24 Spring Mid Q&A

2023-2024学年春季学期期中考试

- 24 Spring Mid Q 中英文试题分离 紧凑版
- 原卷（相片质量）24春线代期中纯享版.pdf

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本试卷共（7）答题，满分（100）分。

1.(15 points, 3 points each) Multiple Choice. Only one choice is correct.

(共15分，每小题3分)选择题，只有一个选项是正确的.

$(1) Let$

$$\alpha_1=\begin{bmatrix}2\\3\\1\end{bmatrix},\alpha_2=\begin{bmatrix}1\\-1\\2\end{bmatrix},\alpha_3=\begin{bmatrix}7\\3\\c\end{bmatrix}.$$

If $\alpha_1,\alpha_2,\alpha_3$ are linearly dependent, then c equals

$(A)5.$

$(B)6.$

$(C)7.$

$(D)8.$

假定

$$\alpha_1 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \alpha_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \alpha_3 = \begin{bmatrix} 7 \\ 3 \\ c \end{bmatrix}.$$

若$\alpha_1, \alpha_2, \alpha_3$线性相关，则$c$的取值为

(A)5.

(B)6.

(C)7.

(D)8.

(2) let A be an $m \times n$ real matrix and b be an $m \times 1$ real column vector. Which of the following statements is correct?

(A) If $A\mathbf{x} = \mathbf{b}$ does not have any solution, then $A\mathbf{x} = \mathbf{0}$ has only the zero solution.

(B) If $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(C) If $m < n$, both $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$ have infinitely many solutions.

(D) If the rank of A is n, then $A\mathbf{x} = \mathbf{0}$ has only the zero solution.

设 A 为 一 个 $m \times n$ 实 矩 阵 , b 为 一 个 m 维 实 列 向 量 , 以 下 说 法 一 定 是 正 确 的 是 ?

(A)若$A\mathbf{x} = \mathbf{b}$无解，则$A\mathbf{x} = \mathbf{0}$只有零解.

$(\mathbf{B})$若$A\mathbf{x} = \mathbf{0}$有无穷多解，则$A\mathbf{x} = \mathbf{b}$有无穷多解。$(\mathbf{C})$若$m < n$,则 $A\mathbf{x} = \mathbf{b}$和$A\mathbf{x} = \mathbf{0}$都有无穷多解。

(D)若A的秩为n,则$A\mathbf{x} = 0$只有零解.

(3) For which value of k does the system

$$\begin{cases} x_1 + 2x_2 - 4x_3 + 3x_4 = 0, \\ x_1 + 3x_2 - 2x_3 - 2x_4 = 0, \\ x_1 + 5x_2 + (5-k)x_3 - 12x_4 = 0, \end{cases}$$

have exactly two free variables?

$(A)5.$

$(B)4.$

$(C)3.$

$(D)2.$

如果以下线性方程组有两个自由变量

$$\begin{cases} x_1 + 2x_2 - 4x_3 + 3x_4 = 0, \\ x_1 + 3x_2 - 2x_3 - 2x_4 = 0, \\ x_1 + 5x_2 + (5-k)x_3 - 12x_4 = 0, \end{cases}$$

k的取值为

$(A)5.$

$(B)4.$

$(C)3.$

$(D)2.$

(4) Let $u, v \in \mathbb{R}^3$ and $\lambda \in \mathbb{R}$. Which of the following statements is false?

(A) If u and v are nonzero vectors satisfying $u^T v = 0$, then u and v are linearly independent.

($\mathbf{B}$) If $u + v$ is orthogonal to $u - v$, then $||u|| = ||v||$.

$(C) u^T v = 0$ if and only if $u = 0$ or $v = 0$.

$(D) \lambda v = 0$ if and only if $v = 0$ or $\lambda = 0$.

设 $u, v \in \mathbb{R}^3, \lambda \in \mathbb{R}$. 以下说法错误的是?

(A)如果u和v为满足$u^T v = 0$的非零向量，则u和v线性无关.

(B)如果$u + v$和$u - v$正交，则$||u|| = ||v||$.

$(C) u^T v = 0$当且仅当 $u = 0$ or $v = 0$.

$(D) \lambda v = 0$当且仅当 $v = 0$ or $\lambda = 0$.

(5) Let A and B be two $n \times n$ matrices. Which of the following assertions is **false**?

(A) If A, B are symmetric matrices, then AB is a symmetric matrix.

$(\mathbf{B})$ If A, B are invertible matrices, then AB is an invertible matrix.

$(\mathbb{C})$ If A, B are permutation matrices, then AB is a permutation matrix.

$(\mathbf{D})$ If A, B are upper triangular matrices, then AB is an upper triangular matrix.

设A和B都为n阶矩阵.以下说法**错误**的是?

$(\mathbf{A})$如果A, B为对称矩阵，则AB也为一个对称矩阵。

$(\mathbf{B})$如果A, B 为可逆矩阵，则 AB 也为一个可逆矩阵.

$(\mathbf{C})$如果A, B为置换矩阵，则AB也为一个置换矩阵.

$(\mathbf{D})$如果A, B为上三角矩阵，则AB也为上三角矩阵。

2.(20 points, 5 points each) Fill in the blanks.

$$(1)\text{Let } A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 3 & 2 \end{bmatrix}, a, b \in \mathbb{R}. \text{ Then } A^{-1} = \_\_\_\_\_\_.$$

$$\text{设 } A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 3 & 2 \end{bmatrix}, a, b \in \mathbb{R}, \text{则} A^{-1} = \_\_\_\_\_\_.$$

(2) Let A be a 4×3 real matrix with rank 2 and $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$.

Then the rank AB is ______.

设A为一个4×3的实矩阵，B为$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$.如果矩阵$A$的秩为 2,

则AB的秩为______.

$$(3)\text{Let} A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 2 \end{bmatrix}. \text{Then} A^{2024} = \underline{\qquad\qquad}.$$

$$\text{设 } A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 2 \end{bmatrix}, \text{则 } A^{2024} = \underline{\qquad\qquad}.$$

(4) Consider the system of linear equations:

$$Ax = b : \begin{cases} x & = & 2 \\ y & = & 3 \\ x + y & = & 6 \end{cases}$$

The least-squares solution for the system is

考虑以下线性方程组:

$$A\mathbf{x} = \mathbf{b} : \begin{cases} x & = & 2 \\ y & = & 3 \\ x + y & = & 6 \end{cases}$$

该线性方程组的最小二乘解为

3.(10points)Let

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 1 \\ 1 & -4 & -7 \end{bmatrix}.$$

Find an LU factorization of A.

设

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 1 \\ 1 & -4 & -7 \end{bmatrix}.$$

求矩阵A的一个LU分解

4.(24 points) Consider the following 4×5 matrix A and 4-dimensional column vector b:

$$A = \begin{bmatrix} 0 & 2 & 4 & 1 & 6 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 4 & 10 & 1 & 2 \\ 0 & -1 & -5 & 1 & 7 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 2 \\ -5 \\ 10 \end{bmatrix}$$

(a) Find a basis for each of the four fundamental subspaces of A.
(b) Find the complete solution to $A\mathbf{x} = \mathbf{b}$.

考虑以下 4× 5 矩阵 A 以及 4 维列向量 $\mathbf{b}$:

$$A = \begin{bmatrix} 0 & 2 & 4 & 1 & 6 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 4 & 10 & 1 & 2 \\ 0 & -1 & -5 & 1 & 7 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ -5 \\ 10 \end{bmatrix}$$

(a)分别求矩阵 A 的四个基本子空间的一组基向量。
(b)求$Ax = b$的所有解.

5.(20 points) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ and T be the linear transformation from $R^{2\times 2}$ to $R^{2\times 2}$ defined by

$$T(X) = XA + AX, \ X \in \mathbb{R}^{2\times 2}.$$

Where $\mathbb{R}^{2\times 2}$ denotes the vector space consisting of all 2×2 real matrices.
(a) Find the matrix representation of T with respect to the following ordered basis

$$v_1=\begin{bmatrix}1&0\\0&0\end{bmatrix},v_2=\begin{bmatrix}0&1\\0&0\end{bmatrix},v_3=\begin{bmatrix}0&0\\1&0\end{bmatrix},v_4=\begin{bmatrix}0&0\\0&1\end{bmatrix}.$$

($\mathbf{b}$) Find a matrix B such that

$$T(B)=\begin{bmatrix}0&0\\0&0\end{bmatrix}.$$

(c) Find a matrix C such that

$$T\left(C\right)=\begin{bmatrix}1&2\\3&4\end{bmatrix}.$$

设 Let $A=\begin{bmatrix}1&1\\0&2\end{bmatrix}$, T为按照以下方式定义的从 $\mathbb{R}^{2\times 2}$到 $\mathbb{R}^{2\times 2}$线性变换:

$$T\left(X\right)=XA+AX,X\in\mathbb{R}^{2\times 2}.$$

其中$\mathbb{R}^{2\times 2}$农示所有2×2实矩阵构成的向量空间.
(a)求T在以下有序基

$$v_1=\begin{bmatrix}1&0\\0&0\end{bmatrix},v_2=\begin{bmatrix}0&1\\0&0\end{bmatrix},v_3=\begin{bmatrix}0&0\\1&0\end{bmatrix},v_4=\begin{bmatrix}0&0\\0&1\end{bmatrix}$$

下的矩阵表示.
(b)求一个矩阵B使得

$$T(B)=\begin{bmatrix}0&0\\0&0\end{bmatrix}.$$

(c)求一个矩阵C使得

$$T(C)=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}.$$

6.(5 points) Let A, B be two $n\times n$ real matrices satisfying $A^2=A$ and $B^2=B$. Show that if $(A+B)^2=A+B$, then $AB=O$. Where O denotes the $n\times n$ zero matrix.
设A, B为满足$A^2=A$和$B^2=B$的n阶实矩阵.证明：如果$(A+B)^2=A+B$,则$AB=O$.其中O 表示n阶零矩阵。

7.(6 points) Let A be a 3×2 matrix, B be a 2×3 matrix such that

$$AB=\begin{bmatrix}8 & 0 & -4\\ -\frac{3}{2} & 9 & -6\\ -2 & 0 & 1\end{bmatrix}.$$

$(a)\mathrm{Compute}(AB)^2.$
(b) Find BA.
设A为 3×2矩阵，B 为 2×3矩阵，并且

$$AB=\begin{bmatrix}8 & 0 & -4\\ -\frac{3}{2} & 9 & -6\\ -2 & 0 & 1\end{bmatrix}.$$

(a)计算$(AB)^2$.
(b)求BA.