Portfolio Optimization for Most-traded Currencies

KELLY CRITERION AND MARKOWITZ PORTFOLIO OPTIMIZATION

Contents

Introduction

In ancient times, people exchanged for goods, and gold once became the only medium in international trade and the only standard of value and wealth.

Bretton Woods system is a new system that combines the gold standard with a fixed exchange rate and can be adjusted appropriately. It is stipulated that countries maintain the price of 1 ounce of gold to 35 U.S. dollars, and the exchange rate of each country can only fluctuate within 1%.

Free-floating exchange rate system meant the foreign exchange rate should be determined by the market of supply and demand, and the fluctuation range of a country's currency exchange rate should not be limited. The difference between this new system and the previous systems is that it abandons the fixed proportional relationship between currencies and gold.

Introduction

The emergence of free-floating exchange rates allows people to predict future currency price fluctuations and make speculative trading operations and risk aversion activities from them. It brings limitless opportunities but does not mean the foreign exchange market is always safe.

Eg: On January 15, 2015, the Swiss National Bank unexpectedly announced a cut in interest rate and abandoned the lower limit of the exchange rate between the euro and the Swiss franc that has been maintained since September 2011. Subsequent the foreign market fluctuated volatility, the euro against the Swiss franc plummeted 21.48% (Lleo & Ziemba, 2015).

The international monetary system is a complex system. The central problem faced by both individual investors and financial institutions is how to optimize the allocation of resources in a volatile and complex environment.

Method-Kelly Criterion

- Kelly criterion points out the optimal proportion of bets that should be placed in each period in a repetitive game or repetitive investment with a positive expected return.
- People play a gambling or investment and most want to make the expected return as much as possible. However, people do not know what the expected return will eventually become, or what value it will converge to.
- According to Kelly's analysis, as long as the number of games is large enough, the logarithmic return will be very close to its expected return.
- Then people naturally want to find a betting ratio that makes the logarithmic return as large as possible. The Kelly formula can just calculate the betting ratio with the largest logarithmic return.
- Rotando and Thorp (1992) concluded that the general statement of Kelly formula is that by finding the capital ratio that maximizes the logarithm expectation of the result, the long-term growth rate can be maximized.

Method-Kelly Criterion

- Kelly introduced an optimal investment position ratio to maximize the long-term cumulative return. The formula of this ratio is expressed as: Kelly% = edge / odds.
- The edge here can be understood as the probability of winning multiplied by the odds and then minus the probability of losing in gambling. When the edge number is positive, the game is worth betting. When the edge is zero or negative, the gambler should not bet.
- Odds is the payoff, which can be understood as the win-loss ratio. For example, if the average profit is 30% and the average loss is 10%, the win-loss ratio will be 3.
- The formula (Kelly%=edge/odds) can only be used for binary bets. It may be suitable for simple races such as horse racing, but it is rarely used for real market investment. An optimized Kelly formula is needed in more complex situations.

Method-Kelly Criterion

Investors can start with 1 unit of wealth, and then bet a fraction of this wealth f on the outcome that appears with probability p and odds b. p is the probability of winning. In this case, $1 + fb$ is the wealth gained. It can be inferred that $1 - p$ is the probability of loss and $1 - f$ is the wealth gained. According to Lo, Orr and Zhang (2018), The core theory of Kelly criterion is to maximize the expected value of the return logarithm. Therefore, as noted by Thorp (2011), the expected value of logarithmic wealth (E) is given by:

$$\log(E) = plog(1 + fb) + (1 - p)\log(1 - f)$$

We differentiate it and set it to 0 to maximize:

$$\frac{d}{df}\log(E) = \frac{d}{df}plog(1 + fb) + (1 - p)\log(1 - f) = 0$$

Solving for f:

And so we have

$$bp(1 - f) = (1 - p)(1 + fb)$$

$$bp - bpf = 1 + fb - p - pfb$$

$$bp = 1 + fb - p$$

Finally we obtain the optimal weight f

$$f = \frac{pb + p - 1}{b}$$

Method-Markowitz Portfolio

Markowitz theory mainly include the method of mean variance analysis and the effective frontier model of the portfolio.

Markowitz starts from the relationship between return and risk, and defines return and risk as mean and variance, which is the mean-variance model. The basic idea of the mean-variance model is to solve the optimal asset allocation ratio coefficient under the condition that investors know how to avoid investment risks and are not satisfied with the return. In the end, the goal of minimizing risk under the constraint of a certain level of return or maximizing return under the constraint of a certain level of risk is achieved.

It can be seen that this theory is mainly based on rational markets, which means that the changes in the risk and return rates of various assets in the market and their influencing factors are in the hands of investors or at least investors can know.

The model also proves that if all its assumptions are met, the point set of the mean-variance combination of all the optimal portfolios of investors is actually a parabola on the two-dimensional plane. In other words, the Markowitz model is a quadratic programming problem with asset weight as a variable. Under restricted conditions, the Lagrange method in differentiation is used to solve the optimal investment ratio that minimizes portfolio risk.

Method-Markowitz Portfolio

From the perspective of economics, investors determine an expected rate of return in advance, and then determine the weight of each asset in the portfolio to minimize the overall investment risk. Therefore, under different expected return levels, the corresponding asset portfolio solution that minimizes the variance is obtained. These solutions constitute the minimum variance combination, which is what we usually call the effective combination.

The curve formed between the expected return rate of the effective portfolio and the corresponding minimum variance is the frontier of the effective portfolio investment.

Investors choose the best investment portfolio plan on the frontier of the effective portfolio (effective curve) according to their own return goals and risk preferences.

Method-Markowitz Portfolio

As found by Markowitz (1952), people can find out the optimal ratio through the following steps

If r_i represents the rate of return for asset i (random variable), for i = 1, 2, . . ., n, we can define the random vector

$$z = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix}$$

Set $u_i = E(r_i)$, $m = (u_1 \quad u_2 \quad \cdots \quad u_n)^T$, and cov(z) = Σ. If $w = (w_1 \quad w_2 \quad \cdots \quad w_n)^T$ is a set of weights associated with a portfolio, then the rate of return of this portfolio $r_p = \sum_{i=1}^{n} r_i w_i$ is also a random variable with mean $m^T w$ and variance $w^T \sum w$:

$$r_p = m^T w$$

$$\sigma_p^2 = w^T \sum w$$

Since w represents the asset allocation vector of the portfolio, it satisfies the following conditions:

$$\sum_{i=1}^{n} w_i = 1$$

Method-Markowitz Portfolio

If u_b is the acceptable baseline expected rate of return, then in the Markowitz theory an optimal portfolio is any portfolio solving the following quadratic program:

$$\min \frac{1}{2} w^T \sum w$$

$$\text{s.t.} \begin{cases} m^T w \geq u_b \\ e^T w = 1 \\ w_i \geq 0 \end{cases}$$

where e always denotes the vector of ones and $w_i \geq 0$ means that short selling is not allowed.

Based on Markowitz theory, Sharpe (1966) proposed to use the Sharpe ratio as a risk adjustment index to measure fund performance:

$$S_p = \frac{E(R_p) - R_f}{\sigma_p}$$

Data and Research Design

- Eight mainstream currencies can constitute seven currency pairs. The currency on the left is the base currency, and the currency on the right is the quote currency.
- The eight major currencies mainly involve the currencies of the United States, European Union, United Kingdom, Australia, New Zealand, Canada, Switzerland, and Japan.
- The article selected seven currency pairs with Japanese Yen as the quote currency. They are USDJPY, EURJPY, GBPJPY, NZDJPY, AUDJPY, CHFJPY, and CADJPY.
- The selected data is the daily closing price of the exchange rate from January 3, 2011 to December 31, 2020. The seven currency pairs have 2609 exchange rate daily closing price data. Hence there are 18263 sample data in total. The data came from the yahoo finance website.

Data and Research Design

- ✓ The Japanese yen is regarded as the quote currency because it has the characteristics of a safe-haven currency in economic fluctuations. The safe-haven currency refers to relatively stable currency that is not easily affected by factors such as politics, wars, and market volatility. When the variability in the market increases, investors will choose to buy safe-haven currencies.

- ✓ Another reason is the low interest rates of yen have attracted investors to carry out arbitrage transactions. Arbitrage trading is that investors in the market usually borrow money from places where interest rates are low and invest in places where interest rates are high, earning intermediate spreads.

Data and Research Design

1. Process the data and calculate the basic statistics. This article mainly analyzes return, it is necessary to first convert price data into return data.
2. Construct a portfolio with equal weights. It is used as a benchmark.
3. Construct an optimized portfolio based on the Kelly method.
4. Generate lots of mean-variance portfolios and get the effective frontier based on the Markowitz theory, find two portfolios with the smallest variance and the largest Sharpe ratio on the effective frontier.
5. Backtest these portfolios and compare the performance.

Data Analysis-Basic Statistical Characteristics

- According to the results in Table 1, the mean and standard deviation of the CHFJPY currency pair is relatively large. This is consistent with the fact that the higher the risk, the higher the return on investment.
- The USDJPY currency pair has the smallest standard deviation, which means that the currency pair has the smallest volatility in the exchange price. This is related to the fact that the USD and the JPY are both currencies with hedging functions.
- The return skewness of the seven currency pairs is not 0, among which the skewness of EURJPY, GBPJPY, NZDJPY, AUDJPY and CADJPY is negative, indicating that their return series are left-skewed. The skewness of USDJPY and CHFJPY is positive, implying that their return series are right-skewed.
- CHFJPY has the largest kurtosis, showing the data characteristics of sharp peaks and thick tails.
- The correlation between the various currency pairs is depicted in Table 2. It can be seen that currency pairs are positively correlated.

Table 1

Basic statistics of currency pairs

	USDJPY	EURJPY	GBPJPY	NZDJPY	AUDJPY	CHFJPY	CADJPY
mean	0.000108	0.000082	0.000067	0.000094	0.000014	0.000142	0.000021
std	0.005635	0.006546	0.007215	0.007836	0.00779	0.007125	0.006987
skew	0.191484	-0.11689	-0.88077	-0.29017	-0.29846	5.692605	-0.10298
kurt	4.04357	4.30624	16.98454	2.575605	2.298002	162.7839	2.396409

Table 2

Correlation of currency pairs

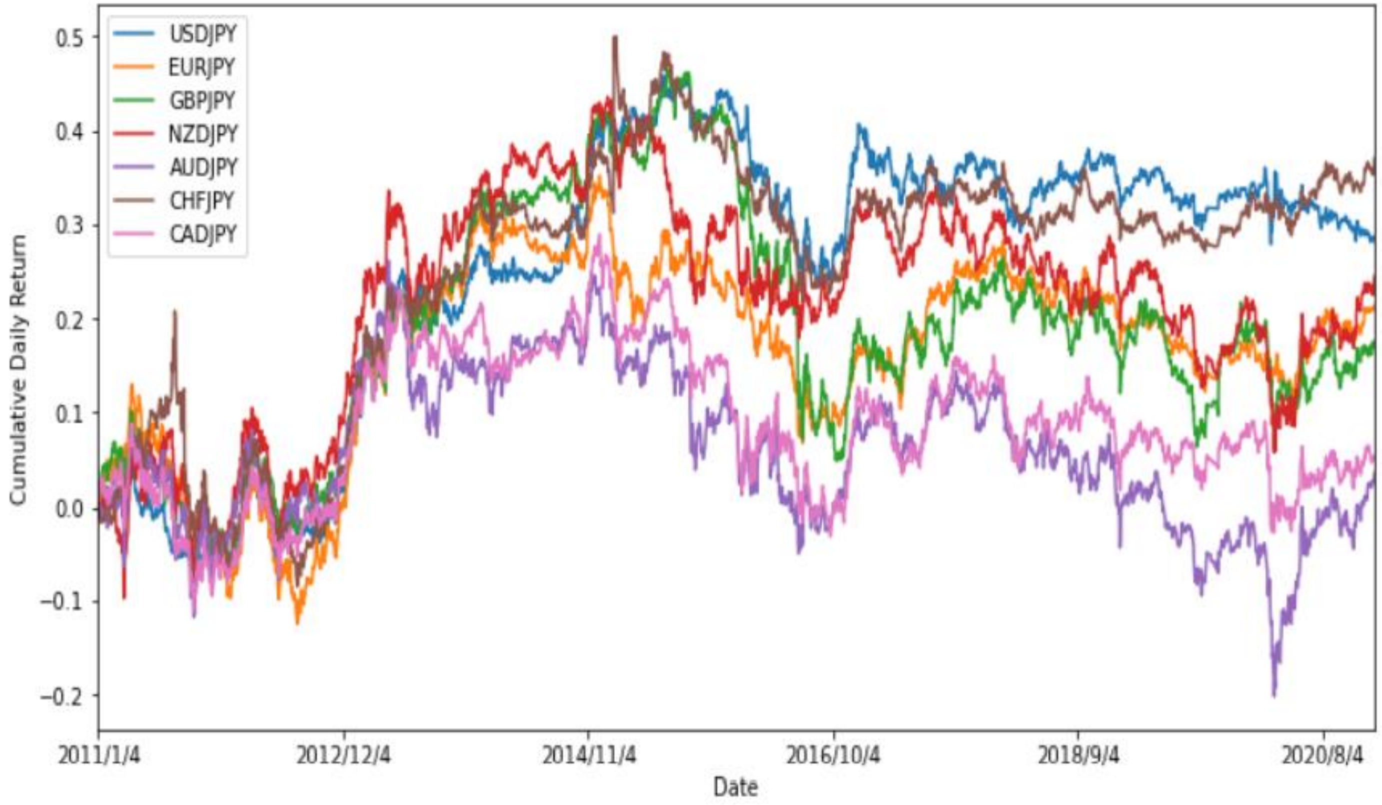
	USDJPY	EURJPY	GBPJPY	NZDJPY	AUDJPY	CHFJPY	CADJPY
USDJPY	1	0.619226	0.655468	0.523277	0.576606	0.44331	0.743254
EURJPY	0.619226	1	0.724023	0.655831	0.676148	0.638604	0.689794
GBPJPY	0.655468	0.724023	1	0.631216	0.670163	0.498513	0.699714
NZDJPY	0.523277	0.655831	0.631216	1	0.854296	0.446613	0.723815
AUDJPY	0.576606	0.676148	0.670163	0.854296	1	0.448293	0.785105
CHFJPY	0.44331	0.638604	0.498513	0.446613	0.448293	1	0.465794
CADJPY	0.743254	0.689794	0.699714	0.723815	0.785105	0.465794	1

Data Analysis-Basic Statistical Characteristics

- As observed in Figure 1, the return time series diagrams of the seven currency pairs all show an obvious growth or decline trend over time.
- For example, looking the fluctuations in return of the GBPJPY currency pair from 2011 to 2020. It declined slightly from 2011 to 2012, and then continued to rise. The highest point of revenue appeared in the first half of 2014. Over the next half of the year, the GBPJPY began to fall sharply, until 2016 earnings fell to the bottom. Since then, the GBPJPY return had gradually increased, but the exchange rate had not been able to return to the high return range of 2014.

Figure 1

Cumulative daily return of currency pairs

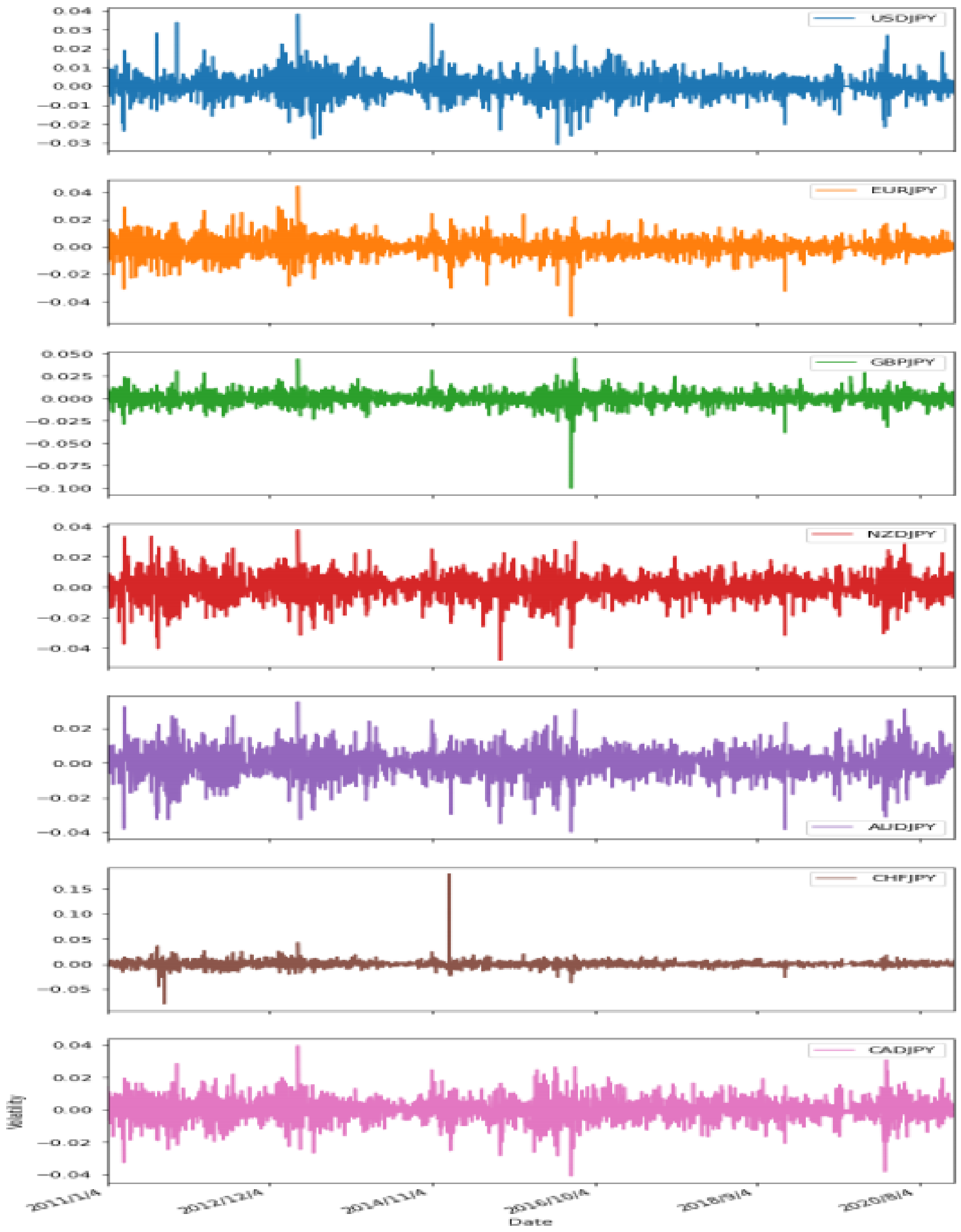


Data Analysis-Basic Statistical Characteristics

Figure 2 represents the volatility. Except for CHFJPY, other currency pairs fluctuate relatively sharp. The uncertainty of the political event of Brexit in 2016 had caused strong volatility in these currency pairs.

Figure 2

Volatility of currency pairs



Data Analysis-Portfolio Weight Determination

- Table 3 contains the portfolio weights. The proportion of the equal weighted portfolio is 1/7. The investment portfolio constructed according to the Kelly criterion has all the weights allocated to CHFJPY.
- According to the above Figure 1, the return of CHFJPY started to rise in 2011 and became the highest in 2020. The Kelly criterion seeks to maximize the logarithm expected return, which is the most direct reason why it invests all the weight in CHFJPY.
- For more specific reasons, we need to pay attention to factors such as the policies and national economy of Switzerland and Japan.
- The line chart of Swiss GDP is presented in Figure 3. It can be inferred that the Swiss economy was in a stable state from 2011 to 2020. Swiss watches have been exported to Asia in large numbers. The Swiss National Bank has great independence in formulating monetary policy, so Switzerland's exchange rate is relatively stable. In addition, Switzerland also pursues a policy of neutrality and non-alignment, so it also has the characteristics of a traditional safe-haven currency.
- Japan's domestic market is narrow, so the export has become a major factor in its economic growth. The Japanese government intervened in the foreign exchange market to prevent the yen exchange rate from being too high, which maintained their export competitiveness.

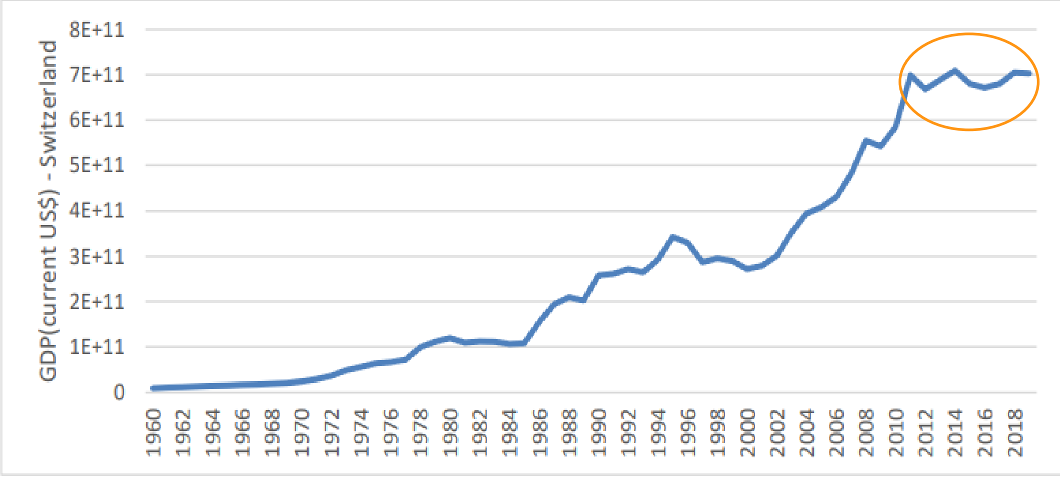
Table 3

Weights of different portfolios

	USDJPY	EURJPY	GBPJPY	NZDJPY	AUDJPY	CHFJPY	CADJPY
Equal-weighted portfolio	1/7	1/7	1/7	1/7	1/7	1/7	1/7
Kelly portfolio	0	0	0	0	0	1	0
Minimum-variance portfolio	0.605	0.098	0	0.065	0	0.232	0
Maximum-Sharpe ratio portfolio	0.534	0	0	0	0	0.466	0

Figure 3

GDP of Switzerland



Data Analysis-Portfolio Weight Determination

Weights of the minimum-variance portfolio are mainly assigned to USDJPY, EURJPY, NZDJPY and CHFJPY. The ratios are 0.605, 0.098, 0.065 and 0.232 respectively. The minimum-variance portfolio reduces risk by diversifying investment.

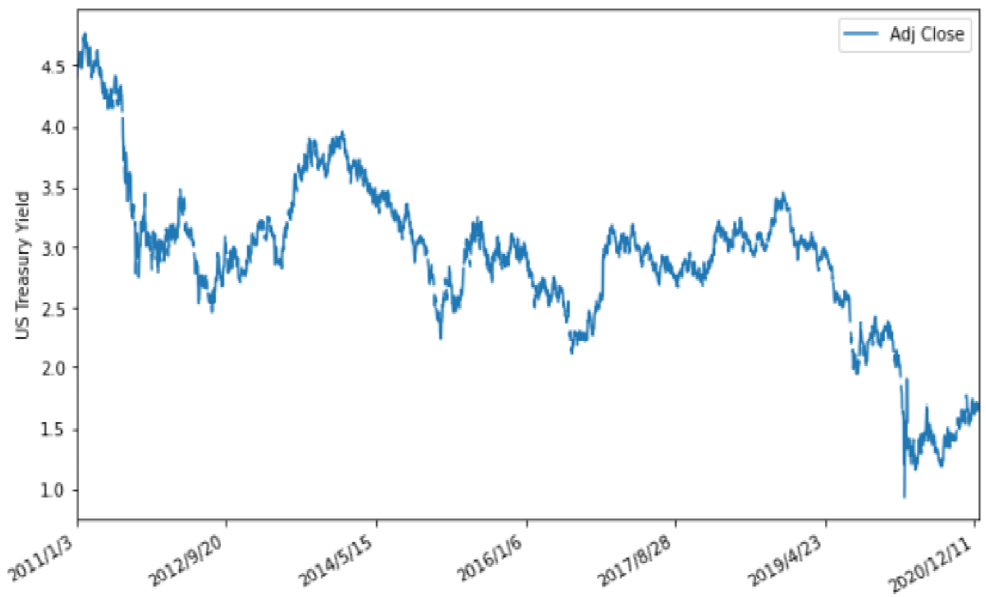
The returns of USDJPY, EURJPY, NZDJPY and CHFJPY are the top four of all currency pairs. It puts 60.5% of the weight on USDJPY. USD is a reserve currency, which can be held by almost all central banks and investment institutions in the world. This is determined by the political and economic status of the United States.

Figure 4 provides the yield trend of US Treasury bonds. Comparing Figure 2 and Figure 4, It can be seen that the rise or fall of the yield of US Treasury bonds has a great impact on the exchange rate of the USD. If US Treasury bond yields rise, it will attract capital inflows and support the exchange rate rise. Conversely, if the yield of Treasury bonds falls, the exchange rate will fall. Hence, investors can refer to the yield of treasury bonds to make foreign exchange investment decisions.

As the political structure of the European Union is relatively decentralized, its ability to influence the exchange rate of the euro is also reduced. The monetary policy implemented by the Central Bank of New Zealand aims to maintain the consumer price index at 1.5%. If banks fail to achieve this goal, they need to adjust their policies. The minimum-variance investment portfolio allocates 16.3% of the weight to EURJPY and NZDJPY, achieving the consequent of reducing risk.

Figure 4

Yield of US Treasury



Data Analysis-Portfolio Weight Determination

Maximum-Sharpe ratio portfolio invested in USDJPY and CHFJPY, with weights of 53.4% and 46.6%.

As it can be observed in Figure 1, the returns of GBPJPY, AUDJPY and CADJPY are the lowest three currency pairs in all investment portfolios. Figure 2 depicts their volatility is also relatively strong.

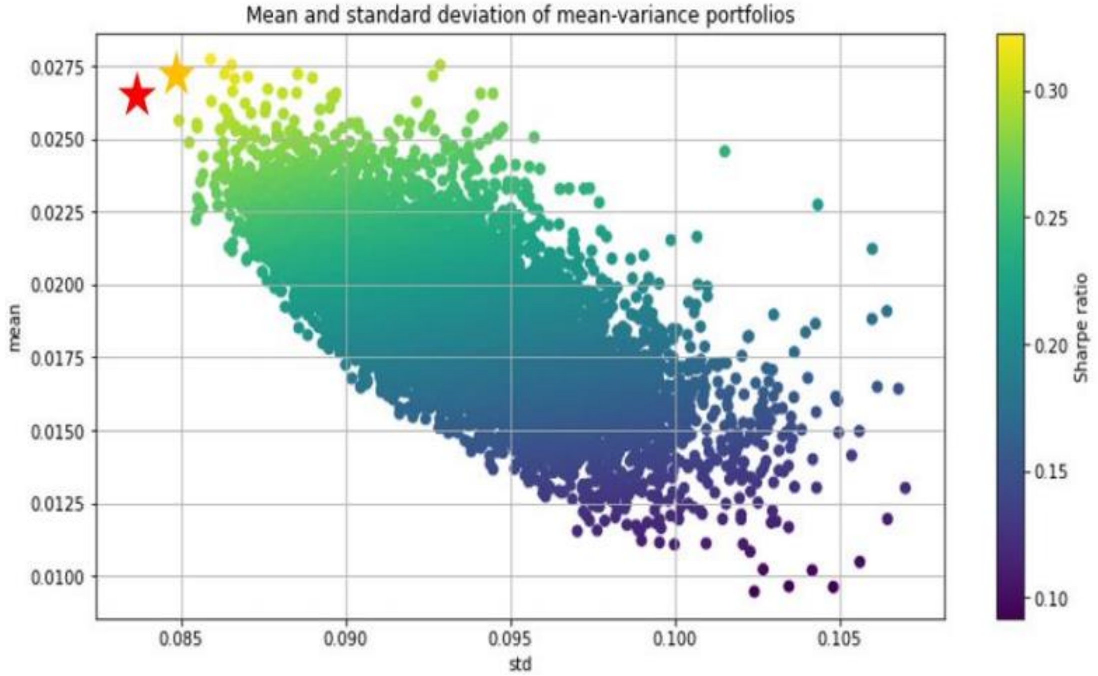
Among them, Canada focuses on major commodities, such as crude oil, non-ferrous metals and basic raw materials for the mineral industry. Its basic price fluctuates very sharply, especially the price of crude oil. The volatility of the Australian dollar is also related to oil.

Therefore, the Kelly portfolio, minimum-variance portfolio and Maximum-Sharpe ratio portfolio have not invested in these currency pairs.

The positions of the minimum-variance and maximum-Sharpe ratio portfolios are presented in Figure 5. The red and orange stars respectively represent portfolios with the smallest variance and the largest Sharpe ratio.

Figure 5

The position of the minimum-variance portfolio and the maximum-Sharpe ratio portfolio



Data Analysis-Portfolio Performance Comparison

Table 4

Performance metrics of different portfolios

	Annual return	Cumulative returns	Annual volatility	Sharpe ratio	Max drawdown
Equal-weighted portfolio	1.50%	16.50%	9.20%	0.21	-24.90%
Kelly portfolio	3.00%	35.70%	11.30%	0.32	-26.40%
Minimum-variance portfolio	2.50%	29.40%	8.30%	0.34	-20.50%
Maximum-Sharpe ratio portfolio	2.80%	33.00%	8.50%	0.37	-21.40%

Table 5

Top 5 worst drawdowns of Kelly portfolio

Worst drawdown periods	Net drawdown in %	Peak date	Valley date	Recovery date	Duration
0	26.41	2011/8/10	2012/7/25	2013/4/11	437
1	25.22	2015/1/19	2016/7/11	NaN	NaN
2	7.29	2014/12/8	2015/1/15	2015/1/16	30
3	5.13	2013/12/31	2014/2/4	2014/11/5	222
4	4.54	2013/4/12	2013/6/17	2013/8/26	97

- The annual return and cumulative returns of the investment portfolio constructed based on the Kelly criterion is the most. The volatility of Kelly portfolio is also the most severe.
- The Maximum drawdown indicates the largest (expressed in %) drop between a peak and a valley. Intuitively speaking, it refers to the losses the strategy has experienced from the base amount of capital which it had at the peak. Therefore, the smaller the maximum drawdown, the better.
- Sharpe ratio is a very popular risk metric. It indicates the amount of excess return (over the risk-free rate) per unit of risk (measured by standard deviation).
- Table 5 presents top 5 worst drawdowns of Kelly portfolio, together with information such as peak, valley date and the duration. From this chart, people can see that the maximum drawdown in Kelly portfolio occurred from August 10, 2011 to July 25, 2012. The recovery date was April 11, 2013, and it went through 437 duration. The second largest drawdown occurred from January 19, 2015 to July 11, 2016. However, it did not recover to the peak until the end of the sample interval.
- The minimum-variance portfolio has the smallest annual variance and maximum drawdown. At the same time, its annualized and cumulative returns are 1% and 12.9% higher than the equal-weighted portfolio. Its Sharpe ratio ranks second among all portfolios.
- Maximum-Sharpe ratio portfolio has the largest Sharpe ratio. The risk and maximum drawdown are 0.9% and 4.4% smaller than the equal-weighted portfolio.

Data Analysis-Portfolio Performance Comparison

- With the annual and monthly return plots, people can see which years and months the algorithm performed the best. For instance, the monthly heatmap plot in Figure 6 depicts this algorithm performed the best in February 2012 (shaded in dark green).
- Furthermore, the distribution of the monthly returns is also instructive in gauging how the algorithm performs in different periods throughout the year and if it is affected by seasonal patterns. However, it can be seen from the data in Figure 6 that the exchange rate is not related to the season.
- Figure 7 shows the rolling Sharpe ratio, which provides more insight into the stability of the portfolio. It is calculated using rolling 6 months of data, not entire sample.

Figure 6

Monthly heatmap plot

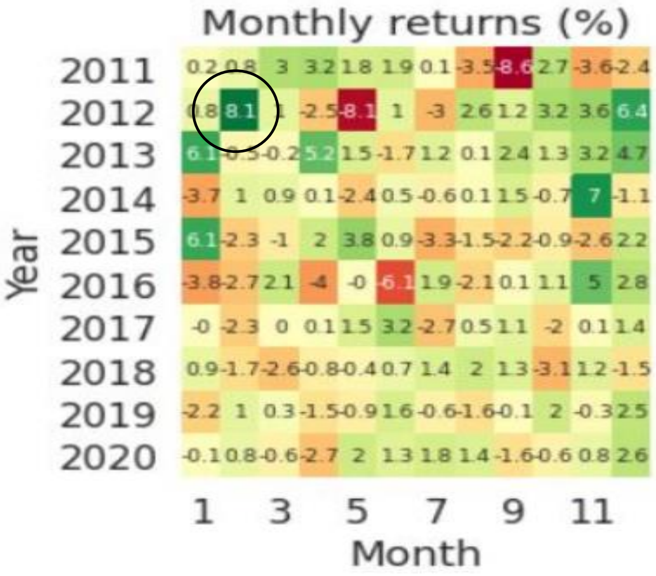
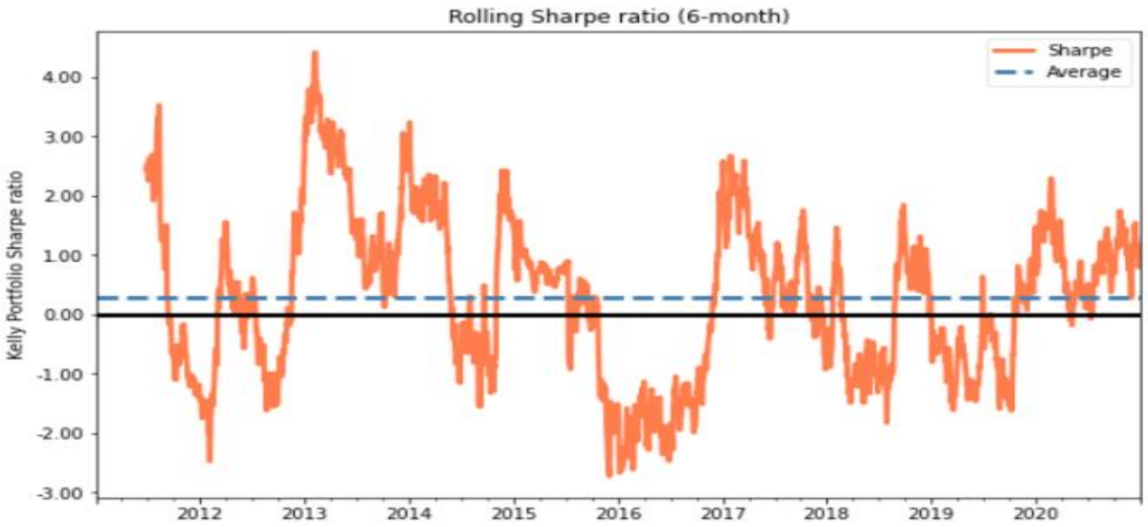


Figure 7

Rolling Sharpe ratio



Data Analysis-Portfolio Performance Comparison

Figure 8

Rolling volatility

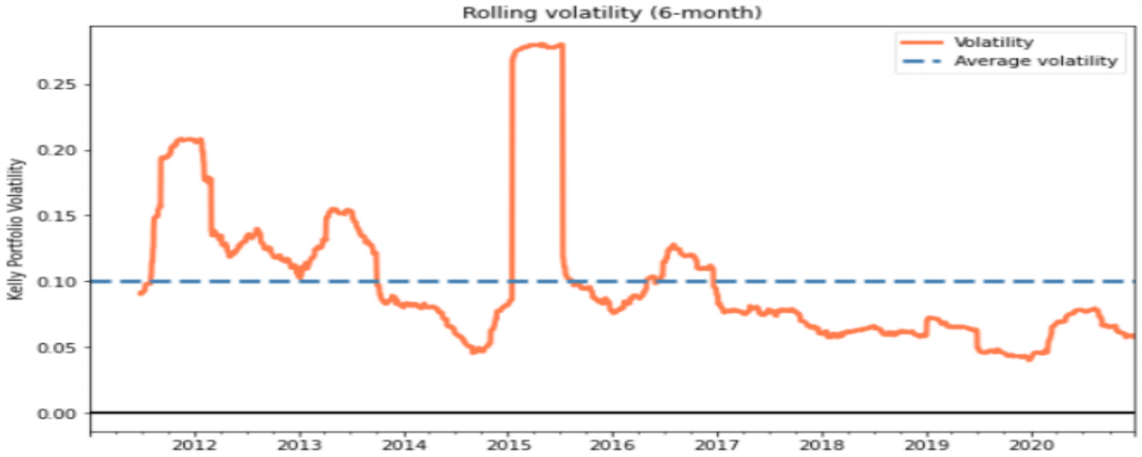
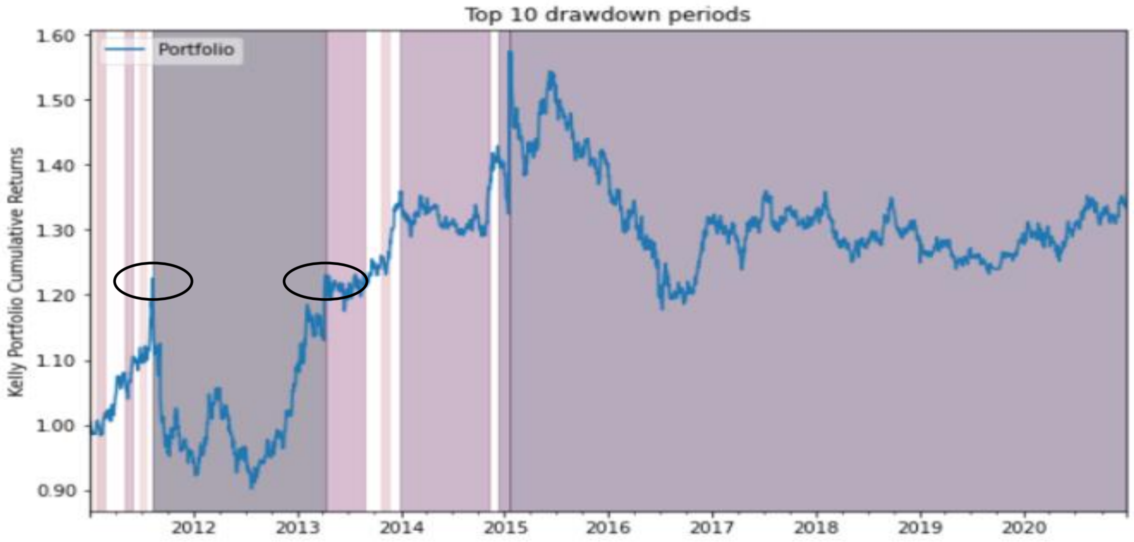


Figure 9

Top 10 drawdown plots



As it can be observed in Figure 8, due to the unexpected removal of exchange rate restrictions by the Swiss National Bank in 2015, rolling volatility fluctuates fiercely.

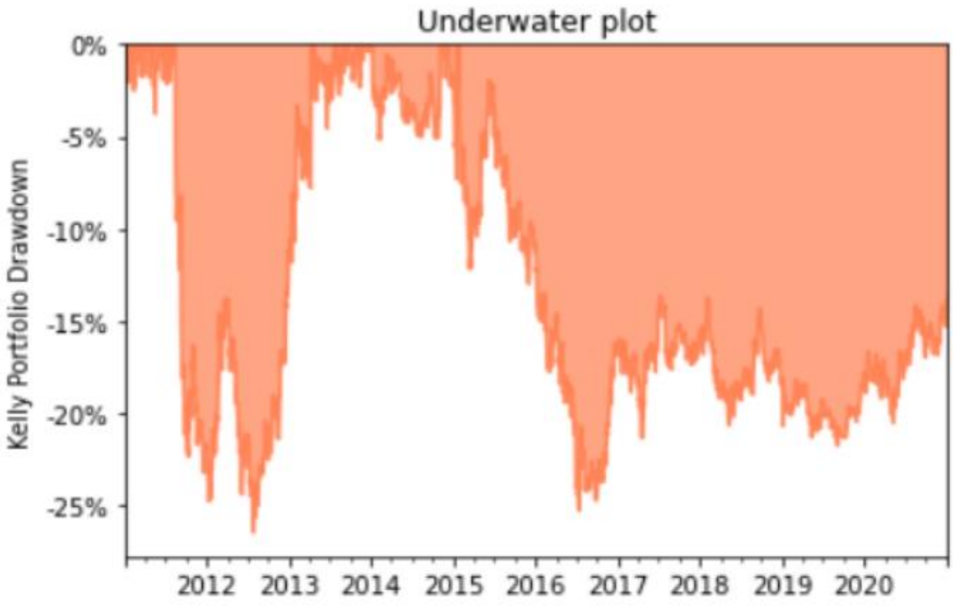
People can use the drawdown plots in Figure 9 to quickly pinpoint the time periods in which the strategy performed the worst. As it can be observed that, returns recovered to the peak in April 2013 after falling in August 2011.

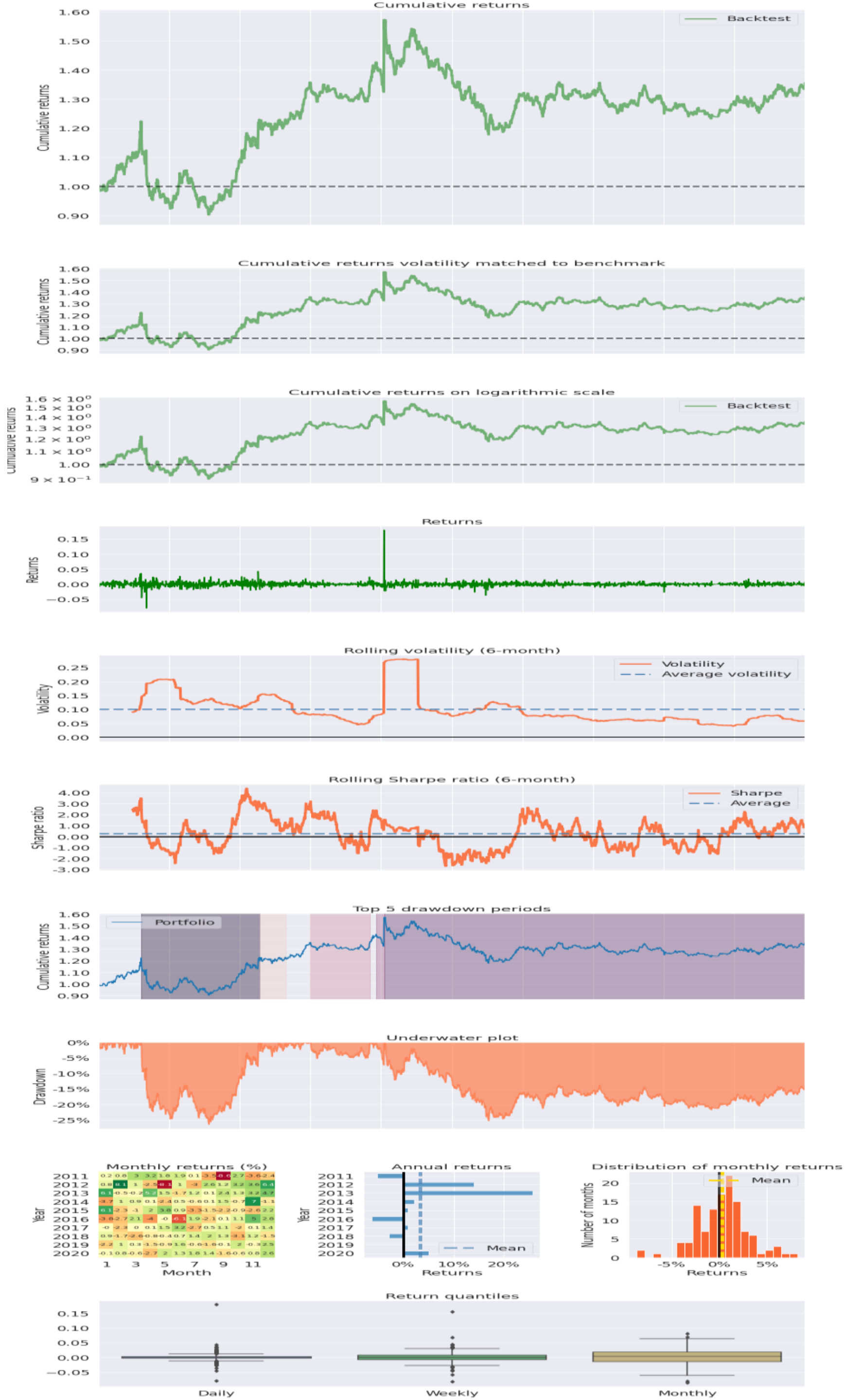
Data Analysis-Portfolio Performance Comparison

Figure 10 shows the underwater plot. It depicts drawdowns and shows how long it took for the portfolio's value to recover to the previous peak, after suffering a loss. It can be inferred that the portfolio is difficult to run after 2015.

Figure 10

Underwater plot




Cumulative returns
Backtest
Cumulative returns
Cumulative returns volatility matched to benchmark
Cumulative returns on logarithmic scale
Returns
Rolling volatility (6-month)
Volatility
Average volatility
Rolling Sharpe ratio (6-month)
Sharpe
Average
Sharpe ratio
Top 5 drawdown periods
Portfolio
Underwater plot
Drawdown
Monthly returns (%)
Year
Month
Annual returns
Mean
Returns
Distribution of monthly returns
Number of months
Return quantiles
Daily
Weekly
Monthly

Cumulative returns
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Cumulative returns
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Cumulative returns on logarithmic scale
Returns
Rolling volatility (6-month)
Volatility
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Underwater plot
Drawdown
Monthly returns (%)
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Monthly returns (%)
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Conclusion

- First, Kelly portfolio pursued to maximize the logarithmic rate of return. it tended to allocate a very small investment ratio to relatively low-yield currency pairs, while allocating a larger proportion of funds to relatively high-yield currency pairs, which is not conducive to better diversification of risks.
- Second, the minimum-variance portfolio overcame the shortcomings of Kelly model of assigning zero weight to low-yield assets. It can reduce the impact of foreign exchange market fluctuations. However, this investment portfolio also sacrificed some returns in order to control risk.
- Third, the maximum-Sharp rate portfolio coordinated both returns and risks balancing them at an appropriate point.
- Therefore, investors need to combine the foundation of the investment model with their own risk preferences, which can obtain a more balanced and optimized portfolio.


Thanks for watching