#### Data-Structure 1

## 1.1 Treap

```
template<class S
       S (*node_pull)(S, S),
S (*node_init)(S),
        class T,
       S (*mapping)(S, T),
T (*tag_pull)(T, T),
T (*tag_init)()>
   struct Treap{
     struct node{
  node *1 = NULL, *r = NULL, *p = NULL;
        int pri = mt(),sz = 1;
       S info;
       T tag = tag_init();
13
       bool rev;
node(S k) : info(k){}
14
15
       ~node(){
          for(auto &i:{1,r})
             delete i;
19
       void all_apply(T t,bool is_rev){
  if(is_rev){
20
21
             swap(1,r);
          info = mapping(info, t);
          tag = tag_pull(tag, t);
26
27
        void push(){
28
          for(auto &i:{1,r})
           if(i)i->all_apply(tag, rev);
31
          tag = tag_init();
          rev = 0:
32
33
       void pull(){
          sz = 1,info = node_init(info);
          for(auto &i:{1,r}){
            if(i){
               sz+=i->sz,i->p = this;
38
               info = node_pull(info,i->info);
40
41
          }
       }
43
     node *root = NULL;
     int size(node *a){
45
       return a?a->sz:0;
46
47
     int size(){
       return size(root);
49
50
51
     node *merge(node *a,node *b){
       if(!a or !b)return a?:b;
52
53
       if(a->pri>b->pri){
          a->push();
          a \rightarrow r = merge(a \rightarrow r, b);
          a->r->p = a;
a->pull();
56
57
58
          return a:
59
60
       else{
          b->push();
          b->l = merge(a,b->l);
b->l->p = b;
b->pull();
62
63
64
65
          return b;
       }
66
67
     void split(node *t, long long k, node *&a, node *&b, bool bst
       if(!t){a = b = NULL;return;}
69
70
        t->push();
        if((bst==0 and size(t->1)+1<=k) or (bst==1 and t->info.key
71
73
          split(t->r), ( bst ? k : k - size(t->l) - 1 ), a->r, b,
          bst);
if(b)b->p = NULL;
          a->pull();
75
       else{
          b = t;
78
          split(t->1, k, a, b->1, bst);
          if(a)a->p = NULL;
80
          b->pull();
81
82
     node *insert(long long idx, S x,bool bst = 0){
       node *a,*b;
85
       split(root, idx, a, b, bst);
node *tmp = new node(x);
86
87
```

```
88
          root = merge(a, merge(tmp, b));
89
          return tmp;
90
91
       void erase(long long l,long long r,bool bst = 0){
          node *a,*b,*c;
93
          split(root, (bst? 1-1 : 1), a, b, bst);
          split(b, (bst? r : r - l + 1), b, c, bst);
95
          delete b;
          root = merge(a,c);
96
97
      f
    operator [](int x){
    node *a, *b, *c;
    split(root, x, a, b, 0);
    split(b, 1, b, c, 0);
    assert(b!=NULL);
100
101
102
          S ans = b->info;
103
          root = merge(a, merge(b, c));
          return ans;
106
       void update(long long l,long long r,T t,bool bst = 0){
  node *a, *b, *c;
  split(root, (bst? l - 1 : l), a, b, bst);
107
108
109
          split(b, (bst? r : r - l + 1), b, c, bst);
110
          if(b)b->all_apply(t, 0);
111
          root = merge(a, merge(b, c));
112
113
       void reverse(long long l,long long r,bool bst = 0){
114
         node *a, *b, *c;

split(root, (bst? l - 1 : l), a, b, bst);

split(b, (bst? r : r - l + 1), b, c, bst);
115
116
          if(b)b->all_apply(tag_init(), 1);
118
          root = merge(a, merge(b, c));
119
120
       int rank(long long k){
  node *a, *b;
  split(root, k - 1, a, b, 1);
121
122
          int ans = size(a);
125
          root = merge(a, b);
          return ans:
126
127
      S* find_next(long long k){
  node *a, *b, *c;
128
129
         split(root, k - 1, a, b, 1);
split(b, 1, b, c, 0);
S* ans = NULL;
if(b)ans = &b->info;
130
131
132
133
          root = merge(a, merge(b, c));
134
          return ans;
135
      137
138
139
140
          if(c)ans = &c->info;
          root = merge(merge(a, c), b);
144
          return ans;
145
       S query(long long l,long long r,bool bst = 0){
  node *a, *b, *c;
  split(root, (bst? l - 1 : l), a, b, bst);
146
147
          split(b, (bst? r : r - l + 1), b, c, bst);
149
150
          assert(b!=NULL);
         S ans = b->info;
root = merge(a, merge(b, c));
151
152
          return ans;
153
154
```

## 1.2 Segtree

```
class T,
         S (*mapping)(S, T),
T (*tag_pull)(T, T),
         T (*tag_init)()>
   struct segment_tree{
     struct node{
       S seg;
10
        T tag = tag_init();
11
        int l,r;
12
13
        node(S \_seg = node\_init(), int \_l = -1, int \_r = -1) : seg(
        __seg), 1(_1), r(_r){}

friend node operator +(const node &lhs,const node &rhs){
    if(lhs.l==-1)return rhs;
    if(rhs.l==-1)return lhs;
15
16
          return node(node_pull(lhs.seg,rhs.seg),lhs.l,rhs.r);
17
20
     vector<node>arr:
     void all_apply(int idx,T t){
21
       arr[idx].seg = mapping(arr[idx].seg, t);
```

```
arr[idx].tag = tag_pull(arr[idx].tag, t);
24
      void push(int idx){
25
        all_apply(idx<<1, arr[idx].tag);</pre>
26
        all_apply(idx<<1|1, arr[idx].tag);</pre>
        arr[idx].tag = tag_init();
29
     inline void build(const vector<S> &v,const int &l,const int &
30
           r, int idx = 1){
        if(idx==1)arr.resize((r-l+1)<<2);
31
        if(l==r){
          arr[idx].seg = v[1];
arr[idx].tag = tag_init();
arr[idx].1 = arr[idx].r = 1;
35
36
          return;
37
        int m = (1+r)>>1;
        build(v,1,m,idx<<1);</pre>
40
        \verb|build(v,m+1,r,idx<<1|1);\\
        arr[idx] = arr[idx<<1]+arr[idx<<1|1];
41
42
      inline void update(const int &ql,const int &qr,T t,int idx =
43
           1){
        assert(ql<=qr);
45
        if(ql<=arr[idx].l and arr[idx].r<=qr){</pre>
46
           all_apply(idx, t);
47
          return;
48
        push(idx);
        int m = (arr[idx].l+arr[idx].r)>>1;
        if(ql<=m)update(ql,qr,t,idx<<1);</pre>
        if(qr>m)update(ql,qr,t,idx<<1|1);
arr[idx] = arr[idx<<1]+arr[idx<<1|1];</pre>
53
55
      inline S query(const int &ql,const int &qr,int idx = 1){
        assert(ql<=qr);
if(ql<=arr[idx].l and arr[idx].r<=qr){
58
          return arr[idx].seg;
        push(idx);
60
        int m = (arr[idx].l+arr[idx].r)>>1;
S ans = node_init(),lhs = node_init(),rhs = node_init();
61
62
        if(ql<=m)lhs = query(ql,qr,idx<<1);
if(qr>m)rhs = query(ql,qr,idx<<1|1);</pre>
63
65
        ans = node_pull(lhs,rhs);
        return ans:
66
67
68 };
```

## 1.3 DsuUndo

```
struct dsu_undo{
     vector<int>sz,p;
     int comps;
     dsu_undo(int n){
       sz.assign(n+5,1);
        p.resize(n+5);
        for(int i = 1;i<=n;++i)p[i] = i;</pre>
        comps = n;
     vector<pair<int,int>>opt;
     int Find(int x){
12
        return x==p[x]?x:Find(p[x]);
     bool Union(int a,int b){
       int pa = Find(a),pb = Find(b);
if(pa==pb)return 0;
15
        if(sz[pa]<sz[pb])swap(pa,pb);</pre>
17
       sz[pa]+=sz[pb];
p[pb] = pa;
        opt.push_back({pa,pb});
        comps - -
22
        return 1;
23
     void undo(){
24
25
             auto [pa,pb] = opt.back();
            opt.pop_back();
p[pb] = pb;
sz[pa]-=sz[pb];
28
29
             comps++;
30
31 };
```

## 1.4 **DSU**

```
struct DSU{
  vector<int>sz;
  int n;
  DSU(int _n):n(_n){
    sz.assign(n+1,-1);
  }
}
```

```
7    int Find(int x){
8        return sz[x]<0?x:sz[x] = Find(sz[x]);
9        bool Union(int a,int b){
10        int pa = Find(a),pb = Find(b);
12        if(pa==pb)return 0;
13        if((-sz[pa])<(-sz[pb]))swap(pa,pb);
14        sz[pa]+=sz[pb];
15        sz[pb] = pa;
16        return 1;
17        }
18        };</pre>
```

## 1.5 Fenwick

```
i template < class T>struct fenwick_tree{
     int n:
     vector<T>arr:
     inline int lowbit(int x){
       return x&(-x);
     fenwick\_tree(int _n) : n(_n){}
       arr.assign(n+5,0);
     T query(int x){
       T ans = 0;
for(int i = x;i>0;i-=lowbit(i)){
11
         ans+=arr[i];
       return ans;
     void update(int x,T y){
17
       for(int i = x;i<=n;i+=lowbit(i)){</pre>
18
         arr[i]+=y;
21
22 };
```

#### 1.6 Persistent DSU

```
i int rk[200001] = {};
   struct Persistent_DSU{
     rope<int>*p;
      int n;
      Persistent_DSU(int _n = 0):n(_n){
        if(n==0)return;
        p = new rope<int>;
int tmp[n+1] = {};
for(int i = 1;i<=n;++i)tmp[i] = i;</pre>
        p->append(tmp,n+1);
11
     Persistent_DSU(const Persistent_DSU &tmp){
  p = new rope<int>(*tmp.p);
12
13
        n = tmp.n;
15
      int Find(int x){
        int px = p \rightarrow at(x)
17
18
        return px==x?x:Find(px);
19
     bool Union(int a,int b){
20
        int pa = Find(a),pb = Find(b);
if(pa==pb)return 0;
21
23
        if(rk[pa]<rk[pb])swap(pa,pb);</pre>
        p->replace(pb,pa);
        if(rk[pa]==rk[pb])rk[pa]++;
        return 1;
27
```

## 1.7 TimingSegtree

```
template < class T, class D>struct timing_segment_tree{
     struct node{
       int 1,r;
       vector<T>opt;
     }:
     vector<node>arr;
     void build(int 1,int r,int idx = 1){
       if(idx==1)arr.resize((r-1+1)<<2);</pre>
       if(l==r){
          arr[idx].1 = arr[idx].r = 1;
          arr[idx].opt.clear();
11
          return;
12
13
       int m = (l+r)>>1;
       build(l,m,idx<<1);</pre>
       build(m+1,r,idx<<1|1);
arr[idx].l = l,arr[idx].r = r;</pre>
16
17
       arr[idx].opt.clear();
```

```
void update(int ql,int qr,T k,int idx = 1){
  if(ql<=arr[idx].l and arr[idx].r<=qr){</pre>
20
21
22
          arr[idx].opt.push_back(k);
          return;
25
        int m = (arr[idx].l+arr[idx].r)>>1;
        if(ql<=m)update(ql,qr,k,idx<<1);</pre>
26
27
        if(qr>m)update(ql,qr,k,idx<<1|1);</pre>
     void dfs(D &d, vector<int>&ans,int idx = 1){
        int cnt = 0;
for(auto [a,b]:arr[idx].opt){
32
          if(d.Union(a,b))cnt++;
33
        if(arr[idx].l==arr[idx].r)ans[arr[idx].l] = d.comps;
35
          dfs(d,ans,idx<<1)
37
          dfs(d,ans,idx<<1|1);</pre>
        while(cnt--)d.undo();
40
41 };
```

11

12

17 18

23

24

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70

## 1.8 AreaOfRectangles

```
\label{long_long_long} \ long \ long \ AreaOfRectangles(vector< tuple< \\ int, int, int, int>>v) \{
     vector<tuple<int,int,int,int>>tmp;
int L = INT_MAX,R = INT_MIN;
     for(auto [x1,y1,x2,y2]:v){
       tmp.push_back({x1,y1+1,y2,1})
       tmp.push_back({x2,y1+1,y2,-1});
R = max(R,y2);
       L = min(L,y1);
     vector<long long>seg((R-L+1)<<2),tag((R-L+1)<<2);</pre>
     sort(tmp.begin(),tmp.end());
     function<void(int,int,int,int,int,int)>update = [&](int ql,
           int qr,int val,int l,int r,int idx){
       if(ql<=l and r<=qr){
  tag[idx]+=val;</pre>
13
14
          if(tag[idx])seg[idx] = r-l+1;
          else if(l==r)seg[idx] = 0;
          else seg[idx] = seg[idx<<1]+seg[idx<<1|1];</pre>
19
20
       int m = (1+r) >> 1;
       if(ql<=m)update(ql,qr,val,l,m,idx<<1);</pre>
21
       if(qr>m)update(q1,qr,va1,m+1,r,idx<<1|1);
if(tag[idx])seg[idx] = r-l+1;</pre>
        else seg[idx] = seg[idx<<1]+seg[idx<<1|1];</pre>
     long long last_pos = 0, ans = 0;
26
     for(auto [pos,l,r,val]:tmp){
27
       ans+=(pos-last_pos)*seg[1];
        update(l,r,val,L,R,1);
       last_pos = pos;
30
31
32
     return ans;
33 }
```

## 1.9 SparseTable

```
template < class T,T (*op)(T,T)>struct sparse_table{
     int n;
     vector<vector<T>>mat:
     sparse_table(): n(0){}
     sparse_table(const vector<T>&v){
       n = (int)(v.size());
       mat.resize(30);
       mat[0] = v;
for(int i = 1;(1<<i)<=n;++i){</pre>
          mat[i].resize(n-(1<<i)+1);
          for(int j = 0; j < n - (1 < i) + 1; ++j){
            mat[i][j] = op(mat[i-1][j], mat[i-1][j+(1<<(i-1))]);
13
       }
15
    T query(int ql,int qr){
  int k = __lg(qr-ql+1);
17
       return op(mat[k][q1],mat[k][qr-(1<<k)+1]);</pre>
20 };
```

## 1.10 VEBTree

```
ı|// Can correctly work with numbers in range [0; MAXN]
 // Supports all std::set operations in O(1) on random queries /
    dense arrays, O(log_64(N)) in worst case (sparce array).
// Count operation works in O(1) always.
  template < unsigned int MAXN >
  class fast_set {
  private:
     static const unsigned int PREF = (MAXN <= 64 ? 0 :</pre>
                         MAXN <= 4096 ? 1 :
                         MAXN <= 262144 ? 1 + 64 :
                         MAXN <= 16777216 ? 1 + 64 + 4096 :
                         MAXN <= 1073741824 ? 1 + 64 + 4096 + 262144 :
    227) + 1;

static constexpr unsigned long long lowest_bitsl[] = {0ULL,

1ULL, 3ULL, 7ULL, 15ULL, 31ULL, 63ULL, 127ULL, 255ULL,

511ULL, 1023ULL, 2047ULL, 4095ULL, 8191ULL, 16383ULL,
            32767ULL, 65535ULL, 131071ULL, 262143ULL, 524287ULL
            1048575ULL, 2097151ULL, 4194303ULL, 8388607ULL, 16777215
           ULL, 33554431ULL, 67108863ULL, 134217727ULL, 268435455ULL, 536870911ULL, 1073741823ULL, 2147483647ULL, 4294967295 ULL, 8589934591ULL, 17179869183ULL, 34359738367ULL,
            68719476735ULL, 137438953471ULL, 274877906943ULL, 549755813887ULL, 1099511627775ULL, 2199023255551ULL
            4398046511103ULL, 8796093022207ULL, 17592186044415ULL
            35184372088831ULĹ, 70368744177663UĹL, 140737488355327ÚLL
             281474976710655ULL, 562949953421311ULL, 1125899906842623
           ULL, 2251799813685247ULL, 4503599627370495ULL, 9007199254740991ULL, 18014398509481983ULL, 36028797018963967ULL, 72057594037927935ULL,
            144115188075855871ULL, 288230376151711743ULL, 576460752303423487ULL, 1152921504606846975ULL,
     2305843009213693951ULL, 4611686018427387903ULL, 9223372036854775807ULL, 18446744073709551615ULL}; static const unsigned int SZ = PREF + (MAXN + 63) / 64 + 1;
     unsigned long long m[SZ] = {0};
     inline unsigned int left(unsigned int v) const {
  return (v - 62) * 64;
     inline unsigned int parent(unsigned int v) const {
       return v / 64 + 62;
     inline void setbit(unsigned int v) {
  m[v >> 6] |= 1ULL << (v & 63);</pre>
     inline void resetbit(unsigned int v) {
       m[v >> 6] &= ~(1ULL << (v & 63));
    inline unsigned int getbit(unsigned int v) const {
  return m[v >> 6] >> (v & 63) & 1;
     inline unsigned long long childs_value(unsigned int v) const
        return m[left(v) >> 6];
     inline int left_go(unsigned int x, const unsigned int c)
           const {
        const unsigned long long rem = x & 63;
unsigned int bt = PREF * 64 + x;
        unsigned long long num = m[bt >> 6] & lowest_bitsll[rem + c
              1;
        if(num) {
           return (x ^ rem) | __lg(num);
        for(bt = parent(bt); bt > 62; bt = parent(bt)) {
          const unsigned long long rem = bt & 63;
num = m[bt >> 6] & lowest_bitsll[rem];
           if(num) {
             bt = (bt ^ rem) | __lg(num);
             break;
          }
       if(bt == 62) {
        while(bt < PREF * 64) {</pre>
          bt = left(bt) | __lg(m[bt - 62]);
        return bt - PREF * 64;
     inline int right_go(unsigned int x, const unsigned int c)
            \quad \text{const } \{
        const unsigned long long rem = x & 63;
unsigned int bt = PREF * 64 + x;
        unsigned long long num = m[bt >> 6] & ~lowest_bitsll[rem +
        if(num) {
           return (x ^ rem) | __builtin_ctzll(num);
        for(bt = parent(bt); bt > 62; bt = parent(bt)) {
```

```
const unsigned long long rem = bt & 63;
num = m[bt >> 6] & ~lowest_bitsll[rem + 1];
73
74
           if(num) {
75
             bt = (bt ^ rem) | __builtin_ctzll(num);
76
77
78
        if(bt == 62) {
79
80
          return -1;
81
        while(bt < PREF * 64) {</pre>
83
          bt = left(bt) | __builtin_ctzll(m[bt - 62]);
        return bt - PREF * 64;
85
     }
86
87
   public:
      fast_set() {
        assert(PREF != 228);
90
91
        setbit(62);
92
93
      bool empty() const {return getbit(63);}
      void clear() {
       fill(m, m + SZ, 0);
setbit(62);
97
99
100
      bool count(unsigned int x) const {
101
        return m[PREF + (x >> 6)] >> (x & 63) & 1;
102
103
104
      void insert(unsigned int x) {
  for(unsigned int v = PREF * 64 + x; !getbit(v); v = parent(
105
106
              v)) {
           setbit(v);
107
108
     }
109
110
     void erase(unsigned int x) {
  if(!getbit(PREF * 64 + x)) {
111
112
113
114
        115
116
           resetbit(v);
11
118
119
120
      int find_next(unsigned int x) const {
121
        return right_go(x, 0);
122
123
      int find_prev(unsigned int x) const {
126
        return left_go(x, 1);
127
128 };
```

## 1.11 DynamicSegtree

```
template < class T > struct dynamic_segment_tree{
    struct node{
      node *1 = NULL,*r = NULL;
      T sum;
      node(T k = 0): sum(k){}
      node(node *p){if(p)*this = *p;}
      ~node(){
         for(auto &i:{1,r})
           if(i)delete i;
      void pull(){
        sum = 0;
for(auto i:{1,r})
          if(i)sum+=i->sum;
    }*root = NULL;
18
     dynamic segment tree(){}
19
     dynamic_segment_tree(const dynamic_segment_tree<T>&tmp){root
20
           new node(tmp.root);}
     void update(node *&t,int pos,T k,int l,int r){
      if(!t)t = new node();
      if(l==r)return t = new node(k),void();
      int m = (1+r)>>1;
      t = new node(t);
      if(pos<=m)update(t->1,pos,k,1,m);
       else update(t->r,pos,k,m+1,r);
       t->pull();
    }void update(int pos,T k,int l = -1e9,int r = 1e9){update(
29
    root,pos,k,l,r);}
T query(node *&t,int ql,int qr,int l,int r){
```

```
if(!t)return 0;
if(q1<=1 and r<=qr)return t->sum;
int m = (1+r)>>1;
T ans = 0;
if(q1<=m)ans+=query(t->1,q1,qr,1,m);
if(qr>m)ans+=query(t->r,q1,qr,m+1,r);
return ans;
}T query(int q1,int qr,int l = -1e9,int r = 1e9){return query (root,q1,qr,1,r);}
}
};
```

## 1.12 ZkwSegtree

template<class S,

```
S (*node_pull)(S, S),
S (*node_init)(),
              class F
             S (*mapping)(S, F),
F (*tag_pull)(F, F),
F (*tag_init)()>
   class segment_tree {
   public:
     11
     explicit segment_tree(const vector<S>& v) : n((int) v.size())
12
        log = std::__lg(2 * n - 1);
size = 1 << log;
        d = vector<S>(size << 1, node_init());</pre>
15
        lz = vector<F>(size, tag_init());
for(int i = 0; i < n; i++) {
   d[size + i] = v[i];</pre>
20
        for(int i = size - 1; i; --i) {
21
          update(i);
        }
23
      void set(int p, S x) {
        assert(0 <= p && p < n);
        p += size;
for(int i = log; i; --i) {
          push(p >> i);
        d[p] = x;
        for(int i = 1; i <= log; ++i) {</pre>
32
          update(p >> i);
33
34
     S get(int p) {
  assert(0 <= p && p < n);</pre>
35
36
        p += size;
        for(int i = log; i; i--) {
39
         push(p >> i);
40
41
        return d[p];
42
     S operator[](int p) {
44
        return get(p);
45
46
     S query(int 1, int r) {
47
        assert(1<=r);
48
        1 += size;
        r += size;
51
        for(int i = log; i; i--) {
          if(((1 >> i) << i) != 1) {
  push(1 >> i);
53
          if(((r >> i) << i) != r) {</pre>
            push(r >> i);
58
        S sml = node_init(), smr = node_init();
59
        while(1 < r) {
   if(1 & 1) {</pre>
60
61
            sml = node_pull(sml, d[1++]);
          if(r & 1) {
            smr = node_pull(d[--r], smr);
          1 >>= 1;
          r >>= 1;
        return node_pull(sml, smr);
70
71
     void apply(int p, F f) {
72
        assert(0 <= p && p < n);
73
        p += size;
        for(int i = log; i; i--) {
          push(p >> i);
        d[p] = mapping(f, d[p]);
        for(int i = 1; i <= log; i++) {</pre>
```

```
80 l
           update(p >> i);
81
        }
82
      void update(int 1, int r, F f) {
83
         assert(l<=r);
86
         1 += size;
         r += size;
87
         for(int i = log; i; i--) {
  if(((1 >> i) << i) != 1) {</pre>
88
89
             push(1 >> i);
91
           if(((r >> i) << i) != r) {
  push((r - 1) >> i);
93
95
         {
           int 12 = 1, r2 = r;
while(1 < r) {</pre>
98
             if(1 & 1) {
99
100
                all_apply(l++, f);
101
              if(r & 1) {
102
                all_apply(--r, f);
103
104
              1 >>= 1;
105
106
              r >>= 1;
107
             = 12;
108
109
           r = r2;
110
         for(int i = 1; i <= log; i++) {
  if(((1 >> i) << i) != 1) {</pre>
111
112
              update(1 >> i);
113
114
115
           if(((r >> i) << i) != r) {</pre>
              update((r - 1) >> i);
116
117
118
        }
119
   private:
120
121
      int n, size, log;
      vector<S> d;
122
      vector<F> lz;
123
      124
125
126
128
           lz[k] = tag_pull(lz[k], f);
129
         }
130
      void push(int k) {
131
        all_apply(k << 1, lz[k]);
all_apply(k << 1 | 1, lz[k]);
132
133
         lz[k] = tag_init();
135
136 };
```

## 1.13 MoAlgo

```
1 struct qry{
    int ql,qr,id;
   };
   template < class T>struct Mo{
    int n,m;
     vector<pii>ans;
    Mo(int _n,int _m): n(_n),m(_m){
  ans.resize(m);
     void solve(vector<T>&v, vector<qry>&q){
       int 1 = 0,r = -1;
vector<int>cnt,cntcnt;
12
13
       cnt.resize(n+5);
       cntcnt.resize(n+5);
14
        int mx = 0;
15
       function<void(int)>add = [&](int pos){
         cntcnt[cnt[v[pos]]]--;
cnt[v[pos]]++;
         cntcnt[cnt[v[pos]]]++;
mx = max(mx,cnt[v[pos]]);
19
20
21
       function<void(int)>sub = [&](int pos){
23
          if(!--cntcnt[cnt[v[pos]]] and cnt[v[pos]]==mx)mx--;
24
          cnt[v[pos]]--;
          cntcnt[cnt[v[pos]]]++;
25
26
         mx = max(mx,cnt[v[pos]]);
       sort(all(q),[&](qry a,qry b){
          static int B = max((int)1,n/max((int)sqrt(m),(int)1));
          if(a.ql/B!=b.ql/B)return a.ql<b.ql;</pre>
          if((a.ql/B)&1)return a.qr>b.qr;
31
          return a.qr<b.qr;</pre>
32
33
```

## 1.14 Hash

```
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 31)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
}
size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM = chrono::steady_clock::
        now().time_since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
}
size_t operator()(pair<uint64_t, uint64_t > x) const {
    static const uint64_t FIXED_RANDOM = chrono::steady_clock::
        now().time_since_epoch().count();
    return splitmix64(3*x.first + x.second + FIXED_RANDOM);
}
topic const uint64_t FIXED_RANDOM = chrono::steady_clock::
        now().time_since_epoch().count();
    return splitmix64(3*x.first + x.second + FIXED_RANDOM);
}
topic const uint64_t FIXED_RANDOM = chrono::steady_clock::
        now().time_since_epoch().count();
    return splitmix64(3*x.first + x.second + FIXED_RANDOM);
}
topic const uint64_t FIXED_RANDOM = chrono::steady_clock::
        now().time_since_epoch().count();
    return splitmix64(3*x.first + x.second + FIXED_RANDOM);
}
topic const uint64_t FIXED_RANDOM = chrono::steady_clock::
        now().time_since_epoch().count();
    return splitmix64(3*x.first + x.second + FIXED_RANDOM);
}
```

#### 1.15 RedBlackTree

# 2 Geometry

## 2.1 Theorem

```
• Pick's Theorem A = I + \frac{B}{2} - 1 A := Area i := PointsInside B := PointsBoundary
```

## 2.2 PointInPolygon

```
template < class T >
int PointInPolygon(const vector < Point < T >> & Poly, const Point < T >> p) {
   int ans = 0;
   for (auto a = --Poly.end(), b = Poly.begin(); b! = Poly.end(); a = b++) {
      if (PointOnSegment(*a,*b,p)) {
        return -1;
      }
      if (seg_intersect(p,p+Point < T > (2e9+7,1),*a,*b)) {
        ans = !ans;
      }
    }
   return ans;
}
```

## 2.3 PointInConvex

```
template < class T >
int PointInConvex (const vector < Point < T > & C, const Point < T > & Point InConvex (const vector < Point < T > & C, const Point < T > & Point InConvex (const vector < Point < T > & C, const Point < T > & Point InConvex (const vector < Point < T > & C, const Point < T > & Point InConvex (const vector < Point < T > & C, const Point < T > & Point < T > & C, const Point < T > & Point < T > & C, const Point <
```

## 2.4 MaximumDistance

```
template < class T >
T MaximumDistance(vector < Point < T >> & p) {
    vector < Point < T >> & p) {
        vector < Point < T >> & p) {
        vector < Point < T >> & p) {
        vector < Point < T >> & p) {
        vector < Point < T >> & p) {
        vector < Point < T >> & p) {
        int n = C.size(), t = 2;
        T ans = 0;
        for (int i = 0; i < n; i ++) {
            while(((C[i] - C[t]) ^ (C[(i+1)%n] - C[t])) < ((C[i] - C[(t+1)%n])) t = (t+1)%n;
            ans = max({ans, abs2(C[i] - C[t]), abs2(C[(i+1)%n] - C[t])
            });
        }
        return ans;
}</pre>
```

## 2.5 PolarAngleSort

```
template < class T >
bool cmp(const Point < T > &a, const Point < T > &b) {
   int lhs = (a, y < 0 || a, y = 0 && a, x > 0) ? 0 : (1 + (a, x != 0 || a, y != 0));
   int rhs = (b, y < 0 || b, y = 0 && b, x > 0) ? 0 : (1 + (b, x != 0 || b, y != 0));
   if(lhs != rhs) {
      return lhs < rhs;
   }
   long long area = (a^b);
   return area ? area > 0 : abs(a, x) + abs(a, y) < abs(b, x) + abs (b, y);
}</pre>
```

## 2.6 MinimumDistance

```
i template < class T>
   T MinimumDistance(vector<Point<T>>&p,int l = -1,int r = -1){
     if(l==-1 and r==-1){
         sort(p.begin(),p.end(),[](Point<T> a,Point<T> b){
           if(a.x!=b.x)return a.x<b.x;</pre>
           return a.y<b.y;</pre>
        });
        p.erase(unique(p.begin(),p.end()),p.end());
         return MinimumDistance(p,0,p.size()-1);
     if(l==r)return numeric_limits<T>::max();
     int m = (l+r)>>1,mid_pos = p[m].x;
T ans = min(MinimumDistance(p,1,m),MinimumDistance(p,m+1,r));
vector<Point<T>>tmp((r-l+1),Point<T>(0,0));
merge(p.begin()+1,p.begin()+m+1, p.begin()+m+1,p.begin()+r+1,
13
              tmp.begin(), [](Point<T> a,Point<T> b){return a.y<b.y;})</pre>
      for(int i = 1;i<=r;++i)p[i] = tmp[i-1];</pre>
     tmp.clear();
for(int i = 1;i<=r;++i){</pre>
        if((p[i].x-mid_pos)*(p[i].x-mid_pos)<ans){</pre>
           tmp.push_back(p[i]);
21
        }
     int n = tmp.size();
for(int i = 0;i<n;++i){
   for(int j = i+1;j<n;++j){</pre>
23
           ans = min(ans,abs2(tmp[i]-tmp[j]));
           if(((tmp[i].y-tmp[j].y)*(tmp[i].y-tmp[j].y))>ans){}
28
              break:
           }
```

```
31  }
32  return ans;
```

## 2.7 ConvexHull

```
i template < class T>
   vector<Point<T>> ConvexHull(vector<Point<T>> v,bool Boundary =
      sort(begin(v),end(v),[&](Point<T> &a,Point<T> &b){
        if(a.x!=b.x)return a.x<b.x:</pre>
        return a.y<b.y;</pre>
      vector<Point<T>>ans;
     int t = 1;
      auto add = [&](Point<T> &p){
        while(ans.size() > t and ((p - ans[ans.size() - 2])^(ans.
back() - ans[ans.size() - 2])) > (Boundary ? 0 : 0-eps)
           ans.pop_back();
        ans.push_back(p);
13
14
     for(int i = 0; i < v.size(); ++i) add(v[i]);</pre>
     t = ans.size();
for(int i = (int)(v.size())-2; i >= 0; --i) add(v[i]);
if(v.size() > 1) ans.pop_back();
15
16
```

## 2.8 Template

```
1 template < class T>
   struct Point{
     T x, y;
     Point(T x = 0,T y = 0) : x(x), y(y) {}
Point operator + (const Point &b) const {
        return Point(x + b.x,y + b.y);
     Point operator - (const Point &b) const {
       return Point(x - b.x,y - b.y);
     Point operator * (T b) const {
       return Point(x*b,y*b);
     Point operator / (T b) const {
       return Point(x/b,y/b);
16
     T operator * (const Point &b) const {
17
       return x * b.x + y * b.y;
     T operator ^ (const Point &b) const {
       return x * b.y - y * b.x;
21
     }
22
   int sign(double a){
     return fabs(a) < eps ? 0 : a > 0 ? 1 : -1;
26
  template < class T>
double abs(const Point < T > & p) {
27
     return sqrtl(p*p);
31
  T abs2(const Point<T>&p){
32
33
     return p*p;
34
   template < class T>
35
  int ori(Point<T> a,Point<T> b,Point<T> c){
     return sign((b-a)^(c-a));
39
   template < class T>
  bool collinearity(Point<T> p1,Point<T> p2,Point<T> p3){
    return sign((p1-p3)^(p2-p3)) == 0;
41
   template < class T>
   bool btw(Point<T> p1,Point<T> p2,Point<T> p3) {
     if(!collinearity(p1, p2, p3)) return 0;
return sign((p1-p3)*(p2-p3)) <= 0;</pre>
46
47
  }
  template < class T>
  bool PointOnSegment(const Point<T> &p1,const Point<T> &p2,
        const Point<T> &p3){
     return collinearity(p1,p2,p3) && btw(p1,p2,p3);
51
   template < class T>
52
  bool seg_intersect(Point<T> p1, Point<T> p2, Point<T> p3, Point
        <T> p4) {
     int a123 = ori(p1, p2, p3);
     int a124 = ori(p1, p2, p4);
int a341 = ori(p3, p4, p1);
     int a342 = ori(p3, p4, p2);
if(a123 == 0 && a124 == 0)
```

```
return btw(p1, p2, p3) || btw(p1, p2, p4) || btw(p3, p4, p1
                                                                                               if(id[v] != -1) low[u] = min(low[u], id[v]);
     ) || btw(p3, p4, p2);
return a123 * a124 <= 0 && a341 * a342 <= 0;
                                                                                                 dfs(v, u);
                                                                                    12
60
                                                                                                 low[u] = min(low[u], low[v]);
                                                                                    13
61
                                                                                                 if(low[v] > id[u]) bridges.EB(u, v);
   double area(vector<Point<T>> v){
                                                                                    15
     if(v.size()<=2)return 0;</pre>
                                                                                    16
64
     double ans = 0;
for(int i = 1;i<v.size()-1;++i){</pre>
65
                                                                                    17
                                                                                          for(int i = 0; i < n; ++i) {
  if(id[i] == -1) dfs(i, -1);</pre>
66
                                                                                    18
       ans+=((v[i]-v[0])^(v[i+1]-v[0]));
67
                                                                                    19
69
     return abs(ans)/2.;
                                                                                          return bridges;
```

# 3 Graph

## 3.1 HLD

```
struct HLD{
     int n,root;
     vector<int>dep,father,sz,mxson,topf,id;
     HLD(int _n,int _root,vector<vector<int>>&g): n(_n),root(_root
       dep.resize(n+5);
       father.resize(n+5);
       sz.resize(n+5);
       mxson.resize(n+5);
       topf.resize(n+5);
       id.resize(n+5);
       function < void(int,int) > dfs = [&](int u,int p){
          dep[u] = dep[p]+1;
13
          father[u] = p;
14
          sz[u] = 1;
         mxson[u] = 0;
for(auto v:g[u]){
15
16
            if(v!=p){
              dfs(v,u);
19
              sz[u]+=sz[v];
20
              if(sz[v]>sz[mxson[u]])mxson[u] = v;
21
           }
         }
22
23
       function<void(int,int)>dfs2 = [&](int u,int top){
          static int idn = 0;
         topf[u] = top;
id[u] = ++idn;
26
27
          if(mxson[u])dfs2(mxson[u],top);
28
          for(auto v:g[u]){
  if(v!=father[u] and v!=mxson[u]){
31
              dfs2(v,v);
32
            }
33
34
         }
35
       dfs(root,0);
       dfs2(root,root);
38
     int query(int u,int v,const auto &qry,const auto &op){
       int ans = 0;
while(topf[u]!=topf[v]){
39
40
          if(dep[topf[u]]<dep[topf[v]])swap(u,v);</pre>
41
          ans = op(ans,qry(id[topf[u]],id[u]));
42
          u = father[topf[u]];
       if(id[u]>id[v])swap(u,v);
45
       ans = op(ans,qry(id[u],id[v]));
46
47
       return ans;
     void update(int u,int v,int val,const auto &upd){
50
       while(topf[u]!=topf[v]){
         if(dep[topf[u]]dep[topf[v]])swap(u,v);
upd(id[topf[u]],id[u],val);
u = father[topf[u]];
51
52
53
       if(id[u]>id[v])swap(u,v);
       upd(id[u],id[v],val);
57
58 };
```

## 3.2 Bridges

```
vector<pii> findBridges(const vector<vector<int>>& g) {
   int n = (int) g.size();
   vector<int> id(n, -1), low(n);
   vector<pii> bridges;
   function</pi>
   function
function tont = 0;
   id[u] = low[u] = cnt++;
   for(auto v : g[u]) {
    if(v == p) continue;
}
```

## 3.3 TwoSat

```
1 struct two_sat{
     SCC s:
     vector<bool>ans;
     int have_ans = 0;
     two_sat(int _n) : n(_n) {
  ans.resize(n+1);
  s = SCC(2*n);
     int inv(int x){
       if(x>n)return x-n;
       return x+n;
13
     void add_or_clause(int u, bool x, int v, bool y){
14
       if(!x)u = inv(u);
15
       if(!y)v = inv(v);
16
       s.add_edge(inv(u), v);
       s.add_edge(inv(v), u);
19
20
     void check(){
       if(have_ans!=0)return;
21
       s.build();
for(int i = 0;i<=n;++i){
          if(s.scc[i]==s.scc[inv(i)]){
25
            have_ans = -1;
26
            return;
          ans[i] = (s.scc[i]<s.scc[inv(i)]);</pre>
       have ans = 1;
31
```

## **3.4** MCMF

```
template < class Cap_t, class Cost_t>
  class MCMF {
  public:
     struct Edge {
       int from;
       int to;
       Cap_t cap;
Cost_t cost;
Edge(int u, int v, Cap_t _cap, Cost_t _cost) : from(u), to(
             v), cap(_cap), cost(_cost) {}
     static constexpr Cap_t EPS = static_cast<Cap_t>(1e-9);
13
     int n;
     vector<Edge> edges;
15
     vector<vector<int>> g;
     vector<Cost_t> d;
vector<bool> in_queue;
18
19
     vector<int> previous_edge;
20
     MCMF() {}
MCMF(int _n) : n(_n+1), g(_n+1), d(_n+1), in_queue(_n+1),
    previous_edge(_n+1) {}
21
23
     void add_edge(int u, int v, Cap_t cap, Cost_t cost) {
  assert(0 <= u && u < n);</pre>
25
       assert(0 <= v && v < n);
       g[u].push_back(edges.size());
       edges.emplace_back(u, v, cap, cost);
       g[v].push_back(edges.size());
       edges.emplace_back(v, u, 0, -cost);
31
     bool spfa(int s, int t) {
       bool found = false;
       fill(d.begin(), d.end(), numeric_limits<Cost_t>::max());
36
       d[s] = 0;
       in queue[s] = true;
37
       queue<int> que;
```

```
que.push(s);
40
       while(!que.empty())
41
          int u = que.front();
42
          que.pop();
          if(u == t) {
             found = true;
45
          in_queue[u] = false;
46
          for(auto& id : g[u]) {
    const Edge& e = edges[id];
    if(e.cap > EPS && d[u] + e.cost < d[e.to]) {
47
               d[e.to] = d[u] + e.cost;
               previous_edge[e.to] = id;
52
               if(!in_queue[e.to]) {
                  que.push(e.to);
                  in_queue[e.to] = true;
               }
57
          }
58
       return found;
59
60
61
     pair<Cap_t, Cost_t> flow(int s, int t, Cap_t f =
62
           numeric_limits<Cap_t>::max()) {
       assert(0 <= s && s < n);
assert(0 <= t && t < n);
63
64
       Cap_t cap = 0;
65
        Cost_t cost = 0;
66
        while(f > 0 && spfa(s, t)) {
          Cap_t send = f;
          int u = t;
while(u != s) {
70
             const Edge& e = edges[previous_edge[u]];
             send = min(send, e.cap);
            u = e.from:
75
          while(u != s) {
   Edge& e = edges[previous_edge[u]];
   e.cap -= send;
76
77
78
             Edge& b = edges[previous_edge[u] ^ 1];
            b.cap += send;
81
            u = e.from;
82
83
          cap += send;
          f -= send;
84
          cost += send * d[t];
85
87
        return make_pair(cap, cost);
89 };
```

## 3.5 LCA

```
vector<vector<int>>g,dp;
   vector<int>deep;
   void build(int root,int n){
  dp.assign(25,vector<int>(n+5));
      deep.assign(n+5,0);
      function < void(int, int, int) > dfs = [&](int u, int p, int dis){
         dp[0][u] = p;
deep[u] = dis;
         for(auto v:g[u]){
  if(v==p)continue;
            dfs(v,u,dis+1);
11
      dfs(root,0,1);
      for(int i = 1;i<=20;++i){
  for(int j = 1;j<=n;++j){</pre>
15
16
            dp[i][j] = dp[i-1][dp[i-1][j]];
17
20
   int LCA(int u.int v){
21
      if(deep[u]<deep[v])swap(u,v);
for(int i = 20;i>=0;--i){
22
        if(deep[dp[i][u]]>=deep[v])
            \dot{u} = dp[i][u];
      if(u==v)return u;
for(int i = 20;i>=0;--i){
   if(dp[i][u]!=dp[i][v])u = dp[i][u],v = dp[i][v];
27
29
30
31
      return dp[0][u];
```

## 3.6 CentroidDecomposition

```
il vector<vector<int>>g;
  vector<int>sz,tmp;
vector<bool>vis;//visit centroid
   int tree_centroid(int u,int n){
     function<void(int,int)>dfs1 = [&](int u,int p){
       sz[u] = 1;
       for(auto v:g[u]){
         if(v==p)continue;
         if(vis[v])continue;
         dfs1(v,u);
         sz[u]+=sz[v];
       }
     function<int(int,int)>dfs2 = [&](int u,int p){
       for(auto v:g[u]){
15
         if(v==p)continue;
         if(vis[v])continue;
         if(sz[v]*2<n)continue;</pre>
19
         return dfs2(v,u);
20
21
       return u;
     dfs1(u,-1);
23
     return dfs2(u,-1);
25
  int cal(int u,int p = -1,int deep = 1){
  int ans = 0;
26
27
28
     tmp.pb(deep);
     sz[u] = 1;
     for(auto v:g[u]){
       if(v==p)continue;
       if(vis[v])continue
33
       ans+=cal(v,u,deep+1);
34
       sz[u]+=sz[v];
35
     //calcuate the answer
37
38
  int centroid_decomposition(int u,int tree_size){
39
    int center = tree_centroid(u,tree_size);
vis[center] = 1;
40
41
     int ans = 0;
     for(auto v:g[center]){
       if(vis[v])continue;
       ans+=cal(v);
45
       for(int i = sz(tmp)-sz[v];i<sz(tmp);++i){</pre>
46
47
         //update
48
     while(!tmp.empty()){
50
51
       //roll_back(tmp.back())
52
       tmp.pop_back();
53
     for(auto v:g[center]){
       if(vis[v])continue;
       ans+=centroid_decomposition(v,sz[v]);
57
58
     return ans;
```

## **3.7** BCC AP

```
1 | struct BCC_AP{
     int dfn_cnt = 0,bcc_cnt = 0,n;
     vector< int>dfn,low,ap,bcc_id;
     stack<int>st;
     vector<bool>vis,is_ap;
     vector<vector<int>>bcc;
     BCC_AP(int _n):n(_n){
       dfn.resize(n+5),low.resize(n+5),bcc.resize(n+5),vis.resize(
             n+5),is_ap.resize(n+5),bcc_id.resize(n+5);
     inline void build(const vector<vector<int>>&g,int u,int p =
10
           -1){
       int child = 0;
       dfn[u] = low[u] = ++dfn_cnt;
12
       st.push(u);
       if(g[u].empty() and p==-1){
  bcc_id[u] = ++bcc_cnt;
  bcc[bcc_cnt].push_back(u);
16
17
18
       for(auto v:g[u]){
21
          if(v==p)continue;
          if(!dfn[v]){
            build(g,v,u);
23
            child++;
            if(dfn[u]<=low[v]){</pre>
               is_ap[u] = 1;
              bcc_id[u] = ++bcc_cnt;
bcc[bcc_cnt].push_back(u);
              while(vis[v]){
29
                 bcc_id[st.top()] = bcc_cnt;
```

```
bcc[bcc_cnt].push_back(st.top());
32
               vis[st.top()] = 0;
33
               st.pop();
34
35
           low[u] = min(low[u],low[v]);
37
         low[u] = min(low[u],dfn[v]);
38
39
       if(p==-1 and child<2)is_ap[u] = 0;
       if(is_ap[u])ap.push_back(u);
42
43 };
```

## 3.8 SCC

```
struct SCC{
     int n, cnt = 0, dfn_cnt = 0;
     vector<vector<int>>g;
     vector<int>sz,scc,low,dfn;
     stack<int>st;
     vector<bool>vis;
     SCC(int _n = 0) : n(_n){
    sz.resize(n+5),scc.resize(n+5),low.resize(n+5),dfn.resize(n
             +5), vis.resize(n+5);
       g.resize(n+5);
     inline void add_edge(int u, int v){
       g[u].push_back(v);
13
     inline void build(){
  function<void(int, int)>dfs = [&](int u,int dis){
   low[u] = dfn[u] = ++dfn_cnt,vis[u] = 1;
15
          st.push(u);
18
          for(auto v:g[u]){
            \textbf{if}(\texttt{!dfn[v]})\{
19
               dfs(v, dis+1);
low[u] = min(low[u],low[v]);
20
21
            else if(vis[v]){
24
               low[u] = min(low[u],dfn[v]);
25
            }
26
          if(low[u]==dfn[u]){
27
             ++cnt;
             while(vis[u]){
               auto v = st.top();
30
31
               st.pop();
               vis[v] = 0;
scc[v] = cnt;
32
33
               sz[cnt]++;
35
          }
37
       for(int i = 0;i<=n;++i){</pre>
38
39
          if(!scc[i]){
            dfs(i, 1);
42
       }
43
44
     vector<vector<int>> compress(){
       vector<vector<int>>ans(cnt+1);
45
       for(int u = 0; u <= n; ++u){
46
47
          for(auto v:g[u]){
            if(scc[u] == scc[v]){
49
               continue;
50
            ans[scc[u]].push_back(scc[v]);
51
          }
52
54
        for(int i = 0;i<=cnt;++i){</pre>
          sort(ans[i].begin(), ans[i].end());
          ans[i].erase(unique(ans[i].begin(), ans[i].end()), ans[i
56
                1.end());
        return ans;
60 };
```

## 3.9 LineContainer

```
template < class T >
T floor_div(T a, T b) {
    return a / b - ((a ^ b) < 0 && a % b != 0);
}

template < class T >
T ceil_div(T a, T b) {
    return a / b + ((a ^ b) > 0 && a % b != 0);
}

return a / b + ((a ^ b) > 0 && a % b != 0);
}
```

```
namespace line_container_internal {
12
   struct line t {
13
14
     mutable long long k, m, p;
15
     inline bool operator<(const line_t& o) const { return k < o.k</pre>
     inline bool operator<(long long x) const { return p < x; }</pre>
17
18
  };
19
  } // line_container_internal
21
  template < bool MAX >
   struct line_container : std::multiset<line_container internal::</pre>
23
     line_t, std::less<>> {
static const long long INF = std::numeric_limits<long long>::
           max();
26
     bool isect(iterator x, iterator y) {
       if(y == end()) {
  x->p = INF;
27
28
29
          return 0;
30
        if(x->k == y->k) {
          x - p = (x - m > y - m ? INF : -INF);
        } else {
          x\rightarrow p = floor_div(y\rightarrow m - x\rightarrow m, x\rightarrow k - y\rightarrow k);
       return x->p >= y->p;
38
39
     void add_line(long long k, long long m) {
       if(!MAX) {
40
          \dot{k} = -\dot{k};
41
        auto z = insert({k, m, 0}), y = z++, x = y;
45
        while(isect(y, z)) {
46
         z = erase(z);
47
48
        if(x != begin() && isect(--x, y)) {
          isect(x, y = erase(y));
51
        while((y = x) != begin() && (--x)->p >= y->p) {
52
          isect(x, erase(y));
53
54
55
     long long get(long long x) {
        assert(!empty());
       auto 1 = *lower_bound(x);
return (1.k * x + 1.m) * (MAX ? +1 : -1);
60
```

## **3.10** Dinic

```
i template < class T>
   struct Dinic{
     struct edge{
        int from, to;
        T cap;
        edge(int _from, int _to, T _cap) : from(_from), to(_to),
              cap(_cap) {}
     int n;
     vector<edge> edges;
     vector<vector<int>> g;
      vector<int> cur, h;
11
     Dinic(int _n) : n(_n+1), g(_n+1) {}
void add_edge(int u, int v, T cap){
        g[u].push_back(edges.size());
15
        edges.push_back(edge(u, v, cap));
        g[v].push_back(edges.size());
edges.push_back(edge(v, u, 0));
16
17
18
      bool bfs(int s,int t){
        h.assign(n, -1);
21
        h[s] = 0;
        queue<int> que;
22
23
        que.push(s);
        while(!que.empty()) {
24
           int u = que.front();
           que.pop();
for(auto id : g[u]) {
26
27
             const edge& e = edges[id];
int v = e.to;
if(e.cap > 0 && h[v] == -1) {
29
30
               h[v] = h[u] + 1;
if(v == t) {
31
                  return 1;
33
                que.push(v);
35
```

```
}
38
        return 0;
     T dfs(int u, int t, T f) {
        if(u == t) {
43
           return f;
44
        Tr = f;
45
        for(int& i = cur[u]; i < (int) g[u].size(); ++i) {</pre>
           int id = g[u][i];
48
           const edge& e = edges[id];
          int v = e.to;
if(e.cap > 0 && h[v] == h[u] + 1) {
  T send = dfs(v, t, min(r, e.cap));
  edges[id].cap -= send;
  edges[id ^ 1].cap += send;
50
52
             r -= send;
if(r == 0) {
55
                return f;
57
58
             }
          }
60
        return f - r;
     T flow(int s, int t, T f = numeric_limits<T>::max()) {
62
        T ans = 0;
while(f > 0 && bfs(s, t)) {
63
64
65
           cur.assign(n, 0);
           T send = d\hat{f}s(s, t, f);
67
           ans += send;
          f -= send;
69
70
        return ans;
71
     vector<pair<int,int>> min_cut(int s) {
        vector<bool> vis(n);
74
        vis[s] = true;
        queue<int> que;
75
        que.push(s);
76
        while(!que.empty()) {
           int u = que.front();
           que.pop();
           for(auto id : g[u]) {
80
             const auto& e = edges[id];
int v = e.to;
if(e.cap > 0 && !vis[v]) {
  vis[v] = true;
81
82
83
                que.push(v);
86
87
          }
88
        vector<pair<int,int>> cut;
        for(int i = 0; i < (int) edges.size(); i += 2) {</pre>
           const auto& e = edges[i];
           if(vis[e.from] && !vis[e.to]) {
93
             cut.push_back(make_pair(e.from, e.to));
        return cut;
  };
```

## Math

#### 4.1 Numbers

Bernoulli numbers

$$B_0 - 1, B_1^{\pm} = \pm \frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0$$

$$\sum_{j=0}^{m} {m+1 \choose j} B_j = 0, \text{ EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^{n} k^m = \frac{1}{m+1} \sum_{k=0}^{m} {m+1 \choose k} B_k^+ n^{m+1-k}$$

$$\begin{split} S(n,k) &= S(n-1,k-1) + kS(n-1,k), S(n,1) = S(n,n) = 1 \\ S(n,k) &= \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} {k \choose i} i^n \\ x^n &= \sum_{i=0}^n S(n,i)(x)_i \end{split}$$

· Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

· Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

```
C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k
```

```
Number of permutations \pi \in S_n in which exactly k elements are greater than the
previous element. k j:s s.t. \pi(j) > \pi(j+1), k+1 j:s s.t. \pi(j) \ge j, k j:s s.t.
E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)
E(n,0) = E(n, n-1) = 1
E(n,k) = \sum_{j=0}^{k} (-1)^{j} {\binom{n+1}{j}} (k+1-j)^{n}
```

## 4.2 ExtendGCD

```
1 // @return $x, y$ s.t. $ax + by = \gcd(a, b)$
2 | 11 ext_gcd(11 a, 11 b, 11& x, 11& y) {
     if(b == 0) {
        x = 1; y = 0;
        return a;
     11 x2, y2;
11 c = a % b;
if(c < 0) c += b;
     11 g = ext_gcd(b, c, x2, y2);
     x = y2;

y = x2 - (a / b) * y2;
     return g;
```

## **BerlekampMassey**

```
| template <typename Tfield>
                                                                                            std::pair<int, std::vector<Tfield>> find_linear_recurrence(
   const std::vector<Tfield> &S) {
   int N = S.size();
   using poly = std::vector<Tfield>;
                                                                                                  poly C_reversed{1}, B{1};
int L = 0, m = 1;
                                                                                                  Tfield b = 1;
                                                                                                  C.resize(std::max(C.size(), B.size() + m));
                                                                                                        Tfield a = d / b;
for (unsigned i = 0; i < B.size(); i++) C[i + m] -= a *
                                                                                                                B[i];
                                                                                                        return C;
                                                                                                  };
                                                                                                  for (int n = 0; n < N; n++) {
    Tfield d = S[n];</pre>
                                                                                         18
                                                                                                        for (int i = 1; i <= L; i++) d += C_reversed[i] * S[n -</pre>
                                                                                                                i];
                                                                                                        if (d == 0)
                                                                                         21
                                                                                                        m++;
else if (2 * L <= n) {
   poly T = C_reversed;
   C_reversed = adjust(C_reversed, B, d, b, m);</pre>
                                                                                                              L = n + 1 - L;
                                                                                                              B = T;
                                                                                                              b = d;
                                                                                                              m = 1;
                                                                                         30
                                                                                                        } else
                                                                                         31
                                                                                                              C_reversed = adjust(C_reversed, B, d, b, m++);
                                                                                                  return std::make_pair(L, C_reversed);
                                                                                         33
                                                                                        33
6 // Calculate $x^N \bmod f(x)$
37 // Known as `Kitamasa method`
38 // Input: f_reversed: monic, reversed (f_reversed[0] = 1)
39 // Complexity: $0(K^2 \log N)$ ($K$: deg. of $f$)
                                                                                         40 // Example: (4, [1, -1, -1]) \rightarrow [2, 3]

(x^4 = (x^2 + x + 2)(x^2 - x - 1) + 3x + 2)

    Stirling numbers of the second kind Partitions of n distinct elements into exactly k 42 // Reference: http://misawa.github.io/others/

                                                                                                   fast_kitamasa_method.html
    http://sugarknri.hatenablog.com/entry
                                                                                                   /2017/11/18/233936
                                                                                             template <typename Tfield>
                                                                                            std::vector<Tfield> monomial_mod_polynomial(long long N, const
                                                                                                  std::vector<ffield> &f_reversed) {
  assert(!f_reversed.empty() and f_reversed[0] == 1);
  int K = f_reversed.size() - 1;
  if (!K) return {};
  int D = 64 - __builtin_clzll(N);
                                                                                                  std::vector<Tfield> ret(K, 0);
                                                                                                  ret[0] = 1;
auto self_conv = [](std::vector<Tfield> x) -> std::vector<
                                                                                                         Tfield> {
                                                                                                        int d = x.size();
```

```
std::vector<Tfield> ret(d * 2 - 1);
              55
56
57
58
59
               return ret;
60
        };
for (int d = D; d--;) {
61
              ret = self_conv(ret);

for (int i = 2 * K - 2; i >= K; i--) {

    for (int j = 1; j <= K; j++) ret[i - j] -= ret[i] *
62
63
64
                            f_reversed[j];
65
              ret.resize(K);
if ((N >> d) & 1) {
66
67
                   ((N >> d) & 1) {
    std::vector<Tfield> c(K);
    c[0] = -ret[K - 1] * f_reversed[K];
    for (int i = 1; i < K; i++) { c[i] = ret[i - 1] -
        ret[K - 1] * f_reversed[K - i]; }</pre>
70
72
              }
73
        return ret;
75
   }
76
   // Guess k-th element of the sequence, assuming linear
77
         recurrence
   // initial_elements: 0-ORIGIN
// Verify: abc198f https://atcoder.jp/contests/abc198/
         submissions/21837815
   template <typename Tfield>
   Tfield guess_kth_term(const std::vector<Tfield> &
81
         initial_elements, long long k) {
assert(k >= 0);
83
         if (k < static_cast<long long>(initial_elements.size()))
         return initial_elements[k];
const auto f = find_linear_recurrence<Tfield>(
   initial_elements).second;
84
        const auto g = monomial_mod_polynomial<Tfield>(k, f);
Tfield ret = 0;
85
86
         for (unsigned i = 0; i < g.size(); i++) ret += g[i] *</pre>
87
               initial_elements[i];
88
         return ret;
89 }
```

## 4.4 InvGCD

```
pair<long long, long long> inv_gcd(long long a, long long b) {
    a %= b;
    if(a < 0) a += b;
    if(a == 0) return {b, 0};
    long long s = b, t = a;
    long long m0 = 0, m1 = 1;
    while(t) {
      long long u = s / t;
s -= t * u;
      m0 -= m1 * u;
      swap(s, t);
      swap(m0, m1);
13
    if(m0 < 0) m0 += b / s;
    return {s, m0};
15
```

#### **Generating Functions** 4.5

• Ordinary Generating Function  $A(x) = \sum_{i>0} a_i x^i$ 

```
-A(rx) \Rightarrow r^n a_n
-A(x) + B(x) \Rightarrow a_n + b_n
-A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}
-A(x)^k \Rightarrow \sum_{i+1+i+2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}
  -\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^{n} a_i
```

• Exponential Generating Function  $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x_i$ 

```
-A(x) + B(x) \Rightarrow a_n + b_n
-A^{(k)}(x) \Rightarrow a_{n+k}
-A(x)B(x) \Rightarrow \sum_{i=0}^{n} {n \choose i} a_i b_{n-i}
-A(x)^{k} \Rightarrow \sum_{i_{1}+i_{2}+\cdots+i_{k}=n}^{n} {n \choose i_{1},i_{2},\ldots,i_{k}} a_{i_{1}} a_{i_{2}} \ldots a_{i_{k}}
-xA(x) \Rightarrow \overline{na_n}^{i_1}
```

· Special Generating Function

$$- (1+x)^{n} = \sum_{i \ge 0} {n \choose i} x^{i}$$

$$- \frac{1}{(1-x)^{n}} = \sum_{i \ge 0} {n \choose i} x^{i}$$

#### 4.6 Theorem

· Cramer's rule

$$\begin{array}{l} ax+by=e\\ cx+dy=f\\ \end{array}\Rightarrow \begin{array}{l} x=\frac{ed-bf}{ad-bc}\\ y=\frac{af-ec}{ad-bc} \end{array}$$

· Kirchhoff's Theorem

Denote L be a  $n \times n$  matrix as the Laplacian matrix of graph G, where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where c is the number of edge (i, j) in G.

- $\begin{array}{ll} & \text{The number of undirected spanning in } G \text{ is } |\text{det}(\tilde{L}_{11})|. \\ & \text{The number of directed spanning tree rooted at } r \text{ in } G \text{ is } |\text{det}(\tilde{L}_{rr})|. \end{array}$
- · Tutte's Matrix

Let D be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if i < j and  $(i,j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{rank(D)}{2}$  is the maximum matching on G.

- · Cayley's Formula
  - Given a degree sequence  $d_1, d_2, \ldots, d_n$  for each labeled vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\cdots(d_n-1)!} \text{ spanning trees.}$  Let  $T_{n,k}$  be the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with k composite the number of labeled forests on n vertices with n vertices where n vertices we have n vertices with n vertices n vertices where n vertices n vert
  - nents, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{-k} = kn^{n-k-1}$
- Erd□s-Gallai theorem

A sequence of nonnegative integers  $d_1 \geq \cdots \geq d_n$  can be represented as the degree sequence of a finite simple graph on n vertices if and only if  $d_1 + \cdots + d_n$ is even and  $\sum_{i=1}^k d_i \le k(k-1) + \sum_{i=k+1}^n \min(d_i,k)$  holds for every  $1 \le k \le n$ .

• Gale-Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \cdots \geq a_n$  and  $b_1, \ldots, b_n$  is bigraphic if and only if  $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} b_i$  and  $\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{n} \min(b_i, k)$  holds for

· Fulkerson-Chen-Anstee theorem

A sequence  $(a_1, b_1), \ldots, (a_n, b_n)$  of nonnegative integer pairs with  $a_1 \ge \cdots \ge a_n$  $a_n \text{ is digraphic if and only if } \sum_{i=1}^n a_i = \sum_{i=1}^n b_i \text{ and } \sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=1}^k a_i \leq \sum_{i=1}^k a_i \leq$  $\sum_{i=1}^{n} \min(b_i, k) \text{ holds for every } 1 \leq k \leq n.$ 

• M bius inversion formula

$$\begin{array}{l} -\ f(n) = \sum_{d\mid n} g(d) \Leftrightarrow g(n) = \sum_{d\mid n} \mu(d) f(\frac{n}{d}) \\ -\ f(n) = \sum_{n\mid d} g(d) \Leftrightarrow g(n) = \sum_{n\mid d} \mu(\frac{d}{n}) f(d) \end{array}$$

- · Spherical cap

  - A portion of a sphere cut off by a plane. r: sphere radius, a: radius of the base of the cap, h: height of the cap,  $\theta$ :

  - $\cos \theta$ )<sup>2</sup>/3. Area =  $2\pi rh = \pi(a^2 + h^2) = 2\pi r^2(1 \cos \theta)$ .

#### 4.7 FloorSum

```
ı|// @param $n < 2^{32}$
   // @param $1 \Leq m < 2^{32}$
   // @return \sum_{i=0}^{n-1} \left( i - 0 \right) 
   pmod\{2^{\lambda}\{64\}\} ull floor_sum_unsigned(ull n, ull m, ull a, ull b) {
     ull ans = 0;
      while(true) {
        if(a >= m) {
  ans += n * (n - 1) / 2 * (a / m);
           a %= m;
        if(b >= m) {
  ans += n * (b / m);
13
          b %= m;
        ull y_max = a * n + b;
        if(y_max < m) break;</pre>
        n = (ull)(y_max / m);
b = (ull)(y_max % m);
        swap(m, a);
      return ans;
  }
23
  11 floor_sum(11 n, 11 m, 11 a, 11 b) {
  assert(0 <= n && n < (1LL << 32));</pre>
```

```
26    assert(1 <= m && m < (1LL << 32));
27    ull ans = 0;
28    if(a < 0) {
29        ull a2 = (a % m + m) % m;
30        ans -= 1ULL * n * (n - 1) / 2 * ((a2 - a) / m);
31        a = a2;
32    }
33    if(b < 0) {
34        ull b2 = (b % m + m) % m;
35        ans -= 1ULL * n * ((b2 - b) / m);
36        b = b2;
37    }
38    return ans + floor_sum_unsigned(n, m, a, b);
39 }
```

## 4.8 Factorizer

```
1 template < class T>
   vector<pair<T, int>> MergeFactors(const vector<pair<T, int>>& a
          , const vector<pair<T, int>>& b) {
      if(j == SZ(b) || (i < SZ(a) && a[i].F < b[j].F)) c.PB(a[i</pre>
11
                ++1);
         else c.PB(b[j++]);
12
13
      return c;
   template < class T>
   vector<pair<T, int>> RhoC(const T& n, const T& c) {
   if(n <= 1) return {};
   if(n % 2 == 0) return MergeFactors({{2, 1}}, RhoC(n / 2, c));</pre>
      if(is_prime_constexpr(n)) return {{n, 1}};
T x = 2, saved = 2, p = 1, lam = 1;
      T = 2, saved = 2, p = 1, lam = 1,
while(true) {
    x = (x * x % n + c) % n;
    T g = __gcd(((x - saved) + n) % n, n);
    if(g != 1) return MergeFactors(RhoC(g, c + 1), RhoC(n / g,
23
25
               c + 1);
         if(p == lam) {
            saved = x;
            p <<= 1;
            lam = 0:
29
30
         lam += 1:
      return {};
34
   template<class T>
vector<pair<T, int>> Factorize(T n) {
  if(n <= 1) return {};
  return RhoC(n, T(1));</pre>
35
36
39
   template < class T>
vector <T> BuildDivisorsFromFactors(const vector<pair<T, int>>&
41
          factors) {
      int total = 1;
      for(int i = 0; i < SZ(factors); ++i) total *= factors[i].</pre>
43
             second + 1;
      vector<T> divisors;
      divisors.reserve(total);
      divisors.PB(1);
for(auto [p, cnt] : factors) {
  int sz = SZ(divisors);
46
         for(int i = 0; i < sz; ++i) {</pre>
            T cur = divisors[i];
            for(int j = 0; j < cnt; ++j) {</pre>
51
              cur *= p;
52
               divisors.PB(cur);
53
           }
      // sort(ALL(divisors));
      return divisors;
```

## 4.9 PowMod

```
constexpr long long Pow(long long x, long long n, int m) {
   if(m == 1) return 0;
   unsigned int _m = (unsigned int)(m);
   unsigned long long r = 1;
   x %= m;
   if(x < 0) x += m;
   unsigned long long y = x;</pre>
```

```
8  while(n) {
9    if(n & 1) r = (r * y) % _m;
10    y = (y * y) % _m;
11    n >>= 1;
12  }
13  return r;
14 }
```

## 4.10 CRT

```
1 // @return
2 // $\text
       $\text{remainder, modulo}$
 3 //
              or
   // $0, 0$ if do not exist
   assert(r.size()==m.size());
      int n = r.size();
      // Contracts: 0 <= r0 < m0
      long long r0 = 0, m0 = 1;
for(int i = 0; i < n; i++) {
         assert(1 <= m[i]);</pre>
        long long r1 = r[i] % m[i];
if(r1 < 0) r1 += m[i];
         long long m1 = m[i];
        if(m0 < m1) {
           swap(r0, r1);
           swap(m0, m1);
        if(m0 % m1 == 0) {
  if(r0 % m1 != r1) return {0, 0};
20
           continue;
21
        long long g, im;
tie(g, im) = inv_gcd(m0, m1);
long long u1 = (m1 / g);
if((r1 - r0) % g) return {0, 0};
long long x = (r1 - r0) / g % u1 * im % u1;
r0 += x * m0;
m0 *= u1;
         if(r0 < 0) r0 += m0;
      return {r0, m0};
```

## 4.11 DiscreteLog

```
i int DiscreteLog(int s, int x, int y, int m) {
      constexpr int K = 0;
      hash_map<int, int> p;
      int \overline{b} = 1;
      for(int i = 0; i < K; ++i) {</pre>
        p[y] = i;
y = 1LL * y * x % m;
         b = 1LL * b * x % m;
      for(int i = 0; i < m + 10; i += K) {
    s = 1LL * s * b % m;</pre>
         if(p.find(s) != p.end()) return i + K - p[s];
12
13
      return -1;
   int DiscreteLog(int x, int y, int m) {
      if(m == 1) return 0;
int s = 1;
      for(int i = 0; i < 100; ++i) {
  if(s == y) return i;
  s = 1LL * s * x % m;</pre>
20
21
      if(s == y) return 100;
      int p = 100 + DiscreteLog(s, x, y, m);
return (pow_mod(x, p, m) != y ? -1 : p);
```

## 4.12 LinearSieve

```
vector<bool> is_prime;
vector<int> primes, phi, mobius, least;
void linear_sieve(int n) {
    n += 1;
    is_prime.resize(n);
    least.resize(n);
    fill(2 + begin(is_prime),end(is_prime), true);
    phi.resize(n); mobius.resize(n);
    phi[1] = mobius[1] = 1;
    least[0] = 0,least[1] = 1;
    for(int i = 2; i < n; ++i) {
        if(is_prime[i]) {
            primes.push_back(i);
        }
}</pre>
```

```
phi[i] = i - 1;
mobius[i] = -1;
15
                       least[i] = i;
16
17
                 for(auto j : primes) {
    if(i * j >= n) break;
    is_prime[i * j] = false;
    least[i * j] = j;
    if(i % j == 0) {
        mobius[i * j] = 0;
        phi[i * j] = phi[i] * j;
    }
}
18
20
21
22
23
25
                            break;
                       } else {
                           mobius[i * j] = mobius[i] * mobius[j];
phi[i * j] = phi[i] * phi[j];
27
                 }
32 }
```

## 5 Misc

## 5.1 FastIO

```
inline char gc() {
    static const int BUF_SIZE = 1 << 22;</pre>
         static int Counts = 1 << 23;
static char Buffer[BUF_SIZE];
static char *Pointer = Buffer, *End = Buffer;</pre>
         if(Pointer == End) {
   if(Counts < BUF_SIZE) {</pre>
                    return EOF;
               Counts = fread(Buffer, 1, BUF_SIZE, stdin);
              Pointer = Buffer;
              End = Buffer + Counts;
12
13
         return *(Pointer++);
15
   }
   template < class T>
17
   inline void read(T& x) {
18
         static char c;
19
         do {
20
         c = gc();
} while(c < '0' && c != '-');
bool neg = (c == '-');
21
23
        if(!neg) {
    x = c - '0';
} else x = 0;
24
25
         while((c = gc()) >= '0') {
            x = (x << 3) + (x << 1) + (c & 15);
29
         if(neg) {
30
31
              x = -x;
32
33
   }
   template < class T, class... U>
inline void read(T& a, U&... b) {
36
         read(a);
37
38
         read(b...);
39
   }
   template < class T>
   inline void write(T temp, char end = '\n') {
    static short digits[20], P;
    if(temp == 0) {
43
44
              putchar_unlocked('0');
45
              putchar_unlocked(end);
47
               return;
48
         if(temp < 0) {
   putchar_unlocked('-');</pre>
49
50
               write(-temp,end);
51
52
               return;
53
         P = -1:
         while(temp) {
55
              digits[++P] = temp % 10;
temp /= 10;
56
57
         while(P >= 0) {
60
              putchar_unlocked(digits[P--] + '0');
61
         putchar_unlocked(end);
62
         return;
63
64 }
```

## 5.2 Debug

## 5.3 Discrete

## 5.4 DuiPai

```
| #include < bits / stdc++.h>
   using namespace std;
   int main(){
      string sol,bf,make;
cout<<"Your solution file name :";</pre>
      cin>>sol;
      cout<<"Brute force file name :";</pre>
      cin>>bf;
      cout<<"Make data file name :";</pre>
      cin>>make;
      system(("g++ "+sol+" -o sol").c_str());
system(("g++ "+bf+" -o bf").c_str());
system(("g++ "+make+" -o make").c_str());
11
12
       for(int t = 0;t<10000;++t){
    system("./make > ./1.in");
15
          double st = clock();
    system("./sol < ./1.in > ./1.ans");
    double et = clock();
16
17
18
            system("./bf < ./1.in > ./1.out");
if(system("diff ./1.out ./1.ans")) {
printf("\033[0;31mWrong Answer\033[0m on test #%d",t);
19
21
22
                     return 0;
23
          else if(et-st>=2000){
24
            printf("\033[0;32mTime limit exceeded\033[0m on test #%d,
                    Time %.0[fms\n",t,et-st);
             return 0;
27
28
          else {
                     printf("\033[0;32mAccepted\033[0m on test #%d, Time
29
                              %.0lfms\n", t, et - st);
               }
31
      }
32 }
```

## 5.5 Timer

```
const clock_t startTime = clock();
inline double getCurrentTime() {
   return (double) (clock() - startTime) / CLOCKS_PER_SEC;
4 }
```

## 5.6 TenarySearch

```
1  // return the maximum of $f(x)$ in $[l, r]$
2  double ternary_search(double 1, double r) {
3    while(r - 1 > EPS) {
4    double m1 = 1 + (r - 1) / 3;
5    double m2 = r - (r - 1) / 3;
6    double f1 = f(m1), f2 = f(m2);
7    if(f1 < f2) 1 = m1;
8    else r = m2;
9    }
10    return f(1);
11 }
12
13    // return the maximum of $f(x)$ in $(l, r]$
14 int ternary_search(int 1, int r) {
15    while(r - 1 > 1) {
```

```
int mid = (1 + r) / 2;
if(f(m) > f(m + 1)) r = m;
ls else l = m;
ly return r;
}
```

## 6 Other

#### 6.1 TouristIO

```
static struct FastInput {
     static constexpr int BUF_SIZE = 1 << 20;</pre>
     char buf[BUF_SIZE];
     size_t chars_read = 0;
size_t buf_pos = 0;
FILE *in = stdin;
     char cur = 0;
     inline char get_char() {
       if(buf_pos >= chars_read) {
  chars_read = fread(buf, 1, BUF_SIZE, in);
10
11
          buf pos = 0;
12
         buf[0] = (chars_read == 0 ? -1 : buf[0]);
13
15
       return cur = buf[buf_pos++];
       // return cur = getchar_unlocked();
17
18
     inline void tie(int) {}
19
     inline explicit operator bool() {
22
       return cur != -1;
23
24
     inline static bool is_blank(char c) {
25
26
       return c <= '
28
29
     inline bool skip_blanks() {
       while(is_blank(cur) && cur != -1) {
30
31
         get_char();
32
       return cur != -1;
34
35
     inline FastInput& operator>>(char& c) {
36
       skip_blanks();
37
       c = cur;
return *this;
38
41
42
     inline FastInput& operator>>(string& s) {
       if(skip_blanks()) {
43
          s.clear();
          do {
47
         } while(!is_blank(get_char()));
48
       return *this;
49
50
51
     template < class T>
53
     inline FastInput& read_integer(T& n) {
54
       // unsafe, doesn't check that characters are actually
            digits
55
       if(skip_blanks()) {
56
          int sign = +1;
if(cur == '-')
  sign = -1;
59
60
            get_char();
61
         do {
62
           n + n + (n << 3) + cur - '0';
          } while(!is_blank(get_char()));
65
         n *= sign;
66
       return *this;
67
68
     template < class T>
71
     inline typename enable_if<is_integral<T>::value, FastInput
          &>::type operator>>(T& n) {
       return read_integer(n);
72
73
     #if!defined(_WIN32) || defined(_WIN64)
     inline FastInput& operator>>(__int128& n) {
77
       return read_integer(n);
     #endif
```

```
80
81
      template < class T>
      inline typename enable if < is floating point < T > :: value,
82
            FastInput&>::type operator>>(T& n) {
         // not sure ifreally fast, for compatibility only
85
         if(skip_blanks()) {
           string s;
(*this) >> s;
86
87
           sscanf(s.c_str(), "%lf", &n);
88
         return *this;
   } fast_input;
92
93
   #define cin fast_input
   static struct FastOutput {
97
      static constexpr int BUF_SIZE = 1 << 20;</pre>
      char buf[BUF_SIZE];
      size t buf pos = 0;
      static constexpr int TMP_SIZE = 1 << 20;</pre>
100
      char tmp[TMP_SIZE];
101
      FILE *out = stdout;
103
104
      inline void put_char(char c) {
        buf[buf_pos++] = c;
if(buf_pos == BUF_SIZE) {
105
106
           fwrite(buf, 1, buf_pos, out);
107
           buf_pos = 0;
109
         // putchar_unlocked(c);
110
111
112
      ~FastOutput() {
113
114
         fwrite(buf, 1, buf_pos, out);
116
      inline FastOutput& operator<<(char c) {</pre>
117
         put char(c);
118
         return *this;
119
120
121
      inline FastOutput& operator<<(const char* s) {</pre>
122
123
         while(*s)
           put_char(*s++);
124
125
         return *this;
126
128
      inline FastOutput& operator<<(const string& s) {</pre>
129
         for(int i = 0; i < (int) s.size(); i++) {</pre>
130
           put_char(s[i]);
131
132
         return *this;
      }
135
136
      template<class T>
      inline char* integer_to_string(T n) {
   // beware of TMP_SIZE
137
138
         char* p = tmp + TMP_SIZE - 1;
139
         if(n == 0) {
140
141
           *--p = '0';
           else {
bool is_negative = false;
142
143
           if(n < 0) {
144
145
              is_negative = true;
147
           while(n > 0) {
  *--p = (char) ('0' + n % 10);
148
149
              n /= 10;
150
           if(is_negative) {
153
               k--p =
           }
154
155
         return p;
156
157
159
      template<class T>
      inline typename enable_if<is_integral<T>::value, char*>::type
    stringify(T n) {
    return integer_to_string(n);
161
162
163
      #if!defined(_WIN32) || defined(_WIN64)
inline char* stringify(__int128 n) {
165
         return integer_to_string(n);
166
167
168
169
170
      template<class T>
      inline typename enable_if<is_floating_point<T>::value, char
   *>::type stringify(T n) {
   sprintf(tmp, "%.17f", n);
172
173
         return tmp;
```

```
174
      }
175
176
      template < class T>
      inline FastOutput& operator<<(const T& n) {</pre>
17
         auto p = stringify(n);
for(; *p != 0; p++) {
178
180
           put_char(*p);
181
         return *this;
182
183
184
   } fast_output;
   #define cout fast_output
```

## 7 Setup

## 7.1 Template

```
#include <bits/extc++.h>
    #include <bits/stdc++.h>
    #pragma GCC optimize("03,unroll-loops")
#pragma GCC target("avx2,bmi,bmi2,lzcnt,popcnt")
#define IOS ios::sync_with_stdio(0),cin.tie(0),cout.tie(0)
    #define int long long
    #define double long double
    #define pb push_back
    #define sz(x) (int)(x).size()
#define all(v) begin(v),end(v)
#define debug(x) cerr<<#x<<" = "<<x<<'\n</pre>
    #define LINE cout<<"\n--
#define endl '\n'</pre>
    #define VI vector<int>
15
    #define F first
    #define S second
    #define MP(a,b) make_pair(a,b)
#define rep(i,m,n) for(int i = m;i<=n;++i)
#define res(i,m,n) for(int i = m;i>=n;--i)
17
    #define gcd(a,b) __gcd(a,b)
#define lcm(a,b) a*b/gcd(a,b)
    #define Case() int _;cin>>_;for(int Case = 1;Case<=_;++Case)
#define pii pair<int,int>
    using namespace __gnu_cxx;
using namespace __gnu_pbds;
using namespace std;
27
    template <typename K, typename cmp = less<K>, typename T =
           thin_heap_tag> using _heap = __gnu_pbds::priority_queue<K,</pre>
    cmp, T>;
template <typename K, typename M = null_type> using _hash =
    gp_hash_table<K, M>;
28
    const int N = 1e6+5,L = 20,mod = 1e9+7;
    const long long inf = 2e18+5;
const double eps = 1e-7,pi = acos(-1);
32
    void solve(){
33
    signed main(){
34
       IOS;
35
       solve();
37 }
```

# 8 String

## 8.1 DynamicKMP

```
template<int ALPHABET, int (*f)(char)>
   class DynamicKMP {
   public:
     DynamicKMP() {}
     DynamicKMP(const string& s) {
        reserve(s.size());
        for(const char& c : s) {
          push(c);
10
     }
11
12
     void push(char c) {
13
        int v = f(c);
        dp.emplace_back();
dp.back()[v] = (int) dp.size();
if(p.empty()) {
15
17
          p.push_back(0);
           return;
        int i = (int) p.size();
for(int j = 0; j < ALPHABET; ++j) {
   if(j == v) {</pre>
22
23
             p.push_back(dp[p[i - 1]][j]);
```

```
} else {
26
           dp.back()[j] = dp[p[i - 1]][j];
27
28
29
30
31
     void pop() {
       p.pop_back();
32
33
       dp.pop_back();
     int query() const {
37
       return p.back();
38
     vector<int> query_all() const {
41
      return p;
43
     void reserve(int sz) {
45
       p.reserve(sz)
46
       dp.reserve(sz);
  private:
49
    vector<int> p;
50
     vector<array<int, ALPHABET>> dp;
51
52 };
```

## 8.2 RollingHash

```
template<int HASH_COUNT, bool PRECOMPUTE_POWERS = false>
   class Hash {
   public:
     static constexpr int MAX_HASH_PAIRS = 10;
        {mul, mod}
     static constexpr const pair<int, int> HASH_PAIRS[] =
           \{\{827167801, 9999999937\}, \{998244353, 9999999929\},
           {146672737, 922722049}, {204924373, 952311013},
           {585761567, 955873937}, {484547929, 901981687},
           {856009481, 987877511}, {852853249, 996724213}, {937381759, 994523539}, {116508269, 993179543}};
     Hash() : Hash("") {}
     Hash(const string& s) : n(s.size()) {
        static_assert(HASH_COUNT > 0 && HASH_COUNT <=</pre>
12
             MAX HASH PAIRS);
        for(int i = 0; i < HASH_COUNT; ++i) {</pre>
13
          const auto& p = HASH_PAIRS[i];
14
          pref[i].resize(n);
          pref[i][0] = s[0];
for(int j = 1; j < n; ++j) {
    pref[i][j] = (1LL * pref[i][j - 1] * p.first + s[j]) %</pre>
17
18
                  p.second:
          }
        if(PRECOMPUTE_POWERS) {
22
          build_powers(n);
23
24
25
26
     void add_char(char c) {
       28
29
       n += 1;
31
        if(PRECOMPUTE_POWERS) {
33
          build_powers(n);
34
        }
35
36
37
     // Return hash values for [l, r)
     array<int, HASH_COUNT> substr(int 1, int r) {
        array<int, HASH_COUNT> res{};
        for(int i = 0; i < HASH_COUNT; ++i) {</pre>
41
          res[i] = substr(i, l, r);
42
43
        return res;
46
     array<int, HASH_COUNT> merge(const vector<pair<int, int>>&
        array<int, HASH_COUNT> res{};
for(int i = 0; i < HASH_COUNT; ++i) {</pre>
48
          const auto& p = HASH_PAIRS[i];
for(auto [1, r] : seg) {
   res[i] = (1LL * res[i] * get_power(i, r - 1) + substr(i)
51
                  , 1, r)) % p.second;
52
53
```

```
return res;
                                                                                    29
55
                                                                                    30
56
                                                                                    31
57
      // build powers up to x^k
                                                                                    32
      void build_powers(int k) {
                                                                                    33
        for(int i = 0; i < HASH_COUNT; ++i) {
  const auto& p = HASH_PAIRS[i];</pre>
60
           int sz = (int) POW[i].size();
if(sz > k) {
61
                                                                                    35
62
                                                                                    36
             continue;
63
                                                                                    37
65
           if(sz == 0) {
                                                                                    39
             POW[i].push_back(1);
                                                                                    40
67
             sz = 1:
                                                                                    41
68
                                                                                    42
           while(sz <= k) {</pre>
69
             POW[i].push_back(1LL * POW[i].back() * p.first % p.
70
             sz += 1;
                                                                                    45
72
           }
                                                                                    46 };
73
        }
      }
75
                                                                                       8.4
      inline int size() const {
78
79
                                                                                     1 template int ALPHABET = 26, char MIN_CHAR = 'a'>
    private:
80
      int n;
81
      static vector<int> POW[MAX_HASH_PAIRS];
83
      array<vector<int>, HASH_COUNT> pref;
      int substr(int k, int l, int r)
85
        assert(0 <= k && k < HASH_COUNT);
assert(0 <= 1 && 1 <= r && r <= n);
86
87
                                                                                          };
         const auto& p = HASH_PAIRS[k];
        if(1 == r) {
                                                                                    11
90
           return 0;
                                                                                    12
91
                                                                                    13
        int res = pref[k][r - 1];
92
        if(1 > 0) {
93
           res -= 1LL * pref[k][l - 1] * get_power(k, r - 1) % p.
                                                                                    16
                second;
96
        if(res < 0) {
          res += p.second;
97
98
                                                                                    20
        return res;
                                                                                    21
100
                                                                                    22
101
                                                                                    23
      int get_power(int a, int b) {
  if(PRECOMPUTE_POWERS) {
102
103
           build_powers(b);
104
                                                                                    26
           return POW[a][b];
105
                                                                                    27
106
         const auto& p = HASH_PAIRS[a];
107
                                                                                    29
108
        return power(p.first, b, p.second);
                                                                                    30
109
      }
110
                                                                                    31
    template<int A, bool B> vector<int> Hash<A, B>::POW[Hash::
111
                                                                                    32
         MAX_HASH_PAIRS];
                                                                                    33
   8.3
           SuffixArray
```

```
struct suffix_array{
     vector<int>SA,Rank,LCP;
     void counting_sort(vector<int>&v,auto getkey){
       int n = 0;
       for(auto i:v)n = max(n,getkey(i)+1);
       vector<int>bucket(n),ans(v.size());
       for(auto i:v)++bucket[getkey(i)];
       partial_sum(begin(bucket),end(bucket),begin(bucket));
       for(auto ite = v.rbegin();ite!=v.rend();++ite)ans[--bucket[
    getkey(*ite)]] = move(*ite);
10
       v.swap(ans);
     suffix_array(string s):n(s.size()){
       SA.resize(n),Rank.resize(n),LCP.resize(n);
for(int i = 0;i<n;++i)SA[i] = i;</pre>
15
16
       sort(SA.begin(),SA.end(),[&](int a,int b){
17
         return s[a]<s[b];</pre>
       for(int i = 0;i<n;++i){</pre>
20
         Rank[SA[i]] = (i?Rank[SA[i-1]]+(s[SA[i]]!=s[SA[i-1]]):SA
21
               [0]);
       for(int k = 0;(1<<k)<=n;++k){</pre>
          vector<int>idx;
          for(int i = n-(1<<k);i<n;++i)idx.push_back(i);</pre>
         for(auto i:SA)if(i>=(1<<k))idx.push_back(i-(1<<k));</pre>
26
         counting_sort(idx,[&](int a){return Rank[a];});
27
         SA.swap(idx);
```

```
vector<int>new_rank(n);
  new_rank[SA[0]] = 0;
for(int i = 1;i<n;++i){</pre>
    auto cmp = [&](int a,int b){
      return Rank[a]!=Rank[b] or a+(1<<k)>=n or Rank[a+(1<<</pre>
           k)]!=Rank[b+(1<<k)];
    new_rank[SA[i]] = new_rank[SA[i-1]]+cmp(SA[i-1],SA[i]);
  Rank.swap(new_rank);
for(int i = 0,k = 0;i<n;++i){</pre>
  if(Rank[i]==0)continue;
  if(k)--k;
  while(i+k<n and SA[Rank[i]-1]+k<n and s[i+k]==s[SA[Rank[i</pre>
       ]-1]+k])++k;
  LCP[Rank[i]] = k;
```

#### Trie

```
class trie {
  public:
   struct Node {
      int go[ALPHABET];
Node() {
        memset(go, -1, sizeof(go));
    trie() {
      newNode();
    inline int next(int p, int v) {
      v1 = newNode();
    inline void insert(const vector<int>& a, int p = 0) {
      for(int v : a) {
        p = next(p, v);
    inline void clear() {
      nodes.clear();
      newNode();
    inline int longest_common_prefix(const vector<int>& a, int p
         = 0) const {
      int ans = 0;
      for(int v : a) {
        if(nodes[p].go[v] != -1) {
          ans += 1
          p = nodes[p].go[v];
        } else {
37
          break;
        }
38
39
40
      return ans;
    }
41
43
  private:
    vector<Node> nodes;
45
46
    inline int newNode()
      nodes.emplace_back();
      return (int) nodes.size() - 1;
```

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## C++ Resource Test

```
#include <hits/stdc++.h>
  using namespace std;
   namespace system_test {
  const size_t KB = 1024;
const size_t MB = KB * 1024;
const size_t GB = MB * 1024;
  size_t block_size, bound;
   void stack_size_dfs(size_t depth = 1) {
    if (depth >= bound)
13
       return;
     int8_t ptr[block_size]; // 若無法編譯將 block_size 改成常數
     memset(ptr, 'a', block_size);
cout << depth << endl;</pre>
     stack_size_dfs(depth + 1);
18
   void stack_size_and_runtime_error(size_t block_size, size_t
        bound = 1024) {
     system_test::block_size = block_size;
     system_test::bound = bound;
stack_size_dfs();
  double speed(int iter_num) {
     const int block_size = 1024;
volatile int A[block_size];
     auto begin = chrono::high_resolution_clock::now();
     while (iter_num--)
  for (int j = 0; j < block_size; ++j)</pre>
         A[j] += j;
     auto end = chrono::high_resolution_clock::now();
     chrono::duration<double> diff = end - begin;
     return diff.count();
```

```
void runtime error 1() {
      // Segmentation fault
      int *ptr = nullptr;
*(ptr + 7122) = 7122;
    void runtime_error_2() {
   // Segmentation fault
   int *ptr = (int *)memset;
       *ptr = 7122;
48 }
   void runtime_error_3() {
   // munmap_chunk(): invalid pointer
   int *ptr = (int *)memset;
       delete ptr;
54
   void runtime_error_4() {
  // free(): invalid pointer
  int *ptr = new int[7122];
       delete[] ptr;
61 }
   void runtime_error_5() {
  // maybe illegal instruction
  int a = 7122, b = 0;
}
       cout << (a / b) << endl;
68
   void runtime_error_6() {
  // floating point exception
  volatile int a = 7122, b = 0;
  cout << (a / b) << endl;
}</pre>
73
    void runtime_error_7() {
  // call to abort.
       assert(false);
80 } // namespace system_test
82 #include <sys/resource.h>
83 void print_stack_limit() { // only work in Linux
     struct r\overline{l}imit \overline{l};
       getrlimit(RLIMIT_STACK, &1);
cout << "stack_size = " << l.rlim_cur << " byte" << endl;</pre>
```