Improved Atomic Norm Based Channel Estimation for Time-varying Narrowband Leaked Channels

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I. Proof of Proposition 1 in the paper that is submitted to IEEE ICASSP 2021

By designing $q_1 = \frac{T}{T_s}$, $q_2 = \frac{m_2}{q_1}$ and $q_3 = \frac{\beta m_2}{q_1}$, we have $q_1 \geq 1$, $m_2 > q_2 \geq 1$, $\beta m_2 > q_3 \geq 1$, $q_1 \in \mathbb{R}$, and $q_2, q_3 \in \mathbb{Z}$ since the condition B is assumed. When m_1 is set to 0 according to the assumption A, the noiseless ratio function $f_k(t_k)$ can be rewritten as

$$\begin{aligned} f_{RRC}(t_k) =& \frac{(-1)^{(q_2+q_2)} t_k (q_1 T_s - 2\beta t_k)}{(q_1 q_2 T_s - t_k)\left(2\beta t_k - (2q_3 - 1)\, q_1 T_s\right)} \\ & \times \frac{(q_1 T_s + 2\beta t_k)}{\left(2\beta t_k - (2q_3 + 1)\, q_1 T_s\right)}, \end{aligned} \tag{1}$$

where $t_k \in (0, MT_s)$ and $t_k \neq \frac{1}{2q_3} m_2 T_s$, $(1 - \frac{1}{2q_3}) m_2 T_s$, $m_2 T_s$, or $(1 + \frac{1}{2q_3}) m_2 T_s$, for all k.

To ensure the uniqueness of the delay estimates, we need to analyze the monotonicity of $f_{RRC}(t_k)$. The first-order derivative of $f_{RRC}(t_k)$ with respect to t_k is given by

$$\begin{aligned} &\frac{\partial f_{RRC}(t_k)}{\partial t_k} \\ &= \frac{(-1)^{q_2+q_3} (t_k - t_k^{(1)})(t_k - t_k^{(2)})(t_k - t_k^{(3)})(t_k - t_k^{(4)})}{\left((t_k - q_1 q_2 T_s)\left(q_1^2 T_s^2 - 4\beta^2 (t_k - q_1 q_2 T_s)^2\right)\right)^2}, \end{aligned} \tag{2}$$

where

$$t_k^{(1)} = \frac{q_1 q_2 T_s}{2}\left(1 - \sqrt{1 + \frac{\sqrt{4q_3^2 - 1}}{\sqrt{3} q_3^2}}\right),$$

$$t_k^{(2)} = \frac{q_1 q_2 T_s}{2}\left(1 - \sqrt{1 - \frac{\sqrt{4q_3^2 - 1}}{\sqrt{3} q_3^2}}\right),$$

$$t_k^{(3)} = \frac{q_1 q_2 T_s}{2}\left(1 + \sqrt{1 - \frac{\sqrt{4q_3^2 - 1}}{\sqrt{3} q_3^2}}\right),$$

and

$$t_k^{(4)} = \frac{q_1 q_2 T_s}{2}\left(1 + \sqrt{1 + \frac{\sqrt{4q_3^2 - 1}}{\sqrt{3} q_3^2}}\right).$$

It can verified that

$$t_k^{(1)} \leq 0 \leq t_k^{(2)} \leq \frac{m_2 T_s}{2} \leq t_k^{(3)} \leq m_2 T_s \leq t_k^{(4)}.$$

Therefore, if $t_k^{(2)} \neq t_k^{(3)}$, the ratio function $f_{RRC}(t_k)$ is not monotonic with respect to t_k when $t_k \in (0, MT_s)$ and multiple delay estimates may exist for a given $f_{RRC}(t_k)$. However, since the assumptions A and B hold, we have $q_3 = 1$ and the following two conditions are satisfied,

1) $t_k^{(2)} = t_k^{(3)}$,
2) $t_k^{(4)} \geq MT_s$.

With the above two conditions, the ratio function in Equation (1) can be simplified into

$$f_{RRC}(t_k) = \frac{(-1)^{q_2} t_k (q_1 T_s + 2\beta t_k)}{(q_1 q_2 T_s - t_k)\left(2\beta t_k - 3 q_1 T_s\right)}. \tag{3}$$

Then, we can derive that when $t_k \in (0, m_2 T_s)$ or $t_k \in (m_2 T_s, MT_s)$, the following inequality holds

$$(-1)^{q_2} \frac{\partial f_{RRC}(t_k)}{\partial t_k} < 0,$$

proving $f_{RRC}(t_k)$ is a monotone function with respect to t_k. Also, we have

$$f_{RRC}(t^{'}) f_{RRC}(t^{''}) < 0, \tag{4}$$

if $t^{'} \in (0, m_2 T_s)$ and $t^{''} \in (m_2 T_s, MTs)$. Therefore, given $f_{RRC}(t_k)$, the delay of all paths can be uniquely estimated.