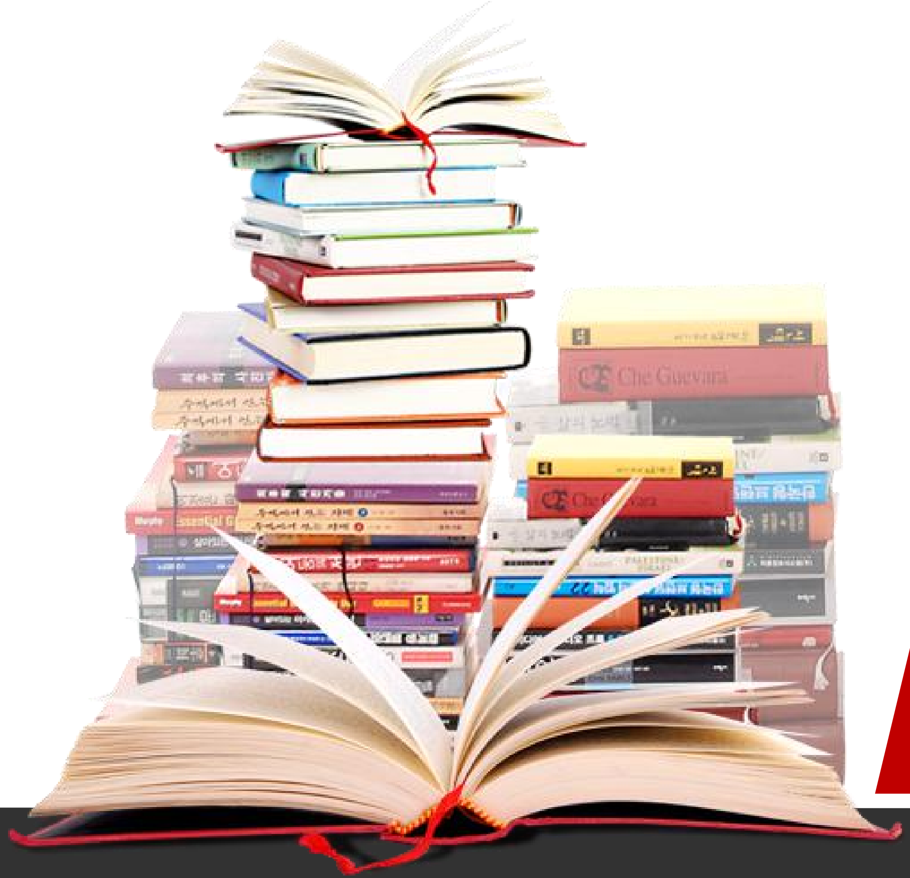


极限环、分岔理论、混沌及庞佳莱映射

制作人：刘清华

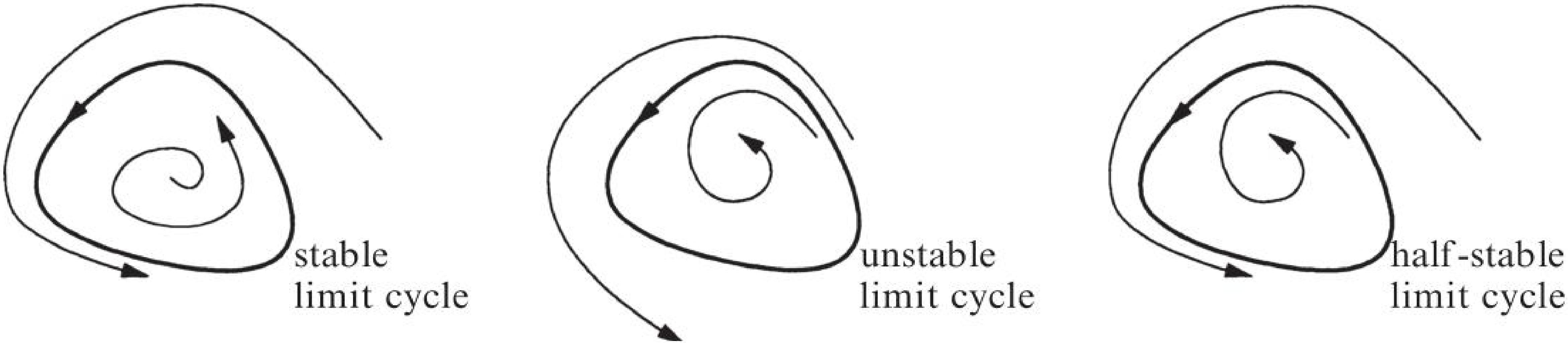
微信公众号：非线性动力学

目录 Content

1. 极限环（Limit cycles）

极限环的定义

A ***limit cycle*** is an isolated closed trajectory. *Isolated* means that neighboring trajectories are not closed; they spiral either toward or away from the limit cycle (Figure 7.0.1).

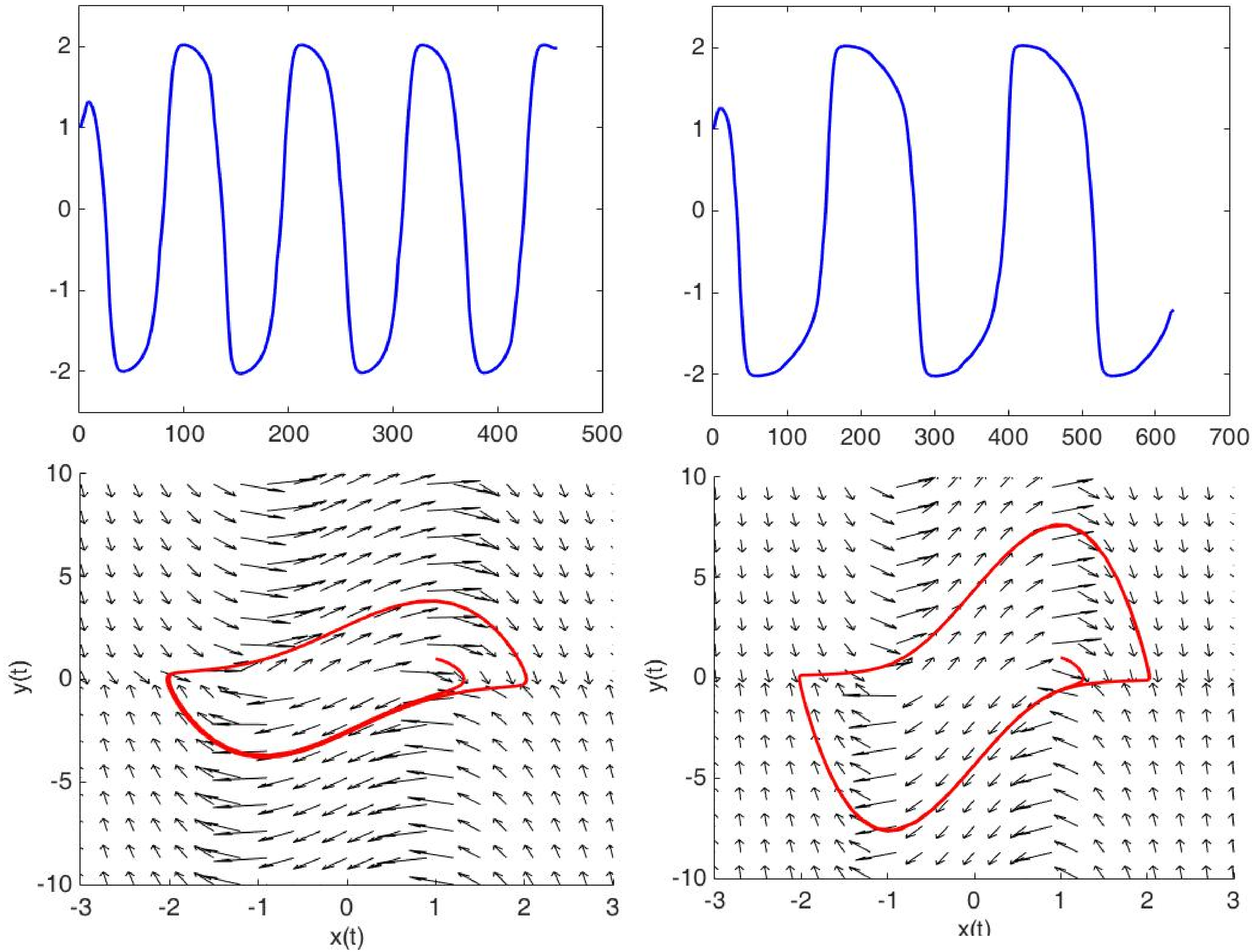


稳定极限环具有很重要的科学意义—它们模拟了具有自发维持的振荡系统。这些系统，在缺少外部周期强制力的情况下也会振荡。譬如：心脏的跳动；飞机机翼上危险的自激振荡；人体激素分泌的日常规律等。下面给一个很多书上实例：范德波尔方程。

真空三极管的自激振荡-范德波尔方程（van der Pol）

$$\ddot{x} + \boxed{\epsilon}\left(x^2 - 1\right)\dot{x} + x = 0,$$

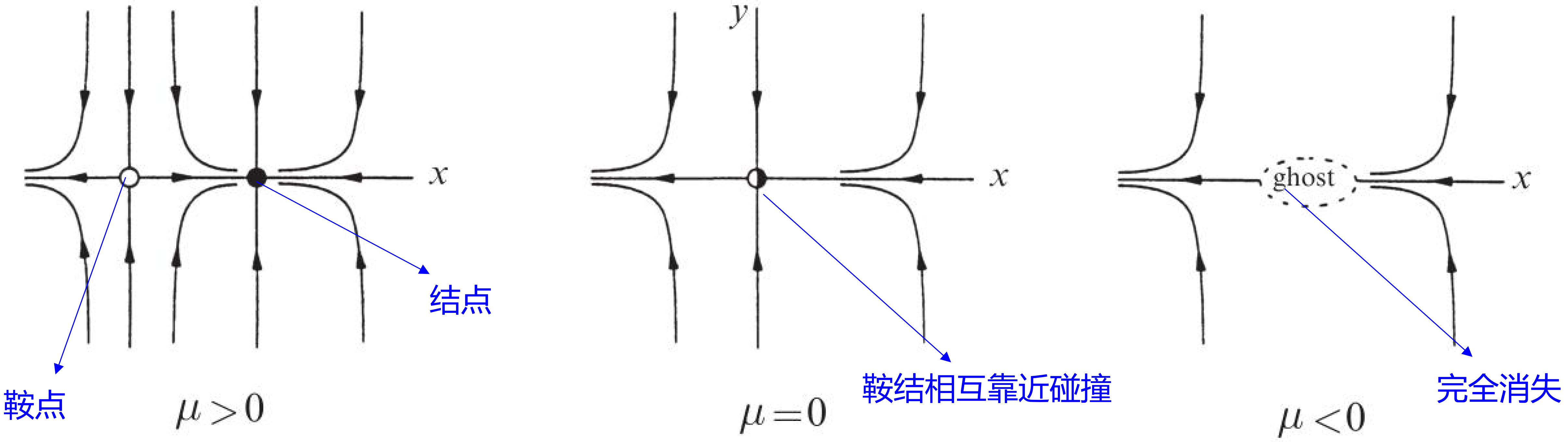
大于零；意味着当x在[-1,1]区间，振荡阻尼为负值，阻尼为负，神奇吧。

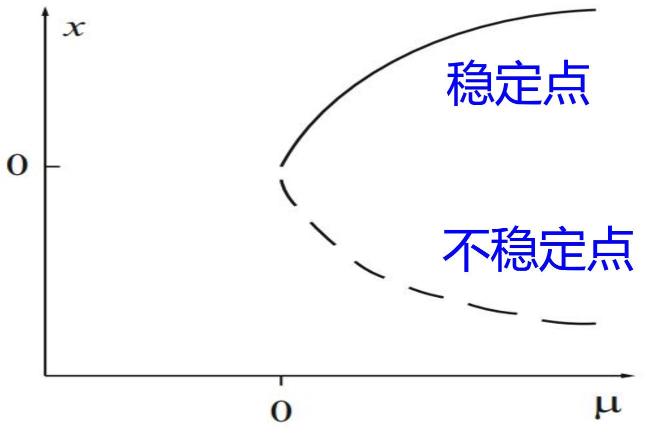



```
clear
hold on
sys = @(t,x) [x(2);-x(1)-5*x(2)*((x(1))^2-1)];
%降阶微分方程
vectorfield(sys,-3:.3:3,-10:1.3:10);
%画周围向量场的
[t,xs] = ode45(sys,[0 30],[1 1]);
%调用ode45解sys;
%plot(xs(:,1),'b','Linewidth',2);%画出时域图
plot(xs(:,1),xs(:,2),'r','Linewidth',2);%画出
hold off
axis([-3 3 -10 10])
fsize=15;
set(gca,'XTick',-3:1:3,'FontSize',fsize)
set(gca,'YTick',-10:5:10,'FontSize',fsize)
xlabel('x(t)','FontSize',fsize)
ylabel('y(t)','FontSize',fsize)
hold off
```

2. 分岔(Bifurcation)

鞍-结分岔（A Saddle-Node Bifurcation） $\dot{x} = \mu - x^2, \quad \dot{y} = -y.$





一个实例

In summary, there are no critical points if μ is negative; there is one nonhyperbolic critical point at the origin if $\mu = 0$; and there are two critical points—one a saddle and the other a node—when μ is positive. The qualitative behavior of the system changes as the parameter μ passes through the bifurcation value $\mu_0 = 0$. The behavior of the critical points can be summarized on a *bifurcation diagram* as depicted in Fig. 13.2.

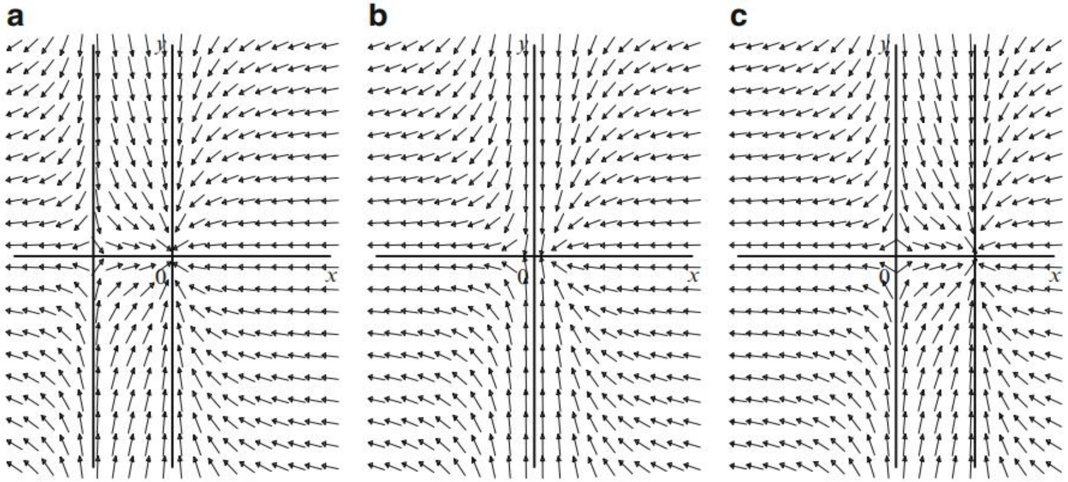
超临界叉式、亚临界叉式和跨临界分岔

$$\dot{x} = \mu x - x^2, \qquad \dot{y} = -y \qquad \text{(transcritical)}$$

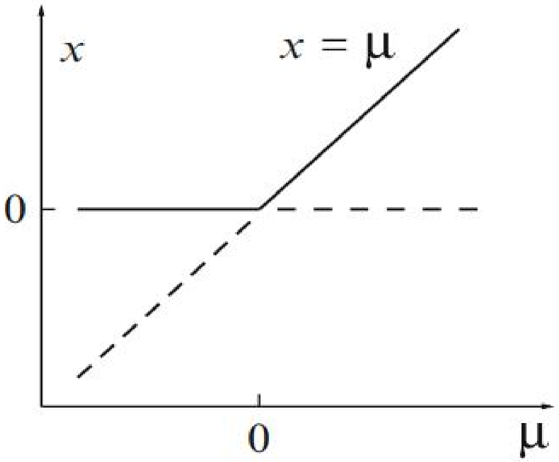
$$\dot{x} = \mu x - x^3, \qquad \dot{y} = -y \qquad \text{(supercritical pitchfork)}$$

$$\dot{x} = \mu x + x^3, \qquad \dot{y} = -y \qquad \text{(subcritical pitchfork)}$$

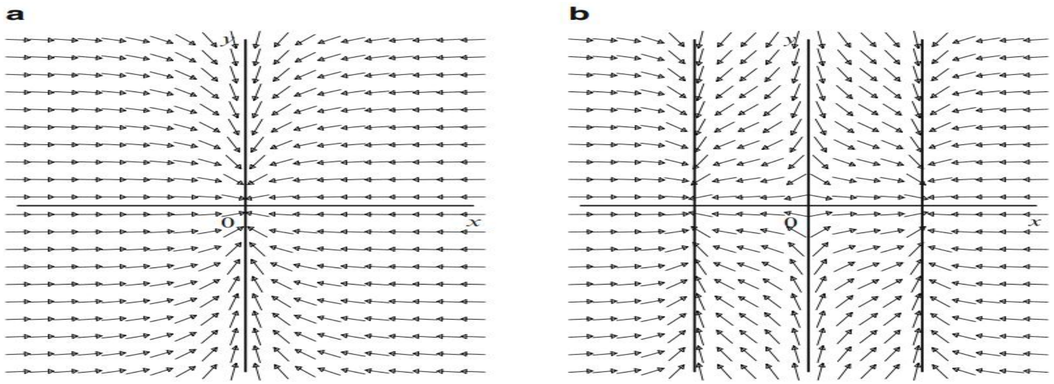
目前有点歧义；不同书里有点不同，暂定。



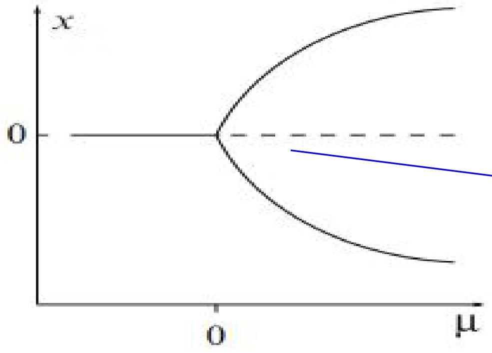
跨临界分岔



跨临界分岔一个实例



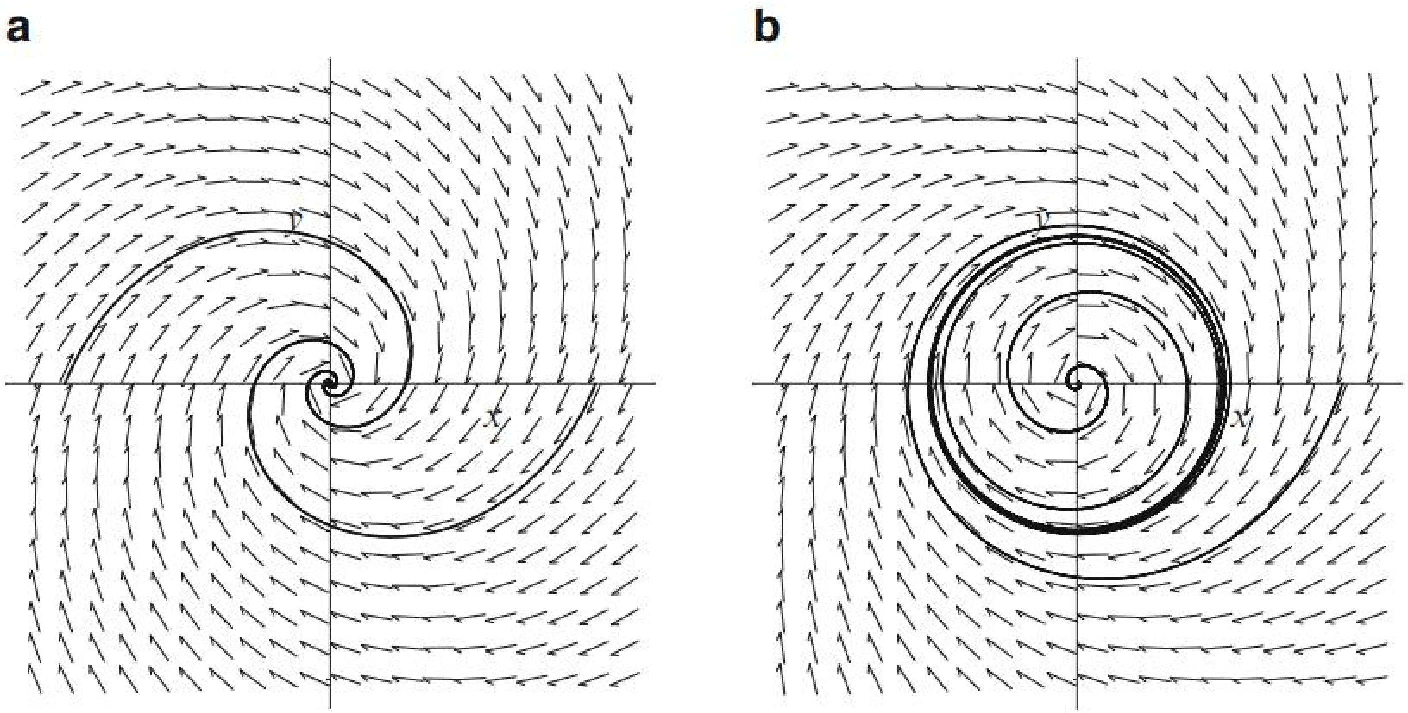
超临界叉式分岔



叉式分岔一个实例

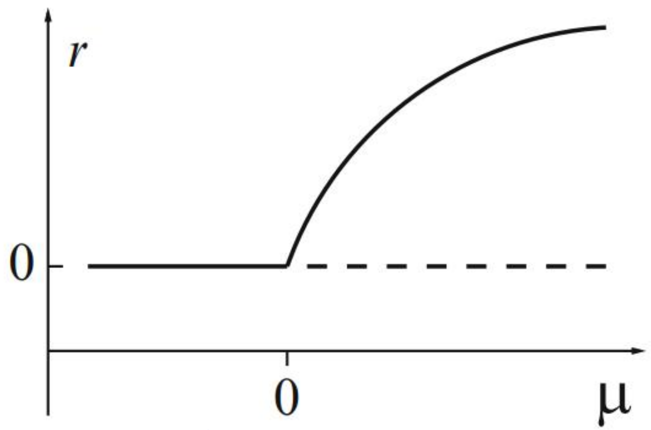
梁的屈曲；双稳系统，除了这个我不熟

霍普夫分岔（Hopf） $\dot{r} = r(\mu - r^2), \quad \dot{\theta} = -1.$

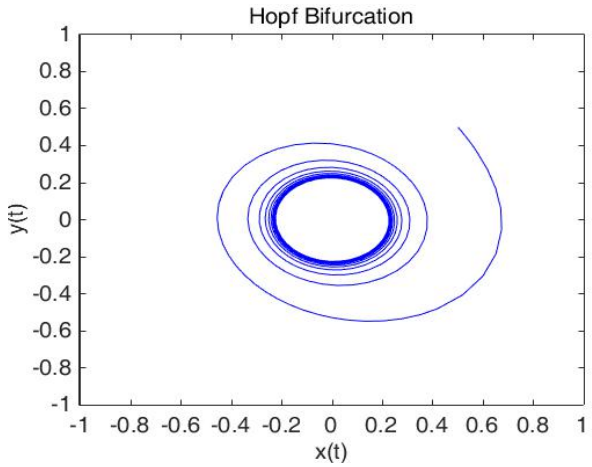



```
function sys=HopfBifurcation(~,x)
global mu
X=x(1,:);
Y=x(2,:);
% Define the system.
P=Y+mu*X-X.*Y.^2;
Q=mu*Y-X-Y.^3;
sys=[P;Q];
```

```
clear
global mu
for j = 1:48
F(j) = getframe;
mu=j/40-1; % mu goes from -1 to 0.2.
options = odeset('RelTol',1e-4,'AbsTol',1e-4);
x0=0.5;y0=0.5;
[t,x]=ode45(@HopfBifurcation,[0 80],[x0 y0],options);
plot(x(:,1),x(:,2),'b');
axis([-1 1 -1 1])
fsize=15;
set(gca,'XTick',-1:0.2:1,'FontSize',fsize)
set(gca,'YTick',-1:0.2:1,'FontSize',fsize)
xlabel('x(t)','FontSize',fsize)
ylabel('y(t)','FontSize',fsize)
title('Hopf Bifurcation','FontSize',15);
F(j) = getframe;
end
movie(F,20)
```



霍普夫分岔一个实例



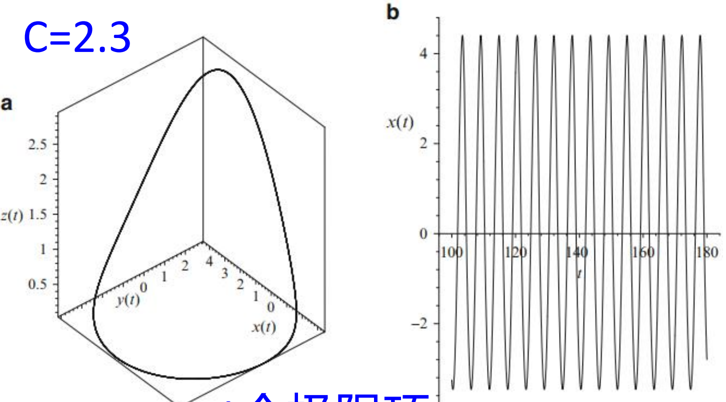
Gif动图

3. 混沌(Chaos)

The Rössler System and Chaos

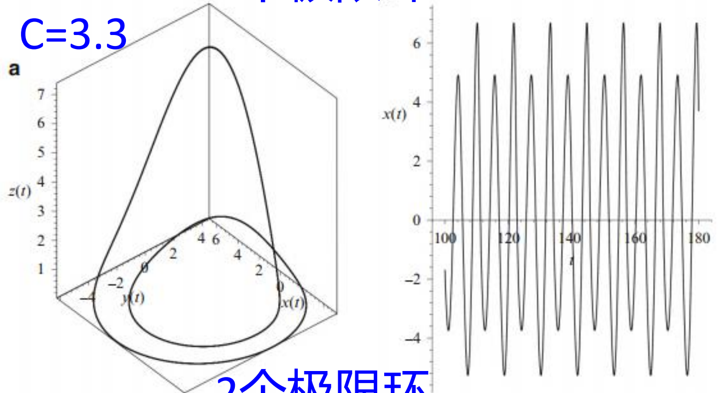
$$\dot{x} = -(y+z), \quad \dot{y} = x + ay, \quad \dot{z} = b + xz - cz,$$

C=2.3



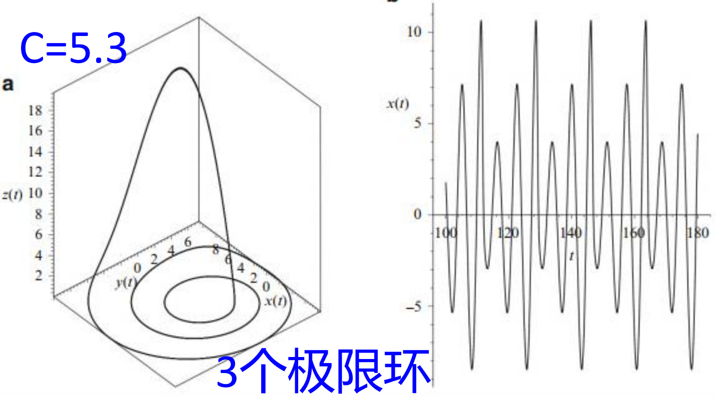
1个极限环

C=3.3



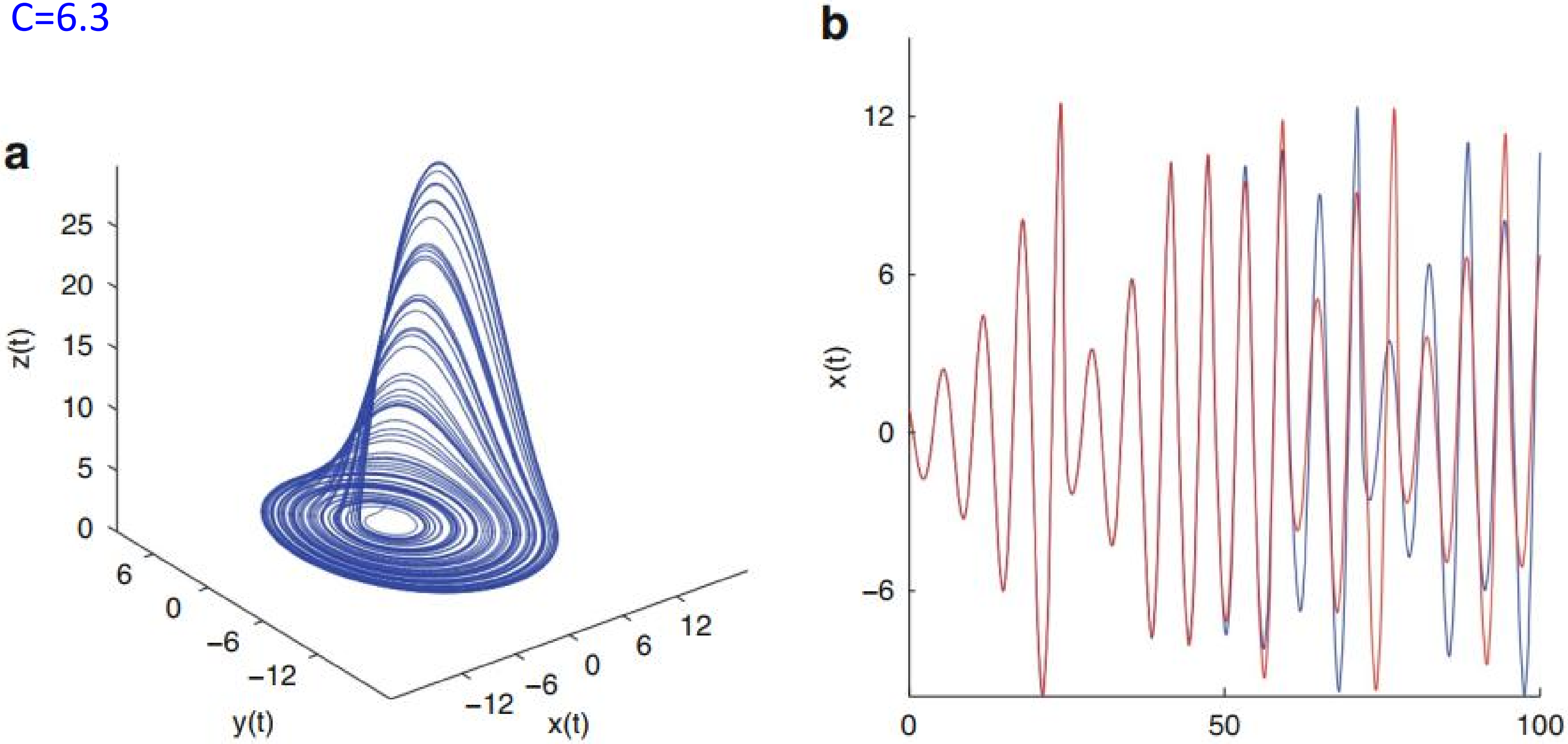
2个极限环

C=5.3



3个极限环

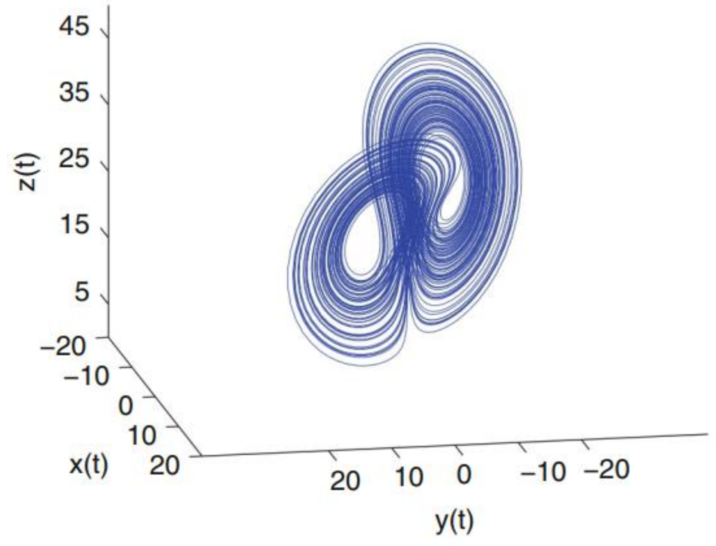
C=6.3



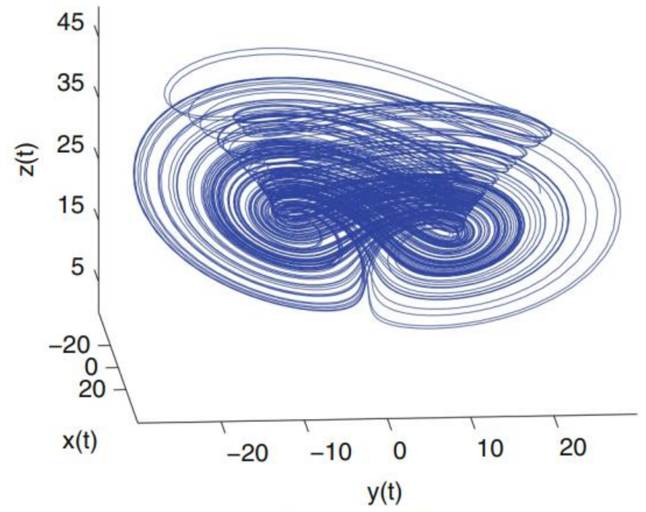
确定性混沌（可以去Matcont官网下载工具箱画图，是一个实例）

There is no universally accepted definition for chaos, but the following characteristics are nearly always displayed by the solutions of chaotic systems:

1. Long-term aperiodic (nonperiodic) behavior
2. Sensitivity to initial conditions
3. Fractal structure.



$$\dot{x} = \sigma(y - x), \quad \dot{y} = rx - y - xz, \quad \dot{z} = xy - bz,$$

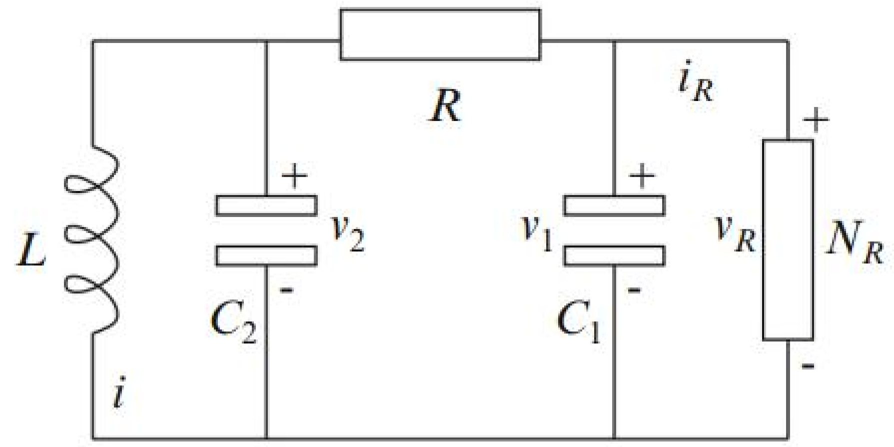


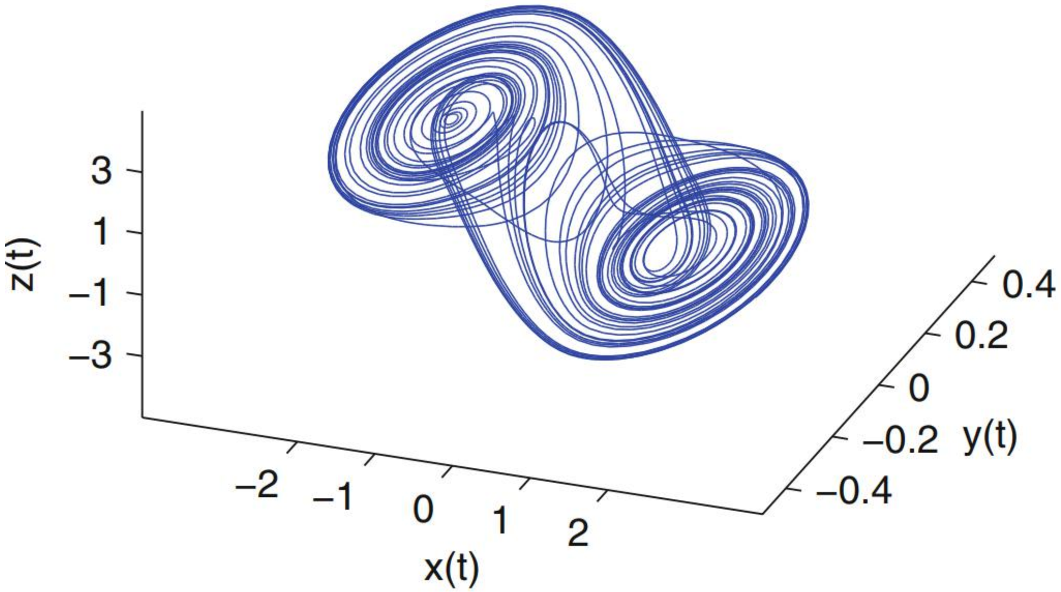
$$\dot{x} = \sigma(y - x), \quad \dot{y} = (r - \sigma)x + ry - xz, \quad \dot{z} = xy - bz.$$

洛伦兹系统及其变异模型：大气现象

```
sigma=10;r=28;b=8/3;
Lorenz=@(t,x) [sigma*(x(2)-x(1));...
 r*x(1)-x(2)-x(1)*x(3);x(1)*x(2)-b*x(3)];
options = odeset('RelTol',1e-4,'AbsTol',1e-4);
[t,xa]=ode45(Lorenz,[0 100],[15,20,30],options);
plot3(xa(:,1),xa(:,2),xa(:,3))
title('The Lorenz Attractor')
fsize=15;
xlabel('x(t)','Fontsize',fsize);
ylabel('y(t)','Fontsize',fsize);
zlabel('z(t)','FontSize',fsize);
```

Chua's Circuit





$$\frac{dv_1}{dt} = \frac{(G(v_2 - v_1) - f(v_1))}{C_1}, \frac{dv_2}{dt} = \frac{(G(v_1 - v_2) + i)}{C_2}, \frac{di}{dt} = -\frac{v_2}{L},$$

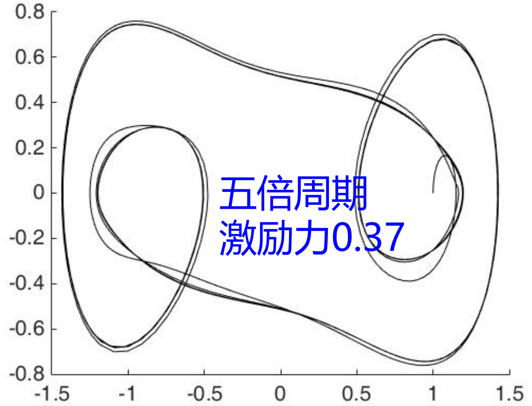
```
Chua=@(t,x) [15*(x(2)-x(1)-(-(5/7)*x(1)+(1/2) ...
*(-(8/7)-(-5/7))*(abs(x(1)+1)-abs(x(1)-1)))); ...
x(1)-x(2)+x(3);-25.58*x(2)];
options = odeset('RelTol',1e-4,'AbsTol',1e-4);
[t,xb]=ode45(Chua,[0 100],[-1.6,0,1.6],options);
plot3(xb(:,1),xb(:,2),xb(:,3))
title('Chua`s Double Scroll Attractor')
fsize=15;
xlabel('x(t)','Fontsize',fsize);
ylabel('y(t)','Fontsize',fsize);
zlabel('z(t)','FontSize',fsize);
```

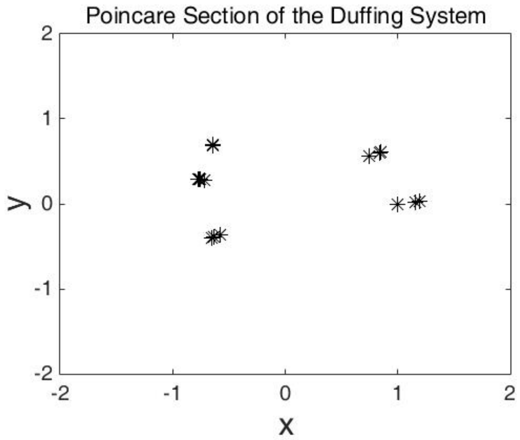
香港城市大学：陈关荣教授团队，对洛伦兹系统、蔡式电路系统研究比较系统，最近听到他基于此电路研究多涡卷，不明觉厉，从他回答学着提到的问题，可以知道他很厉害，可能数学厉害的人才能听得懂。

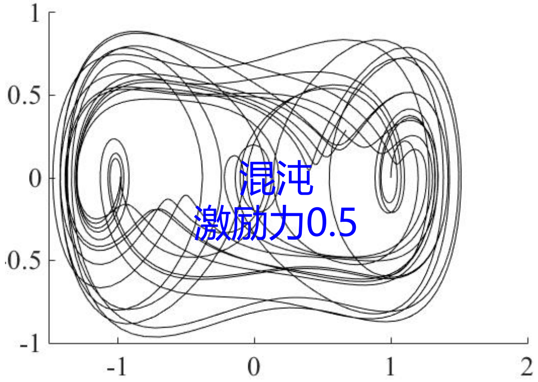
4. 庞佳莱映射

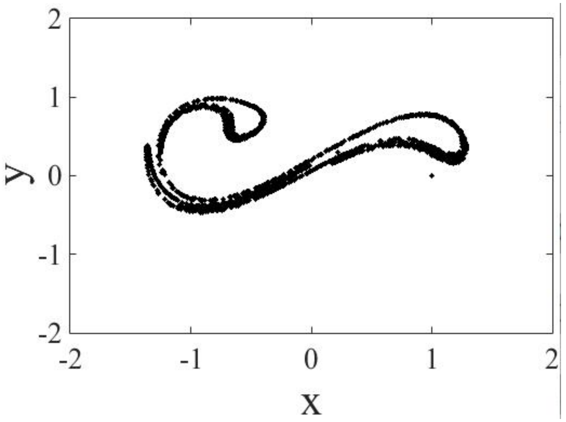
(Poincaré Maps)

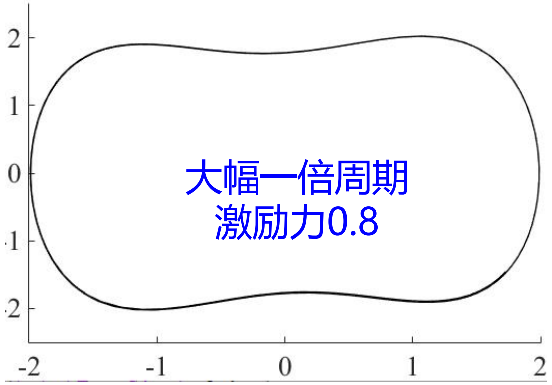
Poincaré Maps(以传说中双稳振子为例) $\dot{x} = y \quad \dot{y} = x - ky - x^3 + \Gamma\cos(\omega t).$

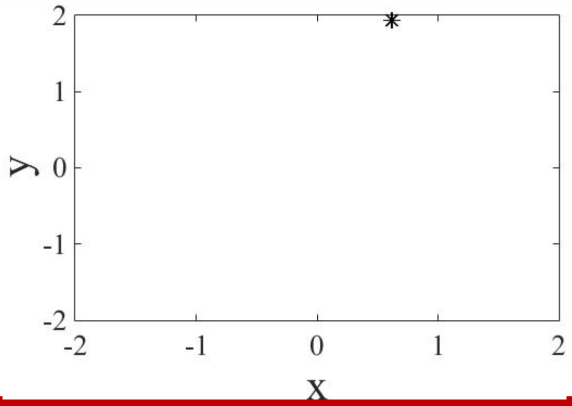






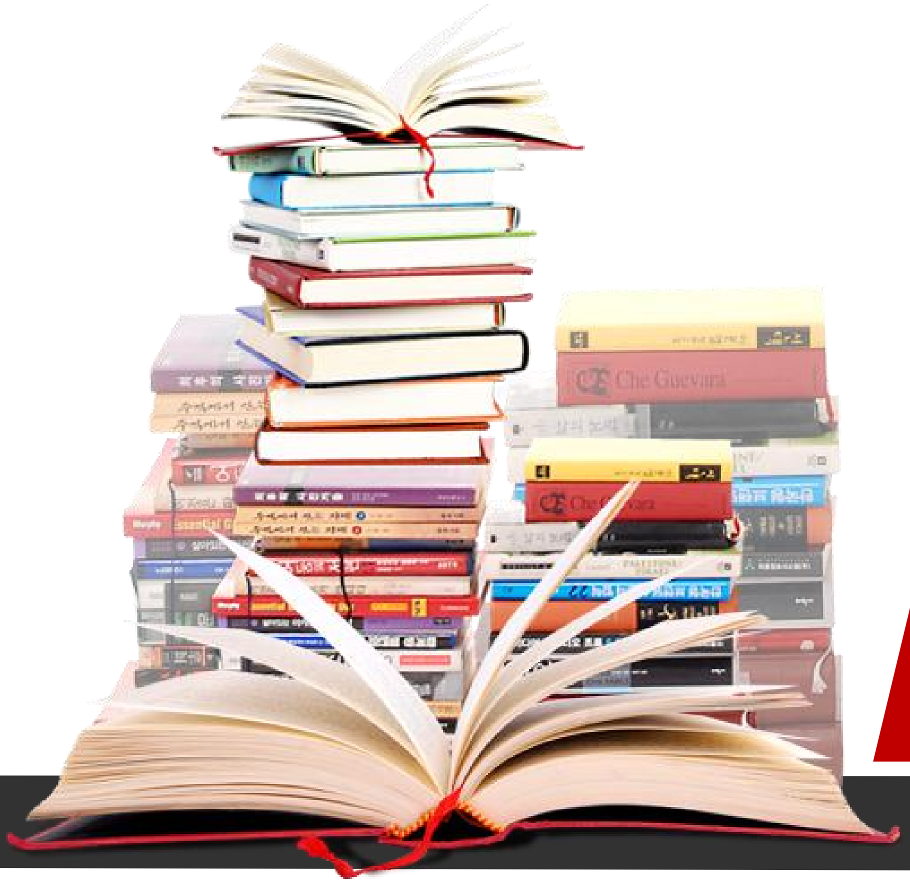







```
clear
Gamma=0.8;
deq=@(t,x) [x(2);x(1)-0.3*x(2)-(x(1))^3+Gamma*cos(1.25*t)];
options=odeset('RelTol',1e-4,'AbsTol',1e-4);
[t,xx]=ode45(deq,0:(2/1.25)*pi:(4000/1.25)*pi,[1,0]);
plot(xx(:,1),'.','MarkerSize',2)
plot(xx(:,1),xx(:,2),'k*','MarkerSize',10);hold on;
fsize=15;
axis([-2 2 -2 2])
xlabel('x','FontSize',fsize)
ylabel('y','FontSize',fsize)
title('Poincare Section of the Duffing System')
```

```
deq=@(t,x) [x(2);x(1)-0.3*x(2)-(x(1))^3+0.8*cos(1.25*t)];
options=odeset('RelTol',1e-4,'AbsTol',1e-4);
[t,xx]=ode45(deq,[0 200],[1,0],options);
plot(xx(1000:end,1),xx(1000:end,2),'b','linewidth','3')
fsize=15;
axis([-2 2 -2 2])
xlabel('x','FontSize',fsize)
ylabel('y','FontSize',fsize)
```



非常感谢收看