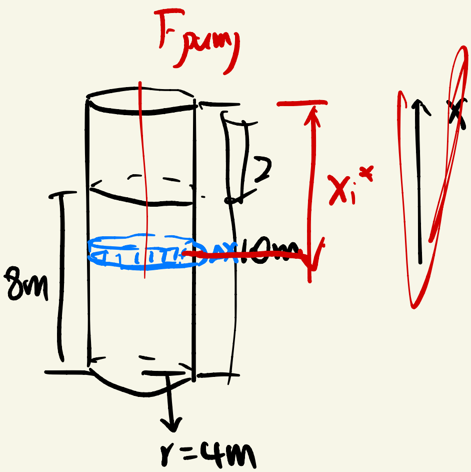
Tutorial 4

Work:

$$W = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx$$

T03 - Ex5

Assume a cylindrical tank of radius 4m and height 10m is filled to a depth of 8m. How much work does it take to pump all the water over the top edge of the tank?



the direction should make x_i^* positive.

$$V_i = \pi r^2 \Delta x$$

$$F_i = \rho V_i g$$

$$= \rho \pi r^2 \Delta x \, g \quad 8 \le x_i^* \le 10$$

$$W_i \approx F_i x_i^* = \rho \pi r^2 g \, x_i^* \Delta x$$

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \rho \pi r^2 g \, x_i^* \Delta x = \int_2^{10} \rho \pi r^2 g \, x dx$$

· Average Value of a Function

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

T04-1

① $f(t) = t^2 - 5t + 6\cos(\pi t)$ on $[-1, \frac{5}{2}]$

② $R(z) = \sin(2z)e^{1-\cos(2z)}$ on $[-\pi, \pi]$

① $$f_{ave} = \frac{1}{\frac{5}{2}-(-1)}\int_{-1}^{\frac{5}{2}} t^2 - 5t + 6\cos(\pi t)\,dt$$

② $$R(z) = \frac{1}{\pi-(-\pi)}\int_{-\pi}^{\pi} R(z)\,dz = 0$$

· Mean Value Thm. for Integrals

There exists $c \in [a,b]$ s.t. $f(c) = f_{ave}$.

T04-2

Determine the number c that satisfies the Mean value Thm. for Integrals for the func. $f(x) = x^2 + 3x + 2$ on $[1, 4]$

① Find f_{ave}

$$f_{ave} = \frac{1}{4-1} \int_1^4 f(x)\,dx = \frac{33}{2}$$

② Find c s.t. $f(c) = \frac{33}{2}$.

$$f(c) = c^2 + 3c + 2 = \frac{33}{2}$$

Δ

$$c = \frac{-3 \pm \sqrt{67}}{2}$$

Arc Length

$$L = \int ds = \begin{cases} \int_a^b \sqrt{1+[f'(x)]^2}\,dx & \text{if } y=f(x) \\ & a \le x \le b \\ \int_c^d \sqrt{1+[g'(y)]^2}\,dy & \text{if } x=g(y) \\ & c \le y \le d. \end{cases}$$

T04-3

Find the length of the curve $f(x) = 2e^x + \frac{1}{8}e^{-x}$ on the interval $[0, \ln 2]$.

① Observe y is a function of x

② $f'(x) = 2e^x - \frac{1}{8}e^{-x}$

$$L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

$$= \int_0^{\ln 2} \underbrace{\sqrt{1+[2e^x - \frac{1}{8}e^{-x}]^2}}_{\overset{\shortparallel}{A}}\,dx$$

$$= \int_0^{\ln 2} 2e^x + \frac{1}{8}\cdot e^{-x}\,dx.$$

$$A := \sqrt{1 + 4e^{2x} - \frac{1}{2} + \frac{1}{64}e^{-2x}}$$

$$= \sqrt{4e^{2x} + \frac{1}{2} + \frac{1}{64}e^{-2x}}$$

$$= \sqrt{\left(2e^{x} + \frac{1}{8}e^{-x}\right)^2}$$

$$= \left|2e^{x} + \frac{1}{8}e^{-x}\right|$$

$$= 2e^{x} + \frac{1}{8}e^{-x}$$

$$S(x) \triangleq \int_a^x \sqrt{1 + [f'(t)]^2}\, dt$$

T04-4

Find the arc length of $g(y) = \frac{1}{8}y^2 - \ln y$

from $y=1$ to $y=2$.

① x is a func. of y

② $g'(y) = \frac{1}{4}y - \frac{1}{y}$

③ $L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy$

$= \int_1^2 \sqrt{1 + (\frac{1}{4}y - \frac{1}{y})^2}\, dy$

$= \int_1^2 \frac{1}{4}y + \frac{1}{y}\, dy$

$= \left[\frac{1}{8}y^2 + \ln|y|\right]_1^2$

Area of a Surface of Revolution

Axis of rotation / The curve described by	x-axis	y-axis
	$S=\int 2\pi y\,ds$	$S=\int 2\pi x\,ds$
$y=f(x)$ $\quad a\le x\le b$ $ds=\sqrt{1+[f'(x)]^2}\,dx$	$S=\int_a^b 2\pi f(x)\,ds$ $=\int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$	
$x=g(y)$ $ds=\sqrt{1+[g'(y)]^2}\,dy$		

T04-5

$y = \sin(x)$, $0 \le x \le \pi$ rotating about x-axis

$$S = \int 2\pi y \, ds$$

$$= \int_0^{\pi} 2\pi y(x) \sqrt{1 + [y'(x)]^2} \, dx$$

$$= \int_0^{\pi} 2\pi \sin(x) \sqrt{1 + [\cos(x)]^2} \, dx$$

$u = \cos(x)$

T04-6

$y=\ln\left(\frac{x+\sqrt{x^2-1}}{2}\right)$. Find the area of the surface generated when the part of the curve between $\left(\frac{5}{4},0\right)$ and $\left(\frac{17}{8},\ln 2\right)$ is revolved about the y-axis.

$$S=\int 2\pi x \, ds$$

Rewrite $y=\ln\left(\frac{x^2+\sqrt{x^2-1}}{2}\right)$

$$e^y=\frac{x+\sqrt{x^2-1}}{2}$$

$$2e^y-x=\sqrt{x^2-1}$$

Take squares on both sides $4e^{2y}-4e^yx+\cancel{x^2}=\cancel{x^2}-1$

$$x=\frac{1+4e^{2y}}{4e^y}$$