Calculus Ⅱ Week6 HW-Questions Release

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12.4#17,21,27,33

17.
$P(2,-2,1), Q(3,-1,2), R(3,-1,1)$
a. Find the area of the triangle determined by the points P, Q,and R.
b. Find a unit vector perpendicular to plane PQR.

21.verify that
$(u\times v)\cdot w=(v\times w)\cdot u=(w\times u)\cdot$
and find the volume of the parallelepiped (box) determined by u, v, and w.

u	v	w
$2i+j$	$2i-j+k$	$i+2k$

27.Which of the following are always true, and which are not always true? Give reasons for your answers.

$\mathbf{a.}\ |\mathbf{u}| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$
$\mathbf{b.}\ \mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|$
$\mathbf{c.}\ \mathbf{u} \times \mathbf{0} = \mathbf{0} \times \mathbf{u} = \mathbf{0}$
$\mathbf{d.}\ \mathbf{u} \times (-\mathbf{u}) = \mathbf{0}$
$\mathbf{e.} \mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$
$\mathbf{f.} \mathbf{u} \left(\mathbf{v} + \mathbf{w}\right) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$
$\mathbf{g.} (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v} = 0$
$\mathbf{h.} (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

33.

$\mathbf{Cancelation\ in\ cross\ products}$:

If $u \times v = u \times w$ and $u \neq 0$,then does $v = w$? Give reasons for your answer.

- Section 12.5， #25,35,45,59

  25.Find equation for the plane through $P_0(2,4,5)$ perpendicular to the line $x = 5 + t, y = 1 + 3t, z = 4t$

35.find the distance from the point to the line.

$(2, 1, 3)$;

$x = 2 + 2t, y = 1 + 6t, z = 3$

45.Find the distance from the plane$x + 2y + 6z = 1$ to the plane $x + 2y + 6z = 10$.

59.Find parametrizations for the lines in which the planes $x-2y+4z=2, x+y-2z=5$

- Section 12.6， #47

47.Show that the volume of the segment cut from the paraboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$$

by the plane $z=h$ equals half the segment's base times its altitude.

Section 13.1， #17,23

17.$r(t)$ is the position of a particle in space at time t.Find the angle between the velocity and acceleration vectors at time$t = 0$.

17. $\mathbf{r}(t) = (\ln(t^2 + 1))\mathbf{i} + (\tan^{-1} t)\mathbf{j} + \sqrt{t^2 + 1}\mathbf{k}$

23.Motion along a circle Each of the following equations in parts(a)-(e) describes the motion of a particle having the same path, namely the unit circle $x^2 + y^2 = 1$. Although the path of each particle in parts (a)-(e) is the same, the behavior, or “dynamics,” of each particle is different. For each particle, answer the following questions.

i) Does the particle have constant speed? If so, what is its constant speed?

ii) Is the particle's acceleration vector always orthogonal to its velocity vector?

iii) Does the particle move clockwise or ounterclockwise around the circle?

iv) Does the particle begin at the point (1,0)?

$$
\begin{aligned}
&\textbf{a. } \mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}, t \geq 0 \\
&\textbf{b. } \mathbf{r}(t) = \cos(2t)\mathbf{i} + \sin(2t)\mathbf{j}, t \geq 0 \\
&\textbf{c. } \mathbf{r}(t) = \cos(t - \pi/2)\mathbf{i} + \sin(t - \pi/2)\mathbf{j}, \quad t \geq 0 \\
&\textbf{d. } \mathbf{r}(t) = (\cos t)\mathbf{i} - (\sin t)\mathbf{j}, \quad t \geq 0 \\
&\textbf{e. } \mathbf{r}(t) = \cos(t^2)\mathbf{i} + \sin(t^2)\mathbf{j}, t \geq 0
\end{aligned}
$$

Section 13.2， #13,23,37

13.Solve the initial value problem for $\mathbf{r}$ as a vector function of t.

Differential equation:

$$\frac{d\mathbf{r}}{dt} = \frac{3}{2}(t + 1)^{1/2}\mathbf{i} + e^{-t}\mathbf{j} + \frac{1}{t + 1}\mathbf{k}$$

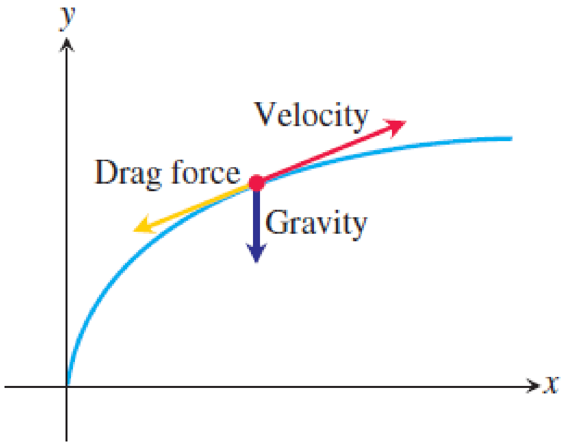
Initial condition:

$$\mathbf{r}(0) = \mathbf{k}$$

23.Projectile flights in the following exercises are to be treated as ideal unless stated otherwise. All launch angles are assumed to be measured from the horizontal. All projectiles are assumed to be launched from the origin over a horizontal surface unless stated otherwise.
23.Firing golf balls A spring gun at ground level fires a golf ball at an angle of 45°. The ball lands 10 m away.
a. What was the ball’s initial speed?
b. For the same initial speed, find the two firing angles that make the range 6 m.

37.Projectile Motion with Linear Drag

The main force affecting the motion of a projectile, other than gravity, is air resistance. This slowing down force is drag force, and it acts in a direction opposite to the velocity of the projectile (see accompanying figure). For projectiles moving through the air at relatively low speeds, however, the drag force is (very nearly) proportional to the speed (to the first power) and so is called linear.



37.Linear drag Derive the equations

$$x = \frac{v_0}{k}\left(1 - e^{-kt}\right)\cos\alpha$$

$$y = \frac{v_0}{k}\left(1 - e^{-kt}\right)(\sin\alpha) + \frac{g}{k^2}\left(1 - kt - e^{-kt}\right)$$

by solving the following initial value problem for a vector r in the plane.

Differential equation:

$$\frac{d^2\mathbf{r}}{dt^2} = -g\mathbf{j} - k\mathbf{v} = -g\mathbf{j} - k\frac{d\mathbf{r}}{dt}$$

Initial conditions: $\mathrm{r}(0) = 0$

$$\left.\frac{d\mathbf{r}}{dt}\right|_{t=0} = \mathbf{v}_0 = (v_0\cos\alpha)\mathbf{i} + (v_0\sin\alpha)\mathbf{j}$$

The drag coefficient k is a positive constant representing resistance due to air density, v_0 and α are the projectile's initial speed and launch angle, and g is the acceleration of gravity.