Tutorial 1

- The fundamental Thm. of Calculus

derivatives definite integral & indefinite integral

Sps f is continuous on $[a,b]$

① $g(x) = \int_a^{\boxed{x}} f(t)\,dt$ then $g'(x) = f(x)$

② $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

① Find the derivative of

$$y = \int_{-4}^{x^2} e^{2t} \cos^2(1-5t)\,dt$$

Step 1: Change the variable to get the same form

Let $u(x) = x^2$

$$y(u) = \int_{-4}^{u} e^{2t}\cos^2(1-5t)\,dt$$

Step 2: By Chain ru

$$y'(x) = y'(u)\cdot u'(x)$$

$$= e^{2u(x)}\cos^2(1-5u(x))\cdot 2x$$

$$= e^{2x^2}\cos^2(1-5x^2)\cdot 2x$$

② Integrate each of the following

(a) $\int_{-2}^{2} (4x^4 - x^2 + 1)\, dx = ①$

$4x^4 + x^3 - x^2 + 1$

Let $g(x) = 4x^4 - x^2 + 1$

$g(-x) = 4(-x)^4 - (-x)^2 + 1$

$= 4x^4 - x^2 + 1 = g(x)$

it is an even function.

$① = 2\int_{0}^{2} 4x^4 - x^2 + 1\, dx$

Method 1 : $= 2\left[\frac{4}{5}x^5 - \frac{1}{3}x^3 + x\right]_0^2$

$= 2\left[\left(\frac{4}{5}2^5 - \frac{1}{3}2^3 + 2\right) - (0)\right]$

Method 2

$$= 8\int_0^2 x^4\,dx - 2\int_0^2 x^2\,dx + 2(2-0)$$

$$= \frac{8}{5}\left[x^5\right]_0^2 - \frac{2}{3}\left[x^3\right]_0^2 + 4$$

$$\int_{-a}^{a} \qquad \int_{-a}^{b} = \underbrace{\int_{-a}^{a}}_{\Delta} + \int_{a}^{b}$$

② $\int_{-10}^{10} x^2 \sin(x^3)\,dx = 0$

$g(x) = x^2 \sin(x^3)$

$g(-x) = (-x)^2 \sin((-x)^3)$

$= -x^2 \sin(x^3) = -g(x)$

odd function

Step 1: Observe $[-a, a]$

Step 2: Check odd/even.

Substitution rule and the substitution rule for definite integrals

① $\int_0^{\ln(1+\pi)} e^x \cos(1-e^x)\,dx = ①$

Method 1

Let $u = 1-e^x$

$du = -e^x\,dx$

$x=0 \Rightarrow u = 1-e^0$
$= 1-1 = 0$

$x = \ln(1+\pi)$
$\Rightarrow u = 1-e^{\ln(1+\pi)}$
$= 1-(1+\pi)$
$= 1-1-\pi$
$= -\pi$

$① = -\int_0^{-\pi} \cos(u)\,du$

$= -\sin(u)\Big|_0^{-\pi}$

$= -\left(\sin(-\pi) - \sin(0)\right) = -(0-0) = 0$

② $\int_{-\pi}^{\frac{\pi}{2}} \cos(x) \cos(\sin(x))\, dx = ③$

Method 2: Substitution rule + F. Thm Calculus

Fundamental

We $\int \cos(x) \cos(\underbrace{\sin(x)}_{u})\, dx = ②$

Let $u = \sin x$

$du = \cos x \, dx$

$② = \int \cos(u)\, du = \sin(u) + C$

$= \sin(\sin(x)) + C.$

Use F. T. C

$③ = \sin(\sin(x)) \Big|_{-\pi}^{\frac{\pi}{2}} = \sin\left(\sin\left(\frac{\pi}{2}\right)\right) -$

$$= \sin\left(\sin\left(\frac{\pi}{2}\right)\right) - \sin\left(\sin(-\pi)\right)$$

$$= \sin(1) - \sin(0)$$

$$= \sin(1)$$

Area between the curves

$$S = \{(x,y) \mid a \le x \le b,\ g(x) \le y \le f(x)\}$$

$$\text{area}(S) = \int_a^b [f(x) - g(x)]\,dx$$

Two things to take care

① a, b ② $f(x) \le \ge g(x)$

① Find the area of region enclosed by $y = 6 - x$ and $y = 9 - \left(\frac{x}{2}\right)^2$

Step 1. Observe whether y is a function of x

or x .. y

y is a function of x

Step 2 : Find the intersection

$$\begin{cases} y_1 = 6-x \\ y_2 = 9-\left(\frac{x}{2}\right)^2 \end{cases}$$

$$6-x = 9-\frac{x^2}{4}$$

$$x^2-4x-12=0$$

$$(x+2)(x-6)=0$$

$$x_1=-2 \quad x_2=6$$

Step 3 Check the sign of $f(x)-g(x)$

① Draw the graph

② Sign chart.

$$y_1-y_2 = (6-x)-\left(9-\left(\frac{x}{2}\right)^2\right) \leq 0$$

$$=$$

Step 4 :

$$\text{Area}(S) = \int_{-2}^{6} \left(y_2(x) - y_1(x) \right) dx$$

$$= \int_{-2}^{6} \left(9 - \left(\frac{x}{2}\right)^2 \right) - (6 - x) \, dx$$

$$= \int_{-2}^{6} \left(-\frac{x^2}{4} + x + 3 \right) dx$$