Casual Inference Introduction

Potential Outcomes

- Framework of causal inference: action (treatment, intervention, or manipulation) is applied to a unit of interest and generates an outcome.
- Potential outcome is the value of the outcome under the treatment status

Ex. Asperin → Person → No Headache

$$\underbrace{\text{No Asperin}}_{\text{action}} \rightarrow \underbrace{\text{Person}}_{\text{unit}} \rightarrow \underbrace{\text{Headache}}_{\text{Potential outcome}}$$

- Each (action, unit) pair is associated with an potential outcome.
- Causal inference is done by comparing potential outcomes for the same unit in the same post treatment period.
- Definition of causal effect does not depend on observed outcome, but on potential outcome.
- Causal statements may not be will defined if the potential outcomes cannot be realized or are unclear (effect of college on occupation, how to rank occupations?).
- Potential outcome model is used for well defined causal statements.

Ex. (action, potential outcome)

① $\left.\begin{array}{l}\text{Asperin}\\ \text{No Asperin}\end{array}\right\}$ Headache

② $\left.\begin{array}{l}\text{Asperin}\\ \text{No Asperin}\end{array}\right\}$ No Headache

③ Asperin → Headache
No Asperin → No Headache

④ Asperin → No Headache
No Asperin → Headache

- Cases 1) and 2) above have no causal effect, but 3) and 4) do have a causal effect.
- "Fundamental problem of causal inference" is that only one of the potential outcomes can be realized and observed.
- Definition of causal effect only requires one unit, but learning about causal effects will usually require comparing multiple units.
- Estimation of causal effects will require comparing observed outcomes.

Stable Unit Treatment Value Assumption (SUTVA)

- Two common informal variation for causal inference is 1) compare outcomes for same unit over time under different actions, and 2) compare outcomes for different units at same time under different actions (control and treatment).

- Individuals may react differently to the same treatment over time, hence 1) above is not ideal. However variation described in 2) is better if different individuals all have same treatment choice (not vary in intensity) and are independent of each other.
- Assumption SUTVA has two components: 1) treatments applied to one unit doesn't effect the outcomes of other units and 2) treatment level is unique so that potential outcomes are well defined.
- Another interpretation of SUTVA is that 1) potential outcomes are independent of treatments status of other units, and 2) the treatment is defined the same for all units, that is the "intensity" of treatment is the same.
- Under the aspirin example, SUTVA would say 1) the effect of one unit taking aspirin does not impact the headache status of another unit, and 2) all aspirin tablets are of the same strength.

Ex. Violates SUTVA

unit1: Asperin / No Asperin

unit2: Asperin / Asperin Old / No Asperin

- The two components of SUTVA are known as 1) "No Interference" or "No spillovers" and 2) "No hidden variation of treatments" or "Homogenous treatment".
- SUTVA is a exclusion restriction used to exclude varying possibilities and make causal inference feasible.
- Exclusion restrictions generally cannot be verified from data but are based on prior knowledge on the subject matter of interest.
- For many economic setting SUTVA may only be feasible when units are modelled as groups such that there can be interaction within group, but independence across groups.
- SUTVA implied potential outcomes are well defined. Under SUTVA multiple units can be used to infer causality.

Assignment mechanism (Potential Outcome Model)

Let D_i be a binary treatment indicator

$$\underbrace{Y_i^{obs}}_{\text{Realized Potential outcome}} = D_i Y_{i1} + (1-D_i) Y_{i0} = \begin{cases} Y_{i1}, & D_i = 1 \\ Y_{i0}, & D_i = 0 \end{cases}$$

$$\underbrace{Y_i^{miss}}_{\text{Missing Potential outcome}} = (1-D_i) Y_{i1} + D_i Y_{i0} = \begin{cases} Y_{i0}, & D_i = 1 \\ Y_{i1}, & D_i = 0 \end{cases}$$

- Three ways to use pre-treatment variables 1) explain variation in the outcome, 2) compute casual effects by subgroups, 3) conditional independence of treatment assignments.

Causal Parameters of Interest

- Unit level causal effects is comparing potential outcome under control and treatment.

$$\text{Unit level Causal Effect} = Y_{i1} - Y_{i0}$$

- Causal effect definition does not depend on which treatment is observed.
- Population level causal effect is the expected outcome under treatment contrasted with expected outcome under control.
- Cannot compute unit TE using data because of the fundamental problem of causal inference.
- Average treatment effect is mean unit level causal effect in population:

$$\text{Average Treatment Effect} = E[Y_{i1} - Y_{i0}]$$

- Conditional average treatment effect is ATE conditioned on a pre-treatment covariate.

$$\text{Conditional ATE} = E[Y_{i1} - Y_{i0} \mid X_i = x]$$

- ATT is the average treatment effect for those initially assigned the treatment

$$ATT = E[Y_{1i} - Y_{0i} \mid D_i = 1]$$

- ATU is the average treatment effect for those assigned to the control group

$$ATU = E[Y_{1i} - Y_{0i} \mid D_i = 0]$$

- ATE can be written as a function (weighted average) of the ATT and ATU

$$\begin{aligned} ATE &= Pr(D_i = 1)\, E(Y_{1i} - Y_{0i} \mid D_i = 1) + Pr(D_i = 0)\, E(Y_{1i} - Y_{0i} \mid D_i = 0) \\ &= \rho\, ATT + (1-\rho)\, ATU \end{aligned}$$

- Mean comparison of outcome across control and treatment is biased by selection and heterogeneous treatment effects.

$$E(Y_i \mid D_i = 1) - E(Y_i \mid D_i = 0) = \underbrace{E(Y_{1i} - Y_{0i})}_{ATE} + \underbrace{(E(Y_{0i} \mid D_i = 1) - E(Y_{0i} \mid D_i = 0))}_{\text{Selection bias}} + \underbrace{(1-\rho)(ATT - ATU)}_{\text{Hetro. bias}}$$

Identification of Causal Parameters

- Under constant treatment effect (causal effect = c) implies 1) ATT = ATU and hence there is heterogeneity bias.

$$Y_{1i} = \rho + Y_{0i} \Rightarrow ATT = ATU$$

- Note that under constant treatment effect the ATT = ATU = ATE.
- Constant treatment effects allows us to reformulate the POM model into a standard regression:

$$Y_i = Y_{0i} + D_i(Y_{1i} - Y_{0i}) \underset{Y_{1i} = \rho + Y_{0i}}{=} Y_{0i} + \rho D_i$$

$$= E(Y_{0i}) + \rho D_i + (Y_{0i} - E(Y_{0i})) = \alpha + \rho D_i + \varepsilon_i$$

- The constant treatment effect assumption is not necessarily required for regression formulation:

$$\text{Hetro. effect: } \begin{cases} Y_{0i} = E(Y_{0i}) + u_{0i} \\ Y_{1i} = E(Y_{1i}) + u_{1i} \end{cases} \Rightarrow E(u_{0i}) = E(u_{1i}) = 0$$

$$\Rightarrow Y_i = Y_{0i} + D_i ATE + D_i(u_{1i} - u_{0i}) = E(Y_{0i}) + D_i ATE + u_{0i} + D_i(u_{1i} - u_{0i})$$

$$\Rightarrow Y_i = \alpha + \rho D_i + \varepsilon_i$$

- Suppose a good teacher perfectly recognizes whether student's needs a tutor or not. Assignment of tutors to student's made by this teacher are not going to be independent of potential outcomes. Hence there will be selection bias.
- If treatment is randomized to individuals, there is no selection bias and also no heterogeneity bias. Comparing across group means is causal and identifies ATE.

$$D_i \perp (Y_{1i}, Y_{0i}) \Rightarrow E[Y_{1i} - Y_{0i}] = E[Y_i | D_i = 1] - E[Y_i | D_i = 0]$$

- Rational choices made by individuals are usually dependant on potential outcomes and hence will violate the independence condition.
- ATE is identified when treatment is randomly assigned to units. This is because now there is 1) no selection bias and 2) no heterogeneity bias.
- Under constant treatment effect and randomly assigned treatment the ATE, ATT, and ATU are all identified and equal to each other.
- The primary assumption for identification for the causal parameters is the randomization of treatment assignment.