 & -7 \end{bmatrix}.$$

Find an LU factorization of A.

4. (24 points) Consider the following 4×5 matrix A and 4-dimensional column vector b:

$$A = \begin{bmatrix} 0 & 2 & 4 & 1 & 6 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 4 & 10 & 1 & 2 \\ 0 & -1 & -5 & 1 & 7 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 2 \\ -5 \\ 10 \end{bmatrix}$$

(a) Find a basis for each of the four fundamental subspaces of A.

(b) Find the complete solution to $A\mathbf{x} = \mathbf{b}$.

5. (20 points) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ and T be the linear transformation from $R^{2\times 2}$ to $R^{2\times 2}$ defined by

$$T(X) = XA + AX,\ X \in \mathbb{R}^{2\times 2}.$$

Where $\mathbb{R}^{2\times 2}$ denotes the vector space consisting of all 2×2 real matrices.

(a) Find the matrix representation of T with respect to the following ordered basis

$$v_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

$(\mathbf{b})$ Find a matrix B such that

$$T(B) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

(c) Find a matrix C such that

$$T\left(C\right) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

6. (5 points) Let A, B be two $n \times n$ real matrices satisfying $A^2 = A$ and $B^2 = B$. Show that if $(A+B)^2 = A+B$, then $AB = O$. Where O denotes the $n \times n$ zero matrix.

7. (6 points) Let A be a 3×2 matrix, B be a 2×3 matrix such that

$$AB = \begin{bmatrix} 8 & 0 & -4 \\ -\frac{3}{2} & 9 & -6 \\ -2 & 0 & 1 \end{bmatrix}.$$

(a)Compute$(AB)^2$.

(b) Find BA.