FIN204-Week7-Ch9-12310903刘华杰

2.An economy has a Cobb-Douglas production function:

$$Y = K^{\alpha}(LE)^{1-\alpha}$$

(For a review of the Cobb-Douglas production function, see Chapter 3.) The economy has a capital share of a third, a saving rate of 24 percent, a depreciation rate of 3 percent, a rate of population growth of 2 percent, and a rate of labor-augmenting technological change of 1 percent. It is in steady state.

$$\alpha = \frac{1}{3}$$
$$s = 0.24$$
$$\delta = 0.03$$
$$n = 0.02$$
$$g = 0.01$$

- a. At what rates do total output, output per worker, and output per effective worker grow?
  Answer:
  在考虑科技进步的Solow model下，科技进步被视作劳动者的乘数E。
  总产出的百分比增速 $= n + g = 0.03$
  人均产出的百分比增速为E的百分比增速 $= g = 0.01$
  有效人均产出的百分比增速为0.
- b. Solve for capital per effective worker, output per effective worker, and the marginal product of capital.

Answer:
在Solow model的稳态下，利用稳态条件和生产函数求解K

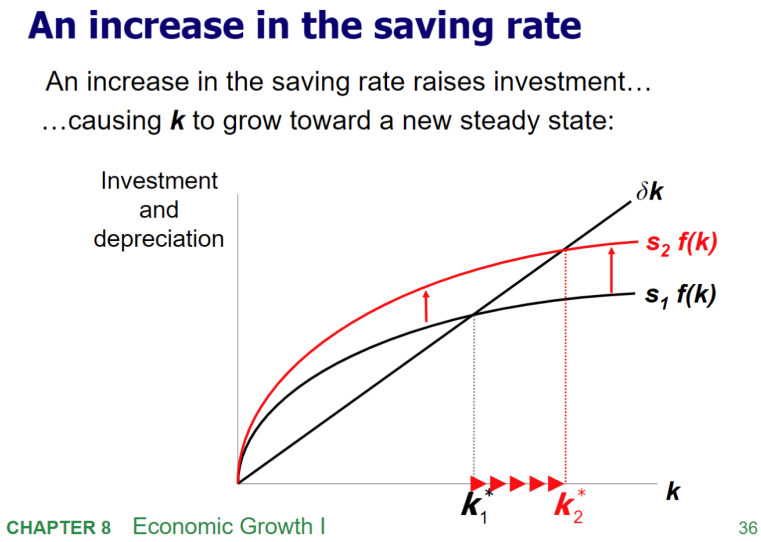
$$
\begin{aligned}
k &= K/LE \\
y &= Y/LE \\
&= k^{1/3} \\
sy &= (n+g+\delta)k \\
0.24k^{1/3} &= (0.06)k \\
k^{2/3} &= 4 \\
k &= 8
\end{aligned}
$$

capital per effective worker：$k = 8$
output per effective worker: $y = 2$
计算资本边际收益：

$$MPK = \frac{d}{dk}\left(k^{1/3}\right) = \frac{1}{3}k^{-2/3} = \frac{1}{12} \approx 0.0833$$

- c. Does the economy have more or less capital than at the Golden Rule steady state? How do you know? To achieve the Golden Rule steady state, does the saving rate need to increase or decrease?
  Answer:
  根据黄金法则，理想$MPK = \delta + n + g = 0.06 < 0.083$
  资本回报率小于黄金稳态，因此，经济体实际运行中资本少于黄金稳态。由于在假设中投资等于储蓄，应当增加储蓄率，以增加投资。
- d. Suppose the change in the saving rate you described in part (c) occurs. During the transition to the Golden Rule steady state, will the growth rate of output per worker be higher or lower than the rate you derived in part (a)? After the economy reaches its new steady state, will the growth rate of output per worker be higher or lower than the rate you derived

in part (a)? Explain your answers.



储蓄率增加，人均资本（此处的人均指per effective worker）提高，根据 $y = k^{1/3}$ ，人均产出增速为技术进步g+资本积累的增速叠加； （a）中人均产出增速仅为技术进步g.

抵达新的稳态后，人均资本增速为零，人均生产函数增速和（a）中相同。

3.In the United States, the capital share of GDP is about 30 percent, the average growth in output is about 3 percent per year, the depreciation rate is about 4 percent per year, and the capital-output ratio is about 2.5. Suppose that the production function is Cobb-Douglas and that the United States has been in a steady state. (For a discussion of the Cobb-Douglas production function, see Chapter 3.)

$$
\begin{aligned}
\alpha &= 0.3 \\
\frac{\delta Y}{Y} &= 0.03 \\
Y &= K^{0.3}L^{0.7} \\
y &= k^{0.3} \\
\delta &= 0.04 \\
\frac{K}{Y} &= 2.5
\end{aligned}
$$

- a. What must the saving rate be in the initial steady state? Hint: Use the steady-state relationship, $s\gamma = (\delta + n + g)k.$]

Answer:
根据提示，储蓄率计算：

$$s = \frac{(\delta + n + g)k}{y} = \frac{(\delta + \delta Y)K}{Y} = 2.5 \times (0.04 + 0.03) = 0.175 = 17.5\%$$

- b. What is the marginal product of capital in the initial steady state?

$$MPK = \frac{d}{dk}(y) = \frac{3}{10}k^{-0.7}$$

- c. Suppose that public policy alters the saving rate so that the economy reaches the Golden Rule level of capital. What will the marginal product of capital be at the Golden Rule steady state? Compare the marginal product at the Golden Rule steady state to the marginal product in the initial steady state. Explain.

黄金法则下$MPK = n + g + \delta = \delta + \frac{\delta Y}{Y} = 0.07$

- d. What will the capital-output ratio be at the Golden Rule steady state? ($Hint$: For the Cobb-Douglas production function, the capital-output ratio is related to the marginal product of capital.)

由于α 就是在Cobb-Douglas函数中资本占总生产函数的比重，

$$\begin{aligned} \alpha Y &= MPK \cdot K \\ \Longrightarrow \quad \frac{K}{Y} &= \frac{\alpha}{MPK} = \frac{0.3}{0.07} \approx 4.29 \end{aligned}$$

- e. What must the saving rate be to reach the Golden Rule steady state?

$$s = (\delta + n + g)\frac{K}{Y} = 0.07 \times 4.29 = 0.3003$$

Steady-state growth rates in the Solow model with tech. progress

Variable	*Symbol*	*Steady-state growth rate*
Capital per effective worker	$k = K/(L \times E)$	0
Output per effective worker	$y = Y/(L \times E)$	0
Output per worker	$(Y/L) = y \times E$	g
Total output	$Y = y \times E \times L$	$n + g$

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