南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目： 概率论与数理统计 开课单位： 数学系

考试时长： 2022/11/12 19:00–21:00 命题教师： 概率统计教学组

题号	1	2	3	4	5	6	7	8
分值	20 分	20 分	10 分	10 分	10 分	10 分	10 分	10 分

本试卷共三大部分，满分（100）分

（考试结束后请将试卷、答题卡一起交给监考老师）

第一部分 选择题（每题4分，总共20分）

Part One – Single Choice (4 marks each question, 20 marks in total)

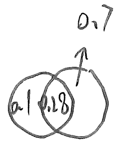
1. 若事件A与事件B互不相容，则以下哪项是正确的？

   Suppose events A and B are mutually exclusive, then which of the following is right? D

   A. $P(\overline{AB}) = 0$ B. $P(AB) = P(A)P(B)$

   C. $P(A) = 1 - P(B)$ D. $P(\overline{A} \cup \overline{B}) = 1$

$\bar{A}\bar{B} = 0.2$



2. 若$P(B) = 0.7$, $P(A \mid B) = 0.4$, $P(A \cup B) = 0.8$, 则以下哪项是错误的？ D,

   Given $P(B) = 0.7$, $P(A \mid B) = 0.4$ and $P(A \cup B) = 0.8$, which of the following is false?

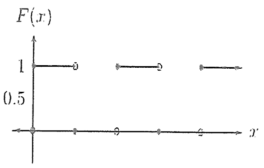
   A. $P(A \cap B) = 0.28$ ✓ B. $P(A) = 0.38$ ✓

   C. $P(AB) > 0$ ✓ D. $P(AB) = P(A)P(B)$ ✗

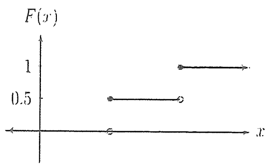
3. 请选出以下四个图像中哪个是离散型随机变量的分布函数.

   From the four given diagrams below, select the diagram that shows a valid c.d.f. for a discrete random variable.

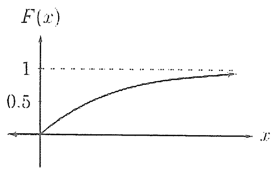
B.



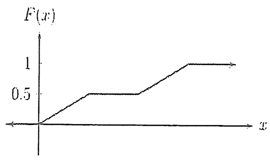
A.



B.



C.



D.

4. 设随机变量X在区间$(2,5)$上服从均匀分布，现对X进行三次独立观测，则至少有两次观测值小于4的概率是多少？ A

$\frac{4}{9}+\frac{8}{27}$

Suppose the r.v. $X \sim U(2,5)$. Select three independent observations from the distribution. What is the probability that at least two observations are less than 4?

A. $\frac{20}{27}$ B. $\frac{12}{27}$ C. $\frac{2}{5}$ D. $\frac{2}{3}$

$C_3^2\,\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}+C_3^3(\frac{2}{3})^3$

5. 设离散型随机变量(X,Y) 的联合频率函数为:

The joint distribution function of random variables X and Y is given by:

$$P(X=x, Y=y)=\frac{x+y}{30}, \qquad x=0,1,2,3;\ y=0,1,2.$$

1 2 3 3 4 5.
6 12

则概率 $P(X>Y)$的值是多少？ C.

What is the value of $P(X>Y)$?

A. $\frac{1}{2}$ B. $\frac{1}{5}$ C. $\frac{3}{5}$ D. $\frac{4}{15}$

$y\backslash x$	0	1	2	3
0		$\frac{1}{30}$	$\frac{2}{30}$	$\frac{3}{30}$
1			$\frac{3}{30}$	$\frac{4}{30}$
2				$\frac{5}{30}$

第二部分 填空题（每空2分，总共20分）

Part Two – Blank Filling (2 marks each blank, 20 marks in total)

1. 将3本书随机的放入4个抽屉，问恰好都放在同一个抽屉中的概率为 $\frac{1}{16}$.

Randomly put 3 books into 4 drawers. What is the probability that they are in the same drawer?

4

0 0 0

$\frac{C_4^1}{4^3}$

$\frac{8}{27}$ $\left(\frac{n}{n+3}\right)^3 = \frac{8}{27}$

$\frac{n}{n+3} = \frac{2}{3}$

$3n = 2n+6.$

2. 袋中装有大小相同的白球3只，黑球若干只，有放回的摸球3次，若至少摸到一只白球的概率为$\frac{19}{27}$，则袋中黑球的个数为___6___.

   There are 3 white balls and several black balls of the same size in a bag, and the balls are touched 3 times. If the probability of touching at least one white ball is $\frac{19}{27}$, what is the number of black balls in the bag?

3. 已知$P(A) = \frac{1}{4}$, $P(B|A) = \frac{1}{3}$, $P(A|B) = \frac{1}{2}$, 求$P(A \cup B) =$ ___$\frac{1}{4} + \frac{1}{6}$___ $- \frac{1}{12} = \frac{1}{3}$.

   Given $P(A) = \frac{1}{4}$, $P(B|A) = \frac{1}{3}$, $P(A|B) = \frac{1}{2}$. What is the value of $P(A \cup B)$?

4. 设随机变量X服从参数为λ的泊松分布，且$P\{X=4\} = P\{X=3\}$，则$\lambda =$ ___4___.

   Let the random variable X follow a Poisson distribution with the parameter λ, and let $P\{X=4\} = P\{X=3\}$, then what is the value of λ?

5. 设随机变量$X \sim U(0,b)$，且$P\{1 < X < 3\} = 1/3$，则$b =$ ___6 or $\frac{3}{2}$___

   Let the r.v. $X \sim U(0,b)$ and $P\{1 < X < 3\} = 1/3$. What is the value of b?

6. 设随机变量$X \sim N(0,1)$，则随机变量$Y = -3X+2$服从的分布为___N(2,9)___

   Let r.v. $X \sim N(0,1)$. What is the distribution of $Y = -3X+2$?

7. 设离散型随机变量X与Y互相独立，且$P\{X=k\} = P\{Y=k\} = \frac{k+1}{3}$，其中 $k = 0, 1$. 则$P\{X=Y\} =$ ___$\frac{5}{9}$___.

   Suppose the discrete random variables X and Y are independent, where $P\{X=k\} = P\{Y=k\} = \frac{k+1}{3}$ for $k = 0, 1$. What is the value of $P\{X=Y\}$?

   A=1

8. 设二维连续型随机变量(X,Y)的联合密度函数为$f(x,y) = \begin{cases} Ae^{-(x+y)}, & x > 0, y > 0; \\ 0, & 其它. \end{cases}$

   则$P\{X < Y\} =$ ___$\frac{1}{2}$___.

   Suppose the joint density function of the continuous random variables (X,Y) is $f(x,y) = \begin{cases} Ae^{-(x+y)}, & x > 0, y > 0; \\ 0, & \text{others.} \end{cases}$ What is the value of $P\{X < Y\}$?

   $\frac{1}{2}$

9. 设平面区域 D 由曲线 $y = 1/x$ 及直线 $y = 0, x = 1, x = e^2$ 所围成，二维连续型随机变量(X,Y) 在 D 上服从均匀分布，求条件密度 $f_{Y|X}(0.25|2) =$ ___2___

   Suppose the plane area D is surrounded by the curve $y = 1/x$ and the straight lines $y = 0, x = 1, x = e^2$. The two-dimensional random variables (X,Y) are in D and follow a uniform distribution. What is the conditional density $f_{Y|X}(0.25|2)$?

10. 设二维随机变量$(X,Y) \sim N(1,0;1,1;0)$, 则$P\{XY-Y<0\}=$ ____$\frac{1}{2}$.____.
Suppose two dimensional random variable $(X,Y) \sim N(1,0;1,1;0)$, then what is the value of $P\{XY-Y<0\}$?

第三部分 解答题（每题10分，总共60分）

Part Three–Question Answering (10 marks each question, 60 marks in total)

1. 一学生接连参加同一课程的两次考试. 第一次及格的概率为p, 若第一次及格则第二次及格的概率也为p; 若第一次不及格则第二次及格的概率为$\frac{p}{2}$.

(1) 若至少有一次及格则他能取得某种资格, 求他取得该资格的概率;

(2) 若已知他第二次已经及格, 求他第一次及格的概率.

A student takes two consecutive exams for the same course. The probability of passing the first exam is p. If the student passes for the first exam, the probability of passing the second exam is p too; if the student fails the first exam, the probability of passing for the second time is $\frac{p}{2}$.

(1) If the student passes the exam at least once, then he can obtain the qualification. Find the probability that he will obtain the qualification;

(2) If it is known that the student passed for the second exam, find the probability that he passed the first exam.

2. 有两个盒子, 第一盒中装有2个红球, 1个白球; 第二盒中装一半红球, 一半白球. 从两盒中各任取一球放在一起, 再从中取一球, 问:

(1) 这个球是红球的概率;

(2) 若发现这个球是红球, 问第一盒中取出的球是红球的概率.

There are two boxes, the first box contains 2 red balls and 1 white ball, and the second box contains half red balls and half white balls. Take a ball from each of the two boxes and put them together, then take a ball from it.

(1) Find the probability that the ball is a red ball;

(2) If the ball is found to be a red ball, find the probability that the ball drawn from the first box is red.

$\frac{1}{3}+\frac{1}{3}+\frac{2}{9}$ $\frac{8}{9}$

3. 一房间有3扇同样大小的窗子, 其中只有一扇是打开的. 有一只鸟自开着的窗子飞入了房间, 它只能从开着的窗子飞出去. 鸟在房子里飞来飞去, 试图飞出房间. 假定鸟是没有记忆的, 它飞向各扇窗子是随机的.

(1) 以X表示鸟为了飞出房间试飞的次数, 求X的频率函数;

(2) 户主声称，他养的一只鸟是有记忆的, 它飞向任一窗子的尝试不多于一次. 以Y表示这只聪明的鸟为了飞出房间试飞的次数. 如户主所说是确实的, 试求Y的频率函数;

(3) 求试飞次数X小于Y的概率.

A room has 3 windows of the same size, and only one of them is open. A bird flew into the room from the open window, it can only fly out from the open window. Birds are flying around the house trying to get out of the room. Assuming the bird has no memory, it flies to the windows randomly.

(1) Denote by X the number of times that the bird has tried in order to fly out of the room, find the frequency function of X;

(2) The owner claims that a bird he keeps has a memory and that it does not attempt to fly to any window more than once. Let Y represent the number of times that this clever bird has tried to fly out of the room. As the owner said, it is indeed, try to find the frequency function of Y;

(3) Find the probability that the number of test flights X is less than Y.

4. 设X的密度函数为$f(x)=\begin{cases} kx^2, & 0\le x<3, \\ 0, & \text{其他}, \end{cases}$. 并且令 $Y=\begin{cases} 2, & X\le 1, \\ X, & 1<X<2, \\ 1, & X\ge 2. \end{cases}$

(1) 求常数k;

(2) 求Y的分布函数;

(3) 求概率$P\{X\le Y\}$.

Let the density function of X be $f(x)=\begin{cases} kx^2, & 0\le x<3, \\ 0, & \text{others}, \end{cases}$. Let $Y=\begin{cases} 2, & X\le 1, \\ X, & 1<X<2, \\ 1, & X\ge 2. \end{cases}$

(1) Find the constant k;

(2) Find the distribution function of Y;

(3) Find the probability of $P\{X \le Y\}$.

5. 设离散型随机变量 X 和 Y 独立, 其中 X 从 $\{0,1,2\}$ 中取值, Y 从 $\{0,1\}$ 中取值, 且已知$P\{X=0\}=\frac{1}{6}$, $P\{X=0,Y=1\}=P\{X=1,Y=0\}=\frac{1}{8}$.

(1) 求X和Y的联合频率函数;

(2) 求$X+Y$的频率函数.

Suppose that the discrete random variables X and Y are independent, where X takes values from $\{0,1,2\}$, and Y takes values from $\{0,1\}$. Moreover, $P\{X=0\}=\frac{1}{6}$, $P\{X=0,Y=1\}=P\{X=1,Y=0\}=\frac{1}{8}$.

(1) Find the joint frequency function of X and Y;

(2) Find the frequency function of $X+Y$.

6. 设(X,Y)的联合密度函数为$f(x,y)=\begin{cases} Ae^{-(x+2y)}, & 0<y<x, \\ 0, & \text{其他}. \end{cases}$

(1) 求常数A;

(2) 求条件密度函数$f_{Y|X}(y|x)$;

(2) 求条件分布函数$F_{X|Y}(x|y)$;

(4) X和Y是否独立? 说明原因.

Let the joint density function of (X,Y) be $f(x,y)=\begin{cases} Ae^{-(x+2y)}, & 0<y<x, \\ 0, & \text{others}. \end{cases}$

(1) Find the constant A;

(2) Find the conditional density function $f_{Y|X}(y|x)$;

(3) Find the conditional distribution function $F_{X|Y}(x|y)$;

(4) Are X and Y independent? Explain the reason.