Tutorial 3

Method of Slicing

① Determine the region $x=a$ and $x=b$
or
$y=c$ and $y=d$

② $A(x)$

③ $V=\int_a^b A(x)\,dx$

Volume by cylindrical shells

* For the solid obtained by rotation only

The volume of the solid obtained

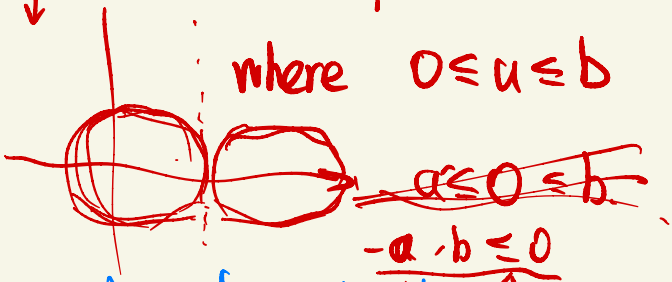
by rotating about the y-axis $(x=0)$

the region under the curve $y=f(x)$

from a to b

$$V=\int_a^b 2\pi \, \text{radius}(x) \cdot \text{height}(x) \, dx$$

where $0 \le a \le b$



~~$a \le 0 \le b$~~

$-a \cdot b \le 0$

Step ①

Determine ① the axis of rotation

② the region

$x=i$ or $y=j$

$(i=0$: y-axis$)$ $(j=0$: x-axis$)$

$[a,b]$ $[c,d]$

2. Determine the radius

height

(Positive!)

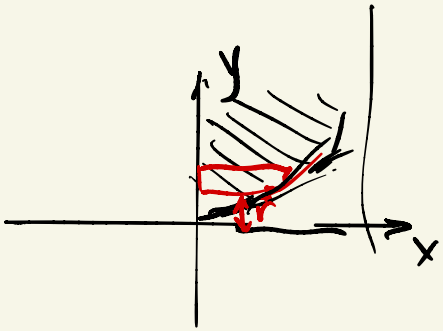
3 Apply the def $V = \int_a^b 2\pi \text{ radius}(x) \text{ height}(x) \, dx$

T03 - Ex 3

Define Q as the region bounded on the right by the graph of $g(y) = 2\sqrt{y}$ and on the left by the y-axis for $y \in [0, 4]$

$x = 2\sqrt{y} \Rightarrow y = \frac{x^2}{4}$

Find the volume of the solid of revolution formed by revolving Q around the x-axis.



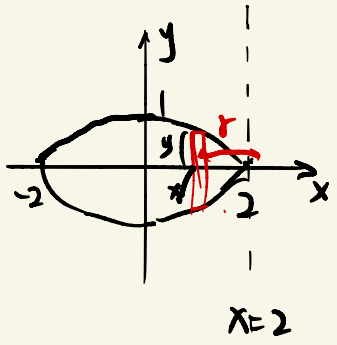
① rotating about x-axis ($y=0$) from $y=0$ to $y=4$

② radius $(y) = y$

height $(y) = g(y) - 0 = 2\sqrt{y}$

$$V = \int_0^4 2\pi y \cdot 2\sqrt{y}\, dy = 4\pi \int_0^4 y^{\frac{3}{2}} dy$$

④ Set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $x^2+4y^2=4$ about the line $x=2$.



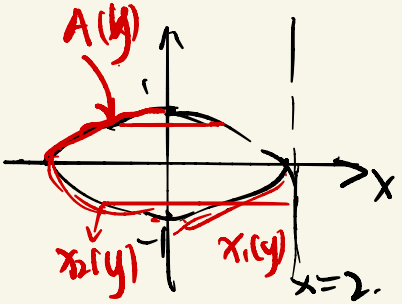
Let $y=0$ in $x^2+4y^2=4 \Rightarrow x=\pm 2$

① axis of rotation: $x=2$. Region: $x=-2$ to $x=2$

② height $(x) = 2y(x) = 2\sqrt{\frac{4-x^2}{4}} = \sqrt{4-x^2}$

radius $(x) = 2-x$

③ $V = \int_{-2}^{2} 2\pi \cdot (2-x) \cdot \sqrt{4-x^2}\, dx$



$x_2(y) = -\sqrt{4-4y^2}$

$x_1(y) = +\sqrt{4-4y^2}$

① Region : $y=-1$ to $y=1$.

② Crossection : Washer

outer radius $(y) = 2-(-\sqrt{4-4y^2})$

inner radius $(y) = 2-\sqrt{4-4y^2}$.

$A(x) = \pi$ (outer radius)^2 $-$ π (inner radius

$(a^2-b^2) = (a-b)(a+b)$

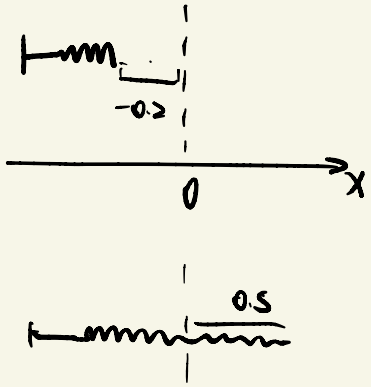
TO 3 - Ex 2

Define R as the region bounded above by ... $f(x) = \sqrt{x}$ below by $g(x) = \frac{1}{x}$ over the interval $[1, 4]$.

Find the volume of .... generated by revolving R around the y-axis.

$W_i =$

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \int_a^b f(x)\,dx$$



$f(x) = kx$

$-10N = k(-0.2)$

$k = 50\ N/m$

$W_i \approx f(x_i^*)\cdot\Delta x = k\cdot x_i^*\cdot\Delta x$

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} W_i = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} kx_i^*\Delta x$$

$$= k\int_0^{0.5} x\,dx$$

T03- Exercise 4

$= \frac{k}{8}$ $= \frac{50}{8}$ J

Sps it takes a force of 10N (in the negative direction) to compress a spring 0.2m from the eqm position.

How much work is done to stretch the spring 0.5m from the eqm position?

Exxise Regia

radius height

T03 - Ex5

Assume a cylindrical tank of radius 4m and height 10m is filled to a depth of 8m. How much work does it take to pump all the water over the top edge of the tank?