Torque: A measure of the force that can cause an object to rotate about an axis, <u>accelerate in angular</u>

Force: is what causes an object to <u>accelerate in linear kinematics</u>.

Thus, equation of motion get force/torque

$$\tau = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q}. \qquad \frac{d}{dt} P \stackrel{\alpha}{=} \frac{(mv)}{dt};$$

force/torque.

$$= M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)$$

Moment of inertia: rotational inertia. Same role as mass does in liear motion. $\frac{\text{torque}}{\text{angular accelation}}$ how hard it is to accelerate

Mass: $\frac{\text{force}}{\text{linear acceleration}}$.

Look Lagrangian again:

$$\text{Kinetic Energy}: T = \frac{1}{2} m v^T v + \frac{1}{2} w^T I w$$

$$= \frac{1}{2} (V^b)^T M V^b$$

where $V^b = \begin{bmatrix} v^b \\ w^b \end{bmatrix}$, I: moment of inertia 3x3

$$M = \begin{bmatrix} m\,\text{Identity}_3 & 0 \\ 0 & I \end{bmatrix}$$

Note that, for a point of mass, it has no rotational inertia unless it is not rotating around an external axis like 

for a 3D object, say a rectangular box, it has rotational inertia even if it does not rotate

so, for a point of mass, $L = \frac{1}{2} m \|V\|^2$

for a rect box $L = \frac{1}{2} m V^T V + \frac{1}{2} w^T I w$

for I, typical form is $\begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}$

when the principal axes of rotation align with the coordinate axes, it will be $\begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$

A point of mass

$T = \frac{1}{2} m \|V\|^2$

2D

$T = \frac{1}{2} m \|V\|^2 + \frac{1}{2} \underbrace{I \dot{\theta}^2}_{\text{scalar}}$

3D

$T = \frac{1}{2} m \|V^2\| + \frac{1}{2} W^T I W$

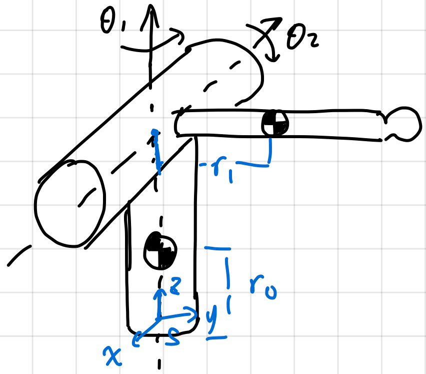
vector

matrix.

Calculate inertia

$\uparrow I \Rightarrow$ angular acceleration $\downarrow$ with same force.

points how far away $\uparrow$ from the center, inertia $\uparrow$



Given θ_1, θ_2

calculate KE $T = \frac{1}{2}(V^b)^T \mathcal{M} V^b$

How to map θ space to velocity space ? Jacobian.

$$V^b = J^b(\theta)\,\dot{\theta}$$

Thus $T = \frac{1}{2}(J^b(\theta)\dot{\theta})^T \mathcal{M} (J^b(\theta)\dot{\theta})$

$$= \frac{1}{2}\dot{\theta}^T (J^b)^T \mathcal{M} J^b \dot{\theta}$$

$$= \frac{1}{2}\sum_{i=0}^{n} (J_i^b \dot{\theta})^T \mathcal{M}_i J_i^b \dot{\theta}$$