南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目：　概率论与数理统计　　　开课单位：　数学系

考试时长：　2 小时　　　命题教师：　概率统计课程组

题号	Part 1	Part 2	Part 3					
			1	2	3	4	5	6
分值								

本试卷共三大部分，满分 100 分（考试结束后请将试卷、答题本、草稿纸一起交给监考老师）

第一部分　选择题（每题 4 分，总共 20 分）

Part One – Single Choice (4 marks each question, 20 marks in total)

1. 设随机变量 X 的概率密度为 $f(x)$，且 $f(-x)=f(x)$, $F(x)$ 是 X 的分布函数，则对任意实数 $a>0$ 有(　　)

(A) $F(-a)=1-\int_0^a f(x)dx$；

(B) $F(-a)=\frac{1}{2}-\int_0^a f(x)dx$；

(C) $F(-a)=F(a)$；

(D) $F(-a)=2F(a)-1$.

Assume the probability density function of random variable X is $f(x)$, and $f(-x)=f(x)$, $F(x)$ is the distribution of the X. Then, given any real number $a>0$, it has (　　)

(A) $F(-a)=1-\int_0^a f(x)dx$；

(B) $F(-a)=\frac{1}{2}-\int_0^a f(x)dx$；

(C) $F(-a)=F(a)$；

(D) $F(-a)=2F(a)-1$.

2. 随机变量 X 的可能值为 $x_1=-1$, $x_2=0$, $x_3=1$，且 $E(X)=0.1$, $D(X)=0.89$，则对应于 x_1,x_2,x_3 的概率 p_1,p_2,p_3 为（　　）

(A) $p_1=0.4$, $p_2=0.1$, $p_3=0.5$；

(B) $p_1=0.1$, $p_2=0.1$, $p_3=0.5$；

(C) $p_1=0.5$, $p_2=0.1$, $p_3=0.4$；

(D) $p_1=0.4$, $p_2=0.5$, $p_3=0.5$.

All the possible values of random variable X are $x_1=-1$, $x_2=0$, $x_3=1$. Furthermore, $E(X)=0.1$, $D(X)=0.89$. What are the corresponding probabilities p_1,p_2,p_3 of the x_1,x_2,x_3 (　　)

(A) $p_1=0.4$, $p_2=0.1$, $p_3=0.5$；

(B) $p_1=0.1$, $p_2=0.1$, $p_3=0.5$；

(C) $p_1=0.5$, $p_2=0.1$, $p_3=0.4$；

(D) $p_1=0.4$, $p_2=0.5$, $p_3=0.5$.

3. 设$X_1,X_2,\cdots$ 为独立随机变量序列，且 X_i 服从参数为λ 的泊松分布，$i=1,2,\cdots$，则(　　)

(A) $\lim\limits_{n\to\infty} P\left\{\dfrac{\sum\limits_{i=1}^{n} X_i - n\lambda}{n\lambda} \le x\right\} = \Phi(x)$；

(B)当 n 充分大时，$\sum\limits_{i=1}^{n} X_i$ 近似服从标准正态分布；

(C)当 n 充分大时，$\sum\limits_{i=1}^{n} X_i$ 近似服从 $N(n\lambda,\ n\lambda)$；

(D)当 n 充分大时，$P(\sum\limits_{i=1}^{n} X_i \le x) \approx \Phi(x)$.

Assume $X_1,X_2,\cdots$ is a sequence of independent random variable. Furthermore, X_i follows Poisson distribution with the parameter of λ，$i=1,2,\cdots$, then (　　)

(A) $\lim\limits_{n\to\infty} P\left\{\dfrac{\sum\limits_{i=1}^{n} X_i - n\lambda}{n\lambda} \le x\right\} = \Phi(x)$；

(B) when n is big enough, $\sum\limits_{i=1}^{n} X_i$ approximately follows standard normal distribution；

(C) when n is big enough，$\sum\limits_{i=1}^{n} X_i$ approximately follows $N(n\lambda,\ n\lambda)$；

(D) when n is big enough，$P(\sum\limits_{i=1}^{n} X_i \le x) \approx \Phi(x)$.

4. $X_1,X_2,\cdots,X_n$是正态分布 $N(0,\sigma^2)$ 的一个样本，若统计量 $K\sum\limits_{i=1}^{n-1}(X_{i+1}-X_i)^2$ 为σ^2 的无偏估计，则 K 的值应该为（　　）

(A) $\dfrac{1}{2n}$;　　(B) $\dfrac{1}{2n-1}$;　　(C) $\dfrac{1}{2n-2}$;　　(D) $\dfrac{1}{n-1}$.

$X_1,X_2,\cdots,X_n$ are the samples from the population $N(0,\sigma^2)$. If the statistics of

$K\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2$ is a unbiased estimation of σ^2, the value of K is (　　)

(A) $\frac{1}{2n}$;　　(B) $\frac{1}{2n-1}$;　　(C) $\frac{1}{2n-2}$;　　(D) $\frac{1}{n-1}$.

5. $X_1, X_2, \cdots, X_n$来自正态总体 $N(0,\sigma^2)$ 的样本,现检验假设$H_0: \sigma^2=1$; $H_1: \sigma^2 \neq 1$, 则选用统计量（　）

(A)$\frac{\bar{X}}{S}\sqrt{n}$;　　(B)$(n-1)S^2$;　　(C) $\sum_{i=1}^{n} {X_i}^2$;　　(D)$\sqrt{n}\bar{X}$.

$X_1, X_2, \cdots, X_n$ are the samples from the population $N(0,\sigma^2)$. If $H_0: \sigma^2=1$; $H_1: \sigma^2 \neq 1$, which of the followings should be selected as the statistic (　　)

(A)$\frac{\bar{X}}{S}\sqrt{n}$;　　(B)$(n-1)S^2$;　　(C) $\sum_{i=1}^{n} {X_i}^2$;　　(D)$\sqrt{n}\bar{X}$.

第二部分 填空题（每空 2 分，总共 20 分）

Part Two – Blank Filling (2 marks each blank, 20 marks in total)

1. 已知$P(A)=\frac{1}{4}$, $P(BC|A)=\frac{1}{8}$, 则事件A,B,C至少有一个不发生的概率是__________.

   Given $P(A)=\frac{1}{4}$ and $P(BC|A)=\frac{1}{8}$, what is the probability that at least one of the events A,B,C doesn't happen __________.

2. 为从 2 个次品、8 个正品的 10 个产品中将 2 个次品挑出，随机地从中逐个测试，则不超过 4 次测试就把 2 个挑出的概率为____________.

   There are 10 products containing 2 defective products. Randomly pick a product each time and check if it is defective or not. What is the probability of getting the two defective products by having less than 4 times picking ________.

3. 若$f(x)=\begin{cases} Ax^2, 1\le x\le 2 \\ Ax,\ 2<x<3 \\ 0,\quad \text{其它} \end{cases}$ 为某随机变量的密度函数，则常数$A=$__________.

   Assume $f(x)=\begin{cases} Ax^2, 1\le x\le 2 \\ Ax,\ 2<x<3 \\ 0,\quad others \end{cases}$ is the density function of a random variable. Then the value of the constant $A=$__________.

4. 设随机变量X和Y的相关系数为 0.9, 若$Z=X-0.4$, 则Y和Z的相关系数为_______.

   The correlation coefficient of random variables X and Y is 0.9. Given $Z=X-0.4$, what is the correlation coefficient of random variables Y and Z _______.

5. 设X服从参数为$\lambda>0$的泊松分布，且已知$E[(X-1)(X-2)]=1$, 则$\lambda=$_______.

   Assume X follows Poisson distribution with the parameter of $\lambda>0$. Given $E[(X-1)(X-2)]=1$, then $\lambda=$_______.

6. 设随机变量X在区间$[-1,2]$上服从均匀分布，随机变量$Y=\begin{cases}1, & 若X>0\\ 0, & 若X=0\\ -1, & 若X<0\end{cases}$，则$Y$的方差$D(Y)=$_______.

Random variable X follows $U[-1,2]$. Random variable $Y=\begin{cases}1, & when\ \ X>0\\ 0, & \text{when}\ \ X=0\\ -1, & when\ \ X<0\end{cases}$, what is the variance $D(Y)=$_______.

7. 假设$X,X_1,X_2,\cdots,X_{10}$是来自正态总体$N(0,\sigma^2)$的样本，$Y^2=\frac{1}{10}\sum_{i=1}^{10}X_i^2$，则$X/Y$服从的分布及参数为_______.

Suppose $X,X_1,X_2,\cdots,X_{10}$ are samples from population $N(0,\sigma^2)$, $Y^2=\frac{1}{10}\sum_{i=1}^{10}X_i^2$, what distribution does X/Y follow _______.

8. 设随机变量X的数学期望$E(X)=\mu$，方差$D(X)=\sigma^2$，则由切比雪夫不等式，有$P(|X-\mu|\geq 3\sigma)\leq$_______.

The expectation of random variable X is $E(X)=\mu$, variance $D(X)=\sigma^2$. In terms of Chebyshev Inequality, then $P(|X-\mu|\geq 3\sigma)\leq$_______.

9. 设随机变量$X_1,X_2,\cdots,X_n,\cdots$相互独立，且$X_i$的密度函数为$f(x)=\begin{cases}\frac{1}{2}e^{-\frac{x}{2}}, & x\geq 0\\ 0, & x<0\end{cases}$，则当$n$充分大时，随机变量$Z_n=\frac{1}{n}\sum_{i=1}^{n}X_i$的概率分布近似服从__________.（请写出分布类型及其参数）

Suppose random variables $X_1,X_2,\cdots,X_n,\cdots$ are independent, Their density functions are $f(x)=\begin{cases}\frac{1}{2}e^{-\frac{x}{2}}, & x\geq 0\\ 0, & x<0\end{cases}$. Thus, when n is big enough, what distribution does the random variable $Z_n=\frac{1}{n}\sum_{i=1}^{n}X_i$ approximately follow _______. (Identify the distribution and its parameters)

10. 设$X_1,X_2,\cdots,X_m$为来自二项分布总体$b(n,p)$的样本，$\bar{X}$和S^2分别为样本均值和样本方差. 记统计量$T=\bar{X}-S^2$，则$E(T)=$_______.

Suppose $X_1, X_2, \cdots, X_m$ are samples from population $b(n,p)$, $\bar{X}$ and S^2 are the sample mean and variance respectively. If a statistic $T = \bar{X} - S^2$, then $E(T) =$_______.

第三部分 大题（每题 10 分，总共 60 分）

Part Three – Question Answering (10 marks each question, 60 marks in total)

1. 将 3 个一样的球随机地放入 4 个杯子中去，求杯中球的最大个数分别为 1，2，3 的概率.

   Throw 3 same balls randomly into 4 cups. What is the probability when the maximum number in any cup is 1, 2 or 3?

2. 离散型随机变量 X 的频率函数为：

X	a	1	4
p	a	$\frac{1}{4}$	b

且 已 知 $E(X)=\frac{3}{2}$.

   a) 求常数 a 和 b 的值；

   b) 设 $Y=2X+1$，求 Y 的方差 $D(Y)$.

The frequent function of discrete random variable X is as follow,

X	a	1	4
p	a	$\frac{1}{4}$	b

and $E(X)=\frac{3}{2}$.

   a) What is the value of the constants a and b?

   b) If $Y=2X+1$，what is $D(Y)$?

3. 已知随机变量 X 和 Y 分别服从正态分布 $N(1,9)$ 和 $N(0,16)$，且 X 和 Y 形成二维正态分布，它的相关系数 ρ_{XY} = -1/2。设 $Z=\frac{X}{3}+\frac{Y}{2}$.

   a) 求 Z 的数学期望 $E(Z)$ 和方差 $D(Z)$；

b) 求 X 与 Z 的相关系数 ρ_{XZ}；

c) 问 X 与 Z 是否相关？它们也相互独立吗？为什么？

Random variables X and Y follow its normal distribution $N(1,9)$ and $N(0,16)$ respectively. Furthermore, X and Y form two dimensional normal and its correlation coefficient of is $\rho_{XY}=$-1/2，and $Z=\dfrac{X}{3}+\dfrac{Y}{2}$,

a) What is the expectation $E(Z)$? What is the variance $D(Z)$?

b) What is the correlation coefficient ρ_{XZ} ?

c) Are the random variables X and Z related? Are they independent? Why?

4. 设 $X_1, X_2, \ldots, X_n$ 为取自总体X的样本，$x_1, x_2, \ldots, x_n$为样本观察值，总体的概率密度函数为

$$f(x)=\begin{cases}\dfrac{\theta}{x^{\theta+1}}, & x>1, \theta>1\\ 0, & x\leq 1\end{cases}$$

求参数θ的矩估计量和最大似然估计量.

Suppose $X_1, X_2, \ldots, X_n$ are samples from a population X ，$x_1, x_2, \ldots, x_n$ are the observed values. The probability density function of the population is as follow.

$$f(x)=\begin{cases}\dfrac{\theta}{x^{\theta+1}}, & x>1, \theta>1\\ 0, & x\leq 1\end{cases}$$

What are Moment Estimation and Maximum Likelihood Estimation of the parameter θ?

5. 车间生产滚珠的直径服从正态分布. 从车间生产的产品中随机取出 9 个，得样本均值为 $\bar{X}$ =15.3，样本方差为s^2 =0.36.

a) 求总体方差 σ^2 的 0.95 双侧置信区间;

b) 在显著性水平为 0.05 下判断是否可以认为该车间生产滚珠直径均值 μ 是 15.

(分位点见试卷最后附表）

A workshop produces rolling balls. The diameter of the rolling balls follows normal distribution. Randomly picked 9 balls from the products, got the measurement of sample mean $\bar{X}$ =15.3 and the variance s^2 =0.36.

a) What is the two-sided Confidence Interval of the population variance σ^2 with the significance level 0.95?

b) with the significance level 0.05, should we decide that the diameter μ of the rolling balls produced by the workshop is 15?

(significance level $\alpha = 0.05$，corresponding quantiles can be found in the following tables)

6. 用两种方法（A 和 B）测定冰自-72 度转变为 0 度的水的融化热（以 cal/g 计）。测得的样本均值和样本方差如下：

A 方法：$n_1 = 10$, 均值$\overline{x}_1 = \frac{1}{10}\sum_{i=1}^{10} x_i = 60$，样本方差$s_1^2 = \frac{1}{9}\sum_{i=1}^{10}(x_i - \overline{x}_1)^2 = 3$；

B 方法：$n_2 = 15$, 均值$\overline{y}_2 = \frac{1}{15}\sum_{i=1}^{15} y_i = 70$，样本方差$s_2^2 = \frac{1}{14}\sum_{i=1}^{15}(y_i - \overline{y}_2)^2 = 2$；

假设 A 方法的数据服从$N(\mu_1, \sigma_1^2)$，B 方法的数据服从$N(\mu_2, \sigma_2^2)$，并这两个样本相互独立，$\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$均未知。问：是否可以认为 A,B 两种方法的总体方差有显著差异？请给出该假设检验问题的检验方法与计算过程。（显著性水平$\alpha = 0.05$，分位点见试卷最后附表）。

There are two measurement methods: A method and B method, for measuring the fusion heat (cal/g) when ice at -72°C transforms into water at 0°C. The sample mean and sample variance obtained by the two methods are as follows:

A method：$n_1 = 10$, $\overline{x}_1 = \frac{1}{10}\sum_{i=1}^{10} x_i = 60$， $s_1^2 = \frac{1}{9}\sum_{i=1}^{10}(x_i - \overline{x}_1)^2 = 3$；

B method：$n_2 = 15$, $\overline{y}_2 = \frac{1}{15}\sum_{i=1}^{15} y_i = 70$， $s_2^2 = \frac{1}{14}\sum_{i=1}^{15}(y_i - \overline{y}_2)^2 = 2$；

Suppose that samples obtained by A method follow $N(\mu_1, \sigma_1^2)$, and by B method follow $N(\mu_2, \sigma_2^2)$. The two sets of samples are independent, $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ are all unknown. Should we。Do the two populations variance σ^2 have difference? State the hypotheses and give the detail of the testing process (significance level $\alpha = 0.05$，corresponding quantiles can be found in the following tables)

附表：

表 1: F分布表$P(F(n_1,n_2)\geq x)=0.975$

(n_1,n_2) 取值	$(n_1,n_2)=(9,14)$	$(n_1,n_2)=(9,15)$	$(n_1,n_2)=(10,14)$	$(n_1,n_2)=(10,15)$	
x取值	0.2633	0.2653	0.2817	0.2840	

表 2: F分布表$P(F(n_1,n_2)\geq x)=0.025$

(n_1,n_2) 取值	$(n_1,n_2)=(9,14)$	$(n_1,n_2)=(9,15)$	$(n_1,n_2)=(10,14)$	$(n_1,n_2)=(10,15)$	
x取值	3.2093	3.1227	3.1469	3.0602	

表 3：t分布表 $P(t(n)\geq x)=0.975$

自由度n取值	8	9	10	14	15	16
x取值	2.3060	2.2621	2.2281	2.1447	2.1314	2.1199

表 4: χ^2 分布表：$P(\chi^2(m)\geq x)=0.025$

自由度 m 取值	6	7	8	9	10
x 取值	14.4493	16.0127	17.5345	19.0227	20.4831

表 5: χ^2 分布表：$P(\chi^2(m)\geq x)=0.975$

自由度 m 取值	6	7	8	9	10
x 取值	1.2373	1.6898	2.1797	2.7003	3.2469