Lixia

Part 1

a. ARCH effect Test for residual sequences. (The residual here is the sequence of residuals produced by the mean equation ARMA)

Dependent Variable: RETURN_PER_DAY
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/18/19 Time: 20:18
Sample: 4/04/1991 9/16/2019
Included observations: 7423
Convergence achieved after 72 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000380	0.000313	1.213160	0.2251
AR(1)	0.869510	0.023100	37.64040	0.0000
MA(1)	-0.835885	0.025058	-33.35852	0.0000
SIGMASQ	0.000446	2.44E-06	182.8739	0.0000

R-squared	0.004616	Mean dependent var	0.000380
Adjusted R-squared	0.004214	S.D. dependent var	0.021176
S.E. of regression	0.021132	Akaike info criterion	-4.875552
Sum squared resid	3.312919	Schwarz criterion	-4.871827
Log likelihood	18099.61	Hannan-Quinn criter.	-4.874272
F-statistic	11.46862	Durbin-Watson stat	1.987373
Prob(F-statistic)	0.000000		

Inverted AR Roots	.87
Inverted MA Roots	.84

Heteroskedasticity Test: ARCH

F-statistic	280.5260	Prob. F(1,7420)	0.0000
Obs*R-squared	270.3794	Prob. Chi-Square(1)	0.0000

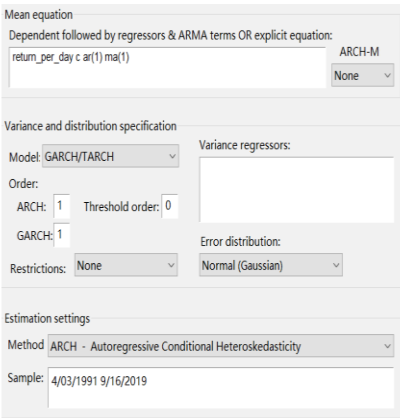
Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 12/18/19 Time: 20:20
Sample (adjusted): 4/05/1991 9/16/2019
Included observations: 7422 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000361	2.26E-05	15.99933	0.0000
RESID^2(-1)	0.190865	0.011396	16.74891	0.0000

R-squared	0.036429	Mean dependent var	0.000446
Adjusted R-squared	0.036300	S.D. dependent var	0.001930
S.E. of regression	0.001895	Akaike info criterion	-9.699202
Sum squared resid	0.026638	Schwarz criterion	-9.697339
Log likelihood	35995.74	Hannan-Quinn criter.	-9.698562
F-statistic	280.5260	Durbin-Watson stat	2.048388
Prob(F-statistic)	0.000000		

The p value is less than 0.05 on the second graph above, so we reject the null hypothesis. That means conditional heteroscedasticity exists. We have the ARCH effect.

b. Parameter estimation of GARCH (1,1) model



Dependent Variable: RETURN_PER_DAY
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 12/18/19 Time: 20:24
Sample (adjusted): 4/05/1991 9/16/2019
Included observations: 7422 after adjustments
Convergence achieved after 115 iterations
Coefficient covariance computed using outer product of gradients
MA Backcast: 4/04/1991
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000331	0.000178	1.856388	0.0634
AR(1)	0.986659	0.003918	251.8549	0.0000
MA(1)	-0.984764	0.004185	-235.3057	0.0000
Variance Equation				
C	3.07E-06	1.80E-07	17.02576	0.0000
RESID(-1)^2	0.066237	0.001650	40.13747	0.0000
GARCH(-1)	0.930829	0.001283	725.2788	0.0000

R-squared	0.001452	Mean dependent var	0.000381
Adjusted R-squared	0.001183	S.D. dependent var	0.021178
S.E. of regression	0.021165	Akaike info criterion	-5.236142
Sum squared resid	3.323420	Schwarz criterion	-5.230554
Log likelihood	19437.32	Hannan-Quinn criter.	-5.234222
Durbin-Watson stat	1.919380		

Inverted AR Roots	.99
Inverted MA Roots	.98

$\mu = 0.000331$, $\alpha = 0.066237$, $\beta = 0.930829$, $\omega = 3.07E\_06$

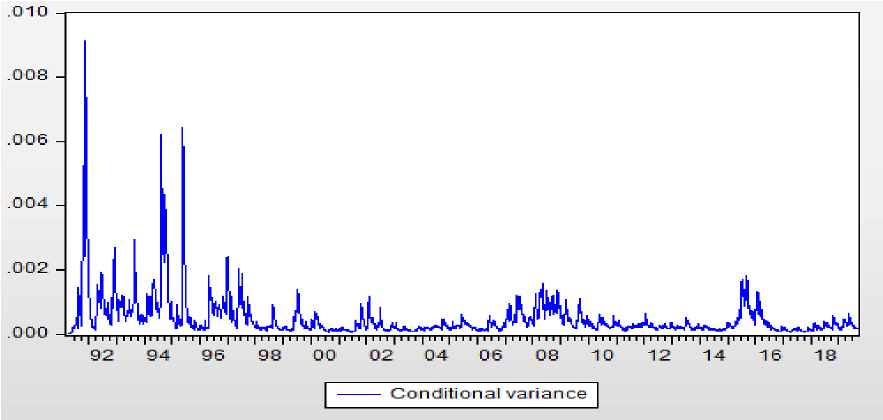
$\alpha + \beta < 1$. This formula satisfies the parameter constraints of the GARCH model.

From the figure, we can get the estimated results of the model:

Mean equation: $y_t = 0.986659 y_{t-1} + u_t - 0.984764 u_{t-1}$

Volitility equation: $\sigma_t^2 = 0.000003 + 0.066237 u_{t-1}^2 + 0.930829 \sigma_{t-1}^2$

c. Draw the conditional variance graph



From the conditional variance graph, in 1991-1992 and 1994-1995, the volatility was high. The volatility in 2008 is also relatively large compared to the years around it. This is consistent with the actual situation of the financial crisis at that time.

d. Do the ARCH test again.

Heteroskedasticity Test: ARCH

F-statistic	2.147575	Prob. F(1,7419)	0.1428
Obs*R-squared	2.147533	Prob. Chi-Square(1)	0.1428

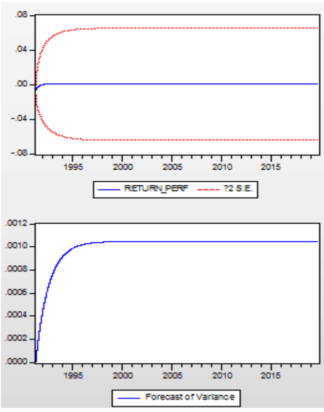
Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 12/18/19 Time: 20:57
Sample (adjusted): 4/08/1991 9/16/2019
Included observations: 7421 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.982886	0.045107	21.78997	0.0000
WGT_RESID^2(-1)	0.017011	0.011608	1.465461	0.1428

R-squared	0.000289	Mean dependent var	0.999897
Adjusted R-squared	0.000155	S.D. dependent var	3.755201
S.E. of regression	3.754911	Akaike info criterion	5.484276
Sum squared resid	104603.1	Schwarz criterion	5.486138
Log likelihood	-20347.40	Hannan-Quinn criter.	5.484916
F-statistic	2.147575	Durbin-Watson stat	2.000507
Prob(F-statistic)	0.142838		

P = 0.1428 > 0.05, we cannot reject the null hypothesis. This indicates that there is no ARCH effect.

f. Forecast

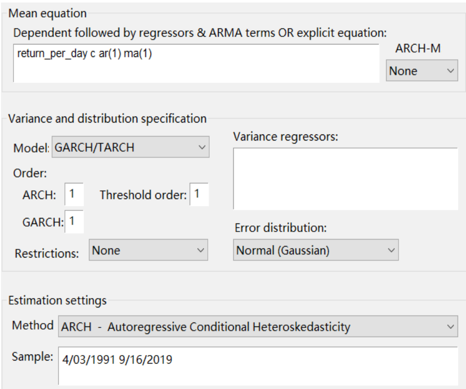


Forecast: RETURN_PERF	
Actual: RETURN_PER_DAY	
Forecast sample: 4/03/1991 9/16/2019	
Adjusted sample: 4/05/1991 9/16/2019	
Included observations: 7422	
Root Mean Squared Error	0.021172
Mean Absolute Error	0.013435
Mean Abs. Percent Error	NA
Theil Inequality Coefficient	0.974205
Bias Proportion	0.000032
Variance Proportion	0.954975
Covariance Proportion	0.044993
Theil U2 Coefficient	NA
Symmetric MAPE	186.3731

From 1991 to 1995, the forecast conditional variance increase. Then it becomes stable after 1995.

Part 2

Parameter estimation of GJR (1,1) model



Dependent Variable: RETURN_PER_DAY
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 12/18/19 Time: 23:27
Sample (adjusted): 4/05/1991 9/16/2019
Included observations: 7422 after adjustments
Convergence achieved after 77 iterations
Coefficient covariance computed using outer product of gradients
MA Backcast: 4/04/1991
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*RESID(-1)^2*(RESID(-1)<0)
+ C(7)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000203	0.000193	1.054904	0.2915
AR(1)	0.986123	0.003962	248.8794	0.0000
MA(1)	-0.984701	0.004126	-238.6818	0.0000
Variance Equation				
C	3.06E-06	1.82E-07	16.78547	0.0000
RESID(-1)^2	0.058551	0.002302	25.43102	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.016927	0.003106	5.450433	0.0000
GARCH(-1)	0.929973	0.001293	719.1527	0.0000

R-squared	0.001176	Mean dependent var	0.000381
Adjusted R-squared	0.000907	S.D. dependent var	0.021178
S.E. of regression	0.021168	Akaike info criterion	-5.237018
Sum squared resid	3.324338	Schwarz criterion	-5.230499
Log likelihood	19441.57	Hannan-Quinn criter.	-5.234778
Durbin-Watson stat	1.917942		
Inverted AR Roots	.99		
Inverted MA Roots	.98		

$\mu = 0.000203$ $\alpha = 0.058551$ $\alpha^- = 0.016927$ $\beta = 0.929973$ $\omega = 3.06E\_06$

GJR equation: $\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \gamma u_{t-1}^2 d_{t-1} + \beta \sigma_{t-1}^2$

d_{t-1} is a dummy variable. $d_{t-1} = 1$ if $u_{t-1} < 0$ (bad news). $d_{t-1} = 0$ if $u_{t-1} > 0$ (good news).

Null hypothesis: $\gamma = 0$ (no leverage effect)

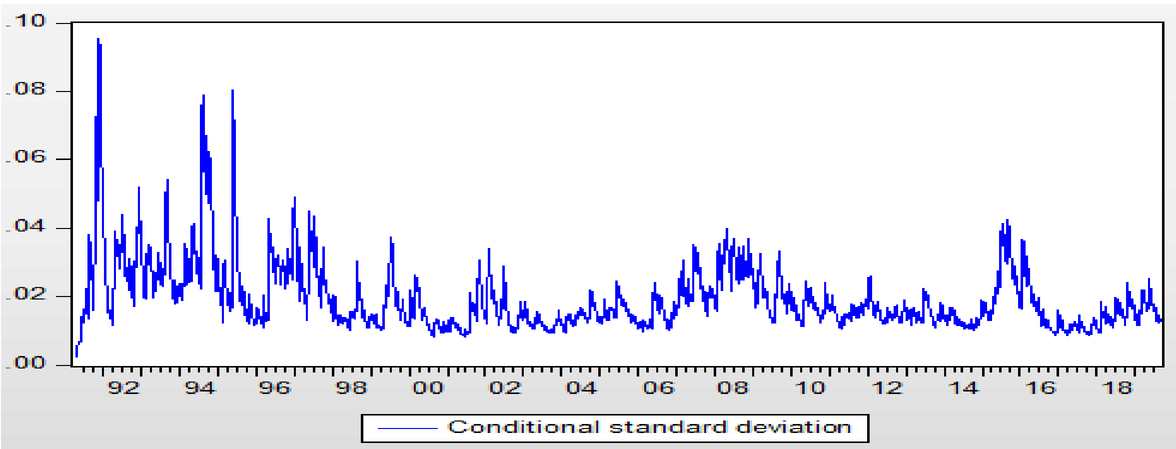
Alternative hypothesis: $\gamma > 0$ (leverage effect)

GJR equation: $\sigma_t^2 = 0.000003 + 0.058551 u_{t-1}^2 + 0.016927 u_{t-1}^2 d_{t-1} + 0.929973 \sigma_{t-1}^2$

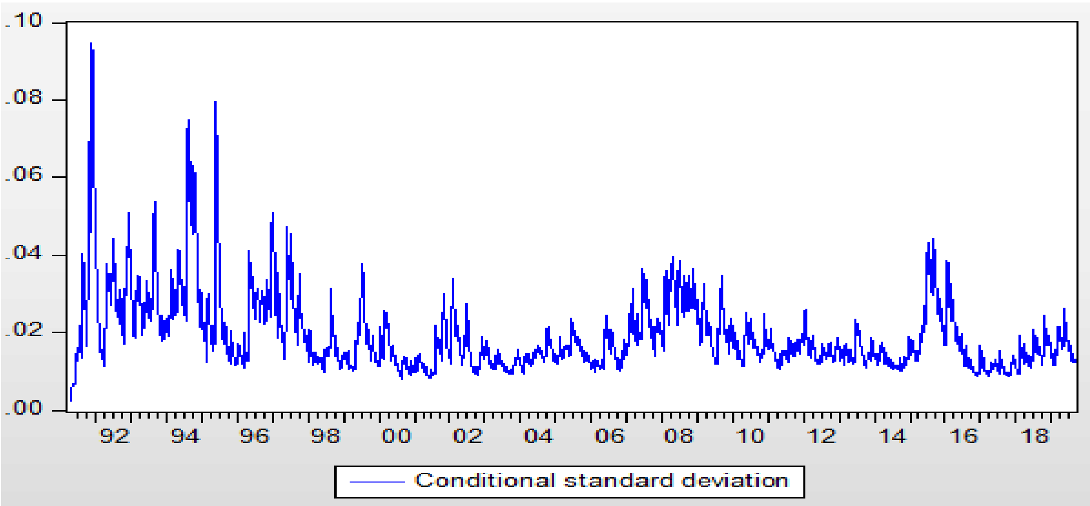
Since $u_{t-1}^2 d_{t-1}$'s p value less than 0.05, we can reject null hypothesis. That means there is leverage effect. Volatility responds differently to good news and bad news.

Part 3

GARCH annualized conditional volatility:



GJR annualized conditional volatility:



The annualized conditional volatility values of GJR model are more intensive.

The GJR model is more sensitive to bad news.