Collaboration and Plagiarism are Not Allowed!

PART 1:

Choose the correct answer(s) or fill in the blank. (2'x 10=20')

1. Choose all correct statements: acd
   a) A system with input-output relation y(t)=2+x(t) is not a linear system, where x and y are the input and output signals, respectively.
   b) A causal system is a memoryless system.
   c) If the input signal to a LTI system is periodic, then the output is also periodic with the same period.
   d) The system y(t)=x(t) sin(t+1) is a causal system.

2. For a CT LTI system, if the linear-constant-coefficient differential equation connecting input x(t) and output y(t) is given, we can also have abcd.
   a) The system function of the LTI system
   b) Frequency response of the LTI system
   c) The Unit impulse response of the LTI system
   d) System output y(t) when the input signal x(t) is given

3. For signal $x(t)=\delta(t)$, its Fourier transform $X(j\omega)=$ 1, and for signal $x(t)=2\delta(t-1)$, its Fourier transform $X(j\omega)=$ $2e^{-jw}$

4. Please 1) describe the frequency response of an ideal lowpass filter, and 2) discuss whether it is a causal system and why:

① The frequency response of an ideal lowpass filter is $H(jw)=\begin{cases}1 & |w|\le w_c\\ 0 & |w|>w_c\end{cases}$

The w_c is the cutoff frequency

② $H(jw)=1$ when $|w|<w_c$, $h(t)=\frac{\sin(w_c t)}{\pi t}$ h(t) is difined when t<0, so noncausal

5. For a CT, time-limited (aperiodic) signal, its Fourier transform is b.
   a) discrete   b) continuous   c) not sure

6. Please describe the steps to get the Fourier transform of a periodic signal.

① caculate $a_k=\frac{1}{T}\int_T x(t)e^{-jk\frac{2\pi}{T}t}dt$ ② Unfolding x(t) to Fourier series $x(t)=\sum_{k=-\infty}^{\infty} a_k\cdot e^{jk\frac{2\pi}{T}t}$ ③ Fourier transform to both sides of above formula

$$F(x(t))=X(jw)=F\left(\sum_{k=-\infty}^{\infty} a_k\cdot e^{jk\frac{2\pi}{T}t}\right)=\sum_{k=-\infty}^{\infty} a_k F\left(e^{jk\frac{2\pi}{T}t}\right)=\sum_{k=-\infty}^{\infty} a_k 2\pi\delta\left(w-k\frac{2\pi}{T}\right)=2\pi\sum_{k=-\infty}^{\infty} a_k\delta\left(w-k\frac{2\pi}{T}\right)$$

The Fourier transform of a CT periodic signal is a ___a___ spectrum.

a) discrete   b) continuous   c) Depends on the input signal

7. For a LTI system with unit impulse response h(t) and input signal x(t), please describe at least 2 ways to determine the output signal y(t).

(1) use Fourier transform to both sides of above formula,
$Y(jw)=X(jw)H(jw)=X(jw)\cdot 1=X(jw)$
(2) use convolution, $y(t)=x(t)*h(t)$

8. $\int_{-\infty}^{3}(2t^2+3t)\delta(\frac{1}{2}t-2)dt =$ (A)

A. 0   B. 27   C. 44   D. 88

9. The Fourier transform of f(t) is $F(j\omega)$, then the Fourier transform of $f'(2t)$ is (B)

A. $j4wF(j2w)$ B. $j\frac{w}{4}F(j\frac{w}{2})$

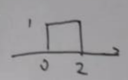
C. $j\frac{w}{2}F(j\frac{w}{2})$ D. $j2wF(j2w)$

10. For a DT LTI system S, the shape of the frequency response of $y[n]=(x[n]+x[n-1]+x[n-2]+x[n-3])/4$ is ___a___; while the shape of the frequency response of $y[n]=x[n]-x[n-1]$ is ___b___.

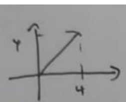
a) Low-pass filtering   b) High-pass filtering

c) Band-pass filtering   d) Dependent on the spectrum of the input signal x[n]

P1 (1) x(t)



h(t)



$y(t)=x(t)*h(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$

when $t\le 0,\ t\ge 6$ $y(t)=0$

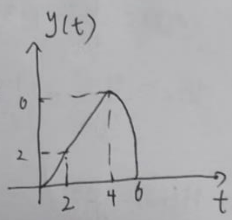
when $0\le t<2$ $y(t)=\frac{t^2}{2}$

when $2\le t<4$ $y(t)=\frac{1}{2}(t+t-2)\times 2=2t-2$

when $4\le t<6$ $y(t)=\frac{t}{2}\times4\times4-\frac{1}{2}(t-2)^2=-\frac{1}{2}t^2+2t+6$

$$y(t)=\begin{cases}\frac{1}{2}t^2, & 0\le t<2\\ 2t-2, & 2\le t<4\\ -\frac{1}{2}t^2+2t+6 & 4\le t<6\\ 0 & \text{otherwise}\end{cases}$$

plot:



(2) using CTFT to transform $x(t)$ to $X(jw)$, $h(t)$ to $H(jw)$,
then $Y(jw)=X(jw)H(jw)$, use inverse fourier transformation to compute $y(t)$.

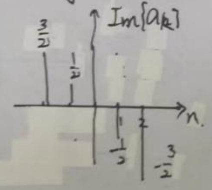
P3 (1) $x[n]=\frac{1}{2}(e^{j\frac{\pi}{4}n}+e^{-j\frac{\pi}{4}n})+\frac{3}{2}(e^{j\frac{\pi}{2}n}+e^{-j\frac{\pi}{2}n})$

P2 (1) $x(t)=\frac{1}{2j}(e^{j\frac{\pi}{4}t}-e^{-j\frac{\pi}{4}t})+\frac{3}{2j}(e^{j2\cdot\frac{\pi}{4}t}-e^{-j2\cdot\frac{\pi}{4}t})$

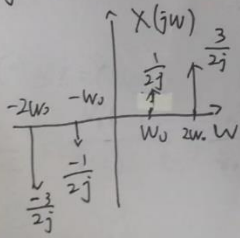
$w_0=\frac{\pi}{4}$ $\quad T=\frac{2\pi}{w}=8$

(2) $x(t)=\frac{1}{2j}e^{j\frac{\pi}{4}t}-\frac{1}{2j}e^{-j\frac{\pi}{4}t}+\frac{3}{2j}e^{j\frac{\pi}{2}t}-\frac{3}{2j}e^{-j\frac{\pi}{2}t}$

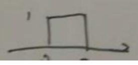
(3) ①



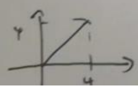
②



P1 (1) x(t)



h(t)



then $Y(jw)=X(jw)H(jw)$, use inverse fourier transformation to compute $y(t)$.

P3 (1) $x[n]=\frac{1}{2}(e^{j\frac{\pi}{4}n}+e^{-j\frac{\pi}{4}n})+\frac{3}{2}(e^{j\frac{\pi}{2}n}+e^{-j\frac{\pi}{2}n})$

$=\frac{1}{2}e^{j\frac{\pi}{4}n}+\frac{1}{2}e^{-j\frac{\pi}{4}n}+\frac{3}{2}e^{j2\cdot\frac{\pi}{4}n}+\frac{3}{2}e^{-j2\cdot\frac{\pi}{4}n}$ $\qquad w_0=\frac{\pi}{4}$, $N=\frac{2\pi}{w_0}=8$

(2) $a_{-2}\sim a_2$: $\{\frac{3}{2},\frac{1}{2},0,\frac{1}{2},\frac{3}{2})$ $\quad a_k=a[k+N]$

$\therefore a_{-5}\sim a_2$: $\{0,0,0,\frac{3}{2},\frac{1}{2},0,\frac{1}{2},\frac{3}{2})$ $\quad a_{37}=0 \quad a_{47}=\frac{1}{2}$

ic input

(3) $H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} = \sum_{n=-\infty}^{\infty} 0.3^n u[n] e^{-j\omega n} = \sum_{n=0}^{\infty}(0.3\times e^{-j\omega})^n.$

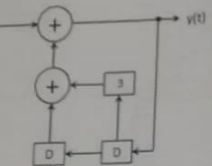
$$H(e^{j\omega}) = \lim_{A\to\infty}\left(1\times\frac{1-(0.3\times e^{-j\omega})^A}{1-0.3e^{-j\omega}}\right) = \frac{1-\lim_{A\to\infty}(0.3e^{-j\omega})^A}{1-0.3\times e^{-j\omega}} = \frac{1}{1-0.3\times e^{-j\omega}}$$

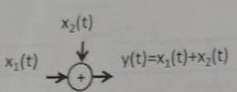
(4) $x[n] \xrightarrow{FT} 2\times\sum_{n=-\infty}^{+\infty} a_k\delta(\omega-\frac{\pi}{4}k)$ $\quad Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = 2\pi\sum_{n=-\infty}^{+\infty} a_k \frac{1}{1-0.3e^{-j\frac{\pi}{4}k}}\delta(\omega-\frac{\pi}{4}k)$

$$Y(e^{j\omega}) \xrightarrow{IFT} y[n] = \frac{e^{j\frac{\pi}{4}n}}{2-0.6e^{j\frac{\pi}{4}}} + \frac{e^{-j\frac{\pi}{4}n}}{2-0.6e^{j\frac{\pi}{4}}} + \frac{3e^{j\frac{\pi}{2}n}}{2-0.6e^{j\frac{\pi}{2}}} + \frac{3e^{-j\frac{\pi}{2}n}}{2-0.6e^{j\frac{\pi}{2}}}$$

P4 (1) $2x(t)+\frac{dx(t)}{dt}+3\frac{dy(t)}{dt}+\frac{d^2y(t)}{dt^2}=y(t)$

TI system





esponse of

$\frac{2j\omega+2}{15-\omega^2+8j\omega}$.

and output $y(t)$ of S.

t)?

P4 (1) $2x(t)+\frac{dx(t)}{dt}+3\frac{dy(t)}{dt}+\frac{d^2y(t)}{dt^2}=y(t)$

(2) $H(j\omega)=\frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega+2}{\omega^2-j\omega+1}$

P5 (1) $H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}$

$Y(j\omega)(15-\omega^2+8j\omega) = X(j\omega)(2j\omega+2)$

$y''(t)+8y(t)+15y(t)=2x'(t)+2x(t)$

(2) $H(j\omega)=\frac{2j\omega+2}{(3+j\omega)(5+j\omega)} = \frac{-2}{3+j\omega}+\frac{4}{5+j\omega}$

$H(j\omega)\xrightarrow{F} -2e^{-3t}u(t)+4e^{-5t}u(t)$

(3) $x(t)=e^{-t}u(t) \xrightarrow{FT} X(j\omega)=\frac{1}{1+j\omega}$

$Y(j\omega)=H(j\omega)X(j\omega)=\frac{1}{j\omega+3}-\frac{1}{j\omega+5}$

$y(t)=e^{-3t}u(t)-e^{-5t}u(t)$