Calculus Ⅱ Week4 HW-Questions

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- 课本作业
  Section 10.9，#18,22,33 Section 10.10，#33,35,38,50
  Section 11.1，#27,32,36 Section 11.2，#11

10.9 #18,22,33

Use power series operations to find the Taylor series at x = 0 for the functions

$$18. sin^2x$$
$$22. \frac{2}{(1-x)^3}$$

Find the first four nonzero terms in the Maclaurin series for the functions

$$33. e^{\sin x}$$

10.10 #33,35,38,50

Indeterminate Forms-Use series to evaluate the limits

$$33. \lim_{y \to 0} \frac{y - \tan^{-1} y}{y^3}$$
$$35. \lim_{x \to \infty} x^2 \left(e^{-1/x^2} - 1 \right)$$
$$38. \lim_{x \to 2} \frac{x^2 - 4}{\ln (x - 1)}$$

Use Table 10.1 to find the sum of each series.

$$50.\ \ x^2 - 2x^3 + \frac{2^2x^4}{2!} - \frac{2^3x^5}{3!} + \frac{2^4x^6}{4!} - \cdots$$

TABLE 10.1 Frequently used Taylor series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, \qquad |x| < 1$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-x)^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n, \qquad |x| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \qquad |x| < \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \qquad |x| < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \qquad |x| < \infty$$

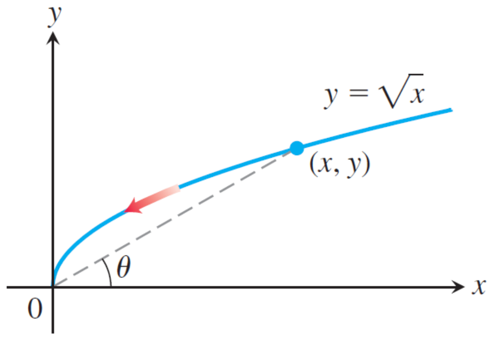
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}, \qquad -1 < x \le 1$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \qquad |x| \le 1$$

11.1 #27,32,36

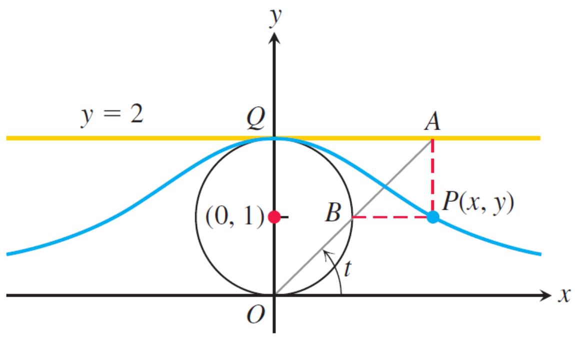
27.Find parametric equations and a parameter interval for the motion of a particle starting at the point (2,0) and tracing the top half of the circle $x^2 + y^2 = 4$ four times.

32.Find a parametrization for the curve $y = \sqrt{x}$ with terminal point (0,0) using the angle θ in the accompanying figure as the parameter.



36.Hypocycloid When a circle rolls on the inside of a fixed circle, any point P on the circumference of the rolling circle describes a $hypocycloid$. Let the fixed circle be $x^2 + y^2 = \alpha^2$, let the radius of the rolling circle be b, and let the initial position of the tracing point P be $A(\alpha, 0)$. Find parametric equations for the hypocycloid, using as the parameter the angle θ from the positive x-axis to the line joining the circles' centers. In particular, if $b = a/4$, as in the accompanying figure, show that the hypocycloid is the astroid

$$x = a\cos^3\theta, \quad y = a\sin^3\theta.$$



11.2 #11

In Exercises 1–14, find an equation for the line tangent to the curve at the point defined by the given value of t. Also, find the value of d^2y/dx^2 at this point.

$$11.\ x = t - sint,\ y = 1 - cost,\ t = \pi/3$$

- Supple HW Assignment 4.pdf

1.Use the Taylor series to compute the following limits.

$$(1) \lim_{x \to 0} \frac{e^{\tan x} - e^{\sin x}}{x \ln(\cos x)}$$

$$(2) \lim_{x \to 0} \frac{x^{\tan x - \sin x} - 1}{x^3 \ln x}$$

(3) $\lim_{n\to\infty}\left(n^{\frac{3}{2}}\left(\sqrt[8]{n^4+4n^3+1}-\sqrt{n+1}\right)\right)$

2.Find the Taylor series at x=0 for the function $f(x) = \arctan \dfrac{1-2x}{1+2x}$ and find $f^{(8)}(0)$. (Hint: $f'(x) = ?$)

3.Find the Taylor series at x=4 for the function $f(x) = \frac{1}{x^2-5x+6}$

4.Find the open intervals of x such that the curve $y = f(x)$ is concave down, where the curve is given by

$$\begin{cases} x = t^3 + 3t + 1 \\ y = t^3 - 3t + 1 \end{cases}$$