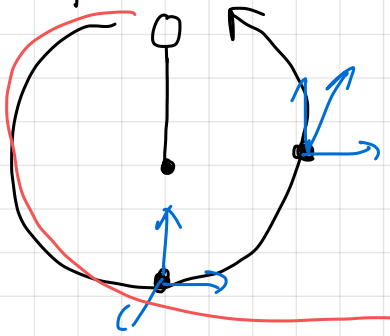
Dynamics : how a robot deals with forces

Inertial frame : A frame that is not accelarating.

i.e. { move at a constant velocity
stationary.

Rotating Reference frame

⊗ Centripetal force is a REAL force



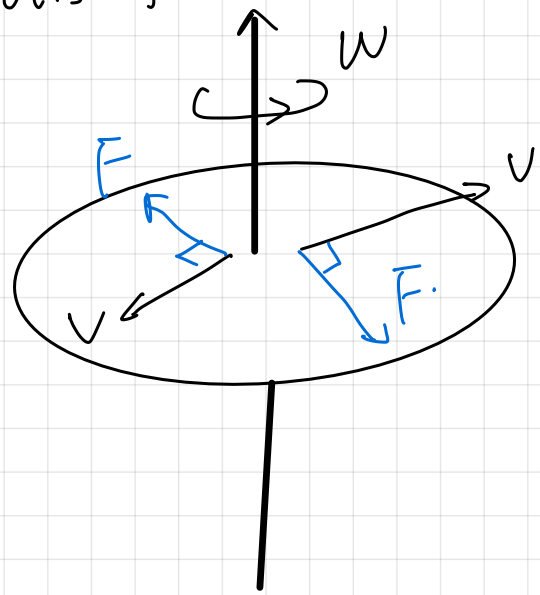
Thus, centripetal force provides acceleration

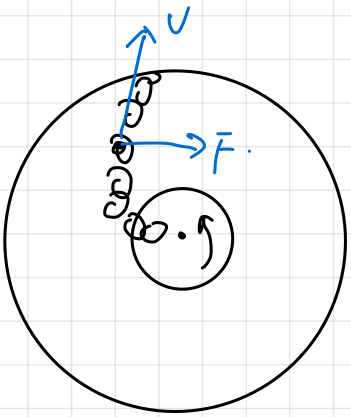
⇒ rotating reference frame is not inertial frame

without centripetal force.

$$F = -mrw^2 = -mr\dot{\theta}^2$$

coriolis force : $F = 2mv \times w$





Complicated to determine Newton-Euler dynamics because of fictious force.

Lagrangian Dynamics

Energy is always conserved.

Find force equations in terms of energy equations.

$KE + PE = \text{Constant}$

how force works in terms of energy. $F \cdot d$

Steps:

① Pick generalized coordinates q (position / angle) minimum info to describe system. only consider changing things

② Find KE, PE.

③ Take Lagrangian $L = T - V$ and find its derivatives.

$$T = \frac{1}{2}[v^T \; w^T]\begin{bmatrix} mI_3 & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix}$$

$$= \frac{1}{2}[v^T \; w^T]\begin{bmatrix} mv \\ Iw \end{bmatrix} = \frac{1}{2}mv^Tv + \frac{1}{2}w^TIw$$

trans KE — rotation KE

④ $$\tau = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q}$$ $\frac{d}{dx}$ Fox.

τ: external forces/torques.

$\frac{\partial L}{\partial \dot q}$: "momentum term" ⇓ the time derivative ⇒ Force $(m\frac{dv}{dt} = ma)$

$\frac{\partial L}{\partial q}$: "work term"

$V_g = mgh \quad V_s = \frac{1}{2}kx^2$

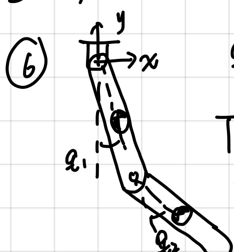
$$m\ddot{x} - mg + kx = 0$$

$+0\dot{x}$ → Coriolis Matrix $C(q)$ (fictitious forces.)

$M(q)$ Inertial Matrix; $G(q)$

equation of motion of the system

⑥



$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$

$T = \frac{1}{2}mV_1^2 + \frac{1}{2}I_1\dot{q}_1^2 + \frac{1}{2}mV_2^2 + \frac{1}{2}I_2(\dot{q}_1+\dot{q}_2)^2$

$V_1^2 = (-\frac{L}{2}\dot{q}_1\sin q_1)^2 + (\frac{L}{2}\dot{q}_1\cos q_1)^2 = \frac{L^2}{4}\dot{q}_1^2$

$V_2^2 = (L\sin q_1 + \frac{L}{2}\sin(q_1+q_2))^2 + (-L\cos q_1 - \frac{L}{2}\cos(q_1+q_2))^2$

$V = -\frac{L}{2}mg\cos q_1 - mg(L\cos q_1 + \frac{L}{2}\cos(q_1+q_2))$

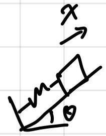
①

$q = [x]$

$L = \frac{1}{2}m\dot{x}^2$

$V = \frac{1}{2}kx^2 - mgx$

②



$q = [x]$

$L = \frac{1}{2}m\dot{x}^2$

$V = \frac{1}{2}kx^2 + mgx\cdot\sin\theta$

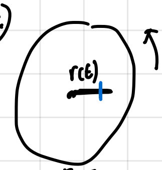
③

$q = [\theta_1 \theta_2 \theta_3]^T$

$L = 0 + \frac{1}{2}I\dot{\theta}_1^2 + \frac{1}{2}m_2V_2^2 + \frac{1}{2}m_2\dot{\theta}_2^2 + \frac{1}{2}m_3V_3^2 + \frac{1}{2}m_3\dot{\theta}_3^2 \quad V = 0$

$V_2^2 = \dot{x}_2^2 + \dot{y}_2^2 = (L\cos\theta_1)^2 + (L\sin\theta_1)^2 = (-L\sin\theta_1\cdot\dot{\theta}_1)^2 + (L\cos\theta_1\dot{\theta}_1)^2 = L^2\dot{\theta}_1^2$

④



$q = \begin{bmatrix} r \\ \theta \end{bmatrix}$

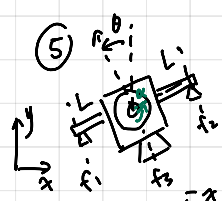
$L = 0 + \frac{1}{2}I\dot{\theta}_1^2 + \frac{1}{2}mV^2$

$V = \dot{x}^2 + \dot{y}^2 = (r(t)\cos\theta)^2 + (r(t)\sin\theta)^2$

$= (\dot{r}\cos\theta - r(t)\sin\theta\,\dot{\theta})^2 + (\dot{r}\sin\theta + r(t)\cos\theta\,\dot{\theta})^2$

$= \dot{r}^2 + r(t)\dot{\theta}^2$

⑤



$q = \begin{bmatrix} x \\ y \\ \theta \\ \alpha \end{bmatrix}$

$L = \frac{1}{2}m_b(\dot{x}^2+\dot{y}^2) + \frac{1}{2}I_b\dot{\theta}^2 + \frac{1}{2}m_r(\dot{x}^2+\dot{y}^2) + \frac{1}{2}I_r(\dot{\theta}+\dot{\alpha})^2$

$\tau_x = -(f_1+f_2+f_3)\sin\theta$

$\tau_y = (f_1+f_2+f_3)\cos\theta$

$\tau_\theta = (f_2 - f_1)L - \tau$

$\tau_\alpha = \tau$

HW

$$r_{i/o} = \begin{bmatrix} -\frac{il}{n}\cdot\cos\theta \\ \frac{il}{n}\cdot\sin\theta \end{bmatrix}$$

$$\dot{r}_{i/o} = \begin{bmatrix} -\frac{il}{n}\sin\theta\,\dot{\theta} \\ \frac{il}{n}\cos\theta\,\dot{\theta} \end{bmatrix}$$

reaction wheel: $\frac{1}{2} I_r (\dot{\theta}+\dot{\alpha})^2 + \frac{1}{2} m_b (\dot{x}^2+\dot{y}^2)$

body: $\frac{1}{2} I_b \dot{\theta}^2 + \frac{1}{2} m_r (\dot{x}^2+\dot{y}^2)$