

LAB4 MATLAB Programming

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Introduction

- 1.Using Matlab to calculate the approximation of Continuous-time Fourier Transform
- 2.Using abs and angle to calculate the magnitude and the phase of the signal.
3. Using lsim(b,a,x,t) to calculate the output of the differential system
- 4.Using freqs(b,a,w) = H to calculate the frequency response of the system
- 5.Using $X = \text{tua} * \text{fft}(x)$ to calculate $X(jw)$

Results and analysis

4.2

(a)

Find an analytic expression for the CTFT of $x(t) = e^{-2|t|}$. Y
think of $x(t) = g(t) + g(-t)$, where $g(t) = e^{-2t}u(t)$.

Let

$$X(t) = e^{-2|t|} = g(t) + g(-t) = e^{-2t}u(t) + e^{2t}u(-2t)$$

The Fourier Transform for $X(t)$ is

$$X(j\omega) = \frac{1}{2 + j\omega} + \frac{1}{2 - j\omega}$$

(b)

Create a vector containing samples of the signal $y(t) = x(t - 5)$ over the range $t = [0 : \tau : T - \tau]$. Since $x(t)$ is effectively zero outside the range $t \in [-5, 5]$, you can calculate the CTFT of the signal $y(t) = x(t - 5)$ from $N = T/\tau$. Your vector y should have length N .

$$Y(j\omega) = e^{j\omega(-5)}X(j\omega) = \frac{e^{j\omega(-5)}}{2 + j\omega} + \frac{e^{j\omega(-5)}}{2 - j\omega}$$

```
tau = 0.01;
T = 10;
t = 0:tau:T-tau;
N = T / tau;
x = exp(-2 * t) .* heaviside(t) + exp(2 * t) .* heaviside(-t);
y = exp(-2 * (t - 5)) .* heaviside(t - 5) + exp(2 * (t - 5)) .* heaviside(-(t - 5));
N
```

$N = 1000$

(c)

Calculate samples $Y(j\omega_k)$ by typing `Y=fftshift(tau*fft(y))`

It outputs a coefficient that has been fftshifted, when the zero frequency would be in the middle of the Y sequence.

```
Y = fftshift(tau*fft(y));
```

(d)

Construct a vector w of frequency samples that correspond to vector Y as follows

```
>> w = -(pi/tau)+(0:N-1)*(2*pi/(N*tau));
```

```
w = -(pi / tau) + (0: N - 1) * (2 * pi / (N * tau));
```

(e)

Since $y(t)$ is related to $x(t)$ through a time shift, the CTFT $X(j\omega)$ is related to $Y(j\omega)$ by a linear phase term of the form $e^{j5\omega}$. Using the frequency samples of $X(j\omega)$ directly from Y , storing the result in the vector X

$y(t) = x(t-5)$ shifted 5 units to the right, So the time domain signal obtained by shifting $y(t)$ left by 5 units is the answer we need to ask for. We just need to multiply inside the frequency domain by a $e^{j5\omega}$

```
X = exp(-1j*5*w).*Y;
```

(f)

Using `abs` and `angle`, plot the magnitude and phase of X versus w specified in w . For the same values of ω , also plot the magnitude and phase of the analytic expression you derived for $X(j\omega)$ in Part (a). Do the results of the CTFT match what you derived analytically? If you plot the magnitude on a logarithmic scale, using `semilogy`, you will notice that the approximation is not as good as at lower frequencies. Since $x(t)$ is a decaying exponential, your approximation will be better for frequencies of the signal that do not vary much over time intervals of length τ .

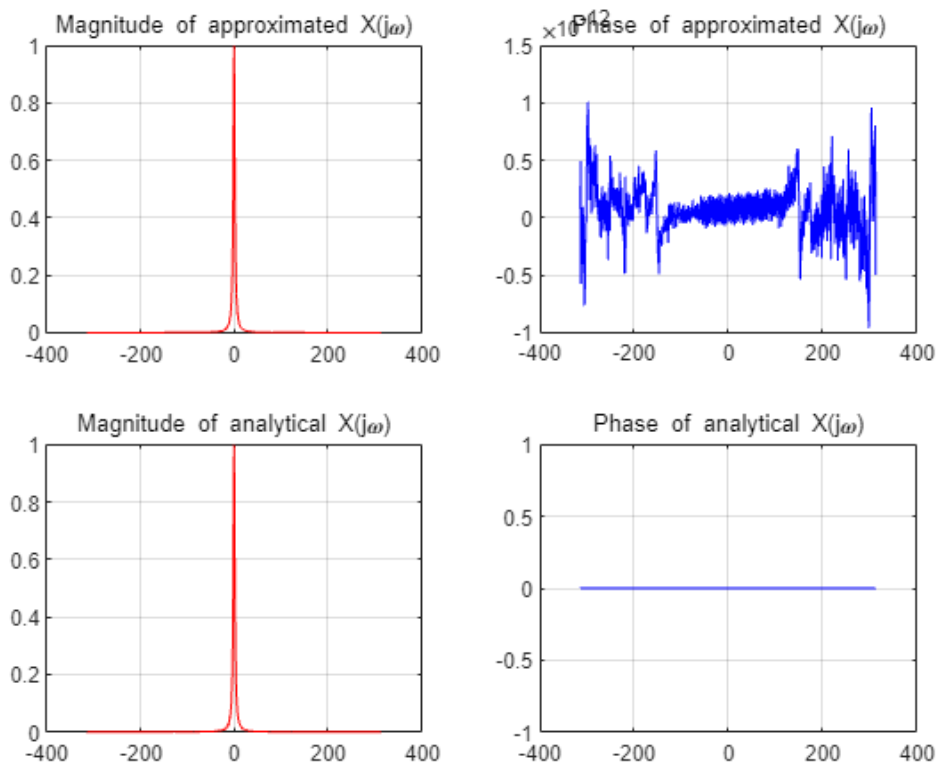
plot

```
X_analytic = 1 ./ (2 + 1j * w) + 1 ./ (2 - 1j * w);
```

```

figure
subplot(2,2,1)
plot(w,abs(X),'r');
title('Magnitude of approximated X(j\omega)');
grid on;
subplot(2,2,2)
plot(w, angle(X), 'b')
title('Phase of approximated X(j\omega)')
grid on;
subplot(2,2,3)
plot(w, abs(X_analytic), 'r')
title('Magnitude of analytical X(j\omega)')
grid on;
subplot(2,2,4)
plot(w, angle(X_analytic), 'b')
title('Phase of analytical X(j\omega)')
grid on;

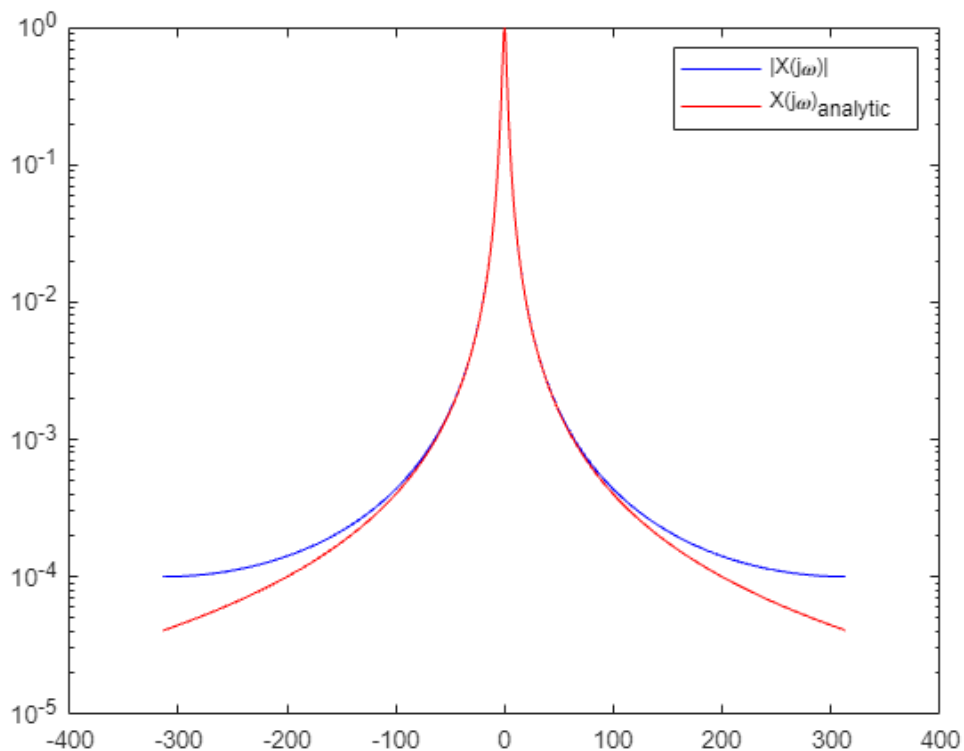
```



```

figure
semilogy(w,abs(X),'b')
hold on
semilogy(w,abs(X_analytic),'r')
hold off
legend('|X(j\omega)|','X(j\omega)_{analytic}')

```



The difference between the value of X and the resolved value is relatively large in the high-frequency region.

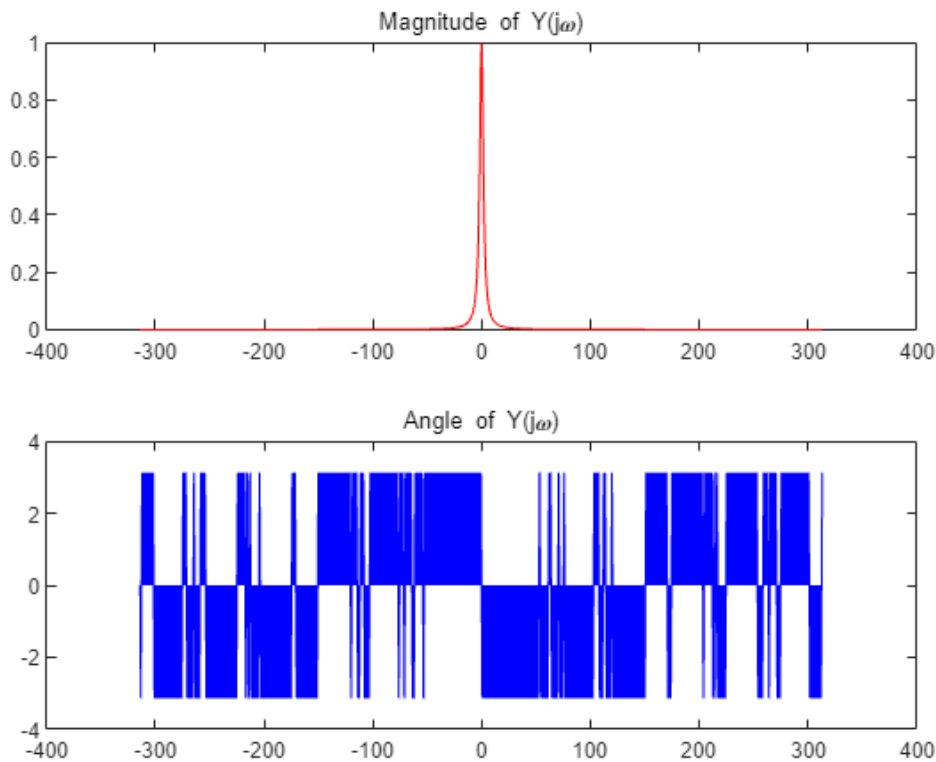
The CTFT provides a more accurate description of frequency components that do not vary much in the τ interval, but the analysis of high frequencies suffers from large errors.

(g)

Plot the magnitude and phase of Y using `abs` and `angle`. How does this compare with X ? Could you have anticipated this result?

Plot the phase and amplitude of Y against X

```
figure
subplot(2, 1, 1)
plot(w, abs(Y), 'r')
title('Magnitude of  $Y(j\omega)$ ')
subplot(2, 1, 2)
plot(w, angle(Y), 'b')
title('Angle of  $Y(j\omega)$ ')
```



X and Y are the same in magnitude but distinct in phase. That is because the time shifting was reflected as a phase shifting in frequency domain.

4.6

```
clear;clc;
load lab4\ctftmod.mat
who
```

您的变量为:

```
af    bf    dash  dot    f1    f2    t     x
```

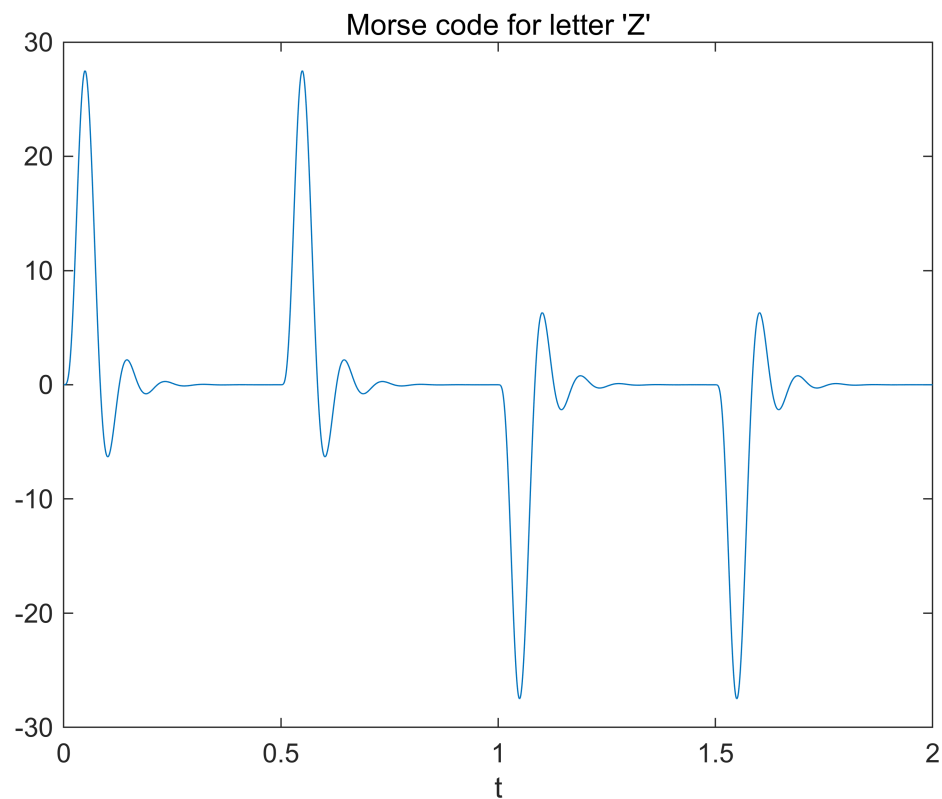
(a)

Using the signals **dot** and **dash**, construct the signal that represents the letter 'Z' in Morse code, and plot it against **t**. As an example, the letter 'A' is represented by typing **c = [dash dot dash dot]**. Store your signal $z(t)$.

Write simple morse code using the material provided in the title

```
z = [dash dash dot dot];
figure
```

```
plot(t,z);
title("Morse code for letter 'Z'")
xlabel("t");
```

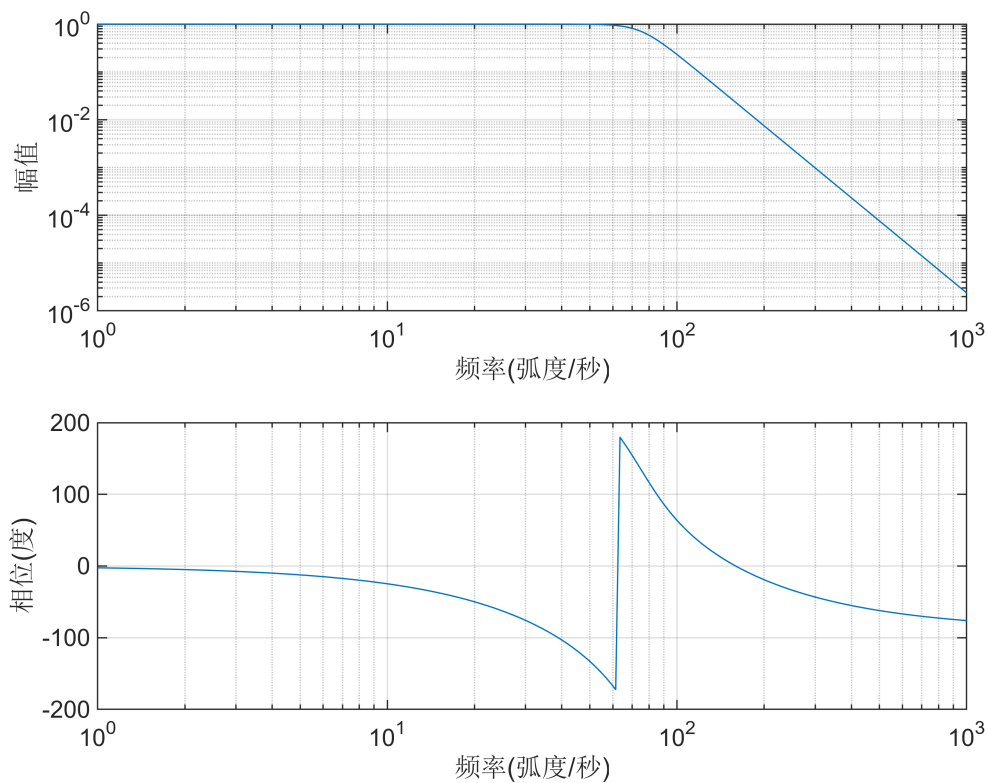


(b)

Plot the frequency response of the filter using `freqs(bf,af)`

plot

```
freqs(bf,af)
```



(c)

The signals `dot` and `dash` are each composed of low frequency components. Their Fourier transforms lie roughly within the passband of the filter. Illustrate this by filtering each of the two signals using

```
>> ydash=lsim(bf,af,dash,t(1:length(dash)));
>> ydot=lsim(bf,af,dot,t(1:length(dot)));
```

Plot the outputs `ydash` and `ydot` along with the original signals.

Input the `dot` and `dash` signals into the low-pass filter and observe the waveforms.

```
ydash = lsim(bf, af, dash, t(1: length(dash)));
ydot = lsim(bf, af, dot, t(1: length(dot)));

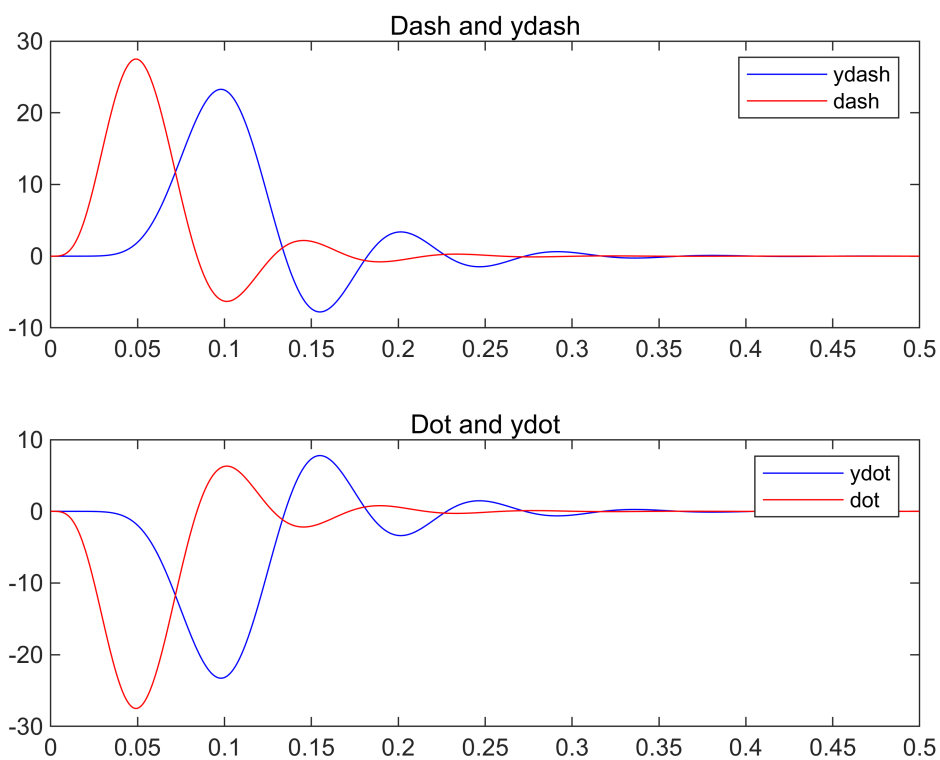
figure
subplot(2, 1, 1)
```



```

plot(t(1: length(dash)), ydash, 'b')
hold on;
plot(t(1: length(dash)), dash, 'r')
title('Dash and ydash')
legend('ydot', 'dot')
subplot(2, 1, 2)
plot(t(1: length(dot)), ydot, 'b')
hold on
plot(t(1: length(dot)), dot, 'r')
title('Dot and ydot')
legend('ydot', 'dot')
hold on
plot(t(1: length(dot)), dot, 'r')
title('Dot and ydot')
legend('ydot', 'dot')

```



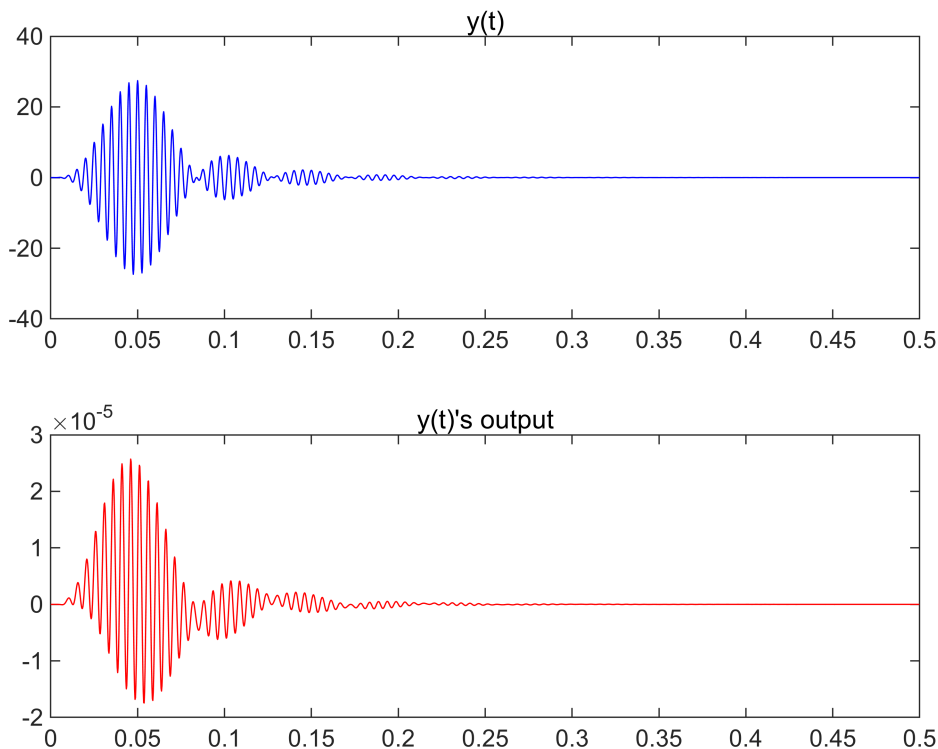
(d)

When the signal `dash` is modulated by $\cos(2\pi f_1 t)$, most of the transform will move outside the passband of the filter. On executing `y=dash*cos(2*pi*f1*t(1:length(dash)))`. Plot the output `yo=lsim(bf,af,y,t)`. Do you get a response as expected?

The dot signal is modulated with the frequency `f1`, while the modulated signal is fed into a low-pass filter. The two outputs are visualised using the `plot` function

```
y = dash .* cos(2 * pi * f1 * t(1: length(dash)));  
yo = lsim(bf, af, y, t(1: length(dash)));
```

```
figure  
subplot(2, 1, 1)  
plot(t(1: length(dash)), y, 'b')  
title('y(t)')  
subplot(2, 1, 2)  
plot(t(1: length(dash)), yo, 'r')  
title('y(t)'s output')
```



As can be seen in the figure, the waveform of the modulated dot signal is basically the same before and after passing through the filter, but the energy is greatly compressed after passing through the low-pass filter.

(e)

Determine analytically the Fourier transform of each of the

$$m(t) \cos(2\pi f_1 t) \cos(2\pi f_1 t),$$

$$m(t) \cos(2\pi f_1 t) \sin(2\pi f_1 t),$$

and

$$m(t) \cos(2\pi f_1 t) \cos(2\pi f_2 t),$$

in terms of $M(j\omega)$, the Fourier transform of $m(t)$.

$$Y_1(j\omega) = \frac{1}{2} \left(\frac{1}{2} M[j(\omega - 4\pi f_1)] + \frac{1}{2} M(j\omega) \right) + \frac{1}{2} \left(\frac{1}{2} M(j\omega) + \frac{1}{2} M[j(\omega + 4\pi f_1)] \right) = \frac{1}{4} M[j(\omega - 4\pi f_1)] + \frac{1}{2} M(j\omega) + \frac{1}{4} M[j(\omega + 4\pi f_1)]$$

$$Y_2(j\omega) = \frac{1}{2j} \left(\frac{1}{2} M[j(\omega - 4\pi f_1)] + \frac{1}{2} M(j\omega) \right) - \frac{1}{2j} \left(\frac{1}{2} M(j\omega) + \frac{1}{2} M[j(\omega + 4\pi f_1)] \right) = \frac{1}{4j} M[j(\omega - 4\pi f_1)] - \frac{1}{4j} M[j(\omega + 4\pi f_1)]$$

$$\begin{aligned} Y_3(j\omega) &= \frac{1}{2} \left(\frac{1}{2} M[j(\omega - 2\pi f_1 - 2\pi f_2)] + \frac{1}{2} M[j(\omega - 2\pi f_1 + 2\pi f_2)] \right) + \frac{1}{2} \left(\frac{1}{2} M[j(\omega + 2\pi f_1 - 2\pi f_2)] + \frac{1}{2} M[j(\omega + 2\pi f_1 + 2\pi f_2)] \right) \\ &= \frac{1}{4} M[j(\omega - 2\pi f_1 - 2\pi f_2)] + \frac{1}{4} M[j(\omega - 2\pi f_1 + 2\pi f_2)] + \frac{1}{4} M[j(\omega + 2\pi f_1 - 2\pi f_2)] + \frac{1}{4} M[j(\omega + 2\pi f_1 + 2\pi f_2)] \end{aligned}$$

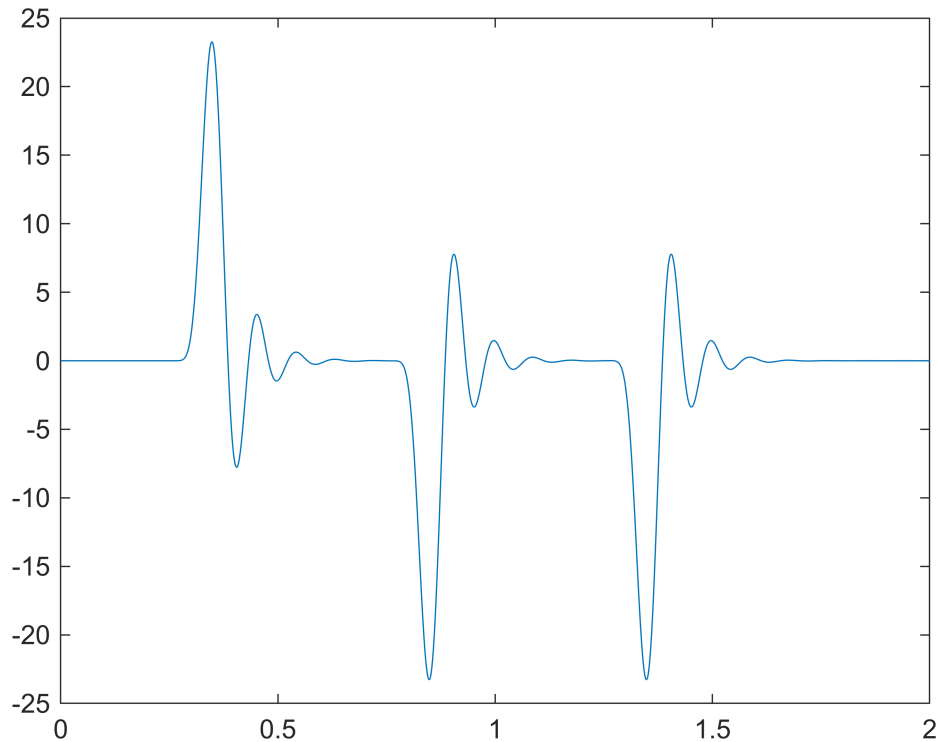
(f)

Using your results from Part (e) and by examining the frequency spectrum as plotted in Part (b), devise a plan for extracting the signal $m_1(t)$ and determine which letter is represented in Message 1.

Demodulate the m_1 component of the x signal. Since the modulating signal of m_1 is a cos-type signal of f_1 , the demodulation should be done by using the same cos-type signal of f_1 to re-translate the original signal to the origin. The components of the signal that are at the origin are low frequency signals and can be passed through

the filter given in the title. Note that the demodulated signal is fed into the filter and multiplied by two to return the energy lost in the demodulation process

```
figure
m1=2*lsim(bf,af,x.*cos(2*pi*f1*t),t);
plot(t,m1);
```



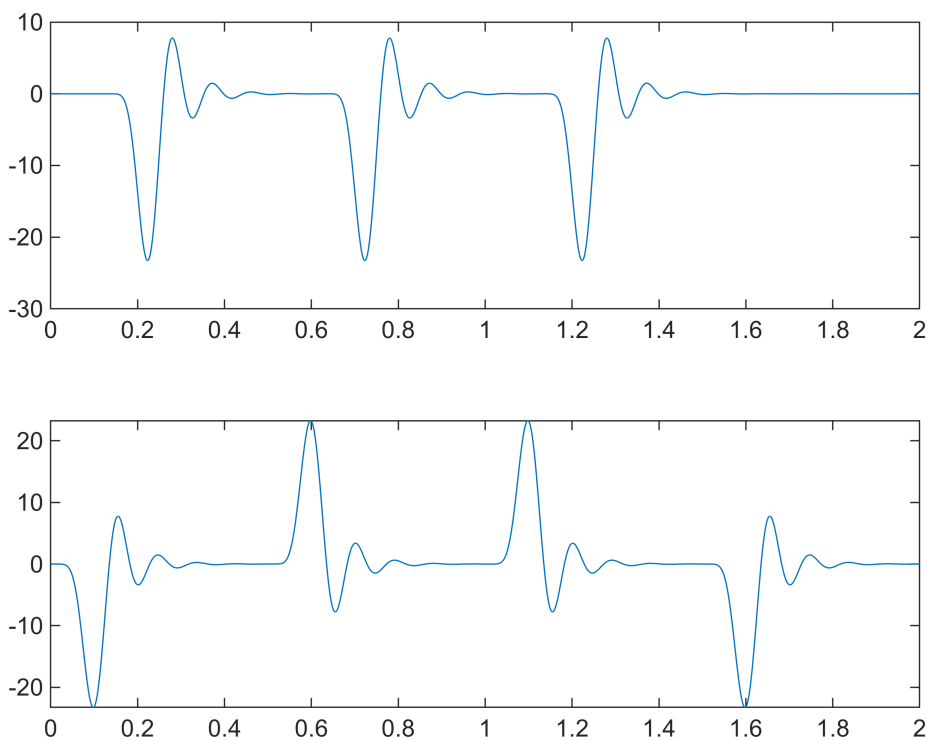
According to the given table , we can know that the letter 1 is 'D'

(g)

Repeat Part (f) for the signals $m_2(t)$ and $m_3(t)$. Agent 008, technology lie?

same as (f)

```
figure
subplot(2,1,1)
m2=2*lsim(bf,af,x.*sin(2*pi*f2*t),t);
plot(t,m2);
subplot(2,1,2)
m3=2*lsim(bf,af,x.*sin(2*pi*f1*t),t);
plot(t,m3);
```

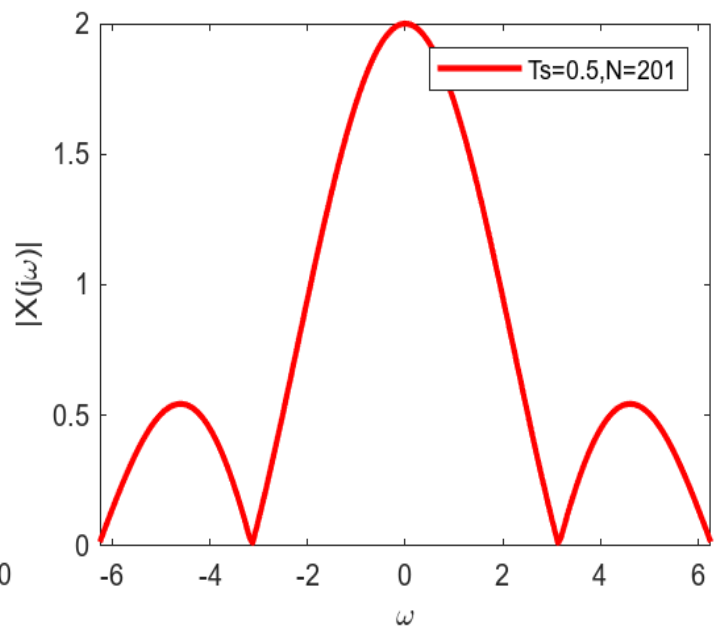
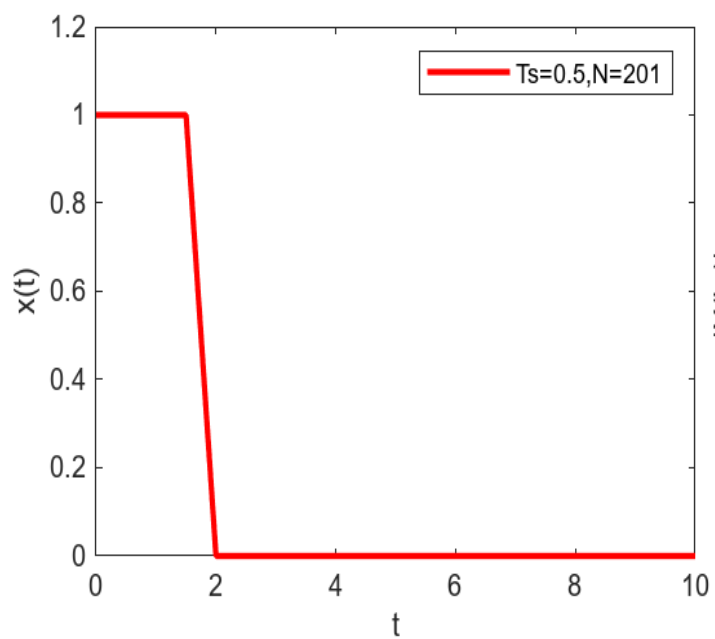


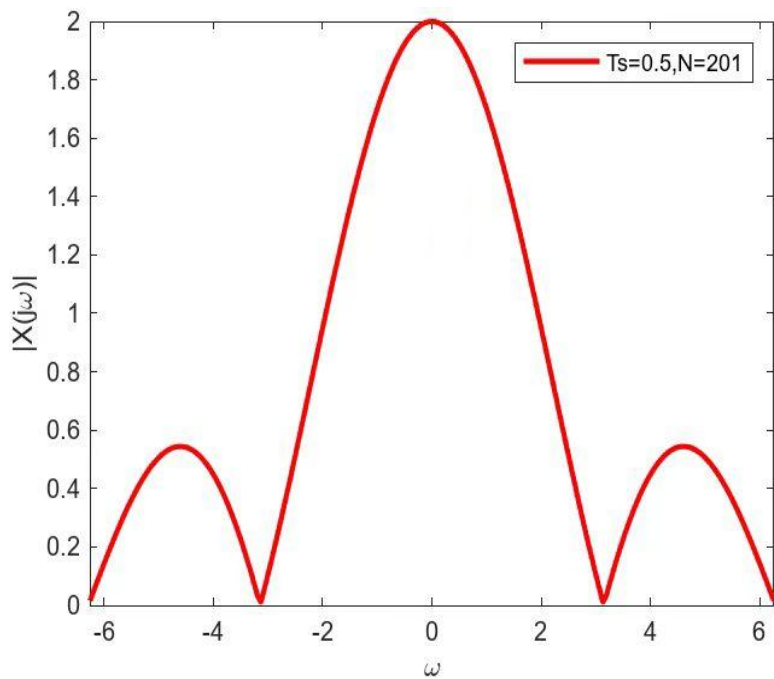
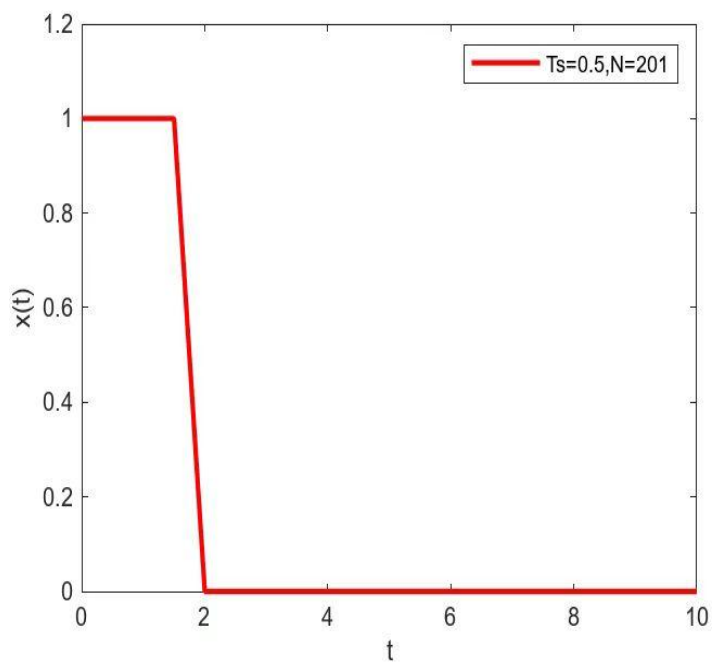
According to the given table , we can know that the letter 2 is 's', the letter 3 is 'P'

the final answer is 'DSP'(Digital Signal Processing)--数字信号处理

Experience

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1. In 4.6, it's fun to use what you've learnt to decode Morse Code!
2. Reading the book carefully before solving the problem.

self—scoring

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