# Answer_Midterm I for Calculus II in Spring Semester, 2018

SUSTC

Midterm I for Calculus II in Spring Semester, 2018

1.(30 pts)(Mid-18) Determine which of the following series converges absolutely, converges or diverges. Use any method, and give reasons for your answers.

$$(1) \sum_{n=1}^{\infty} \frac{2^n + 4^n}{3^n + 4^n}$$

$$\text{Solution: } \lim_{n\to\infty} a_n = \frac{(\frac{1}{2})^n + 1}{(\frac{3}{4})^{n+1} + 1} \to 1, \sum_{n=1}^{\infty} a_n \text{ diverges}$$

$$(2) \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

Solution:The Integral Test (for function continuous, positive, decreasing)

$$f(x) = \frac{1}{x(\ln x)^2}, f(x) > 0,$$

$$f'(x) = -\frac{2 + \ln x}{x^2(\ln x)^3} < 0 \text{ for } x \geq 2,$$

$$\int_2^{\infty} f(x)dx = \int_2^{\infty} \frac{1}{x(\ln x)^2}dx = \int_2^{\infty} (\ln x)^{-2}\frac{dx}{x},$$

$$\text{Substitute } u = \ln x, du = \frac{dx}{x},$$

$$\int_{\ln 2}^{\infty} \frac{1}{u^2}du \text{ converges,}$$

$$\text{so } \sum a_n \text{ converges.}$$

$$(3) \sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}};$$

Solution:Limit Comparison Test (for nonnegative terms)

$$a_n = \frac{1}{n\sqrt[n]{n}}, b_n = \frac{1}{n}(a_n, b_n > 0)$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{n}} = 1$$

because $\sum_{n=1}^{\infty} b_n$ diverges , $\sum_{n=1}^{\infty} a_n$ diverges

$$(4) \sum_{n=1}^{\infty} \frac{n!(n+1)!(n+2)!}{(3n)!}$$

Solution:The Ratio Test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| =$$

$$= \lim_{n\to\infty} \frac{(n+1)!(n+2)!(n+3)!}{(3n+3)!} \cdot \frac{(3n)!}{n!(n+1)!(n+2)!}$$

$$= \lim_{n\to\infty} \frac{(n+1)(n+2)(n+3)}{(3n+1)(3n+2)(3n+3)}$$

$$= \frac{1}{27} < 1$$

so $\sum_{n=1}^{\infty} a_n$ converges.

$$(5) \sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2+1} - n)$$

Solution:Leibniz's Test

$$u_n = \sqrt{n^2+1} - n > 0$$

$$u_n = \frac{1}{\sqrt{n^2+1}+n} \text{ decreases for } n \geq 1$$

$$\lim_{n\to\infty} u_n = 0,$$

so it's convergent by alternating series test.

$$\sum_{n=1}^{\infty} \frac{u_n}{b_n} \left(b_n = \frac{1}{n}\right) \to \frac{1}{2}$$

so $\sum u_n \& \sum b_n$ both diverge since $\sum b_n$ diverges

In conclusion, $\sum_{n=1}^{\infty} (-1)^n u_n$ conditionally converges

2.(15 pts)(Mid-18)

(1)Find the radius and interval of convergence of the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{\sqrt{n^2+3}}$$

$$a_n = \frac{x^n}{\sqrt{n^2+3}}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|x\cdot\sqrt{\frac{(n+1)^2+3}{n^2+3}}\right| \to |x|$$

so the radius of convergence r $= 1$

(2)For what values of $x$ does the series converge absolutely, or conditionally?

$$x = \ 1:\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n^2+3}}\text{converges by alternating series test}$$

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+3}}\text{diverges since }\lim_{n\to\infty}\frac{\frac{1}{\sqrt{n^2+3}}}{\frac{1}{n}}\to 1\&\sum_{n=1}^{\infty}\frac{1}{n}\text{diverges}$$

$$x = -1:\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+3}}\text{diverges}$$

3.(10 pts) (Mid-18)Find the Maclaurin series of the function $f(x) = (x+1)e^x$.

4.(10 pts)(Mid-18) Use series to evaluate the limit

$$\lim_{x\to 0}\frac{\ln(1+x^2)}{1-\cos x}$$

5.(10 pts)(Mid-18) Find the length of astroid

$$x = \cos^3 t, y = \sin^3 t, 0 \le t \le 2\pi$$

6.(10 pts)(Mid-18) Find the area of the region bounded by the circle $r = 2\sin\theta$ for $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$.

7.(5 pts) (Mid-18)Find the first four terms of the binomial series for the function

$$(1+x)^{\frac{1}{2}}$$

8.(10 pts) (Mid-18)Does the following sequence converge? If so, to what value?

$$x_1 = 1, x_{n+1} = \frac{x_n}{2 + x_n}, n = 1, 2, 3, \ldots$$