23 Fall Midterm A Release

#试题清除计划

线代23秋期中试题答案 发布版

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- OCR协助：SimpleTex

- 反馈意见：

Q1 (1)A (2)B（存疑, 一说D） (3)D (4)B (5)D

Q2

(1)21

(2)$-A^{-1}CB^{-1}$

(3)1or-3 （存疑，一说 1 or -2）

(4)$\begin{bmatrix} -\frac{19}{11} \\ -\frac{5}{11} \end{bmatrix}$

(5)$\frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \text{or} -\frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Q3

A basis for $C(A)$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix} \right\}.$$

A basis for $C(A^T)$

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} \text{or} \left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 3 \\ 1 \end{bmatrix} \right\}$$

A basis for $N(A)$

$$\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 0 \\ -1 \end{bmatrix} \right\}.$$

A basis for $N(A^T)$

$$\left\{ \begin{bmatrix} -5 \\ 13 \\ -3 \\ 1 \end{bmatrix} \right\}.$$

$$(b) \begin{bmatrix} 5 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

Q4

Gaussian Eliminations give:

$$\begin{bmatrix} 1 & -1 & -1 & \vdots & 2 & 2 \\ 2 & a & 1 & \vdots & 1 & a \\ -1 & 1 & a & \vdots & -a-1 & -2 \end{bmatrix}$$
$$\to \begin{bmatrix} 1 & -1 & -1 & \vdots & 2 & 2 \\ 0 & a+2 & 3 & \vdots & -3 & a-4 \\ 0 & 0 & a-1 & \vdots & 0 & 0 \end{bmatrix}$$

If $a=-2$, then $rankA = 2 \neq 3 = rank(A \vdots B)$,$AX=B$ has no solution.

If $a \neq 1$ and $a \neq -1, AX=B$ has a unique solution.

$$\begin{bmatrix} 1 & -1 & -1 & 1 & 2 \\ 0 & a+2 & 3 & 1 & -3 \\ 0 & 0 & a-1 & 1 & 1-a \end{bmatrix} \Rightarrow x = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & -1 & -1 & \vdots & 2 \\ 0 & a+2 & 3 & \vdots & -3 \\ 0 & 0 & a-1 & \vdots & 1-a \end{bmatrix} \Rightarrow x = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & -1 & -1 & \vdots & z \\ 0 & a+2 & 3 & \vdots & a-4 \\ 0 & 0 & a-1 & \vdots & 0 \end{bmatrix} \Rightarrow X = \begin{bmatrix} \frac{3a}{a+2} \\ \frac{a-4}{a+2} \\ 0 \end{bmatrix}.$$

$$X = \begin{bmatrix} 1 & \frac{3a}{a+2} \\ 0 & \frac{a-4}{a+2} \\ -1 & 0 \end{bmatrix}$$

If $a = 1$, $Ax = B$ has infinitely many solutions

$$\begin{bmatrix} 1 & -1 & -1 & -2 & \\ 0 & 3 & 3 & -3 & \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + k_1 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -1 & 2 & \\ 0 & 3 & 3 & 1 & -3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow x = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 1 & 1 \\ -k_1 - 1 & -k_2 - 1 \\ k_1 & k_2 \end{bmatrix}, \quad k_1, k_2 \quad \text{anbitrary constants.}$$

Q5

(a)Let $X, Y \in M_{2\times 2}(R)$ and $C \in R$, than we have

$$
\begin{aligned}
T\left(CX+Y\right) &= \begin{bmatrix} tr & A^T\left(CX+Y\right) \\ tr & B^T\left(CX+Y\right) \\ tr & C^T\left(CX+Y\right) \end{bmatrix} \\
&= c\begin{bmatrix} tr(A^TX) \\ tr(B^TX) \\ tr(C^TX) \end{bmatrix} + \begin{bmatrix} tr(A^TX) \\ tr(B^TY) \\ tr(C^TY) \end{bmatrix} \\
&= cT(X)+T(Y)
\end{aligned}
$$

(b)

$$
\begin{aligned}
T\left(V_1\right) &= \begin{bmatrix}1\\0\\0\end{bmatrix} = 1\underbrace{\begin{bmatrix}1\\0\\0\end{bmatrix}}_{=w_1} + 0\underbrace{\begin{bmatrix}0\\1\\0\end{bmatrix}}_{=w_2} + 0\underbrace{\begin{bmatrix}0\\0\\1\end{bmatrix}}_{=w_3} \\
T\left(V_2\right) &= \begin{bmatrix}-1\\0\\0\end{bmatrix} = -1\begin{bmatrix}1\\0\\1\end{bmatrix} + 0\begin{bmatrix}0\\1\\0\end{bmatrix} + 0\cdot\begin{bmatrix}0\\0\\1\end{bmatrix} \\
T\left(V_3\right) &= \begin{bmatrix}0\\1\\1\end{bmatrix} = 0\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\cdot\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\cdot\begin{bmatrix}0\\0\\1\end{bmatrix} \\
T\left(V_4\right) &= \begin{bmatrix}0\\1\\-1\end{bmatrix} = 0\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\cdot\begin{bmatrix}0\\0\\0\end{bmatrix} + -1\cdot\begin{bmatrix}0\\0\\1\end{bmatrix}
\end{aligned}
$$

Therefore,the matix representation of T with respect to V_1, V_2, V_3, V_4, V_4, and W_1, V_2, W_3 , is:

$$
\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}.
$$

$(c)\quad \text{Since}\quad T(A)=\begin{bmatrix}2\\0\\0\end{bmatrix}, T(B)=\begin{bmatrix}0\\1\\0\end{bmatrix}, T(C)=\begin{bmatrix}0\\0\\1\end{bmatrix},$

We can tarke X to be

$$\frac{1}{2}A-2B+C$$
$$=\begin{bmatrix}y_2 & 0\\0 & -y_2\end{bmatrix}-\begin{bmatrix}0 & 2\\-0 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}y_2 & -2\\1 & -y_2\end{bmatrix}.$$

Q6

Apply Elementary Row and Column Operations to A and C to obtain $D_1=\begin{bmatrix}I_1 & 0\\0 & 0\end{bmatrix}$ for A and $D_2=\begin{bmatrix}I_3 & 0\\0 & 0\end{bmatrix}$ for C.

Where $r=rankA, s=rankC$.
Let $M=\begin{bmatrix}A & B\\0 & C\end{bmatrix}$. Then M can be converted to $M_1=\begin{bmatrix}D_1 & C_1\\0 & D_2\end{bmatrix}$ via elementary row and column operations.

Furthermore, the pivots in D_1 and D_2 can be used to eliminate the nonzero entries in C_1, to obtain

$$M_2=\begin{bmatrix}I_r & 0 & 0 & 0\\0 & 0 & 0 & C_2\\0 & 0 & I_s & 0\\0 & 0 & 0 & 0\end{bmatrix}.$$

In conclusion,

$$\begin{aligned}rankM &= rankM_1=rankM_2=r+s+rankC_2\\ &\geq r+s=rankA+rankC\end{aligned}$$

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Email: （邮箱已更换）huajiebridge34@gmail.com
Github repo: LIUBINfighter/Open_Notes_SUSTech: 南方科技大学一位23级本科生的学习笔记，论文和项目 (github.com)
目前的工程文件以及草稿不定期上传到仓库线性代数栏目。你也可以下载往期结项的文件了解我的工作方式，欢迎来戳。

同时，本人以个人身份向各位同学和高年级助教征求如下表格中留空的材料，包括照片，扫描件，手写件，演示文稿等文件，二版时会将您加入贡献者栏并赠与免费样书，如果你是愿意帮助的热心人，助教或互助课堂的主讲人，能够予以OCR，排版，校对，答案审核一类的协助就更好了：

目前项目进度如下

	原卷	**答案**	**完整度**
20Fall	可用	可用	☑
21Spring	可用	可用	☑
21Fall	可用	无	
22Spring	无	无	

	原卷	**答案**	**完整度**
22Fall	相片质量	无	
23Spring	无选择题	手写答案	
23Fall	Released	Released	☑
24Spring	Released	Released	☑