23 Spring Midterm Answer

线性代数 23春季 期中试题答案 发布版

Q1 (1)A (2)D (3)C (4)B (5)B

Q2

(1)4

(2)$-\frac{1}{2}A-I$

(3)1

(4)3

(5)$\begin{bmatrix}\frac{5}{3}\\-\frac{1}{3}\end{bmatrix}$

Q3

(a) $\alpha\neq 0,1,-3$

(b) $\alpha=0,1,-3$

(c) $\alpha=2$

$$A_\alpha^{-1}=\begin{bmatrix}-\frac{1}{2} & -\frac{1}{20} & -\frac{4}{5}\\ 0 & \frac{1}{5} & \frac{1}{5}\\ -\frac{1}{2} & \frac{3}{20} & \frac{2}{5}\end{bmatrix}$$

Q4

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 9 & 1 & 0 \\ -1 & -\frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -12 & -8 \\ 0 & 0 & 1 \end{bmatrix}$$

Q5

$$k \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, k \neq 0.$$

Q6 (a)

- linear independent
- E spans $\mathbf{R}^{2\times 2}$.

(b)

$$\cdot T(X+Y) = T(X) + T(Y),$$
$$\cdot T(\lambda X) = \lambda T(X).$$

(c)

$$M = \begin{bmatrix} a & c & 0 & 0 \\ b & d & 0 & 0 \\ 0 & 0 & a & c \\ 0 & 0 & 0 & d \end{bmatrix}.$$

Q7(a)

Pivot columns of A: $a_1, a_2, \cdots, a_r$;

Pivot columns of B : $b_1, b_2, \cdots, b_s$;

$rankA = r, rankB = s.$

$$
\begin{aligned}
V &= span\left(a_1, \cdots, a_s, b_1, \cdots, b_s\right). dimV \leq r+s \\
&= span\left(a_1, \cdots, a_s, b_1, \cdots, b_s\right) \supseteq C\left(A+B\right) \\
&\implies dimC(A+B) \leq dimV \\
&\implies rank(A+B) \leq rank(A) + rank(B)
\end{aligned}
$$

(b)

$A+B-B=A$

$rank(A+B-B) \leq rank(A+B) + rank(-B) \ldots \text{by (a)}$

$rank(A+B) + rank(-B) = rank(B)$

$\implies rankA - rankB \leq rank(A+B)$

Q8 P_1, Q_1 invertible.

$$
\begin{aligned}
A &= P_1 \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} Q_1 \\
&= \underbrace{P_1 \begin{bmatrix} I_r \\ 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} I_r & 0 \end{bmatrix} Q_1}_{C}
\end{aligned}
$$