1. each x_i is an independent Bernoulli random variable with $p \in [0,1]$, then:

$$\begin{cases} P(X_i=1) = p \\ p(X_i=0) = 1-p \end{cases}$$

(a) $P(X_1=1) = p$

(b) $P(X_1=1, X_2=1) = P(X_1=1) \cdot P(X_2=1) = p \cdot p = p^2$

(c) $P(X_1=1, X_2=1, X_3=0) = P(X_1=1) \cdot P(X_2=1) \cdot P(X_3=0)$

$= p \cdot p \cdot (1-p)$

$= p^2(1-p)$

(d) $P(X_1=1, X_2=1, \cdots X_n=1, X_{n+1}=0) = P(X_1=1)P(X_2=1)\cdots P(X_n=1)P(X_{n+1}=0)$

$= p \cdot p \cdot \cdots p \cdot (1-p)$

$= p^n(1-p)$

(e) (i) If we flip tail immediately, then $Y = 0$

For H, H, T, $Y = 1$

then $Y \in \{0, 1, 2, \dots\}$ i.e. Natural Numbers

(ii) For (H, H, T)

$P(Y=2) = P(\text{heads, heads, tail})$

$= p \cdot p \ (1-p)$

$= p^2(1-p)$

So for $P(Y=y) = p^y (1-p)$

(f) Positive return $p = 0.53$

Negative return $1-p = 0.47$

P(first 5 days are all positive) + P(first 6 days are positive)

+ P(first 7 days are positive) + ⋯

$$= \sum_{k=5}^{\infty} p^k (1-p) + \lim_{k \to \infty} p^k.$$

But, we cannot sum infinitely, then.

$$p^5(1-p) + p^6(1-p) + \cdots + p^n(1-p) + p^{n+1}$$

Let n be any finite cutoff, like 10, 20 or others.

$$\text{then Probability} = \sum_{k=5}^{n} p^k(1-p) + p^{n+1}$$