南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目：概率论与数理统计　　开课单位：数学系
考试时长：2023/11/18 19:00–21:00　　命题教师：概率统计教学组

本试卷共三大部分，满分（100）分
（考试结束后请将试卷、答题卡一起交给监考老师）

题号	1	2	3	4	5	6	7	8
分值	20 分	20 分	10 分	10 分	10 分	10 分	10 分	10 分

第一部分　选择题（每题4分，总共20分）
Part One – Single Choice (4 marks each question, 20 marks in total)

1. 设两个事件A与B是独立的，已知A, B中至少有一个发生的概率为8/9，且A发生B不发生的概率与A不发生B发生的概率相等，则概率$P(A)$等于(　　)。

   Event A and event B are independent, and the probability that at least one of A, B occurs is 8/9. It is known that the probability that A occurs and B does not occur is equal to the probability that A does not occur and B occurs, then the probability of A, i.e., $P(A)$, equals (　　).

   (A) $\frac{2}{3}$　　(B) $\frac{1}{3}$　　(C) $\frac{4}{9}$　　(D) $\frac{5}{9}$

2. 若函数$F(x) = \frac{2}{2+x^2}$为随机变量X的分布函数，则X可能的取值范围为(　　)。

   If $F(x) = \frac{2}{2+x^2}$ is the cumulative distribution function of random variable X, then the value range of X is (　　).

   (A) $(-\infty, \infty)$　　(B) $(0, \infty)$　　(C) $(-\infty, 0)$　　(D) $(0, 1)$

3. 设随机变量X服从参数为1的指数分布，即$X \sim \mathrm{Exp}(1)$，a为一个大于零的常数，则$P(X \le a+1 | X > a)$等于(　　)。

   Random variable X follows an exponential distribution with parameter 1, i.e., $X \sim \mathrm{Exp}(1)$, a is a constant greater than zero, then $P(X \le a+1 | X > a)$ equals (　　).

   (A) $1 - e^{-1}$　　(B) e^{-1}　　(C) $1 - e^{-a}$　　(D) e^{-a}

4. 设平面区域D由曲线$y=\dfrac{1}{x}$及直线$y=0,x=1,x=e^2$所围成。二维随机变量(X,Y)在区域D上服从均匀分布，则(X,Y)关于X的边缘密度函数在$x=2$处的值$f_X(2)=$(

D is a flat area enclosed by the curve $y=\dfrac{1}{x}$ and straight lines $y=0,x=1,x=e^2$. The two-dimensional random variable (X,Y) follows a uniform distribution on D, then the value of the marginal density function of (X,Y) for X at $x=2$, i.e., $f_X(2)=($ $)$.

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{e}$ (D) $\dfrac{1}{e^2}$

5. 设随机变量X与Y独立，$X\sim N(0,1)$，$Y\sim N(1,1)$，则以下结论正确的是 ，

Random variables X and Y are independent, $X\sim N(0,1)$, $Y\sim N(1,1)$, then which of the following statements is correct ( ).

(A) $P(X-Y\leq 0)=0.5$ (B) $P(X-Y\leq 1)=0.5$
(C) $P(X+Y\leq 0)=0.5$ (D) $P(X+Y\leq 1)=0.5$

第二部分 填空题（每空2分，总共20分）

Part Two – Blank Filling (2 marks each blank, 20 marks in total)

1. 设事件A,B,C 相互独立, 且$P(A)=P(B)=1/2$, $P(C)=1/3$. 则事件A, B, C 中至多发生一件的概率为________.
Let A,B,C be three mutually independent events, with $P(A)=P(B)=1/2$, $P(C)=1/3$. Then the probability that at most one of the events A,B,C happens is ________.

2. 把3双不同的筷子打乱随机分给3个人. 则至少有一个人拿到原本一双中的两只筷子的概率为________.
Distribute randomly 6 chopsticks from 3 different pairs to three persons. Then the probability of at least one person getting two chopsticks from the same pair is ________.

3. 从3 个红球、2 个黑球和1 个白球中不放回地依次取出2 个球, 则取出的两个球均为红球的概率为____
Take two balls without replacement from 3 red balls, 2 black balls and 1 white ball. Then the probability that both balls are red is ________.

4. 设X 服从参数为$\lambda = \ln 2$ 的泊松分布. 令$F(x)$ 为X 的分布函数. 则$F(1) =$ ___.
Let X be a Poisson random variable with parameter $\lambda = \ln 2$ and let $F(x)$ be its cumulative distribution function. Then $F(1) =$ ________.

5. 抛10次正反面的概率均为1/2 的硬币. 令A 为恰好出现2 次正面的事件, B 为前5 次均出现反面的事件. 则$P(B \mid A) =$ ________.
Toss a fair coin 10 times. Let A be the event of getting exactly 2 heads in total, and let B be the event of getting all tails in the first 5 tosses. Then $P(B \mid A) =$ ________.

6. 假设某随机变量X 的分布函数为$F(x) = ae^{bx}/(1+2e^x)$, 则$a =$ ________, $b =$ ________.
Suppose that X is a random variable with cumulative distribution function $F(x) = ae^{bx}/(1+2e^x)$. Then $a =$ ________, $b =$ ________.

7. 假设某次60分及格的考试中, A 同学的成绩服从正态分布$N(65, a^2)$, B 同学的成绩服从正态分布$N(60+a, 10^2)$. 若A 及格的概率不低于B 及格的概率, 则a 的取值范围为________.
In an exam that requires 60 to pass, suppose that the score of student A has the normal distribution $N(65, a^2)$ and the score of student B has the normal distribution $N(60+a, 10^2)$. If the probability of A passing the exam is at least as high as B doing so, then the range for a is ________.

8. 设X 服从$(0,1)$ 上的均匀分布, 在X 给定时, Y 服从$(0,X)$ 上的均匀分布. 考虑事件$A = \{Y \geq 1/3\}$, $B = \{X \leq 2/3\}$. 则$P(B \mid A) =$ ________.
Suppose that X has the uniform distribution over $(0,1)$, and given X, Y has the uniform distribution over $(0, X)$. Consider the events $A = \{Y \geq 1/3\}$ and $B = \{X \leq 2/3\}$. Then $P(B \mid A) =$ ________.

9. 已知两个独立离散型随机变量X 和Y 满足$X, Y \in \{0,1\}$, 且$P(X=0, Y=1) = 1/4$, $P(X=1, Y=0) = 1/6$. 则$P(X=0, Y=0) =$ ________.
Two independent random variables X and Y satisfy that $X, Y \in \{0,1\}$ and $P(X=0, Y=1) = 1/4$, $P(X=1, Y=0) = 1/6$. Then $P(X=0, Y=0) =$ ________.

10. 设X 和Y相互独立且分别服从参数为1 和2 的指数分布. 令$Z = \max(X, Y)$. 则当$z \geq 0$ 时, Z 的密度函数为$f_Z(z) =$
Let X and Y be independent and be exponential random variables with parameters 1 and 2, respectively. Let $Z = \max(X, Y)$. Then for $z \geq 0$, the density function for Z is $f_Z(z) =$ ________.

第三部分 解答题（每题10分，总共60分）

Part Three–Question Answering (10 marks each question, 60 marks in total)

1. 记样本空间为Ω，现有两个随机事件A和B, 它们发生的概率分别为0.4和0.7。

(1) 事件A和B是否可能互不相容（互斥），证明你的回答；

(2) 如果$A \cup B = \Omega$，事件A和B是否独立，证明你的回答；

(3) 现在考虑一个几何概型，设样本空间为$\Omega = [0,1]$，即实数轴上的单位区间，并且$A = [0, 0.4]$，$B = [a, b]$，$0 \le a < b \le 1$。如果A和B独立，请确定a和b的取值。

Let the sample space be Ω. Now consider two events A and B, with the probability of occurrence being respectively 0.4 and 0.7.

(1) Are A and B disjoint? Prove your answer;

(2) If $A \cup B = \Omega$, are A and B independent? Prove your answer;

(3) Now consider a geometric model of probability. Let $\Omega = [0,1]$ be the unit interval in $\mathbb{R}$, and suppose $A = [0, 0.4]$, $B = [a, b]$, $0 \le a < b \le 1$. If A and B independent, find the values of a and b.

2. 某保险公司把被保险人分为三类“谨慎的”，“一般的”，“冒失的”。统计资料表明，这三种人在一年内发生事故的概率分别为0.05，0.15和0.3。如果“谨慎的”被保险人占20%，“一般的”占50%，“冒失的”占30%。问：

(1) 一个被保险人在一年内出事故的概率是多大？

(2) 若已知某被保险人出了事故，求他是“谨慎的”类型的概率。

An insurance company divides the insured into three categories: “cautious”, “average” and “rash”. Statistics show that the probabilities of accidents for these three types of people within a year are 0.05, 0.15 and 0.3 respectively. If “cautious” insured people account for 20%, “average” people account for 50%, and “rash” people 30%.

(1) What is the probability that an insured person will be involved in an accident within a year?

(2) If it is known that an insured person has been involved in an accident, find the probability that he belongs top the “cautious” category.

3. 设随机变量X 服从正态分布，$X \sim N(5, 25)$.

(1) 计算$P(3 < X < 7)$;

(2) 求出满足$P(X > x) = 0.39$ 的x;

(3) 求$Y = e^X$ 的取值范围和密度函数.

[标准正态分布表: $\Phi(0.26) = 0.6$; $\Phi(0.28) = 0.61$; $\Phi(0.30) = 0.62$; $\Phi(0.33) = 0.63$; $\Phi(0.36) = 0.64$; $\Phi(0.39) = 0.65$; $\Phi(0.4) = 0.66$; $\Phi(0.44) = 0.67$]

Suppose that the random variable X follows normal distribution, $X \sim N(5, 25)$.

(1) Compute $P(3 < X < 7)$;

(2) Find x such that $P(X > x) = 0.39$;

(3) Find the value range and the density function of $Y = e^X$.

[Note: Standard normal distribution table $\Phi(0.26) = 0.6$; $\Phi(0.28) = 0.61$; $\Phi(0.30) = 0.62$; $\Phi(0.33) = 0.63$; $\Phi(0.36) = 0.64$; $\Phi(0.39) = 0.65$; $\Phi(0.4) = 0.66$; $\Phi(0.44) = 0.67$]

4. 设二维随机变量(X, Y)的联合频率函数为

$X \setminus Y$	0	1	2
0	0.06	0.15	α
1	β	0.35	0.21

(1) 常数α和β需要满足什么条件?

(2) 若X和Y互相独立，求α和β的值;

(3) 若X和Y互相独立，求X和Y的边际频率函数.

Let the joint frequency function of the two-dimensional random variable (X, Y) be

$X \setminus Y$	0	1	2
0	0.06	0.15	α
1	β	0.35	0.21

(1) What conditions do you need to meet for the constant α and β?

(2) If X and Y are independent, please calculus the values of α and β;

(3) If X and Y are independent, please find the marginal frequency functions of X and Y, respectively.

5. 设两个随机变量X 和Y 的联合概率密度函数为

$$f(x,y)=\begin{cases}1, & |x|<y, 0<y<1\\ 0, & \text{其他}\end{cases}$$

(1) 求边际密度函数$f_X(x)$ 和$f_Y(y)$。
(2) X 和Y 是否独立。
(3) 求条件密度$f_{X|Y}(x|y)$以及$f_{Y|X}(y|x)$。

Let X and Y have the joint density function:

$$f(x,y)=\begin{cases}1, & |x|<y, 0<y<1\\ 0, & \text{others}\end{cases}$$

(1) Find the marginal densities $f_X(x)$ and $f_Y(y)$.
(2) Are X and Y independent?
(3) Find the conditional densities function $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$.

6. 设随机变量X和Y相互独立，$X\sim U(0,1)$，$Y\sim U(0,2)$。
(1) 求$P(X\leq Y)$。
(2) 求$X+Y$的分布函数。

Suppose that the random variables X and Y are independent, where $X\sim U(0,1)$, $Y\sim U(0,2)$.
(1) Find the probability of $P(X\leq Y)$.
(2) Find the distribution function of $X+Y$.