1 Data-Structure

1.1 Treap

```
template<class S,
    S (*node_pull)(S, S),
    S (*node_init)(S),
    class T,
    S (*mapping)(S, T),
    T (*tag_pull)(T, T),
    T (*tag_init)()>
struct Treap{
  struct node{
    node *l = NULL,*r = NULL,*p = NULL;
    const int pri = rand();
    int sz = 1;
    S info;
    T tag = tag_init();
    bool rev;
    node(S k) : info(k){}
    ~node(){
      for(auto &i:{l,r})
        delete i;
    }
    void all_apply(T t,bool is_rev){
      if(is_rev){
        swap(l,r);
        rev^=1;
      }
      info = mapping(info, t);
      tag = tag_pull(tag, t);
    }
    void push(){
      for(auto &i:{l,r})
        if(i)i->all_apply(tag, rev);
      tag = tag_init();
      rev = 0;
    }
    void pull(){
      sz = 1,info = node_init(info);
      for(auto &i:{l,r}){
        if(i){
          sz+=i->sz,i->p = this;
          info = node_pull(info,i->info);
        }
      }
    }
  };
  node *root = NULL;
  int size(node *a){
    return a?a->sz:0;
  }
  int size(){
    return size(root);
  }
  node *merge(node *a,node *b){
    if(!a or !b)return a?:b;
    if(a->pri>b->pri){
      a->push();
      a->r = merge(a->r,b);
      a->r->p = a;
      a->pull();
      return a;
    }
    else{
      b->push();
      b->l = merge(a,b->l);
      b->l->p = b;
      b->pull();
      return b;
    }
  }
  void split(node *t, long long k, node *&a, node *&b, const
      bool &bst){
    if(!t){a = b = NULL;return;}
    t->push();
    if((bst==0 and size(t->l)+1<=k) or (bst==1 and t->info.key
        <=k)){
      a = t;
      split(t->r, ( bst ? k : k - size(t->l) - 1 ), a->r, b,
          bst);
      if(b)b->p = NULL;
      a->pull();
    }
    else{
      b = t;
      split(t->l, k, a, b->l, bst);
      if(a)a->p = NULL;
      b->pull();
    }
  }
  node *insert(long long idx, S x,bool bst = 0){
    node *a,*b;
    split(root, idx, a, b, bst);
    node *tmp = new node(x);
    root = merge(a, merge(tmp, b));
    return tmp;
  }
  void erase(long long l,long long r,bool bst = 0){
    node *a,*b,*c;
    split(root, (bst? l-1 : l), a, b, bst);
    split(b, (bst? r : r - l + 1), b, c, bst);
    delete b;
    root = merge(a,c);
  }
  S operator [](int x){
    node *a, *b, *c;
    split(root, x, a, b, 0);
    split(b, 1, b, c, 0);
    assert(b!=NULL);
    S ans = b->info;
    root = merge(a, merge(b, c));
    return ans;
  }
  int rank(long long k){
    node *a, *b;
    split(root, k - 1, a, b, 1);
    int ans = size(a);
    root = merge(a, b);
    return ans;
  }
  S* find_next(long long k){
    node *a, *b, *c;
    split(root, k - 1, a, b, 1);
    split(b, 1, b, c, 0);
    S* ans = NULL;
    if(b)ans = &b->info;
    root = merge(a, merge(b, c));
    return ans;
  }
  S* find_prev(long long k){
    node *a, *b, *c;
    split(root, k , a, b, 1);
    split(a, size(a) - 1, a, c, 0);
    S* ans = NULL;
    if(c)ans = &c->info;
    root = merge(merge(a, c), b);
    return ans;
  }
  void update(long long l,long long r,T t,bool bst = 0){
    node *a, *b, *c;
    split(root, (bst? l - 1 : l), a, b, bst);
    split(b, (bst? r : r - l + 1), b, c, bst);
    if(b)b->all_apply(t, 0);
    root = merge(a, merge(b, c));
  }
  void reverse(long long l,long long r,bool bst = 0){
    node *a, *b, *c;
    split(root, (bst? l - 1 : l), a, b, bst);
    split(b, (bst? r : r - l + 1), b, c, bst);
    if(b)b->all_apply(tag_init(), 1);
    root = merge(a, merge(b, c));
  }
  S query(long long l,long long r,bool bst = 0){
    node *a, *b, *c;
    split(root, (bst? l - 1 : l), a, b, bst);
    split(b, (bst? r : r - l + 1), b, c, bst);
    S ans;
    if(b)ans = b->info;
    root = merge(a, merge(b, c));
    return  ans;
  }
};
```

1.2 Segtree

```
template<class S,
     S (*node_pull)(S, S),
     S (*node_init)(),
     class T,
     S (*mapping)(S, T),
     T (*tag_pull)(T, T),
     T (*tag_init)()>
struct segment_tree{
  struct node{
    S seg;
    T tag = tag_init();
    int l,r;
    node(S _seg = node_init(),int _l = -1,int _r = -1) : seg(
        _seg), l(_l), r(_r){}
    friend node operator +(const node &lhs,const node &rhs){
      if(lhs.l==-1)return rhs;
      if(rhs.l==-1)return lhs;
      return node(node_pull(lhs.seg,rhs.seg),lhs.l,rhs.r);
    };
  };
  vector<node>arr;
  void all_apply(int idx,T t){
```

```
    arr[idx].seg = mapping(arr[idx].seg, t);
    arr[idx].tag = tag_pull(arr[idx].tag, t);
  }
  void push(int idx){
    all_apply(idx<<1, arr[idx].tag);
    all_apply(idx<<1|1, arr[idx].tag);
    arr[idx].tag = tag_init();
  }
  inline void build(const vector<S> &v,const int &l,const int &r,int idx = 1){
    if(idx==1)arr.resize((r-l+1)<<2);
    if(l==r){
      arr[idx].seg = v[l];
      arr[idx].tag = tag_init();
      arr[idx].l = arr[idx].r = l;
      return;
    }
    int m = (l+r)>>1;
    build(v,l,m,idx<<1);
    build(v,m+1,r,idx<<1|1);
    arr[idx] = arr[idx<<1]+arr[idx<<1|1];
  }
  inline void update(const int &ql,const int &qr,T t,int idx = 1){
    assert(ql<=qr);
    if(ql<=arr[idx].l and arr[idx].r<=qr){
      all_apply(idx, t);
      return;
    }
    push(idx);
    int m = (arr[idx].l+arr[idx].r)>>1;
    if(ql<=m)update(ql,qr,t,idx<<1);
    if(qr>m)update(ql,qr,t,idx<<1|1);
    arr[idx] = arr[idx<<1]+arr[idx<<1|1];
  }
  inline S query(const int &ql,const int &qr,int idx = 1){
    assert(ql<=qr);
    if(ql<=arr[idx].l and arr[idx].r<=qr){
      return arr[idx].seg;
    }
    push(idx);
    int m = (arr[idx].l+arr[idx].r)>>1;
    S ans = node_init(),lhs = node_init(),rhs = node_init();
    if(ql<=m)lhs = query(ql,qr,idx<<1);
    if(qr>m)rhs = query(ql,qr,idx<<1|1);
    ans = node_pull(lhs,rhs);
    return ans;
  }
};
```

1.3 DsuUndo

```
struct dsu_undo{
  vector<int>sz,p;
  int comps;
  dsu_undo(int n){
    sz.assign(n+5,1);
    p.resize(n+5);
    for(int i = 1;i<=n;++i)p[i] = i;
    comps = n;
  }
  vector<pair<int,int>>opt;
  int Find(int x){
    return x==p[x]?x:Find(p[x]);
  }
  bool Union(int a,int b){
    int pa = Find(a),pb = Find(b);
    if(pa==pb)return 0;
    if(sz[pa]<sz[pb])swap(pa,pb);
    sz[pa]+=sz[pb];
    p[pb] = pa;
    opt.push_back({pa,pb});
    comps--;
    return 1;
  }
  void undo(){
        auto [pa,pb] = opt.back();
        opt.pop_back();
        p[pb] = pb;
        sz[pa]-=sz[pb];
        comps++;
  }
};
```

1.4 DSU

```
struct DSU{
  vector<int>sz;
  int n;
  DSU(int _n):n(_n){
    sz.assign(n+1,-1);
  }
  int Find(int x){
    return sz[x]<0?x:sz[x] = Find(sz[x]);
  }
  bool Union(int a,int b){
    int pa = Find(a),pb = Find(b);
    if(pa==pb)return 0;
    if((-sz[pa])<(-sz[pb]))swap(pa,pb);
    sz[pa]+=sz[pb];
    sz[pb] = pa;
    return 1;
  }
};
```

1.5 Fenwick

```
template<class T>struct fenwick_tree{
  int n;
  vector<T>arr;
  inline int lowbit(int x){
    return x&(-x);
  }
  fenwick_tree(int _n) : n(_n){
    arr.assign(n+5,0);
  }
  T query(int x){
    T ans = 0;
    for(int i = x;i>0;i-=lowbit(i)){
      ans+=arr[i];
    }
    return ans;
  }
  void update(int x,T y){
    for(int i = x;i<=n;i+=lowbit(i)){
      arr[i]+=y;
    }
  }
};
```

1.6 Persistent DSU

```
int rk[200001] = {};
struct Persistent_DSU{
  rope<int>*p;
  int n;
  Persistent_DSU(int _n = 0):n(_n){
    if(n==0)return;
    p = new rope<int>;
    int tmp[n+1] = {};
    for(int i = 1;i<=n;++i)tmp[i] = i;
    p->append(tmp,n+1);
  }
  Persistent_DSU(const Persistent_DSU &tmp){
    p = new rope<int>(*tmp.p);
    n = tmp.n;
  }
  int Find(int x){
    int px = p->at(x);
    return px==x?x:Find(px);
  }
  bool Union(int a,int b){
    int pa = Find(a),pb = Find(b);
    if(pa==pb)return 0;
    if(rk[pa]<rk[pb])swap(pa,pb);
    p->replace(pb,pa);
    if(rk[pa]==rk[pb])rk[pa]++;
    return 1;
  }
};
```

1.7 TimingSegtree

```
template<class T,class D>struct timing_segment_tree{
  struct node{
    int l,r;
    vector<T>opt;
  };
  vector<node>arr;
  void build(int l,int r,int idx = 1){
    if(idx==1)arr.resize((r-l+1)<<2);
    if(l==r){
      arr[idx].l = arr[idx].r = l;
      arr[idx].opt.clear();
      return;
    }
    int m = (l+r)>>1;
    build(l,m,idx<<1);
    build(m+1,r,idx<<1|1);
    arr[idx].l = l,arr[idx].r = r;
```

```
    arr[idx].opt.clear();
  }
  void update(int ql,int qr,T k,int idx = 1){
    if(ql<=arr[idx].l and arr[idx].r<=qr){
      arr[idx].opt.push_back(k);
      return;
    }
    int m = (arr[idx].l+arr[idx].r)>>1;
    if(ql<=m)update(ql,qr,k,idx<<1);
    if(qr>m)update(ql,qr,k,idx<<1|1);
  }
  void dfs(D &d,vector<int>&ans,int idx = 1){
    int cnt = 0;
    for(auto [a,b]:arr[idx].opt){
      if(d.Union(a,b))cnt++;
    }
    if(arr[idx].l==arr[idx].r)ans[arr[idx].l] = d.comps;
    else{
      dfs(d,ans,idx<<1);
      dfs(d,ans,idx<<1|1);
    }
    while(cnt--)d.undo();
  }
};
```

1.8 AreaOfRectangles

```
long long AreaOfRectangles(vector<tuple<int,int,int,int>>v){
  vector<tuple<int,int,int,int>>tmp;
  int L = INT_MAX,R = INT_MIN;
  for(auto [x1,y1,x2,y2]:v){
    tmp.push_back({x1,y1+1,y2,1});
    tmp.push_back({x2,y1+1,y2,-1});
    R = max(R,y2);
    L = min(L,y1);
  }
  vector<long long>seg((R-L+1)<<2),tag((R-L+1)<<2);
  sort(tmp.begin(),tmp.end());
  function<void(int,int,int,int,int,int)>update = [&](int ql,
       int qr,int val,int l,int r,int idx){
    if(ql<=l and r<=qr){
      tag[idx]+=val;
      if(tag[idx])seg[idx] = r-l+1;
      else if(l==r)seg[idx] = 0;
      else seg[idx] = seg[idx<<1]+seg[idx<<1|1];
      return;
    }
    int m = (l+r)>>1;
    if(ql<=m)update(ql,qr,val,l,m,idx<<1);
    if(qr>m)update(ql,qr,val,m+1,r,idx<<1|1);
    if(tag[idx])seg[idx] = r-l+1;
    else seg[idx] = seg[idx<<1]+seg[idx<<1|1];
  };
  long long last_pos = 0,ans = 0;
  for(auto [pos,l,r,val]:tmp){
    ans+=(pos-last_pos)*seg[1];
    update(l,r,val,L,R,1);
    last_pos = pos;
  }
  return ans;
}
```

1.9 SparseTable

```
template<class T,T (*op)(T,T)>struct sparse_table{
  int n;
  vector<vector<T>>mat;
  sparse_table(): n(0){}
  sparse_table(const vector<T>&v){
    n = (int)(v.size());
    mat.resize(30);
    mat[0] = v;
    for(int i = 1;(1<<i)<=n;++i){
      mat[i].resize(n-(1<<i)+1);
      for(int j = 0;j<n-(1<<i)+1;++j){
        mat[i][j] = op(mat[i-1][j],mat[i-1][j+(1<<(i-1))]);
      }
    }
  }
  T query(int ql,int qr){
    int k = __lg(qr-ql+1);
    return op(mat[k][ql],mat[k][qr-(1<<k)+1]);
  }
};
```

1.10 VEBTree

```
// Can correctly work with numbers in range [0; MAXN]
// Supports all std::set operations in O(1) on random queries /
     dense arrays, O(log_64(N)) in worst case (sparce array).
// Count operation works in O(1) always.
template<unsigned int MAXN>
class fast_set {
private:
  static const unsigned int PREF = (MAXN <= 64 ? 0 :
                MAXN <= 4096 ? 1 :
                MAXN <= 262144 ? 1 + 64 :
                MAXN <= 16777216 ? 1 + 64 + 4096 :
                MAXN <= 1073741824 ? 1 + 64 + 4096 + 262144 :
                     227) + 1;
  static constexpr unsigned long long lowest_bitsll[] = {0ULL,
       1ULL, 3ULL, 7ULL, 15ULL, 31ULL, 63ULL, 127ULL, 255ULL,
       511ULL, 1023ULL, 2047ULL, 4095ULL, 8191ULL, 16383ULL,
       32767ULL, 65535ULL, 131071ULL, 262143ULL, 524287ULL,
       1048575ULL, 2097151ULL, 4194303ULL, 8388607ULL, 16777215
       ULL, 33554431ULL, 67108863ULL, 134217727ULL, 268435455ULL
       , 536870911ULL, 1073741823ULL, 2147483647ULL, 4294967295
       ULL, 8589934591ULL, 17179869183ULL, 34359738367ULL,
       68719476735ULL, 137438953471ULL, 274877906943ULL,
       549755813887ULL, 1099511627775ULL, 2199023255551ULL,
       4398046511103ULL, 8796093022207ULL, 17592186044415ULL,
       35184372088831ULL, 70368744177663ULL, 140737488355327ULL,
        281474976710655ULL, 562949953421311ULL, 1125899906842623
       ULL, 2251799813685247ULL, 4503599627370495ULL,
       9007199254740991ULL, 18014398509481983ULL,
       36028797018963967ULL, 72057594037927935ULL,
       144115188075855871ULL, 288230376151711743ULL,
       576460752303423487ULL, 1152921504606846975ULL,
       2305843009213693951ULL, 4611686018427387903ULL,
       9223372036854775807ULL, 18446744073709551615ULL};
  static const unsigned int SZ = PREF + (MAXN + 63) / 64 + 1;
  unsigned long long m[SZ] = {0};

  inline unsigned int left(unsigned int v) const {
    return (v - 62) * 64;
  }

  inline unsigned int parent(unsigned int v) const {
    return v / 64 + 62;
  }

  inline void setbit(unsigned int v) {
    m[v >> 6] |= 1ULL << (v & 63);
  }

  inline void resetbit(unsigned int v) {
    m[v >> 6] &= ~(1ULL << (v & 63));
  }

  inline unsigned int getbit(unsigned int v) const {
    return m[v >> 6] >> (v & 63) & 1;
  }

  inline unsigned long long childs_value(unsigned int v) const
       {
    return m[left(v) >> 6];
  }

  inline int left_go(unsigned int x, const unsigned int c)
       const {
    const unsigned long long rem = x & 63;
    unsigned int bt = PREF * 64 + x;
    unsigned long long num = m[bt >> 6] & lowest_bitsll[rem + c
         ];
    if(num) {
      return (x ^ rem) | __lg(num);
    }
    for(bt = parent(bt); bt > 62; bt = parent(bt)) {
      const unsigned long long rem = bt & 63;
      num = m[bt >> 6] & lowest_bitsll[rem];
      if(num) {
        bt = (bt ^ rem) | __lg(num);
        break;
      }
    }
    if(bt == 62) {
      return -1;
    }
    while(bt < PREF * 64) {
      bt = left(bt) | __lg(m[bt - 62]);
    }
    return bt - PREF * 64;
  }

  inline int right_go(unsigned int x, const unsigned int c)
       const {
    const unsigned long long rem = x & 63;
    unsigned int bt = PREF * 64 + x;
    unsigned long long num = m[bt >> 6] & ~lowest_bitsll[rem +
         c];
    if(num) {
      return (x ^ rem) | __builtin_ctzll(num);
    }
    for(bt = parent(bt); bt > 62; bt = parent(bt)) {
```

```
      const unsigned long long rem = bt & 63;
      num = m[bt >> 6] & ~lowest_bitsll[rem + 1];
      if(num) {
        bt = (bt ^ rem) | __builtin_ctzll(num);
        break;
      }
    }
    if(bt == 62) {
      return -1;
    }
    while(bt < PREF * 64) {
      bt = left(bt) | __builtin_ctzll(m[bt - 62]);
    }
    return bt - PREF * 64;
  }

public:
  fast_set() {
    assert(PREF != 228);
    setbit(62);
  }

  bool empty() const {return getbit(63);}

  void clear() {
    fill(m, m + SZ, 0);
    setbit(62);
  }

  bool count(unsigned int x) const {
    return m[PREF + (x >> 6)] >> (x & 63) & 1;
  }

  void insert(unsigned int x) {
    for(unsigned int v = PREF * 64 + x; !getbit(v); v = parent(v)) {
      setbit(v);
    }
  }

  void erase(unsigned int x) {
    if(!getbit(PREF * 64 + x)) {
      return;
    }
    resetbit(PREF * 64 + x);
    for(unsigned int v = parent(PREF * 64 + x); v > 62 && !childs_value(v); v = parent(v)) {
      resetbit(v);
    }
  }

  int find_next(unsigned int x) const {
    return right_go(x, 0);
  }

  int find_prev(unsigned int x) const {
    return left_go(x, 1);
  }
};
```

1.11 DynamicSegtree

```

template<class T>struct dynamic_segment_tree{
  struct node{
    node *l = NULL,*r = NULL;
    T sum;
    node(T k = 0): sum(k){}
    node(node *p){if(p)*this = *p;}
    ~node(){
      for(auto &i:{l,r})
        if(i)delete i;
    }
    void pull(){
      sum = 0;
      for(auto i:{l,r})
        if(i)sum+=i->sum;
    }
  }*root = NULL;
  int n;
  dynamic_segment_tree(){}
  dynamic_segment_tree(const dynamic_segment_tree<T>&tmp){root = new node(tmp.root);}
  void update(node *&t,int pos,T k,int l,int r){
    if(!t)t = new node();
    if(l==r)return t = new node(k),void();
    int m = (l+r)>>1;
    t = new node(t);
    if(pos<=m)update(t->l,pos,k,l,m);
    else update(t->r,pos,k,m+1,r);
    t->pull();
  }void update(int pos,T k,int l = -1e9,int r = 1e9){update(root,pos,k,l,r);}
  T query(node *&t,int ql,int qr,int l,int r){
    if(!t)return 0;
    if(ql<=l and r<=qr)return t->sum;
    int m = (l+r)>>1;
    T ans = 0;
    if(ql<=m)ans+=query(t->l,ql,qr,l,m);
    if(qr>m)ans+=query(t->r,ql,qr,m+1,r);
    return ans;
  }T query(int ql,int qr,int l = -1e9,int r = 1e9){return query(root,ql,qr,l,r);}
};
```

1.12 ZkwSegtree

```
template<class S,
         S (*node_pull)(S, S),
         S (*node_init)(),
         class F,
         S (*mapping)(S, F),
         F (*tag_pull)(F, F),
         F (*tag_init)()>
class segment_tree {
public:
  segment_tree() : segment_tree(0) {}
  explicit segment_tree(int _n) : segment_tree(vector<S>(_n, node_init())) {}
  explicit segment_tree(const vector<S>& v) : n((int) v.size()) {
    log = std::__lg(2 * n - 1);
    size = 1 << log;
    d = vector<S>(size << 1, node_init());
    lz = vector<F>(size, tag_init());
    for(int i = 0; i < n; i++) {
      d[size + i] = v[i];
    }
    for(int i = size - 1; i; --i) {
      update(i);
    }
  }
  void set(int p, S x) {
    assert(0 <= p && p < n);
    p += size;
    for(int i = log; i; --i) {
      push(p >> i);
    }
    d[p] = x;
    for(int i = 1; i <= log; ++i) {
      update(p >> i);
    }
  }
  S get(int p) {
    assert(0 <= p && p < n);
    p += size;
    for(int i = log; i; i--) {
      push(p >> i);
    }
    return d[p];
  }
  S operator[](int p) {
    return get(p);
  }
  S query(int l, int r) {
    r++;
    assert(l<=r);
    l += size;
    r += size;
    for(int i = log; i; i--) {
      if(((l >> i) << i) != l) {
        push(l >> i);
      }
      if(((r >> i) << i) != r) {
        push(r >> i);
      }
    }
    S sml = node_init(), smr = node_init();
    while(l < r) {
      if(l & 1) {
        sml = node_pull(sml, d[l++]);
      }
      if(r & 1) {
        smr = node_pull(d[--r], smr);
      }
      l >>= 1;
      r >>= 1;
    }
    return node_pull(sml, smr);
  }
  void apply(int p, F f) {
    assert(0 <= p && p < n);
    p += size;
    for(int i = log; i; i--) {
      push(p >> i);
    }
    d[p] = mapping(f, d[p]);
    for(int i = 1; i <= log; i++) {
```

```
      update(p >> i);
    }
  }
  void update(int l, int r, F f) {
    r++;
    assert(l<=r);
    l += size;
    r += size;
    for(int i = log; i; i--) {
      if(((l >> i) << i) != l) {
        push(l >> i);
      }
      if(((r >> i) << i) != r) {
        push((r - 1) >> i);
      }
    }
    {
      int l2 = l, r2 = r;
      while(l < r) {
        if(l & 1) {
          all_apply(l++, f);
        }
        if(r & 1) {
          all_apply(--r, f);
        }
        l >>= 1;
        r >>= 1;
      }
      l = l2;
      r = r2;
    }
    for(int i = 1; i <= log; i++) {
      if(((l >> i) << i) != l) {
        update(l >> i);
      }
      if(((r >> i) << i) != r) {
        update((r - 1) >> i);
      }
    }
  }
private:
  int n, size, log;
  vector<S> d;
  vector<F> lz;
  inline void update(int k) { d[k] = node_pull(d[k << 1], d[k
      << 1 | 1]); }
  void all_apply(int k, F f) {
    d[k] = mapping(d[k], f);
    if(k < size) {
      lz[k] = tag_pull(lz[k], f);
    }
  }
  void push(int k) {
    all_apply(k << 1, lz[k]);
    all_apply(k << 1 | 1, lz[k]);
    lz[k] = tag_init();
  }
};
```

1.13 MoAlgo

```
struct qry{
  int ql,qr,id;
};
template<class T>struct Mo{
  int n,m;
  vector<pii>ans;
  Mo(int _n,int _m): n(_n),m(_m){
    ans.resize(m);
  }
  void solve(vector<T>&v,vector<qry>&q){
    int l = 0,r = -1;
    vector<int>cnt,cntcnt;
    cnt.resize(n+5);
    cntcnt.resize(n+5);
    int mx = 0;
    function<void(int)>add = [&](int pos){
      cntcnt[cnt[v[pos]]]--;
      cnt[v[pos]]++;
      cntcnt[cnt[v[pos]]]++;
      mx = max(mx,cnt[v[pos]]);
    };
    function<void(int)>sub = [&](int pos){
      if(!--cntcnt[cnt[v[pos]]] and cnt[v[pos]]==mx)mx--;
      cnt[v[pos]]--;
      cntcnt[cnt[v[pos]]]++;
      mx = max(mx,cnt[v[pos]]);
    };
    sort(all(q),[&](qry a,qry b){
      static int B = max((int)1,n/max((int)sqrt(m),(int)1));
      if(a.ql/B!=b.ql/B)return a.ql<b.ql;
      if((a.ql/B)&1)return a.qr>b.qr;
      return a.qr<b.qr;
    });
    for(auto [ql,qr,id]:q){
      while(l>ql)add(--l);
      while(r<qr)add(++r);
      while(l<ql)sub(l++);
      while(r>qr)sub(r--);
      ans[id] = {mx,cntcnt[mx]};
    }
  }
};
```

1.14 Hash

```
struct custom_hash {
  static uint64_t splitmix64(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM = chrono::steady_clock::
        now().time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
  }
  size_t operator()(pair<uint64_t,uint64_t> x) const {
    static const uint64_t FIXED_RANDOM = chrono::steady_clock::
        now().time_since_epoch().count();
    return splitmix64(3*x.first + x.second + FIXED_RANDOM);
  }
};
template<class T,class U>using hash_map = gp_hash_table<T,U,
    custom_hash>;
```

1.15 RedBlackTree

```
template<class T, typename cmp=less<>>struct _tree{//#include<
    bits/extc++.h>
  tree<pair<T,int>,null_type,cmp,rb_tree_tag,
      tree_order_statistics_node_update>st;
  int id = 0;
  void insert(T x){st.insert({x,id++});}
  void erase(T x){st.erase(st.lower_bound({x,0}));}
  int order_of_key(T x){return st.order_of_key(*st.lower_bound
      ({x,0}));}
  T find_by_order(int x){return st.find_by_order(x)->first;}
  T lower_bound(T x){return st.lower_bound({x,0})->first;}
  T upper_bound(T x){return st.upper_bound({x,(int)1e9+7})->
      first;}
  T smaller_bound(T x){return (--st.lower_bound({x,0}))->first
      ;}
};
```

2 Geometry

2.1 Theorem

- Pick' s Theorem

  $A = I + \frac{B}{2} - 1$

  $A := Area$

  $i := PointsInside$

  $B := PointsBoundary$

2.2 PointInPolygon

```
template<class T>
int PointInPolygon(const vector<Point<T>> &Poly, const Point<T>
    p){
  int ans = 0;
  for(auto a = --Poly.end(),b = Poly.begin();b!=Poly.end();a =
      b++){
    if(PointOnSegment(*a,*b,p)){
      return -1;
    }
    if(seg_intersect(p,p+Point<T>(2e9+7,1),*a,*b)){
      ans = !ans;
    }
  }
  return ans;
}
```

2.3 PointInConvex

```cpp
template<class T>
int PointInConvex(const vector<Point<T>>&C,const Point<T>&p){
  if(btw(C[0],C[1],p) || btw(C[0],C.back(),p))return -1;
  int l = 0,r = (int)C.size()-1;
  while(l<=r){
    int m = (l+r)>>1;
    auto a1 = (C[m]-C[0])^(p-C[0]);
    auto a2 = (C[(m+1)%C.size()]-C[0])^(p-C[0]);
    if(a1>=0 and a2<=0){
      auto res = (C[(m+1)%C.size()]-C[m])^(p-C[m]);
      return res > 0 ? 1 : (res >= 0 ? -1 : 0);
    }
    if(a1 < 0) r = m-1;
    else l = m+1;
  }
  return 0;
}
```

2.4 MaximumDistance

```cpp
template<class T>
T MaximumDistance(vector<Point<T>>&p){
  vector<Point<T>>C = ConvexHull(p,0);
  int n = C.size(),t = 2;
  T ans = 0;
  for(int i = 0;i<n;i++){
    while(((C[i] - C[t]) ^ (C[(i+1)%n] - C[t])) < ((C[i] - C[(t
        +1)%n]) ^ (C[(i+1)%n] - C[(t+1)%n]))) t = (t + 1)%n;
    ans = max({ans, abs2(C[i] - C[t]), abs2(C[(i+1)%n] - C[t])
        });
  }
  return ans;
}
```

2.5 PolarAngleSort

```cpp
template<class T>
bool cmp(const Point<T> &a,const Point<T> &b){
  int lhs = (a.y < 0 || a.y==0 && a.x > 0) ? 0 : (1 + (a.x != 0
      || a.y != 0));
  int rhs = (b.y < 0 || b.y==0 && b.x > 0) ? 0 : (1 + (b.x != 0
      || b.y != 0));
  if(lhs != rhs) {
    return lhs < rhs;
  }
  long long area = (a^b);
  return area ? area > 0 : abs(a.x) + abs(a.y) < abs(b.x) + abs
      (b.y);
}
```

2.6 MinimumDistance

```cpp
template<class T>
T MinimumDistance(vector<Point<T>>&p,int l = -1,int r = -1){
  if(l==-1 and r==-1){
    sort(p.begin(),p.end(),[](Point<T> a,Point<T> b){
      if(a.x!=b.x)return a.x<b.x;
      return a.y<b.y;
    });
    p.erase(unique(p.begin(),p.end()),p.end());
    return MinimumDistance(p,0,p.size()-1);
  }
  if(l==r)return numeric_limits<T>::max();
  int m = (l+r)>>1,mid_pos = p[m].x;
  T ans = min(MinimumDistance(p,l,m),MinimumDistance(p,m+1,r));
  vector<Point<T>>tmp((r-l+1),Point<T>(0,0));
  merge(p.begin()+l,p.begin()+m+1, p.begin()+m+1,p.begin()+r+1,
      tmp.begin(), [](Point<T> a,Point<T> b){return a.y<b.y;})
      ;
  for(int i = l;i<=r;++i)p[i] = tmp[i-l];
  tmp.clear();
  for(int i = l;i<=r;++i){
    if((p[i].x-mid_pos)*(p[i].x-mid_pos)<ans){
      tmp.push_back(p[i]);
    }
  }
  int n = tmp.size();
  for(int i = 0;i<n;++i){
    for(int j = i+1;j<n;++j){
      ans = min(ans,abs2(tmp[i]-tmp[j]));
      if(((tmp[i].y-tmp[j].y)*(tmp[i].y-tmp[j].y))>ans){
        break;
      }
    }
  }
  return ans;
}
```

2.7 ConvexHull

```cpp
template<class T>
vector<Point<T>> ConvexHull(vector<Point<T>> v,bool Boundary =
    1){
  sort(begin(v),end(v),[&](Point<T> &a,Point<T> &b){
    if(a.x!=b.x)return a.x<b.x;
    return a.y<b.y;
  });
  vector<Point<T>>ans;
  int t = 1;
  auto add = [&](Point<T> &p){
    while(ans.size() > t and ((p - ans[ans.size() - 2])^(ans.
        back() - ans[ans.size() - 2])) > (Boundary ? 0 : 0-eps)
        )
      ans.pop_back();
    ans.push_back(p);
  };
  for(int i = 0; i < v.size(); ++i) add(v[i]);
  t = ans.size();
  for(int i = (int)(v.size())-2; i >= 0; --i) add(v[i]);
  if(v.size() > 1) ans.pop_back();
  return ans;
}
```

2.8 Template

```cpp
template<class T>
struct Point{
  T x,y;
  Point(T x = 0,T y = 0) : x(x), y(y) {}
  Point operator + (const Point &b) const {
    return Point(x + b.x,y + b.y);
  }
  Point operator - (const Point &b) const {
    return Point(x - b.x,y - b.y);
  }
  Point operator * (T b) const {
    return Point(x*b,y*b);
  }
  Point operator / (T b) const {
    return Point(x/b,y/b);
  }
  T operator * (const Point &b) const {
    return x * b.x + y * b.y;
  }
  T operator ^ (const Point &b) const {
    return x * b.y - y * b.x;
  }
};
int sign(double a){
  return fabs(a) < eps ? 0 : a > 0 ? 1 : -1;
}
template<class T>
double abs(const Point<T>&p){
  return sqrtl(p*p);
}
template<class T>
T abs2(const Point<T>&p){
  return p*p;
}
template<class T>
int ori(Point<T> a,Point<T> b,Point<T> c){
  return sign((b-a)^(c-a));
}
template<class T>
bool collinearity(Point<T> p1,Point<T> p2,Point<T> p3){
  return sign((p1-p3)^(p2-p3)) == 0;
}
template<class T>
bool btw(Point<T> p1,Point<T> p2,Point<T> p3) {
  if(!collinearity(p1, p2, p3)) return 0;
  return sign((p1-p3)*(p2-p3)) <= 0;
}
template<class T>
bool PointOnSegment(const Point<T> &p1,const Point<T> &p2,
    const Point<T> &p3){
  return collinearity(p1,p2,p3) && btw(p1,p2,p3);
}
template<class T>
bool seg_intersect(Point<T> p1, Point<T> p2, Point<T> p3, Point
    <T> p4) {
  int a123 = ori(p1, p2, p3);
  int a124 = ori(p1, p2, p4);
  int a341 = ori(p3, p4, p1);
  int a342 = ori(p3, p4, p2);
  if(a123 == 0 && a124 == 0)
```

```
    return btw(p1, p2, p3) || btw(p1, p2, p4) || btw(p3, p4, p1
        ) || btw(p3, p4, p2);
  return a123 * a124 <= 0 && a341 * a342 <= 0;
}
template<class T>
double area(vector<Point<T>> v){
  if(v.size()<=2)return 0;
  double ans = 0;
  for(int i = 1;i<v.size()-1;++i){
    ans+=((v[i]-v[0])^(v[i+1]-v[0]));
  }
  return abs(ans)/2.;
}
```

3 Graph

3.1 HLD

```
struct heavy_light_decomposition{
  int n;
  vector<int>dep,father,sz,mxson,topf,id;
  vector<vector<int>>g;
  heavy_light_decomposition(int _n = 0) : n(_n) {
    g.resize(n+5);
    dep.resize(n+5);
    father.resize(n+5);
    sz.resize(n+5);
    mxson.resize(n+5);
    topf.resize(n+5);
    id.resize(n+5);
  }
  void add_edge(int u, int v){
    g[u].push_back(v);
    g[v].push_back(u);
  }
  void dfs(int u,int p){
    dep[u] = dep[p]+1;
    father[u] = p;
    sz[u] = 1;
    mxson[u] = 0;
    for(auto v:g[u]){
      if(v==p)continue;
      dfs(v,u);
      sz[u]+=sz[v];
      if(sz[v]>sz[mxson[u]])mxson[u] = v;
    }
  }
  void dfs2(int u,int top){
    static int idn = 0;
    topf[u] = top;
    id[u] = ++idn;
    if(mxson[u])dfs2(mxson[u],top);
    for(auto v:g[u]){
      if(v!=father[u] and v!=mxson[u]){
        dfs2(v,v);
      }
    }
  }
  void build(int root){
    dfs(root,0);
    dfs2(root,root);
  }
  vector<pair<int, int>> path(int u,int v){
    vector<pair<int, int>>ans;
    while(topf[u]!=topf[v]){
      if(dep[topf[u]]<dep[topf[v]])swap(u,v);
      ans.push_back({id[topf[u]], id[u]});
      u = father[topf[u]];
    }
    if(id[u]>id[v])swap(u,v);
    ans.push_back({id[u], id[v]});
    return ans;
  }
};
```

3.2 Bridges

```
vector<pii> findBridges(const vector<vector<int>>& g) {
  int n = (int) g.size();
  vector<int> id(n, -1), low(n);
  vector<pii> bridges;
  function<void(int, int)> dfs = [&](int u, int p) {
    static int cnt = 0;
    id[u] = low[u] = cnt++;
    for(auto v : g[u]) {
      if(v == p) continue;
      if(id[v] != -1) low[u] = min(low[u], id[v]);
      else {
        dfs(v, u);
        low[u] = min(low[u], low[v]);
        if(low[v] > id[u]) bridges.EB(u, v);
      }
    }
  };
  for(int i = 0; i < n; ++i) {
    if(id[i] == -1) dfs(i, -1);
  }
  return bridges;
}
```

3.3 TwoSat

```
struct two_sat{
  SCC s;
  vector<bool>ans;
  int have_ans = 0;
  int n;
  two_sat(int _n) : n(_n) {
    ans.resize(n+1);
    s = SCC(2*n);
  }
  int inv(int x){
    if(x>n)return x-n;
    return x+n;
  }
  void add_or_clause(int u, bool x, int v, bool y){
    if(!x)u = inv(u);
    if(!y)v = inv(v);
    s.add_edge(inv(u), v);
    s.add_edge(inv(v), u);
  }
  void check(){
    if(have_ans!=0)return;
    s.build();
    for(int i = 0;i<=n;++i){
      if(s.scc[i]==s.scc[inv(i)]){
        have_ans = -1;
        return;
      }
      ans[i] = (s.scc[i]<s.scc[inv(i)]);
    }
    have_ans = 1;
  }
};
```

3.4 MCMF

```
template<class Cap_t, class Cost_t>
class MCMF {
public:
  struct Edge {
    int from;
    int to;
    Cap_t cap;
    Cost_t cost;
    Edge(int u, int v, Cap_t _cap, Cost_t _cost) : from(u), to(
        v), cap(_cap), cost(_cost) {}
  };

  static constexpr Cap_t EPS = static_cast<Cap_t>(1e-9);

  int n;
  vector<Edge> edges;
  vector<vector<int>> g;
  vector<Cost_t> d;
  vector<bool> in_queue;
  vector<int> previous_edge;

  MCMF() {}
  MCMF(int _n) : n(_n+1), g(_n+1), d(_n+1), in_queue(_n+1),
      previous_edge(_n+1) {}

  void add_edge(int u, int v, Cap_t cap, Cost_t cost) {
    assert(0 <= u && u < n);
    assert(0 <= v && v < n);
    g[u].push_back(edges.size());
    edges.emplace_back(u, v, cap, cost);
    g[v].push_back(edges.size());
    edges.emplace_back(v, u, 0, -cost);
  }

  bool spfa(int s, int t) {
    bool found = false;
    fill(d.begin(), d.end(), numeric_limits<Cost_t>::max());
    d[s] = 0;
    in_queue[s] = true;
    queue<int> que;
    que.push(s);
    while(!que.empty()) {
      int u = que.front();
```

```
      que.pop();
      if(u == t) {
        found = true;
      }
      in_queue[u] = false;
      for(auto& id : g[u]) {
        const Edge& e = edges[id];
        if(e.cap > EPS && d[u] + e.cost < d[e.to]) {
          d[e.to] = d[u] + e.cost;
          previous_edge[e.to] = id;
          if(!in_queue[e.to]) {
            que.push(e.to);
            in_queue[e.to] = true;
          }
        }
      }
    }
    return found;
  }

  pair<Cap_t, Cost_t> flow(int s, int t, Cap_t f =
      numeric_limits<Cap_t>::max()) {
    assert(0 <= s && s < n);
    assert(0 <= t && t < n);
    Cap_t cap = 0;
    Cost_t cost = 0;
    while(f > 0 && spfa(s, t)) {
      Cap_t send = f;
      int u = t;
      while(u != s) {
        const Edge& e = edges[previous_edge[u]];
        send = min(send, e.cap);
        u = e.from;
      }
      u = t;
      while(u != s) {
        Edge& e = edges[previous_edge[u]];
        e.cap -= send;
        Edge& b = edges[previous_edge[u] ^ 1];
        b.cap += send;
        u = e.from;
      }
      cap += send;
      f -= send;
      cost += send * d[t];
    }
    return make_pair(cap, cost);
  }
};
```

3.5 LCA

```
vector<vector<int>>g,dp;
vector<int>deep;
void build(int root,int n){
  dp.assign(25,vector<int>(n+5));
  deep.assign(n+5,0);
  function<void(int,int,int)>dfs = [&](int u,int p,int dis){
    dp[0][u] = p;
    deep[u] = dis;
    for(auto v:g[u]){
      if(v==p)continue;
      dfs(v,u,dis+1);
    }
  };
  dfs(root,0,1);
  for(int i = 1;i<=20;++i){
    for(int j = 1;j<=n;++j){
      dp[i][j] = dp[i-1][dp[i-1][j]];
    }
  }
}
int LCA(int u,int v){
  if(deep[u]<deep[v])swap(u,v);
  for(int i = 20;i>=0;--i){
    if(deep[dp[i][u]]>=deep[v])
      u = dp[i][u];
  }
  if(u==v)return u;
  for(int i = 20;i>=0;--i){
    if(dp[i][u]!=dp[i][v])u = dp[i][u],v = dp[i][v];
  }
  return dp[0][u];
}
```

3.6 CentroidDecomposition

```
vector<vector<int>>g;
vector<int>sz,tmp;
vector<bool>vis;//visit_centroid
int tree_centroid(int u,int n){
  function<void(int,int)>dfs1 = [&](int u,int p){
    sz[u] = 1;
    for(auto v:g[u]){
      if(v==p)continue;
      if(vis[v])continue;
      dfs1(v,u);
      sz[u]+=sz[v];
    }
  };
  function<int(int,int)>dfs2 = [&](int u,int p){
    for(auto v:g[u]){
      if(v==p)continue;
      if(vis[v])continue;
      if(sz[v]*2<n)continue;
      return dfs2(v,u);
    }
    return u;
  };
  dfs1(u,-1);
  return dfs2(u,-1);
}
int cal(int u,int p = -1,int deep = 1){
  int ans = 0;
  tmp.pb(deep);
  sz[u] = 1;
  for(auto v:g[u]){
    if(v==p)continue;
    if(vis[v])continue;
    ans+=cal(v,u,deep+1);
    sz[u]+=sz[v];
  }
  //calcuate the answer
  return ans;
}
int centroid_decomposition(int u,int tree_size){
  int center = tree_centroid(u,tree_size);
  vis[center] = 1;
  int ans = 0;
  for(auto v:g[center]){
    if(vis[v])continue;
    ans+=cal(v);
    for(int i = sz(tmp)-sz[v];i<sz(tmp);++i){
      //update
    }
  }
  while(!tmp.empty()){
    //roll_back(tmp.back())
    tmp.pop_back();
  }
  for(auto v:g[center]){
    if(vis[v])continue;
    ans+=centroid_decomposition(v,sz[v]);
  }
  return ans;
}
```

3.7 BCC AP

```
struct BCC_AP{
  int dfn_cnt = 0,bcc_cnt = 0,n;
  vector<int>dfn,low,ap,bcc_id;
  stack<int>st;
  vector<bool>vis,is_ap;
  vector<vector<int>>bcc;
  BCC_AP(int _n):n(_n){
    dfn.resize(n+5),low.resize(n+5),bcc.resize(n+5),vis.resize(
        n+5),is_ap.resize(n+5),bcc_id.resize(n+5);
  }
  inline void build(const vector<vector<int>>&g,int u,int p =
      -1){
    int child = 0;
    dfn[u] = low[u] = ++dfn_cnt;
    st.push(u);
    vis[u] = 1;
    if(g[u].empty() and p==-1){
      bcc_id[u] = ++bcc_cnt;
      bcc[bcc_cnt].push_back(u);
      return;
    }
    for(auto v:g[u]){
      if(v==p)continue;
      if(!dfn[v]){
        build(g,v,u);
        child++;
        if(dfn[u]<=low[v]){
          is_ap[u] = 1;
          bcc_id[u] = ++bcc_cnt;
          bcc[bcc_cnt].push_back(u);
          while(vis[v]){
            bcc_id[st.top()] = bcc_cnt;
            bcc[bcc_cnt].push_back(st.top());
            vis[st.top()] = 0;
            st.pop();
          }
```

```
        }
        low[u] = min(low[u],low[v]);
      }
      low[u] = min(low[u],dfn[v]);
    }
    if(p==-1 and child<2)is_ap[u] = 0;
    if(is_ap[u])ap.push_back(u);
  }
};
```

3.8 SCC

```
struct SCC{
  int n,cnt = 0,dfn_cnt = 0;
  vector<vector<int>>g;
  vector<int>sz,scc,low,dfn;
  stack<int>st;
  vector<bool>vis;
  SCC(int _n = 0) : n(_n){
    sz.resize(n+5),scc.resize(n+5),low.resize(n+5),dfn.resize(n+5),vis.resize(n+5);
    g.resize(n+5);
  }
  inline void add_edge(int u, int v){
    g[u].push_back(v);
  }
  inline void build(){
    function<void(int, int)>dfs = [&](int u,int dis){
      low[u] = dfn[u] = ++dfn_cnt,vis[u] = 1;
      st.push(u);
      for(auto v:g[u]){
        if(!dfn[v]){
          dfs(v, dis+1);
          low[u] = min(low[u],low[v]);
        }
        else if(vis[v]){
          low[u] = min(low[u],dfn[v]);
        }
      }
      if(low[u]==dfn[u]){
        ++cnt;
        while(vis[u]){
          auto v = st.top();
          st.pop();
          vis[v] = 0;
          scc[v] = cnt;
          sz[cnt]++;
        }
      }
    };
    for(int i = 0;i<=n;++i){
      if(!scc[i]){
        dfs(i, 1);
      }
    }
  }
  vector<vector<int>> compress(){
    vector<vector<int>>ans(cnt+1);
    for(int u = 0;u<=n;++u){
      for(auto v:g[u]){
        if(scc[u] == scc[v]){
          continue;
        }
        ans[scc[u]].push_back(scc[v]);
      }
    }
    for(int i = 0;i<=cnt;++i){
      sort(ans[i].begin(), ans[i].end());
      ans[i].erase(unique(ans[i].begin(), ans[i].end()), ans[i].end());
    }
    return ans;
  }
};
```

3.9 LineContainer

```
template<class T>
T floor_div(T a, T b) {
  return a / b - ((a ^ b) < 0 && a % b != 0);
}

template<class T>
T ceil_div(T a, T b) {
  return a / b + ((a ^ b) > 0 && a % b != 0);
}

namespace line_container_internal {

struct line_t {
  mutable long long k, m, p;

  inline bool operator<(const line_t& o) const { return k < o.k; }
  inline bool operator<(long long x) const { return p < x; }
};

} // line_container_internal

template<bool MAX>
struct line_container : std::multiset<line_container_internal::line_t, std::less<>> {
  static const long long INF = std::numeric_limits<long long>::max();

  bool isect(iterator x, iterator y) {
    if(y == end()) {
      x->p = INF;
      return 0;
    }
    if(x->k == y->k) {
      x->p = (x->m > y->m ? INF : -INF);
    } else {
      x->p = floor_div(y->m - x->m, x->k - y->k);
    }
    return x->p >= y->p;
  }

  void add_line(long long k, long long m) {
    if(!MAX) {
      k = -k;
      m = -m;
    }
    auto z = insert({k, m, 0}), y = z++, x = y;
    while(isect(y, z)) {
      z = erase(z);
    }
    if(x != begin() && isect(--x, y)) {
      isect(x, y = erase(y));
    }
    while((y = x) != begin() && (--x)->p >= y->p) {
      isect(x, erase(y));
    }
  }

  long long get(long long x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return (l.k * x + l.m) * (MAX ? +1 : -1);
  }
};
```

3.10 Dinic

```
template<class T>
struct Dinic{
  struct edge{
    int from, to;
    T cap;
    edge(int _from, int _to, T _cap) : from(_from), to(_to), cap(_cap) {}
  };
  int n;
  vector<edge> edges;
  vector<vector<int>> g;
  vector<int> cur, h;
  Dinic(int _n) : n(_n+1), g(_n+1) {}
  void add_edge(int u, int v, T cap){
    g[u].push_back(edges.size());
    edges.push_back(edge(u, v, cap));
    g[v].push_back(edges.size());
    edges.push_back(edge(v, u, 0));
  }
  bool bfs(int s,int t){
    h.assign(n, -1);
    h[s] = 0;
    queue<int> que;
    que.push(s);
    while(!que.empty()) {
      int u = que.front();
      que.pop();
      for(auto id : g[u]) {
        const edge& e = edges[id];
        int v = e.to;
        if(e.cap > 0 && h[v] == -1) {
          h[v] = h[u] + 1;
          if(v == t) {
            return 1;
          }
          que.push(v);
        }
      }
    }
    return 0;
  }
```

```
  T dfs(int u, int t, T f) {
    if(u == t) {
      return f;
    }
    T r = f;
    for(int& i = cur[u]; i < (int) g[u].size(); ++i) {
      int id = g[u][i];
      const edge& e = edges[id];
      int v = e.to;
      if(e.cap > 0 && h[v] == h[u] + 1) {
        T send = dfs(v, t, min(r, e.cap));
        edges[id].cap -= send;
        edges[id ^ 1].cap += send;
        r -= send;
        if(r == 0) {
          return f;
        }
      }
    }
    return f - r;
  }
  T flow(int s, int t, T f = numeric_limits<T>::max()) {
    T ans = 0;
    while(f > 0 && bfs(s, t)) {
      cur.assign(n, 0);
      T send = dfs(s, t, f);
      ans += send;
      f -= send;
    }
    return ans;
  }
  vector<pair<int,int>> min_cut(int s) {
    vector<bool> vis(n);
    vis[s] = true;
    queue<int> que;
    que.push(s);
    while(!que.empty()) {
      int u = que.front();
      que.pop();
      for(auto id : g[u]) {
        const auto& e = edges[id];
        int v = e.to;
        if(e.cap > 0 && !vis[v]) {
          vis[v] = true;
          que.push(v);
        }
      }
    }
    vector<pair<int,int>> cut;
    for(int i = 0; i < (int) edges.size(); i += 2) {
      const auto& e = edges[i];
      if(vis[e.from] && !vis[e.to]) {
        cut.push_back(make_pair(e.from, e.to));
      }
    }
    return cut;
  }
};
```

4 Math

4.1 Numbers

- Bernoulli numbers

  $B_0 - 1, B_1^{\pm} = \pm\frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$

  $\sum_{j=0}^{m} \binom{m+1}{j} B_j = 0$, EGF is $B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}$.

  $S_m(n) = \sum_{k=1}^{n} k^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k^+ n^{m+1-k}$

- Stirling numbers of the second kind Partitions of n distinct elements into exactly k groups.

  $S(n,k) = S(n-1,k-1) + kS(n-1,k), S(n,1) = S(n,n) = 1$

  $S(n,k) = \frac{1}{k!} \sum_{i=0}^{k} (-1)^{k-i} \binom{k}{i} i^n$

  $x^n = \sum_{i=0}^{n} S(n,i)(x)_i$

- Pentagonal number theorem

  $$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left(x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

  $C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$

  $C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$

- Eulerian numbers

  Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

  $E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$

  $E(n,0) = E(n,n-1) = 1$

  $E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$

4.2 ExtendGCD

```
// @return $x, y$ s.t. $ax + by = \gcd(a, b)$
ll ext_gcd(ll a, ll b, ll& x, ll& y) {
  if(b == 0) {
    x = 1; y = 0;
    return a;
  }
  ll x2, y2;
  ll c = a % b;
  if(c < 0) c += b;
  ll g = ext_gcd(b, c, x2, y2);
  x = y2;
  y = x2 - (a / b) * y2;
  return g;
}
```

4.3 list

4.4 InvGCD

```
pair<long long, long long> inv_gcd(long long a, long long b) {
  a %= b;
  if(a < 0) a += b;
  if(a == 0) return {b, 0};
  long long s = b, t = a;
  long long m0 = 0, m1 = 1;
  while(t) {
    long long u = s / t;
    s -= t * u;
    m0 -= m1 * u;
    swap(s, t);
    swap(m0, m1);
  }
  if(m0 < 0) m0 += b / s;
  return {s, m0};
}
```

4.5 GeneratingFunctions

- Ordinary Generating Function $A(x) = \sum_{i \geq 0} a_i x^i$
  - $A(rx) \Rightarrow r^n a_n$
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^{n} a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\cdots+i_k=n} a_{i_1} a_{i_2} \cdots a_{i_k}$
  - $xA(x)' \Rightarrow na_n$
  - $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^{n} a_i$

- Exponential Generating Function $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x_i$
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A^{(k)}(x) \Rightarrow a_{n+k}$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\cdots+i_k=n} \binom{n}{i_1, i_2, \ldots, i_k} a_{i_1} a_{i_2} \cdots a_{i_k}$
  - $xA(x) \Rightarrow na_n$

- Special Generating Function
  - $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
  - $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{i}{n-1} x^i$

4.6 Theorem

- Modular Arithmetic

$$(a+b) \bmod m = (a \bmod m + b \bmod m) \bmod m$$

$$(a-b) \bmod m = (a \bmod m - b \bmod m) \bmod m$$

$$(a \cdot b) \pmod m = ((a \bmod m) \cdot (b \bmod m)) \bmod m$$

$$a^b \bmod m = (a \bmod m)^{b \bmod m - 1} \bmod m$$

- Cramer's rule

$$\begin{matrix} ax+by=e \\ cx+dy=f \end{matrix} \Rightarrow \begin{matrix} x = \dfrac{ed-bf}{ad-bc} \\ y = \dfrac{af-ec}{ad-bc} \end{matrix}$$

- Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i,j) in G.

  - The number of undirected spanning in G is $|\det(\tilde{L}_{11})|$.
  - The number of directed spanning tree rooted at r in G is $|\det(\tilde{L}_{rr})|$.

- Tutte's Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniformly at random) if $i < j$ and $(i,j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

- Cayley's Formula

  - Given a degree sequence $d_1, d_2, \ldots, d_n$ for each labeled vertices, there are $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!}$ spanning trees.
  - Let $T_{n,k}$ be the number of labeled forests on n vertices with k components, such that vertex $1, 2, \ldots, k$ belong to different components. Then $T_{n,k} = kn^{n-k-1}$.

- Erd□s–Gallai theorem

A sequence of nonnegative integers $d_1 \geq \cdots \geq d_n$ can be represented as the degree sequence of a finite simple graph on n vertices if and only if $d_1 + \cdots + d_n$ is even and $\sum_{i-1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k)$ holds for every $1 \leq k \leq n$.

- Gale–Ryser theorem

A pair of sequences of nonnegative integers $a_1 \geq \cdots \geq a_n$ and $b_1, \ldots, b_n$ is bigraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{n} \min(b_i, k)$ holds for every $1 \leq k \leq n$.

- Fulkerson–Chen–Anstee theorem

A sequence $(a_1, b_1), \ldots, (a_n, b_n)$ of nonnegative integer pairs with $a_1 \geq \cdots \geq a_n$ is digraphic if and only if $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$ and $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{k} \min(b_i, k-1) + \sum_{i=k+1}^{n} \min(b_i, k)$ holds for every $1 \leq k \leq n$.

- M□bius inversion formula

  - $f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$
  - $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$

- Spherical cap

  - A portion of a sphere cut off by a plane.
  - r: sphere radius, a: radius of the base of the cap, h: height of the cap, θ: $\arcsin(a/r)$.
  - Volume $= \pi h^2(3r-h)/3 = \pi h(3a^2+h^2)/6 = \pi r^3(2+\cos\theta)(1-\cos\theta)^2/3$.
  - Area $= 2\pi rh = \pi(a^2+h^2) = 2\pi r^2(1-\cos\theta)$.

4.7 FloorSum

```cpp
// @param $n < 2^{32}$
// @param $1 \leq m < 2^{32}$
// @return $\sum_{i=0}^{n-1} \lfloor \frac{ai + b}{m} \rfloor \
    pmod{2^{64}}$
ull floor_sum_unsigned(ull n, ull m, ull a, ull b) {
  ull ans = 0;
  while(true) {
    if(a >= m) {
      ans += n * (n - 1) / 2 * (a / m);
      a %= m;
    }
    if(b >= m) {
      ans += n * (b / m);
      b %= m;
    }
    ull y_max = a * n + b;
    if(y_max < m) break;
    n = (ull)(y_max / m);
    b = (ull)(y_max % m);
    swap(m, a);
  }
  return ans;
}

ll floor_sum(ll n, ll m, ll a, ll b) {
  assert(0 <= n && n < (1LL << 32));
  assert(1 <= m && m < (1LL << 32));
  ull ans = 0;
  if(a < 0) {
    ull a2 = (a % m + m) % m;
    ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
    a = a2;
  }
  if(b < 0) {
    ull b2 = (b % m + m) % m;
    ans -= 1ULL * n * ((b2 - b) / m);
    b = b2;
  }
  return ans + floor_sum_unsigned(n, m, a, b);
}
```

4.8 Factorizer

```cpp
template<class T>
vector<pair<T, int>> MergeFactors(const vector<pair<T, int>>& a
    , const vector<pair<T, int>>& b) {
  vector<pair<T, int>> c;
  int i = 0, j = 0;
  while(i < SZ(a) || j < SZ(b)) {
    if(i < SZ(a) && j < SZ(b) && a[i].F == b[j].F) {
      c.EB(a[i].F, a[i].S + b[j].S);
      ++i, ++j;
      continue;
    }
    if(j == SZ(b) || (i < SZ(a) && a[i].F < b[j].F)) c.PB(a[i
        ++]);
    else c.PB(b[j++]);
  }
  return c;
}
template<class T>
vector<pair<T, int>> RhoC(const T& n, const T& c) {
  if(n <= 1) return {};
  if(n % 2 == 0) return MergeFactors({{2, 1}}, RhoC(n / 2, c));
  if(is_prime_constexpr(n)) return {{n, 1}};
  T x = 2, saved = 2, p = 1, lam = 1;
  while(true) {
    x = (x * x % n + c) % n;
    T g = __gcd(((x - saved) + n) % n, n);
    if(g != 1) return MergeFactors(RhoC(g, c + 1), RhoC(n / g,
        c + 1));
    if(p == lam) {
      saved = x;
      p <<= 1;
      lam = 0;
    }
    lam += 1;
  }
  return {};
}
template<class T>
vector<pair<T, int>> Factorize(T n) {
  if(n <= 1) return {};
  return RhoC(n, T(1));
}
template<class T>
vector<T> BuildDivisorsFromFactors(const vector<pair<T, int>>&
    factors) {
  int total = 1;
  for(int i = 0; i < SZ(factors); ++i) total *= factors[i].
      second + 1;
```

```
  vector<T> divisors;
  divisors.reserve(total);
  divisors.PB(1);
  for(auto [p, cnt] : factors) {
    int sz = SZ(divisors);
    for(int i = 0; i < sz; ++i) {
      T cur = divisors[i];
      for(int j = 0; j < cnt; ++j) {
        cur *= p;
        divisors.PB(cur);
      }
    }
  }
  // sort(ALL(divisors));
  return divisors;
}
```

4.9 GuessKth

```
template <typename Tfield>
std::pair<int, std::vector<Tfield>> find_linear_recurrence(const std::vector<Tfield> &S) {
    int N = S.size();
    using poly = std::vector<Tfield>;
    poly C_reversed{1}, B{1};
    int L = 0, m = 1;
    Tfield b = 1;

    // adjust: C(x) <- C(x) - (d / b) x^m B(x)
    auto adjust = [](poly C, const poly &B, Tfield d, Tfield b, int m) -> poly {
        C.resize(std::max(C.size(), B.size() + m));
        Tfield a = d / b;
        for (unsigned i = 0; i < B.size(); i++) C[i + m] -= a * B[i];
        return C;
    };

    for (int n = 0; n < N; n++) {
        Tfield d = S[n];
        for (int i = 1; i <= L; i++) d += C_reversed[i] * S[n - i];

        if (d == 0)
            m++;
        else if (2 * L <= n) {
            poly T = C_reversed;
            C_reversed = adjust(C_reversed, B, d, b, m);
            L = n + 1 - L;
            B = T;
            b = d;
            m = 1;
        } else
            C_reversed = adjust(C_reversed, B, d, b, m++);
    }
    return std::make_pair(L, C_reversed);
}

// Calculate $x^N \bmod f(x)$
// Known as `Kitamasa method`
// Input: f_reversed: monic, reversed (f_reversed[0] = 1)
// Complexity: $O(K^2 \log N)$ ($K$: deg. of $f$)
// Example: (4, [1, -1, -1]) -> [2, 3]
//          ( x^4 = (x^2 + x + 2)(x^2 - x - 1) + 3x + 2 )
// Reference: http://misawa.github.io/others/fast_kitamasa_method.html
//            http://sugarknri.hatenablog.com/entry/2017/11/18/233936
template <typename Tfield>
std::vector<Tfield> monomial_mod_polynomial(long long N, const std::vector<Tfield> &f_reversed) {
    assert(!f_reversed.empty() and f_reversed[0] == 1);
    int K = f_reversed.size() - 1;
    if (!K) return {};
    int D = 64 - __builtin_clzll(N);
    std::vector<Tfield> ret(K, 0);
    ret[0] = 1;
    auto self_conv = [](std::vector<Tfield> x) -> std::vector<Tfield> {
        int d = x.size();
        std::vector<Tfield> ret(d * 2 - 1);
        for (int i = 0; i < d; i++) {
            ret[i * 2] += x[i] * x[i];
            for (int j = 0; j < i; j++) ret[i + j] += x[i] * x[j] * 2;
        }
        return ret;
    };
    for (int d = D; d--;) {
        ret = self_conv(ret);
        for (int i = 2 * K - 2; i >= K; i--) {
            for (int j = 1; j <= K; j++) ret[i - j] -= ret[i] * f_reversed[j];
        }
        ret.resize(K);
        if ((N >> d) & 1) {
            std::vector<Tfield> c(K);
            c[0] = -ret[K - 1] * f_reversed[K];
            for (int i = 1; i < K; i++) { c[i] = ret[i - 1] - ret[K - 1] * f_reversed[K - i]; }
            ret = c;
        }
    }
    return ret;
}

// Guess k-th element of the sequence, assuming linear recurrence
// initial_elements: 0-ORIGIN
// Verify: abc198f https://atcoder.jp/contests/abc198/submissions/21837815
template <typename Tfield>
Tfield guess_kth_term(const std::vector<Tfield> &initial_elements, long long k) {
    assert(k >= 0);
    if (k < static_cast<long long>(initial_elements.size()))
        return initial_elements[k];
    const auto f = find_linear_recurrence<Tfield>(initial_elements).second;
    const auto g = monomial_mod_polynomial<Tfield>(k, f);
    Tfield ret = 0;
    for (unsigned i = 0; i < g.size(); i++) ret += g[i] * initial_elements[i];
    return ret;
}
```

4.10 PowMod

```
constexpr long long Pow(long long x, long long n, int m) {
  if(m == 1) return 0;
  unsigned int _m = (unsigned int)(m);
  unsigned long long r = 1;
  x %= m;
  if(x < 0) x += m;
  unsigned long long y = x;
  while(n) {
    if(n & 1) r = (r * y) % _m;
    y = (y * y) % _m;
    n >>= 1;
  }
  return r;
}
```

4.11 ModInt

```
template<int id>
struct modint {
public:
  static constexpr int mod() { return id; }

  constexpr modint() : value(0) {}
  modint(long long x) : value(x % mod()) {
    if(value < 0) value += mod();
  }

  constexpr int val() const { return value; }

  constexpr modint inv() const {
    return Pow(value, mod()-2, mod());
  }

  constexpr modint& operator+=(const modint& rhs) & {
    value += rhs.value;
    if(value >= mod()) {
      value -= mod();
    }
    return *this;
  }

  constexpr modint& operator-=(const modint& rhs) & {
    value -= rhs.value;
    if(value < 0) {
      value += mod();
    }
    return *this;
  }

  constexpr modint& operator*=(const modint& rhs) & {
    value = 1LL * value * rhs.value % mod();
    return *this;
  }

  constexpr modint& operator/=(const modint& rhs) & {
    return *this *= rhs.inv();
  }
```

```

  friend constexpr modint operator+(modint lhs, modint rhs) { return lhs += rhs; }
  friend constexpr modint operator-(modint lhs, modint rhs) { return lhs -= rhs; }
  friend constexpr modint operator*(modint lhs, modint rhs) { return lhs *= rhs; }
  friend constexpr modint operator/(modint lhs, modint rhs) { return lhs /= rhs; }
  constexpr modint operator+() const { return *this; }
  constexpr modint operator-() const { return modint() - *this; }
  constexpr bool operator==(const modint& rhs) const { return value == rhs.value; }
  constexpr bool operator!=(const modint& rhs) const { return value != rhs.value; }


  int value;
};
using mint = modint<mod>;
```

4.12 CRT

```
// @return
//  $\text{remainder, modulo}$
//      or
//  $0, 0$ if do not exist
pair<long long, long long> crt(const vector<long long>& r, const vector<long long>& m) {
  assert(r.size()==m.size());
  int n = r.size();
  // Contracts: 0 <= r0 < m0
  long long r0 = 0, m0 = 1;
  for(int i = 0; i < n; i++) {
    assert(1 <= m[i]);
    long long r1 = r[i] % m[i];
    if(r1 < 0) r1 += m[i];
    long long m1 = m[i];
    if(m0 < m1) {
      swap(r0, r1);
      swap(m0, m1);
    }
    if(m0 % m1 == 0) {
      if(r0 % m1 != r1) return {0, 0};
      continue;
    }
    long long g, im;
    tie(g, im) = inv_gcd(m0, m1);
    long long u1 = (m1 / g);
    if((r1 - r0) % g) return {0, 0};
    long long x = (r1 - r0) / g % u1 * im % u1;
    r0 += x * m0;
    m0 *= u1;
    if(r0 < 0) r0 += m0;
  }
  return {r0, m0};
}
```

4.13 DiscreteLog

```
//give you $a, b, m$ find $x$ such that $a^x \equiv m (\mod m)$
#line 2 "library/math/discrete-log.hpp"
#include <vector>
#include <cmath>
#include <cassert>
#line 2 "library/data-structure/pbds.hpp"
#include <ext/pb_ds/assoc_container.hpp>
#line 2 "library/random/splitmix64.hpp"
#include <chrono>

namespace felix {

namespace internal {

struct splitmix64_hash {
  // http://xoshiro.di.unimi.it/splitmix64.c

  static unsigned long long splitmix64(unsigned long long x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }

  unsigned long long operator()(unsigned long long x) const {
    static const unsigned long long FIXED_RANDOM = std::chrono::steady_clock::now().time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
  }
};

} // namespace internal


} // namespace felix

#line 4 "library/data-structure/pbds.hpp"

namespace felix {

template<class T, class U, class H = internal::splitmix64_hash> using hash_map = __gnu_pbds::gp_hash_table<T, U, H>;
template<class T, class H = internal::splitmix64_hash> using hash_set = hash_map<T, __gnu_pbds::null_type, H>;

template<class T, class Comp = std::less<T>> using ordered_set = __gnu_pbds::tree<T, __gnu_pbds::null_type, Comp, __gnu_pbds::rb_tree_tag, __gnu_pbds::tree_order_statistics_node_update>;
template<class T> using ordered_multiset = ordered_set<T, std::less_equal<T>>;

} // namespace felix
#line 2 "library/modint/barrett.hpp"

namespace felix {

namespace internal {

// Fast modular multiplication by barrett reduction

// Reference: https://en.wikipedia.org/wiki/Barrett_reduction

struct barrett {
  unsigned int m;
  unsigned long long im;

  explicit barrett(unsigned int _m) : m(_m), im((unsigned long long)(-1) / _m + 1) {}

  unsigned int umod() const { return m; }

  unsigned int mul(unsigned int a, unsigned int b) const {
    unsigned long long z = a;
    z *= b;
#ifdef _MSC_VER
    unsigned long long x;
    _umul128(z, im, &x);
#else
    unsigned long long x = (unsigned long long)(((unsigned __int128)(z) * im) >> 64);
#endif
    unsigned long long y = x * m;
    return (unsigned int)(z - y + (z < y ? m : 0));
  }
};

} // namespace internal


} // namespace felix

#line 2 "library/math/binary-gcd.hpp"

namespace felix {

template<class T>
inline T binary_gcd(T a, T b) {
  if(a == 0 || b == 0) {
    return a | b;
  }
  int8_t n = __builtin_ctzll(a);
  int8_t m = __builtin_ctzll(b);
  a >>= n;
  b >>= m;
  while(a != b) {
    T d = a - b;
    int8_t s = __builtin_ctzll(d);
    bool f = a > b;
    b = f ? b : a;
    a = (f ? d : -d) >> s;
  }
  return a << (n < m ? n : m);
}

} // namespace felix

#line 8 "library/math/discrete-log.hpp"

namespace felix {

int discrete_log(int a, int b, int m) {
  assert(b < m);
  if(b == 1 || m == 1) {
    return 0;
  }
  int n = (int) std::sqrt(m) + 1, e = 1, f = 1, j = 1;
```

```
  hash_map<int, int> baby;
  internal::barrett bt(m);
  while(j <= n && (e = f = bt.mul(e, a)) != b) {
    baby[bt.mul(e, b)] = j++;
  }
  if(e == b) {
    return j;
  }
  if(binary_gcd(m, e) == binary_gcd(m, b))  {
    for(int i = 2; i < n + 2; i++) {
      e = bt.mul(e, f);
      if(baby.find(e) != baby.end()) {
        return n * i - baby[e];
      }
    }
  }
  return -1;
}

} // namespace felix
```

4.14 LinearSieve

```
vector<bool> is_prime;
vector<int> primes, phi, mobius, least;
void linear_sieve(int n) {
  n += 1;
  is_prime.resize(n);
  least.resize(n);
  fill(2 + begin(is_prime),end(is_prime), true);
  phi.resize(n); mobius.resize(n);
  phi[1] = mobius[1] = 1;
  least[0] = 0,least[1] = 1;
  for(int i = 2; i < n; ++i) {
    if(is_prime[i]) {
      primes.push_back(i);
      phi[i] = i - 1;
      mobius[i] = -1;
      least[i] = i;
    }
    for(auto j : primes) {
      if(i * j >= n) break;
      is_prime[i * j] = false;
      least[i * j] = j;
      if(i % j == 0) {
        mobius[i * j] = 0;
        phi[i * j] = phi[i] * j;
        break;
      } else {
        mobius[i * j] = mobius[i] * mobius[j];
        phi[i * j] = phi[i] * phi[j];
      }
    }
  }
}
```

5 Misc

5.1 FastIO

```
inline char gc() {
    static const int BUF_SIZE = 1 << 22;
    static int Counts = 1 << 23;
    static char Buffer[BUF_SIZE];
    static char *Pointer = Buffer, *End = Buffer;
    if(Pointer == End) {
        if(Counts < BUF_SIZE) {
            return EOF;
        }
        Counts = fread(Buffer, 1, BUF_SIZE, stdin);
        Pointer = Buffer;
        End = Buffer + Counts;
    }
    return *(Pointer++);
}

template<class T>
inline void read(T& x) {
    static char c;
    do {
        c = gc();
    } while(c < '0' && c != '-');
    bool neg = (c == '-');
    if(!neg) {
        x = c - '0';
    } else x = 0;
    while((c = gc()) >= '0') {
        x = (x << 3) + (x << 1) + (c & 15);
    }
    if(neg) {
        x = -x;
    }
}

template<class T, class... U>
inline void read(T& a, U&... b) {
    read(a);
    read(b...);
}

template<class T>
inline void write(T temp, char end = '\n') {
    static short digits[20], P;
    if(temp == 0) {
        putchar_unlocked('0');
        putchar_unlocked(end);
        return;
    }
    if(temp < 0) {
        putchar_unlocked('-');
        write(-temp,end);
        return;
    }
    P = -1;
    while(temp) {
        digits[++P] = temp % 10;
        temp /= 10;
    }
    while(P >= 0) {
        putchar_unlocked(digits[P--] + '0');
    }
    putchar_unlocked(end);
    return;
}
```

5.2 Debug

```
#ifdef LOCAL
  #define eprintf(...) { fprintf(stderr, __VA_ARGS__); fflush(
      stderr); }
#else
  #define eprintf(...) 42
#endif
```

5.3 Discrete

```
template<class T>
vector<int> Discrete(const vector<T>&v){
  vector<int>ans;
  vector<T>tmp(v);
  sort(begin(tmp),end(tmp));
  tmp.erase(unique(begin(tmp),end(tmp)),end(tmp));
  for(auto i:v)ans.push_back(lower_bound(begin(tmp),end(tmp),i)
      -tmp.begin()+1);
  return ans;
}
```

5.4 DuiPai

```
#include<bits/stdc++.h>
using namespace std;
int main(){
  string sol,bf,make;
  cout<<"Your solution file name :";
  cin>>sol;
  cout<<"Brute force file name :";
  cin>>bf;
  cout<<"Make data file name :";
  cin>>make;
  system(("g++ "+sol+" -o sol").c_str());
  system(("g++ "+bf+" -o bf").c_str());
  system(("g++ "+make+" -o make").c_str());
  for(int t = 0;t<10000;++t){
    system("./make > ./1.in");
    double st = clock();
        system("./sol < ./1.in > ./1.ans");
        double et = clock();
        system("./bf < ./1.in > ./1.out");
        if(system("diff ./1.out ./1.ans")) {
      printf("\033[0;31mWrong Answer\033[0m on test #%d",t);
            return 0;
        }
    else if(et-st>=2000){
      printf("\033[0;32mTime limit exceeded\033[0m on test #%d,
            Time %.0lfms\n",t,et-st);
      return 0;
    }
    else {
```

```
            printf("\033[0;32mAccepted\033[0m on test #%d, Time
                 %.0lfms\n", t, et - st);
        }
    }
}
```

5.5 Timer

```
const clock_t startTime = clock();
inline double getCurrentTime() {
  return (double) (clock() - startTime) / CLOCKS_PER_SEC;
}
```

5.6 TenarySearch

```
// return the maximum of $f(x)$ in $[l, r]$
double ternary_search(double l, double r) {
  while(r - l > EPS) {
    double m1 = l + (r - l) / 3;
    double m2 = r - (r - l) / 3;
    double f1 = f(m1), f2 = f(m2);
    if(f1 < f2) l = m1;
    else r = m2;
  }
  return f(l);
}

// return the maximum of $f(x)$ in $(l, r]$
int ternary_search(int l, int r) {
  while(r - l > 1) {
    int mid = (l + r) / 2;
    if(f(m) > f(m + 1)) r = m;
    else l = m;
  }
  return r;
}
```

6 Other

6.1 TouristIO

```
static struct FastInput {
  static constexpr int BUF_SIZE = 1 << 20;
  char buf[BUF_SIZE];
  size_t chars_read = 0;
  size_t buf_pos = 0;
  FILE *in = stdin;
  char cur = 0;

  inline char get_char() {
    if(buf_pos >= chars_read) {
      chars_read = fread(buf, 1, BUF_SIZE, in);
      buf_pos = 0;
      buf[0] = (chars_read == 0 ? -1 : buf[0]);
    }
    return cur = buf[buf_pos++];
    // return cur = getchar_unlocked();
  }

  inline void tie(int) {}

  inline explicit operator bool() {
    return cur != -1;
  }

  inline static bool is_blank(char c) {
    return c <= ' ';
  }

  inline bool skip_blanks() {
    while(is_blank(cur) && cur != -1) {
      get_char();
    }
    return cur != -1;
  }

  inline FastInput& operator>>(char& c) {
    skip_blanks();
    c = cur;
    return *this;
  }

  inline FastInput& operator>>(string& s) {
    if(skip_blanks()) {
      s.clear();
      do {
        s += cur;
      } while(!is_blank(get_char()));
    }
    return *this;
  }

  template<class T>
  inline FastInput& read_integer(T& n) {
    // unsafe, doesn't check that characters are actually
        digits
    n = 0;
    if(skip_blanks()) {
      int sign = +1;
      if(cur == '-') {
        sign = -1;
        get_char();
      }
      do {
        n += n + (n << 3) + cur - '0';
      } while(!is_blank(get_char()));
      n *= sign;
    }
    return *this;
  }

  template<class T>
  inline typename enable_if<is_integral<T>::value, FastInput
      &>::type operator>>(T& n) {
    return read_integer(n);
  }

  #if!defined(_WIN32) || defined(_WIN64)
  inline FastInput& operator>>(__int128& n) {
    return read_integer(n);
  }
  #endif

  template<class T>
  inline typename enable_if<is_floating_point<T>::value,
      FastInput&>::type operator>>(T& n) {
    // not sure ifreally fast, for compatibility only
    n = 0;
    if(skip_blanks()) {
      string s;
      (*this) >> s;
      sscanf(s.c_str(), "%lf", &n);
    }
    return *this;
  }
} fast_input;

#define cin fast_input

static struct FastOutput {
  static constexpr int BUF_SIZE = 1 << 20;
  char buf[BUF_SIZE];
  size_t buf_pos = 0;
  static constexpr int TMP_SIZE = 1 << 20;
  char tmp[TMP_SIZE];
  FILE *out = stdout;

  inline void put_char(char c) {
    buf[buf_pos++] = c;
    if(buf_pos == BUF_SIZE) {
      fwrite(buf, 1, buf_pos, out);
      buf_pos = 0;
    }
    // putchar_unlocked(c);
  }

  ~FastOutput() {
    fwrite(buf, 1, buf_pos, out);
  }

  inline FastOutput& operator<<(char c) {
    put_char(c);
    return *this;
  }

  inline FastOutput& operator<<(const char* s) {
    while(*s) {
      put_char(*s++);
    }
    return *this;
  }

  inline FastOutput& operator<<(const string& s) {
    for(int i = 0; i < (int) s.size(); i++) {
      put_char(s[i]);
    }
    return *this;
  }

  template<class T>
  inline char* integer_to_string(T n) {
    // beware of TMP_SIZE
```

```
    char* p = tmp + TMP_SIZE - 1;
    if(n == 0) {
      *--p = '0';
    } else {
      bool is_negative = false;
      if(n < 0) {
        is_negative = true;
        n = -n;
      }
      while(n > 0) {
        *--p = (char) ('0' + n % 10);
        n /= 10;
      }
      if(is_negative) {
        *--p = '-';
      }
    }
    return p;
  }

  template<class T>
  inline typename enable_if<is_integral<T>::value, char*>::type
       stringify(T n) {
    return integer_to_string(n);
  }

  #if!defined(_WIN32) || defined(_WIN64)
  inline char* stringify(__int128 n) {
    return integer_to_string(n);
  }
  #endif

  template<class T>
  inline typename enable_if<is_floating_point<T>::value, char
      *>::type stringify(T n) {
    sprintf(tmp, "%.17f", n);
    return tmp;
  }

  template<class T>
  inline FastOutput& operator<<(const T& n) {
    auto p = stringify(n);
    for(; *p != 0; p++) {
      put_char(*p);
    }
    return *this;
  }
} fast_output;

#define cout fast_output
```

7 Setup

7.1 Template

```
#include <bits/extc++.h>
#include <bits/stdc++.h>
#pragma GCC optimize("O3,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#define IOS ios::sync_with_stdio(0),cin.tie(0),cout.tie(0)
#define int long long
#define double long double
#define pb push_back
#define sz(x) (int)(x).size()
#define all(v) begin(v),end(v)
#define debug(x) cerr<<#x<<" = "<<x<<'\n'
#define LINE cout<<"\n----------------\n"
#define endl '\n'
#define VI vector<int>
#define F first
#define S second
#define MP(a,b) make_pair(a,b)
#define rep(i,m,n) for(int i = m;i<=n;++i)
#define res(i,m,n) for(int i = m;i>=n;--i)
#define gcd(a,b) __gcd(a,b)
#define lcm(a,b) a*b/gcd(a,b)
#define Case() int _;cin>>_;for(int Case = 1;Case<=_;++Case)
#define pii pair<int,int>
using namespace __gnu_cxx;
using namespace __gnu_pbds;
using namespace std;
template <typename K, typename cmp = less<K>, typename T =
     thin_heap_tag> using _heap = __gnu_pbds::priority_queue<K,
     cmp, T>;
template <typename K, typename M = null_type> using _hash =
     gp_hash_table<K, M>;
const int N = 1e6+5,L = 20,mod = 1e9+7;
const long long inf = 2e18+5;
const double eps = 1e-7,pi = acos(-1);
void solve(){
}
signed main(){
  IOS;
  solve();
}
```

8 String

8.1 DynamicKMP

```
template<int ALPHABET, int (*f)(char)>
class DynamicKMP {
public:
  DynamicKMP() {}

  DynamicKMP(const string& s) {
    reserve(s.size());
    for(const char& c : s) {
      push(c);
    }
  }

  void push(char c) {
    int v = f(c);
    dp.emplace_back();
    dp.back()[v] = (int) dp.size();
    if(p.empty()) {
      p.push_back(0);
      return;
    }
    int i = (int) p.size();
    for(int j = 0; j < ALPHABET; ++j) {
      if(j == v) {
        p.push_back(dp[p[i - 1]][j]);
      } else {
        dp.back()[j] = dp[p[i - 1]][j];
      }
    }
  }

  void pop() {
    p.pop_back();
    dp.pop_back();
  }

  int query() const {
    return p.back();
  }

  vector<int> query_all() const {
    return p;
  }

  void reserve(int sz) {
    p.reserve(sz);
    dp.reserve(sz);
  }

private:
  vector<int> p;
  vector<array<int, ALPHABET>> dp;
};
```

8.2 RollingHash

```
template<int HASH_COUNT>
struct RollingHash {

  static const int MAX_HASH_PAIRS = 10;

  // {mul, mod}
  const vector<pair<int, int>> HASH_PAIRS = {{827167801,
       999999937}, {998244353, 999999929}, {146672737,
       922722049}, {204924373, 952311013}, {585761567,
       955873937}, {484547929, 901981687}, {856009481,
       987877511}, {852853249, 996724213}, {937381759,
       994523539}, {116508269, 993179543}};

  int n;
  vector<int> POW[MAX_HASH_PAIRS];
  array<vector<int>, HASH_COUNT> pref;

  int substr(int k, int l, int r) {
    const auto& p = HASH_PAIRS[k];
    if(l == r) {
      return 0;
    }
    int res = pref[k][r - 1];
    if(l > 0) {
      res -= 1LL * pref[k][l - 1] * get_power(k, r - l) % p.
           second;
```

```
    }
    if(res < 0) {
      res += p.second;
    }
    return res;
  }

  // build powers up to x^k
  void build_powers(int k) {
    for(int i = 0; i < HASH_COUNT; ++i) {
      const auto& p = HASH_PAIRS[i];
      int sz = (int) POW[i].size();
      if(sz > k) {
        continue;
      }
      if(sz == 0) {
        POW[i].push_back(1);
        sz = 1;
      }
      while(sz <= k) {
        POW[i].push_back(1LL * POW[i].back() * p.first % p.second);
        sz += 1;
      }
    }
  }

  int get_power(int a, int b) {
    build_powers(b);
    return POW[a][b];
  }

  RollingHash() : RollingHash("") {}

  RollingHash(const string& s) : n(s.size()) {
    //static_assert(HASH_COUNT > 0 && HASH_COUNT <= MAX_HASH_PAIRS);
    for(int i = 0; i < HASH_COUNT; ++i) {
      const auto& p = HASH_PAIRS[i];
      pref[i].resize(n);
      pref[i][0] = s[0];
      for(int j = 1; j < n; ++j) {
        pref[i][j] = (1LL * pref[i][j - 1] * p.first + s[j]) % p.second;
      }
    }
    build_powers(n);
  }

  void add_char(char c) {
    for(int i = 0; i < HASH_COUNT; ++i) {
      const auto& p = HASH_PAIRS[i];
      pref[i].push_back((1LL * (n == 0 ? 0 : pref[i].back()) * p.first + c) % p.second);
    }
    n += 1;
    build_powers(n);
  }

  // Return hash values for [l, r)
  array<int, HASH_COUNT> substr(int l, int r) {
    array<int, HASH_COUNT> res{};
    for(int i = 0; i < HASH_COUNT; ++i) {
      res[i] = substr(i, l, r);
    }
    return res;
  }

  array<int, HASH_COUNT> merge(const vector<pair<int, int>>& seg) {
    array<int, HASH_COUNT> res{};
    for(int i = 0; i < HASH_COUNT; ++i) {
      const auto& p = HASH_PAIRS[i];
      for(auto [l, r] : seg) {
        res[i] = (1LL * res[i] * get_power(i, r - l) + substr(i, l, r)) % p.second;
      }
    }
    return res;
  }

  inline int size() const {
    return n;
  }
};
```

8.3 SuffixArray

```
struct suffix_array{
  int n;
  vector<int>SA,Rank,LCP;
  void counting_sort(vector<int>&v,auto getkey){
    int n = 0;
    for(auto i:v)n = max(n,getkey(i)+1);
    vector<int>bucket(n),ans(v.size());
    for(auto i:v)++bucket[getkey(i)];
    partial_sum(begin(bucket),end(bucket),begin(bucket));
    for(auto ite = v.rbegin();ite!=v.rend();++ite)ans[--bucket[getkey(*ite)]] = move(*ite);
    v.swap(ans);
    return;
  }
  suffix_array(string s):n(s.size()){
    SA.resize(n),Rank.resize(n),LCP.resize(n);
    for(int i = 0;i<n;++i)SA[i] = i;
    sort(SA.begin(),SA.end(),[&](int a,int b){
      return s[a]<s[b];
    });
    for(int i = 0;i<n;++i){
      Rank[SA[i]] = (i?Rank[SA[i-1]]+(s[SA[i]]!=s[SA[i-1]]):SA[0]);
    }
    for(int k = 0;(1<<k)<=n;++k){
      vector<int>idx;
      for(int i = n-(1<<k);i<n;++i)idx.push_back(i);
      for(auto i:SA)if(i>=(1<<k))idx.push_back(i-(1<<k));
      counting_sort(idx,[&](int a){return Rank[a];});
      SA.swap(idx);
      vector<int>new_rank(n);
      new_rank[SA[0]] = 0;
      for(int i = 1;i<n;++i){
        auto cmp = [&](int a,int b){
          return Rank[a]!=Rank[b] or a+(1<<k)>=n or Rank[a+(1<<k)]!=Rank[b+(1<<k)];
        };
        new_rank[SA[i]] = new_rank[SA[i-1]]+cmp(SA[i-1],SA[i]);
      }
      Rank.swap(new_rank);
    }
    for(int i = 0,k = 0;i<n;++i){
      if(Rank[i]==0)continue;
      if(k)--k;
      while(i+k<n and SA[Rank[i]-1]+k<n and s[i+k]==s[SA[Rank[i]-1]+k])++k;
      LCP[Rank[i]] = k;
    }
  }
};
```

8.4 Trie

```
template<int ALPHABET = 26, char MIN_CHAR = 'a'>
class trie {
public:
  struct Node {
    int go[ALPHABET];
    Node() {
      memset(go, -1, sizeof(go));
    }
  };

  trie() {
    newNode();
  }

  inline int next(int p, int v) {
    return nodes[p].go[v] != -1 ? nodes[p].go[v] : nodes[p].go[v] = newNode();
  }

  inline void insert(const vector<int>& a, int p = 0) {
    for(int v : a) {
      p = next(p, v);
    }
  }

  inline void clear() {
    nodes.clear();
    newNode();
  }

  inline int longest_common_prefix(const vector<int>& a, int p = 0) const {
    int ans = 0;
    for(int v : a) {
      if(nodes[p].go[v] != -1) {
        ans += 1;
        p = nodes[p].go[v];
      } else {
        break;
      }
    }
    return ans;
  }

private:
  vector<Node> nodes;

```

```
46    inline int newNode() {
47      nodes.emplace_back();
48      return (int) nodes.size() - 1;
49    }
50  };
```

ACM ICPC Team Reference - LeeJiaHuaPlayMinecraft

Contents

ACM ICPC Judge Test - LeeJiaHuaPlayMinecraft

C++ Resource Test

```cpp
#include <bits/stdc++.h>
using namespace std;

namespace system_test {

const size_t KB = 1024;
const size_t MB = KB * 1024;
const size_t GB = MB * 1024;

size_t block_size, bound;
void stack_size_dfs(size_t depth = 1) {
  if (depth >= bound)
    return;
  int8_t ptr[block_size]; // 若無法編譯將 block_size 改成常數
  memset(ptr, 'a', block_size);
  cout << depth << endl;
  stack_size_dfs(depth + 1);
}

void stack_size_and_runtime_error(size_t block_size, size_t
     bound = 1024) {
  system_test::block_size = block_size;
  system_test::bound = bound;
  stack_size_dfs();
}

double speed(int iter_num) {
  const int block_size = 1024;
  volatile int A[block_size];
  auto begin = chrono::high_resolution_clock::now();
  while (iter_num--)
    for (int j = 0; j < block_size; ++j)
      A[j] += j;
  auto end = chrono::high_resolution_clock::now();
  chrono::duration<double> diff = end - begin;
  return diff.count();
}

void runtime_error_1() {
  // Segmentation fault
  int *ptr = nullptr;
  *(ptr + 7122) = 7122;
}

void runtime_error_2() {
  // Segmentation fault
  int *ptr = (int *)memset;
  *ptr = 7122;
}

void runtime_error_3() {
  // munmap_chunk(): invalid pointer
  int *ptr = (int *)memset;
  delete ptr;
}

void runtime_error_4() {
  // free(): invalid pointer
  int *ptr = new int[7122];
  ptr += 1;
  delete[] ptr;
}

void runtime_error_5() {
  // maybe illegal instruction
  int a = 7122, b = 0;
  cout << (a / b) << endl;
}

void runtime_error_6() {
  // floating point exception
  volatile int a = 7122, b = 0;
  cout << (a / b) << endl;
}

void runtime_error_7() {
  // call to abort.
  assert(false);
}

} // namespace system_test

#include <sys/resource.h>
void print_stack_limit() { // only work in Linux
  struct rlimit l;
  getrlimit(RLIMIT_STACK, &l);
  cout << "stack_size = " << l.rlim_cur << " byte" << endl;
}
```