Tutorial 5

- Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$$\boxed{\int x \, d\cos x}$$

$$\boxed{\int t^2 \, de^t}$$

$$\int t^4 e^t \, dt$$

T05-1 $\displaystyle\int t^7 \sin(2t^4)\, dt = A$

Let $\underbrace{u = 2t^4}_{4}$ $\quad du = 8t^3 dt$

$$A = \frac{1}{16}\int (2t^4) \cdot \sin(2t^4) \cdot (8t^3)\, dt.$$

$$= \frac{1}{16}\int u \sin u \, du$$

$$= \frac{-1}{16}\int u \, d\cos u$$

$$= -\frac{1}{16}\left[u\cos u - \int \cos u \, du\right]$$

$$= -\frac{1}{16}\left[u\cos u - \sin u\right] + C$$

$$= -\frac{1}{16}\left[2t^4\cos(2t^4) - \sin(2t^4)\right] + C$$

$$\int 6 \tan^{-1}\left(\frac{8}{w}\right) dw = A$$

$$u = \tan^{-1}\left(\frac{8}{w}\right) \longrightarrow du = \frac{1}{1+\left(\frac{8}{w}\right)^2} \cdot 8 \cdot (-1) \frac{1}{w^2} dw \quad \frac{-8}{w^2+64} dw$$

$$dv = 1 \cdot dw \longrightarrow v = w$$

$$A = 6 \int \tan^{-1}\left(\frac{8}{w}\right) dw$$

$$= 6\left[\tan^{-1}\left(\frac{8}{w}\right) \cdot w - \int w \cdot \frac{-8}{w^2+64} dw\right]$$

$$= 6 \tan^{-1}\left(\frac{8}{w}\right) w + \frac{48}{2} \int \frac{1}{w^2+64} d(w^2+64)$$

$$= \quad '' \quad + 24 \ln|w^2+64| + C$$

$$= \quad '' \quad + 24 \ln(w^2+64) + C$$

$$\int (\ln x)^2 \, dx$$

$$u = (\ln x)^2 \longrightarrow du = 2 \ln x \cdot \frac{1}{x} \, dx$$

$$dv = dx \longrightarrow v = x$$

$$A := (\ln x)^2 \cdot x - \int \cancel{x} \left(2 \cdot \ln x \cdot \cancel{\frac{1}{x}}\right) dx$$

$$= x (\ln x)^2 - 2 \int \ln x \, dx$$

$$s = \ln x \longrightarrow ds = \frac{1}{x} \, dx$$

$$dt = dx \longrightarrow t = x$$

$$= x (\ln x)^2 - 2 \left[\ln x \cdot x - \int x \cdot \frac{1}{x} \, dx \right]$$

$$= x (\ln x)^2 - 2x \ln x + 2x + C$$

- Trigonalmetric Integrals

$$\int \sin^m x \cos^n x \, dx$$

$$\sin^2 x + \cos^2 x = 1$$

$$\int$$

$\sin^2 x \cos^2 x$ $\quad$ $\sin^3 x \cos^2 x$ $\quad$ $\sin^2 x \cos^3 x$

save $\sin x$

$\cos^3 x \sin^3 x$

$$\int \tan^m x \sec^n x \, dx$$

$$\sec^2 x = \tan^2 x + 1$$

$$\sec^2 x \, dx = d\tan x$$

$$\tan x \sec x \, dx = d\sec x$$

$\tan^2 x \underline{\sec^2 x}$ $\quad$ $\tan^3 x \underline{\sec^2 x}$

$(\sec^2 x)$ $(\tan x \sec x)$

$\underline{\tan^3 x \sec^3 x}$ $\quad$ $\underline{\tan^3 x \sec^3 x}$

$(\tan x \sec x)$

$$\int \sin mx \cos nx \, dx$$

$$\int \sin mx \sin nx \, dx$$

$$\int \cos \quad \cos \quad dx.$$

Math1014 Exam Formula Sheet

Trigonometric Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$

$$\sin(A-B) = \sin A\cos B - \sin B\cos A$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\sin A\cos B = \frac{1}{2}\Big(\sin(A+B) + \sin(A-B)\Big)$$

$$\cos A\cos B = \frac{1}{2}\Big(\cos(A+B) + \cos(A-B)\Big)$$

$$\sin A\sin B = \frac{1}{2}\Big(\cos(A-B) - \cos(A+B)\Big)$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x dx$$

$$\int \sin^n x dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x dx$$

$$\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2}x\tan x + \frac{n-2}{n-1}\int \sec^{n-2}x dx$$

$$\int \tan^n x dx = \frac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x dx$$

$$\int \sec x dx$$

$$\int +$$

$$\int \tan^3(6x)\sec^{10}(6x)\,dx$$

Let $u = 6x$ $\quad du = 6\,dx$

$$A = 6\int \tan^3(u)\sec^{10}(u)\,du$$

$$= 6\int \tan^2(u)\sec^9(u)\ \underbrace{\left(\tan(u)\sec(u)\right)du}_{d\sec(u)}$$

$$= 6\int (\sec^2 u - 1)\sec^9(u) \qquad d\sec(u)$$

Let $t = \sec u = \sec(6x)$

$$A = 6\int (t^2-1)\ t^9 \quad dt$$

$$\int_1^3 \sin(8x)\sin x \, dx$$

$$= \int_1^3 \frac{1}{2}\left[\cos(8x-x) - \cos(8x+x)\right] dx$$

$$= \frac{1}{2}\int_1^3 \cos(7x)\, dx - \frac{1}{2}\int_1^3 \cos(9x)\, dx$$

$$= \frac{1}{\dots}\int_1^3 \cos(7x)\, d(7x)$$

$$\int \frac{2+7\sin^3(z)}{\cos^2(z)}\, dz$$

$$= 2\int \sec^2(z)\, dz + 7\int \frac{\sin^3(z)}{\cos^2(z)}\, dz$$

$$= 2\tan(z) - 7\int \frac{1-\cos^2(z)}{\cos^2(z)}\, d\cos(z)$$

Let $u = \cos(z)$

$$= 2\tan(z) - 7\int \frac{1-u^2}{u^2}\, du$$

$$= 2\tan(z) - 7\left[-\frac{1}{u} - u\right] + C$$

$$= 2\tan(z) + 7\cdot\sec(z) + 7\cos(z) + C$$