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T1 ________	**87611**	F1 ________
T2 ________		F2 ________
T3 ________	Problem Chosen	F3 ________
T4 ________	**D**	F4 ________

2018
MCM/ICM
Summary Sheet

Modeling the Future of Electric Vehicle

Summary

The problem we are trying to solve in this paper is to help a nation plan the final network of charging stations as well as the development of charging station network, when this country begins its switch from gasoline cars to electric vehicles. The following points summarize our main work:

1. Calculate the amount of the destination chargers in residential area by introduce a Monte Carlo prediction model to simulate random electric vehicle charging demand of residents. Simulant discrete functions are constructed to reflect the daily usage habit.

2. Estimate the number of the supercharger on roads by using linear road model, ensuring the power supply of the charging station will meet traffic consumption.

3. Find the distribution of charging stations between different areas.

4. Forecast the sales of electric vehicles over time based on the extension of Bass model. Sensitivity analysis indicates the effect of infrastructure and economic subside factors.

5. Predict the incremental sale rate of EVs from Gompertz model under the political impact. This will also give some information about how polite implement determines the growing speed.

Keywords: Electric Vehicle, Monte Carlo Method, Gompertz Model, Bass Model

HANDOUT

Nowadays, countries around the world are facing the threat of energy crisis. In the near future, fossil fuels will gradually dry up, and the system that burns fossil fuels will gradually change.
In order to deal with the energy crisis, we have to look to renewable energy, electric cars consume electricity which can be provided by renewable energy, rather than traditional cars which use gasoline.
It is foreseeable that electric cars will gradually replace traditional cars in the near future and become the main mode of transportation.
Now, electric cars are still less popular than traditional cars because of the high price of electric cars and the small number of charging stations.
Therefore, in order to popularize electric vehicles, some suggestions are put forward for your reference:

1. Preferential policies should be provided to electric vehicle producers such as lower tax rates and subsidies to motivate electric car production.
2. Preferential policies such as tax credit should be given to electric vehicle owner to stimulate the demand for electric car
3. Improvement of ancillary facility like charging stations should be accelerated
4. The early investment in charging stations should be in city areas
5. Electric-vehicle-friendly technology like autopilot should be supported

Ultimately according to the Gompertz Model, it is suggested that the gas vehicle-ban date should be at year 2050 when most cars are electric-driven.

Contents

1 Abstract

The problem we are trying to solve in this paper is to help a nation plan the final network of charging stations as well as the development of charging station network, when this country begins its switch from gasoline cars to electric vehicles. The following points summarize our main work:

1. Calculate the amount of the destination chargers in residential area by introduce a Monte Carlo prediction model to simulate random electric vehicle charging demand of residents. Simulant discrete functions are constructed to reflect the daily usage habit.
2. Estimate the number of the supercharger on roads by using linear road model, ensuring the power supply of the charging station will meet traffic consumption.
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2 Task One

2.1 Modeling for City Roads

This is a model to calculate the demand of electric energy in densely populated areas such as urban, suburban and rural areas. Short-distance travel is mostly occurred in these areas, with characteristics of long charging time and low charging frequency.

2.1.1 Assumptions and simplification

As in populated areas, electric vehicle are basically used for commuting travel with characteristics mentioned above, hypothesis are made as:

1. The electric vehicle user will not charge until the last time he/she arrives at parking lot in a day
2. Charge duration is proportional to the travel distance

3. Each electric vehicle is charged only once a day
4. Electric vehicle owners have the same behavior pattern as the fuel vehicle owners

2.1.2 Model Principle

Assuming there is a virtue world with n electric vehicle.The kth car starts to charge at the time of S_k lasting T_k. Thus, at a random time of t, we have the state function of the kth car:

$$C_k(t) = \begin{cases} 1, & S_k \leq t \leq S_k + T_k \\ 0, & \text{otherwise} \end{cases}$$

The purpose of the model is to calculate total number of charging electric vehicle at any particular moment in the world, which is defined as $N_c(t)$.

2.1.3 Model Building

Retrieving the probability density function[1]:

$$f_s(x) = \begin{cases} \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left[-\dfrac{(x-\mu)^2}{2\sigma^2}\right], & \mu - 12 < x \leq 24 \\ \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left[-\dfrac{(x+24-\mu)^2}{2\sigma^2}\right], & 0 < x \leq \mu - 12 \end{cases}$$

Where

$$\mu = 17.6$$

$$\sigma = 3.4$$

We have the distribution of start-charging time shown in Figure 1

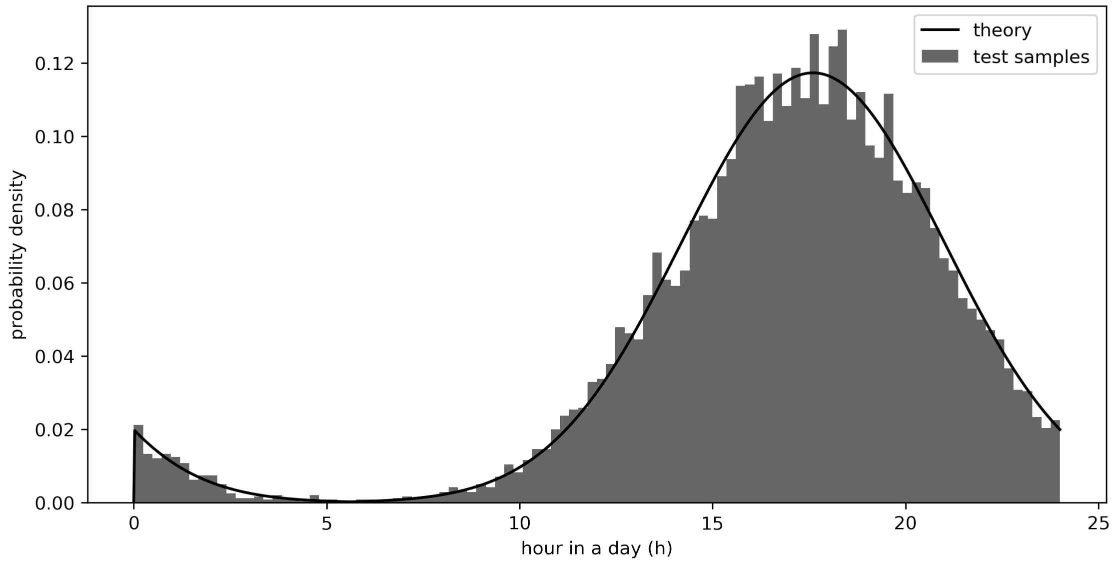

Figure 1

Retrieving the probability density function from xxx:

$$f_t(x) = ae^{bx} + ce^{dx}, \qquad x \geq 0$$

Where $a = -0.8225, b = -976.06, c = 0.8919, d = -0.8912$
We have the distribution of charging duration shown in Figure 2

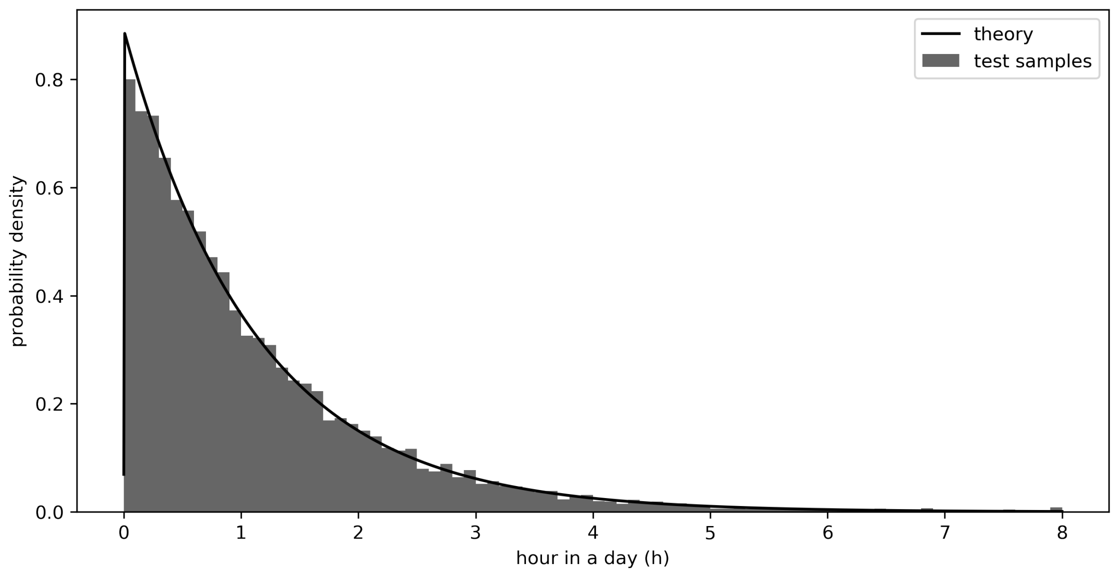

Figure 2

Having had the distribution of start-charging time and charging duration, the Monte Carlo Method can be applied to stimulate the situation in virtue worlds with 10,000 electric vehicle. Repeat this process for 60 times and take the average value in Figure 3

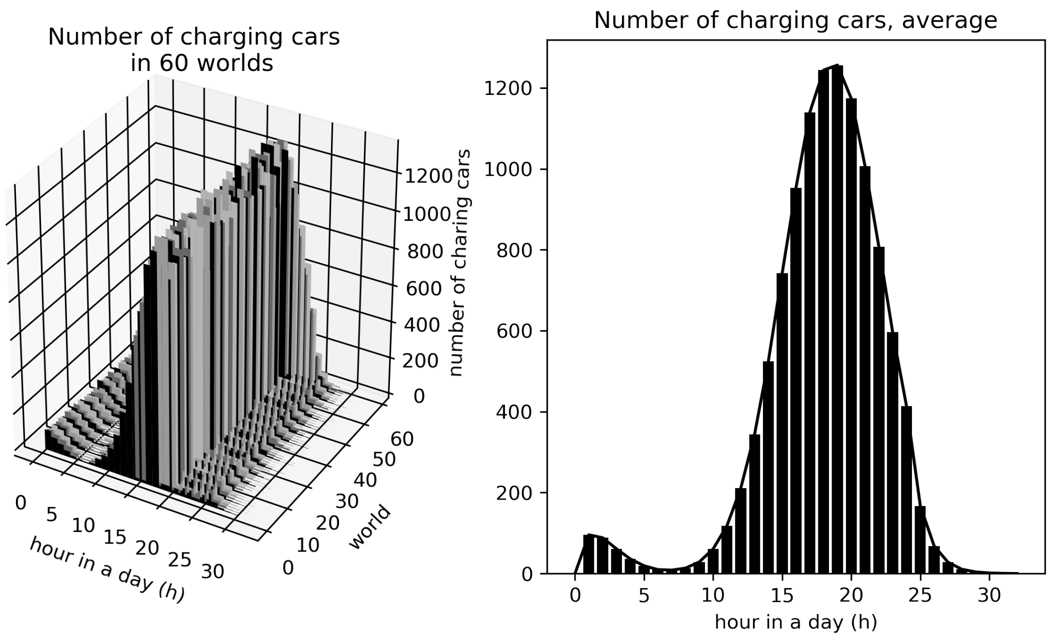

Figure 3

The outcome of the programmee indicates that in a virtue with 10,000 car, the peak number of electric vehicle charging at the same time(Around 20:00) is 1259.

2.2 Modeling for Liner Roads

A rudimentary model is introduced here. As the basis of the use of electric vehicle, the power provided by the charging station must be at least equal the aggregated power consumption of electric vehicle. This model is designed for the charging stations set alongside liner roads such as intercity roads and high way.

2.2.1 Assumptions and Simplification

1. All charging station are standardized and same attracting to drivers
2. All electric vehicle have similar performance
3. All batteries are evenly consumed under different pavement condition
4. All vehicle will go to the closest charging station when warned by the vehicle

2.2.2 Notations

Notations	Defination
R	A certain section
T	Traffic volume on a certain section
Q	Average power consumption/100km
R	Probability of charging
W	Battery Capacity
P	Probability for a car to charge
a	Working time ratio

2.2.3 Derivation process of the model

Take the section R as the research object. Considering the situation that one single car running on the road. Its warning battery capacity is $w(kWh)$ and full battery capacity is $W(kWh)$.
It is certain that at the entrance of road L, its battery capacity W(x), is in a situation of $w < W(x) < W$. At the farside of the road R, the remaining battery capacity is $W(x) - W(R)$ where $W(R)$ is the power consumed in R. When $W(x) - W(R) < w$, the car must charge by $W - w$ in a charge station in R. Thus, when leaving the road, the battery capacity of the car is :

$$W_s = W(x) - W(L) + W - w$$

As the battery capacity declines homogeneously[2] with the rise of travel distance, $W(x)$ would be evenly distributed between w and W, and the probability for a car to charge is:

$$P = \frac{W(R)}{W - w}$$

The power to be charged to the vehicle is:

$$(W - w) \times \frac{W(R)}{W - w} = W(R)$$

According to the derivation process, we get the conclusion that power charged to vehicle equals to the power sonsumption.

2.2.4 Model Building

The power consumption of cars W_c on road R_1 is calculated as:

$$W_c = Q_i \times L_j \times V_j$$

Total power consumption of all vehicle W_c on road L_j is calculated as:

$$W_c = \sum_{i=1}^{N} (Q_i \times L_j \times V_{i,j}) = L_j \times \sum_{i=1}^{N} (Q_i \times V_{i,j})$$

Total power consumption of all vehicle on all roads L is calculated as:

$$W_c = \sum_{j=1}^{N}(L_j \times \sum_{i=1}^{N}(Q_i \times V_{i,j}))$$

When $W(0)$ represents the charging power of one charging station, the total amounts of charge stations on the roads Z should be the quotient of W_q and W_0:

$$Z_{max} = \frac{W_c}{W_0} = \frac{\sum_{j=1}^{N}(L_j \times \sum_{i=1}^{N}(Q_i \times V_{i,j}))}{W_0}$$

2.3 Fitting the model to data

If everyone switched toTesla Model 3 in the US, to explore the network of Tesla charging stations in the United States, we use statistics of Tesla Model 3, Tesla superchargre and Tesla destination charger.

2.3.1 Model for City Areas

In the process of building the model, we have already use the data of Tesla destination charger. The peak number of electric vehicle charging at the same time is 1259 every 10,000. If the maximum demand is met, destion charger will be able to meet electric demand at any time of the day. Because the total car ownership in the USA is 220,000,000, we anticipate that the maximum value of electric vehicle charging at the same time would be $1259 \times \frac{220{,}000{,}000}{10000} = 27,698,000$. In the same waythe maximum value of electric vehicle charging at the same time in the real world would be 37770

2.3.2 Model for Liner Road

Here Q_i is considered to be a constant 18, $\sum_{j=1}^{N}(L_j \times \sum_{i=1}^{N}(V_{i,j}))$ can be substituted by the motor Vehicle travel milage on liner roads in the United States[],which is 2954.4 billion miles and $W_0 = 100$, the power of Tesla supercharge, a=0.3. Z=2,023,561.
Considering every supercharging station contains 7.5 supercharger on average, there are supposed to be 269,808 supercharging station in America. As the actual number of Tesla vehicle running on road now is 300,000, while total car ownership in the USA is 220,000,000, the ideal number of supercharging station is supposed to be 368 according to the model.

2.4 Conclusion

The predicted number of two kinds of charger are listed in the following table:

	Number of supercharging station	Number of destination charger
Virtue World	269,808	27,698,000
Real World	368	37,770

The real number of Tesla supercharging station is 465[3] and Tesla destination is 45886[4], which is not far from our predicted number. Finally, we can conclude that Tesla is on track to allow a complete switch to all-electric in the US.

To determine the distribution of charging stations between urban, suburban, and rural areas, population and electric vehicle distribution are supposed to be taken into consideration. NTHS survey shows that 3.9 billions milage are covered in rural roads while 1.2 billion in urban and suburban areas. Another survey show the average daily travel distance in rural areas.[5] It is easy to understand that $the\ Number\ of\ vehicle = \frac{Total\ milage}{average\ daily\ traveldistance \times 365}$. The charging stations are considered to be distributed in accordance with the distribution of electric vehicle, and the results are listed in the following table:

Region	Number of destination charger	Proportion
Urban	16,341,820	59%
Suburban	7,478,460	27%
Rural	3,877,720	14%

3 Task Two

3.0.1 Applying model to task 2a

According to the model mentioned in task one, fitting the traffic data in Korean, we can quickly get the result:(See in appendix)

Region	Number of destination charger	Proportion
Urban	1,372,000	58%
Suburban	637,000	26%
Rural	4,410,000	18%

	Number of supercharging station	Number of destination charger
Virtue World	24,462	2,450,000

The key factors that number of charger are the population density and traffic volume.

3.0.2 Bass Model for task 2b

Using the extension of bass model based on the theory of innovation diffusion to estimate the sales of electric vehicles in Korea.

1.1 The extension of bass diffusion model
The extension of bass diffusion model is a model based on the bass model and the influence factors of the marketing strategy. Bass model is often used to predict changes in the sales of innovative goods over time. Bass model proposed the concept of innovative adopter and imitator. The innovator p is a consumer that buys innovative products because he is influenced by external influences such as mass media, and the innovator p will not be affected by people who already use the product. The imitator q is a consumer who buy innovative products because he is influenced by the adopters within the social system. It is assumed that the number of product adopters increased at time t is linear with the total number of adopters at time 0 t. The basic bass model is:

$$n(t) = \frac{dN(t)}{dt} = p[m - N(t)] + \frac{q}{m}N(t)[m - N(t)]$$

$$n(t) = m[\frac{p(p+q)^2 e^{-(p+q)t}}{[p + qe^{-(p+q)t}]^2}]$$

In the formula, $n(t)$ is the increasing number of product adopters at time t; p is the innovation coefficient; $q$is the imitation coefficient; m is the maximum capacity of the market; $N(t)$ is the cumulative number of adopters from 0 to t. On the basis of the tradition Bass model, the extension of bass model takes the influence coefficient into consideration, so that the merchants can accelerate or postpone the adoption of new products by changing the investment of advertisement and price. The formulation of the extension of bass model is:

$$n(t) = \frac{dN(t)}{dt} = p[m - N(t)] + \frac{q}{m}N(t)[m - N(t)] \times x(t)$$

$X(t)$ is the influence coefficient at time of t, and its expression is:

$$x(t) = 1 + [\frac{\delta Pr(t)}{Pr(t-1)}\beta_1 + [\frac{\delta ADV(t)}{ADV(t-1)}]\beta_2$$

In formulation (3),$Pr(t)$ is price of product at time t. $Pr(t)Pr(t)Pr(t-1)$; $ADV(t)$ is the advertising input of innovative product at time of t. $ADV(t)ADV(t)ADV(t-1)1$ is the influence coefficient of the price, and @2 @is the influence coefficient of the advertisement.

1.2 Constructing the model of electric vehicle sale based on extension of bass model

In this case, we are aiming to promote the model analysis of electric vehicles in the country, so we need to modify the function $X(t)$. In the price function of an electric vehicle, the purchaser estimates the cost of the purchase based on the current cost level and the cost of using it over the next 10 years. This includes the cost of charging and the purchase price of the electric vehicle itself. Meanwhile, subsidy provided by the country will reduce the price further. Consumers will compare the total cost with the cost of buying a gasoline car. In addition, considering the development of the electric car is still in its infancy, infrastructure such as charging stations clearly have a greater influence on the function $x(t)$ than advertisement. As a result, we replace advertisement with number of charging station. So the modified function $x(t)$ is:

$$x(t) = 1 - \beta_1[\frac{C_1(t+1) - C_1(t)}{C_1(t)} - \frac{C(t+1) - C_2(t)}{C_2(t)}] + \beta_2 \frac{C_s(t+1) - C_s(t)}{C_s(t)}$$

$$C_1(t) = P(vehicle) - subsidy + 10 \times \frac{AVMT}{EMPG} \times P(electric)$$

$$C_2(t) = P(vehicle) + 10 \times \frac{AVMT}{MPG} \times P(gasoline)$$

In formula (5), $C1(t)$ and $C2(t)$ are the cost of electric vehicles and gasoline cars at time t. 1 is influence coefficient of the price. In equation (5) and (6), $P(vehicle)$ is car current purchasing price, $subsidy$ is policy subsidy, $AVMT$ is average vehicle miles travelled, $EMPG$ is power consumption per 100 miles, $P(electric)$ is the current electricity price, $MPG$is oil consumption per 100 miles, $P(gasoline)$ is the current price of gasoline; $CS(t)$ is the number of public charging stations at time t.

1.3 Estimating the parameters of Bass model

According to the parameter of formula (5) and (6), We should choose the suitable electric vehicles and the gasoline car to represent formula C1(t) and C2(t).So we use the Toyota Prius and the Toyota Corolla to represent electric cars and gasoline cars. What more, according to the relevant subsidy policy, We found that the government will compensate consumers who buy electric vehicles[6], So we can reduce the price of electric cars directly.[7]

Considering that there were few electric cars in Korea between 2000 and 2008, we use alternative data from the United States to represent n(t). We use non-linear least square to estimate parameters of the model and use Matlab to calculate parameters. We get the parameters that: $p = 0.0088,\ \ = 0.456,\ \beta_1 = -0.076,\ \beta_2 = 0.3636$, according to the parameters, compared with imitation coefficient, innovation coefficient p has less effect. As the number of consumers accumulates, imitation coefficient will play the leading role and the growth curve goes into a period of rapid growth. This situation accords with the general law of innovation product.

Predicting sales of electric vehicles in South Korea

Assuming the data in the model

(1) According to a market survey of cars in South Korea, South Korea has about 20 million cars. Thus, We analyze the population of South Korea and concluded that if all cars are transmitted to electric vehicles, maximum number of electric cars $m = 20million$. (2) We take electric vehicles-Modern IONIQ as an example, which are the best-seller in 2015, the power consumption is 17.6Kwh per 100km. We take gasoline vehicles-Modern Sonata 8 as an example, the oil consumption 15L per 100km. (3) The statistics from 2001 of electricity price in South Korea is 0.0840.0710.0700.074$/kw.h$. The growth rate was -0.001. So we can predict the trend of electricity price with this rate. While the oil price is affected by energy issues and international politics, it is too complex to estimate its trend. So we assume growth rate was +0.0651$/L$.
(See in appendix) we will get the figure about the sales of EVs in South Korea:

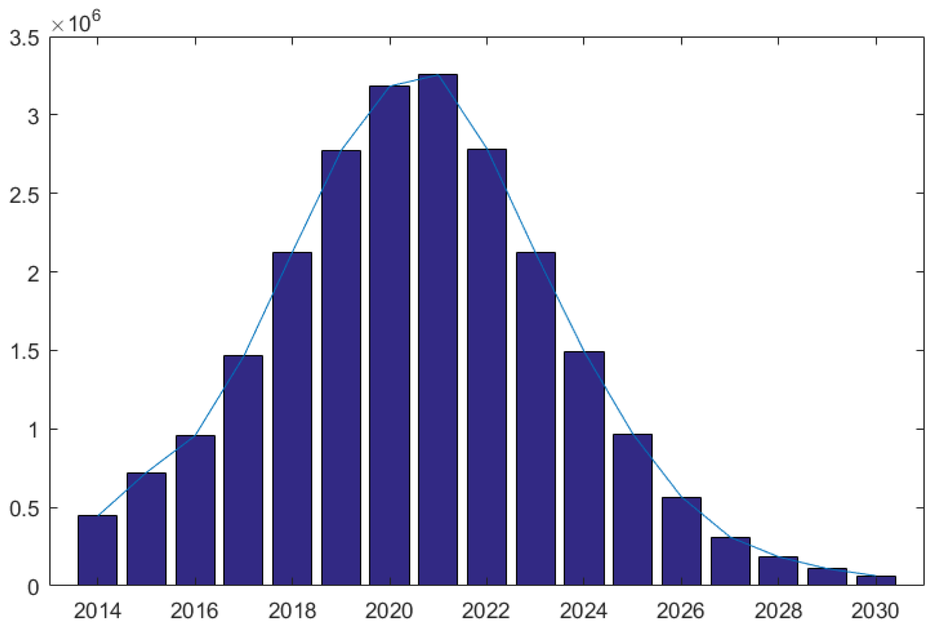

Figure 4

Sensitivity analysis of priceβ_1 and infrastructureβ_2. In order to study the impact brought by price fluctuations and infrastructure construction, sensitivity analysis was performed on these two parameters and the result is shown in the figure below:

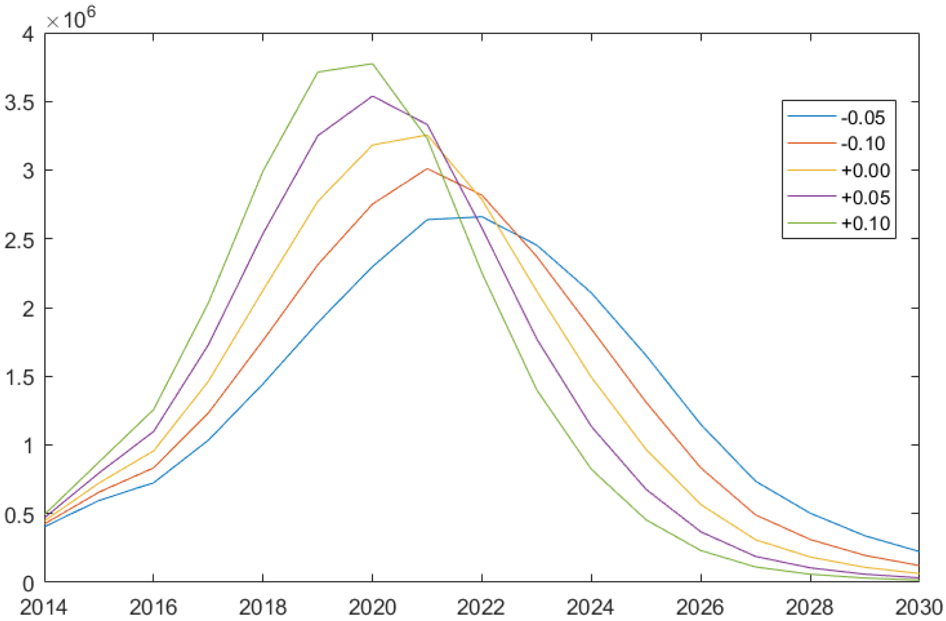

Figure 5: How the change of β_1 affect the sales

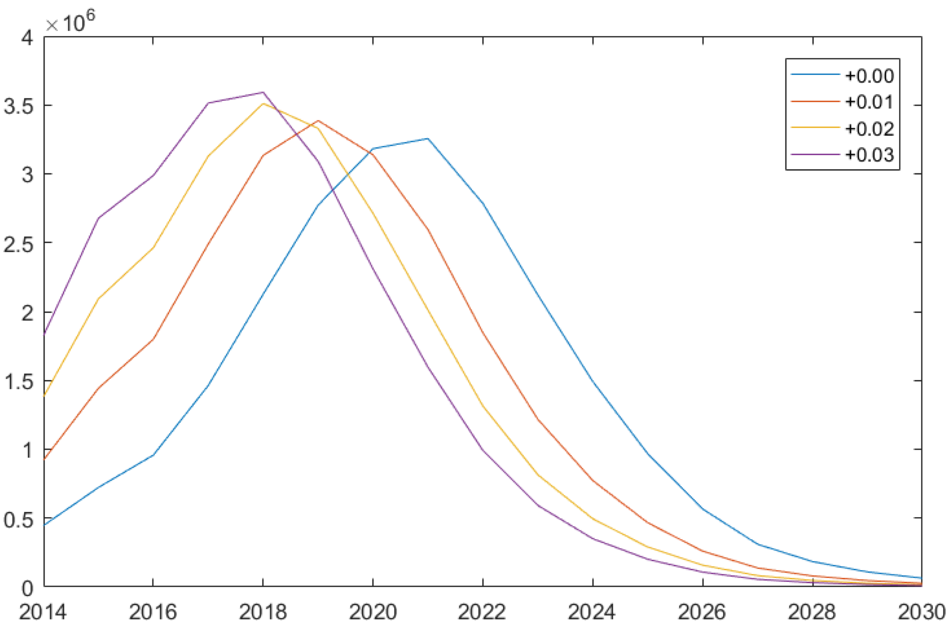

Figure 6: How the change of β_2 affect the sales

According to the figure, β_1 has the similar effect as imitation coefficient q. The larger the value of β_1, the larger the sales volume of electric vehicles, and the quicker will m get its maximize value. β_2 has the similar effect as innovation coefficient p. The larger the value of β_2, the quicker will m get its maximize value. Both β_1 and β_2 accelerate the overall acceleration.

The analysis above shows that in the early stage of the development of electric vehicle market, after charging pile reaching a certain quantity value, β_2 will experience a rapid increase. Thus, it is suggested that some charging piles should be build first to make people more desirable to buy electric vehicle. According to the analysis about β_1, when the government increases subsidies, an obvious increase of β_1 and m would occur. In a word, we believe that Korean government should establish charging stations in the city first, with city government providing more subsidies, to bring about a greater diffusion rate.
1.4 Conclusion:
South Korea's investment plan should be: build a certain number of charging piles first to increase the attraction of electric vehicle; Build charging stations in cities first and increase subsidies for car purchases. The main factors that affect our proposal are price factors (i.e., subsidies) and infrastructure factors.

3.0.3 Gompertz Model for task 2c

A Gompertz curve or Gompertz function, named after Benjamin Gompertz, is a sigmoid function. It is a type of mathematical model for a time series, where growth is slowest at the start and end of a time period. []An example of the model mobile phone uptake, where costs were initially high (so uptake was slow), followed by a period of rapid growth, followed by a slowing of uptake as saturation was reached.
Gompertz model is considered appropriate to anticipate the growth of electric vehicle. In the early stage of electric vehicle development, according to the Path-Dependence theory, the expansion must be slow. followed by a period of rapid growth, benefited from the support of national policies and other factors, followed by a slowing of growth as saturation was reached.
The formula of Gompertz is :

$$Y_t = ke^{-ae^{-bt}}$$

Where a and b is parameters to be estimated t is the time variable.

3.0.4 Fitting the Model with Data

k is the maximize of Y_t, and it is considered to be the car ownership of the country. To determine the value of a and b, we need to do some deformation on the formula: Taking logarithm on both sides of the formula:

$$\ln \frac{k}{Y} = ae^{-bt}$$

Taking logarithm again:

$$\ln \ln |\frac{k}{Y}| = \ln a - bt$$

It can be considered as a linear regression model.Fitting the data:

Year	2011	2012	2013	2014
EV ownership in Korea	1.1	1.5	1.7	2.0
$\ln\ln\|\frac{k}{Y}\|$	1	8	26	100

The result of liner regression is $a = 8.7557, b = 0.1033$, and k is the number of car ownership in Korea, 20,000,000. The Gompertz is determined as:

$$Y_t = 20000000e^{-8.7557e^{-0.1033t}},$$

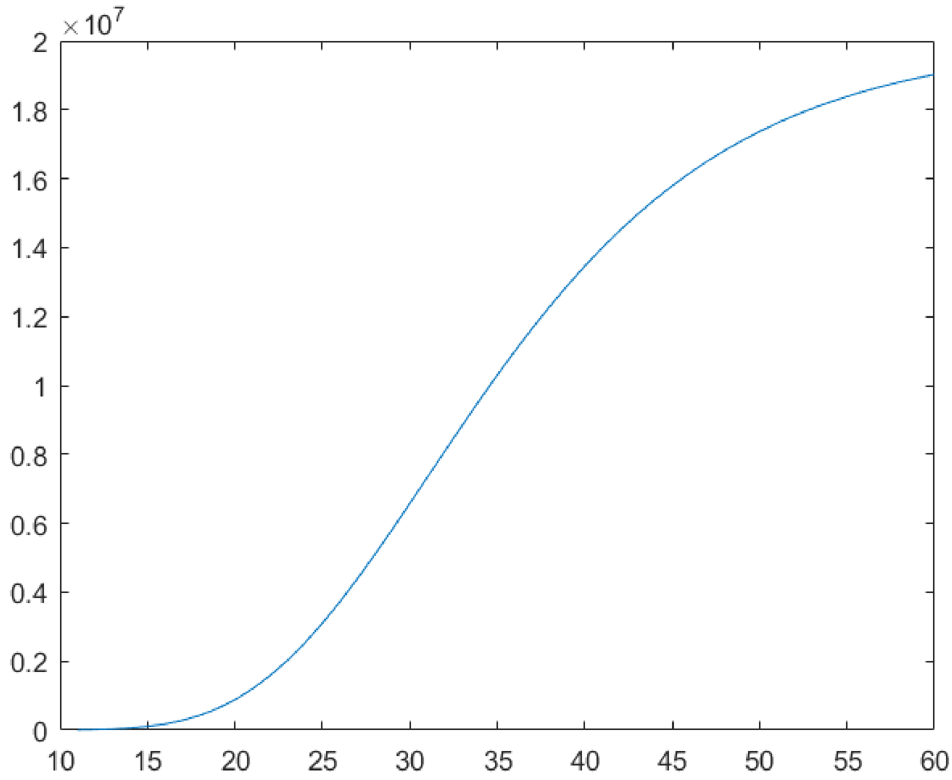

Figure 7

and we can calculate that there will be 10% electric vehicle in year 2024, 30% in 2030, 50% in 2036 and 70% in 2042. The key factors are the starting time and strength of policy support. According to the sensitivity analysis, the change of a leads to a shift of the curve and the change of b influence the slope of the curve, indicating the change of increasing rate.(See in appendix)

4 Task Three

All model proposed above apply to countries with very different geographies, population density distributions, and wealth distributions. Despite the fact that

countries have different national conditions, electric vehicle still experiences similar stages of rapid increase, slow increase and saturation and this is there Gompertz Model can be applied to.
The key factor is considered to be the start time and strength of state policy support, at least at the starting stage, as is analyzed in task two.
Such a classification system does exist, but very complex. Many factors, including terrain, population density, GDP etc. should be considered in classification. Taking population density as an example, most of the population of a country are often concentrated along coast, river and in the plain area. There are not many population living in the mountains, desert and jungle terrain. Therefore, more charging stations should be set in former areas than in latter areas.

5 Task Four

With the development of science and technology, many new forms of transportation are bound to have an impact on the developing electric car market.
Some new ways of travelling like car - share and ride - share services can also reduce carbon emissions. These new competitors can influence the electric vehicle market share, which in Gompertz growth model, affect the parameter k, reducing the maximize number of electric cars.
As to autopilot and rapid battery-swap stations, a symbiotic relationship would occur between these two technology and electric vehicle. Take autopilot as an example, this technology would allow electric vehicle to seek for charging station intelligently, making the distribution of start-charging time more smooth, indicating a smaller peak value of number of charging cars at the same time. It is a great progress that brings a sharp decline to the demand of charging station.

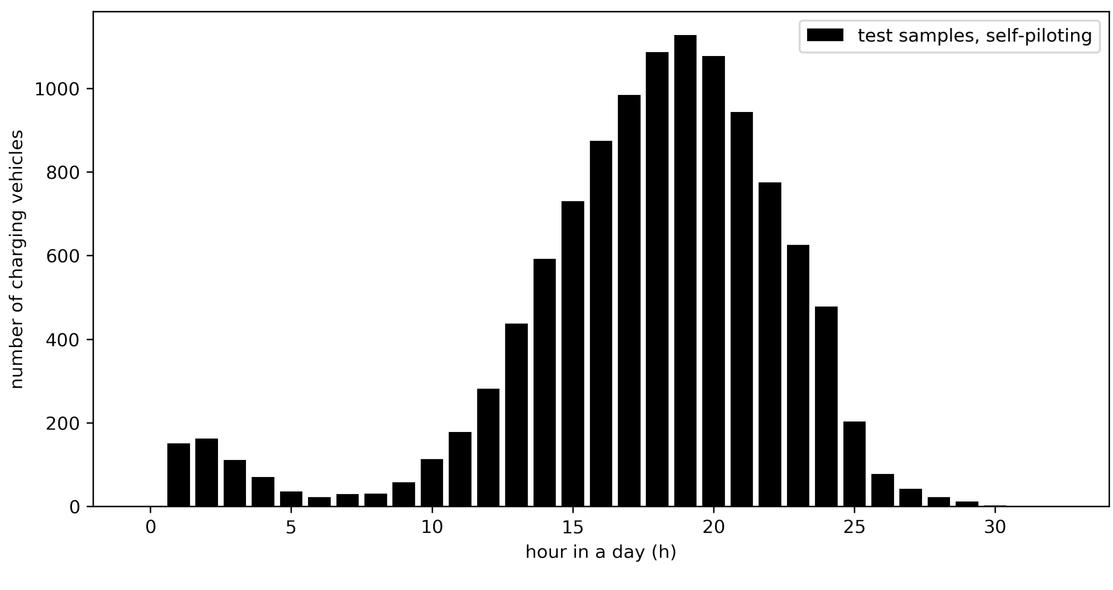

Figure 8

Flying cars and Hyperloops are still at the experimental stage, unable to enter the market in a short time. But in the long run they may be potential competitors against electric vehicle. The invest in these new technology will also weaken the support to electric vehicle, leading to a bigger β in the model:

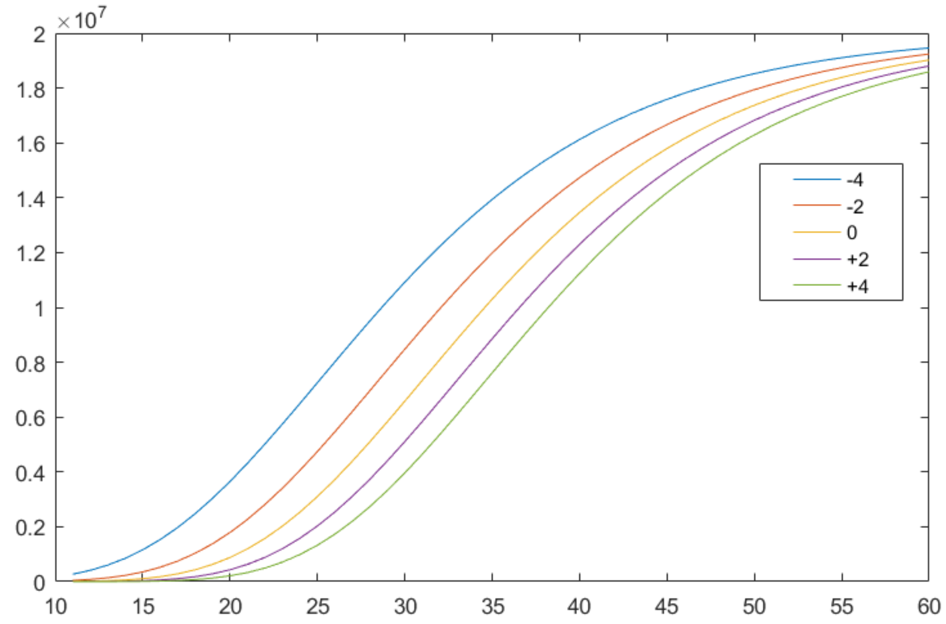

Figure 9

In general, self-driving, rapid battery-swap stations for electric cars would increase the use of electric vehicle. Flying cars and a Hyperloop may be potential competitors and car-share and ride-share services may not have much effect on electric cars.

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Appendices

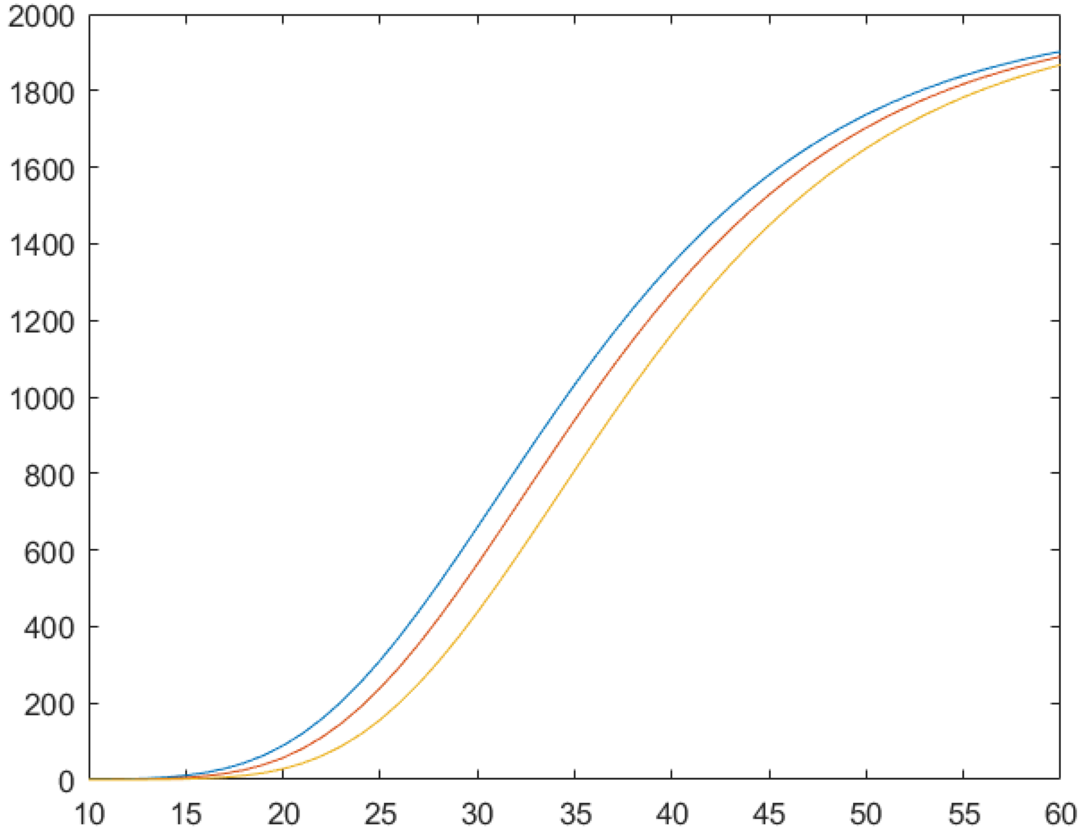

Figure 10: Sensitivity Analysis

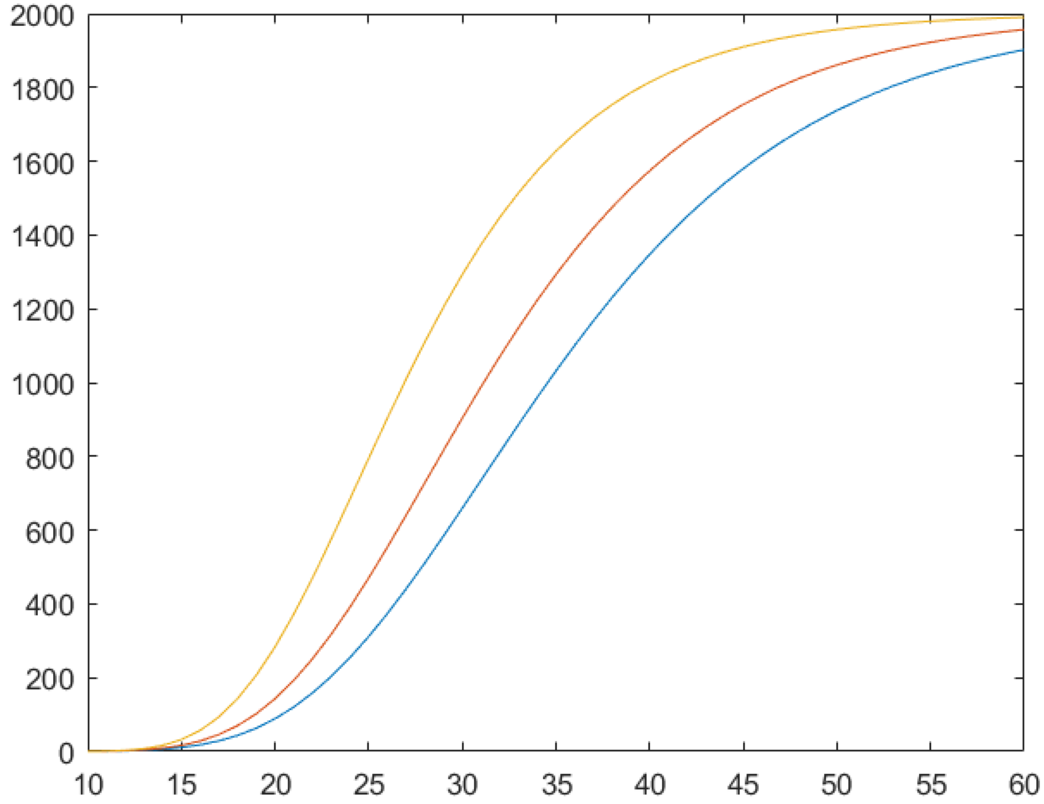

Figure 11: Sensitivity Analysis

Key Program in Task one:

```
import sympy.stats as stats
import scipy.stats

sigma, mu, x = symbols("sigma mu x", real=True)
fs = Piecewise([1 / (sigma * sqrt(2 * pi)) * exp(- (x - mu)**2 / (2 * sigma**2)), And(mu - 12 < x, x <= 24)],
               [1 / (sigma * sqrt(2 * pi)) * exp(- (x + 24 - mu)**2 / (2 * sigma**2)), And(0 < x, x <=  mu - 12)],
               [0, True]).subs({
    mu: 17.6,
    sigma: 3.4
})
fsH = lambdify(x, fs, "numpy")
# f

# X = stats.ContinuousRV(x, f, Interval.Lopen(0, 24))
# stats.sample(X)
# plt.plot(X, lambdify(x, f)(X))
```

Figure 12: Key Program

```
class StartTimeDistribution(scipy.stats.rv_continuous):
#     def _pdf(self, x):
#         return fH(x)
    def _cdf(self, x):
        return FsH(x)

startTimeDistribution = StartTimeDistribution(a=0, b=24)
startTimeSamples = startTimeDistribution.rvs(size=10000)
# startTimeSamples
```

Figure 13: Key Program

```
X = np.linspace(0, 8, 1000)

fig = plt.figure(figsize=(10, 5), dpi=300)
ax = fig.add_subplot(111)

ax.plot(X, ftH(X), label="theory")
ax.hist(timeSamples, bins=80, density=True, label="test samples")

ax.set_xlabel("hour in a day (h)")
ax.set_ylabel("probability density")

ax.legend()
```

Figure 14: Key Program

```
X = np.linspace(0, 24, 1000)

fig = plt.figure(figsize=(10, 5), dpi=300)
ax = fig.add_subplot(111)

ax.plot(X, fsH(X), label="theory")
ax.hist(startTimeSamples, bins=100, density=True, label="test samples")

# ax.set_title("Probability ")
ax.set_xlabel("hour in a day (h)")
ax.set_ylabel("probability density")

ax.legend()
```

Figure 15: Key Program

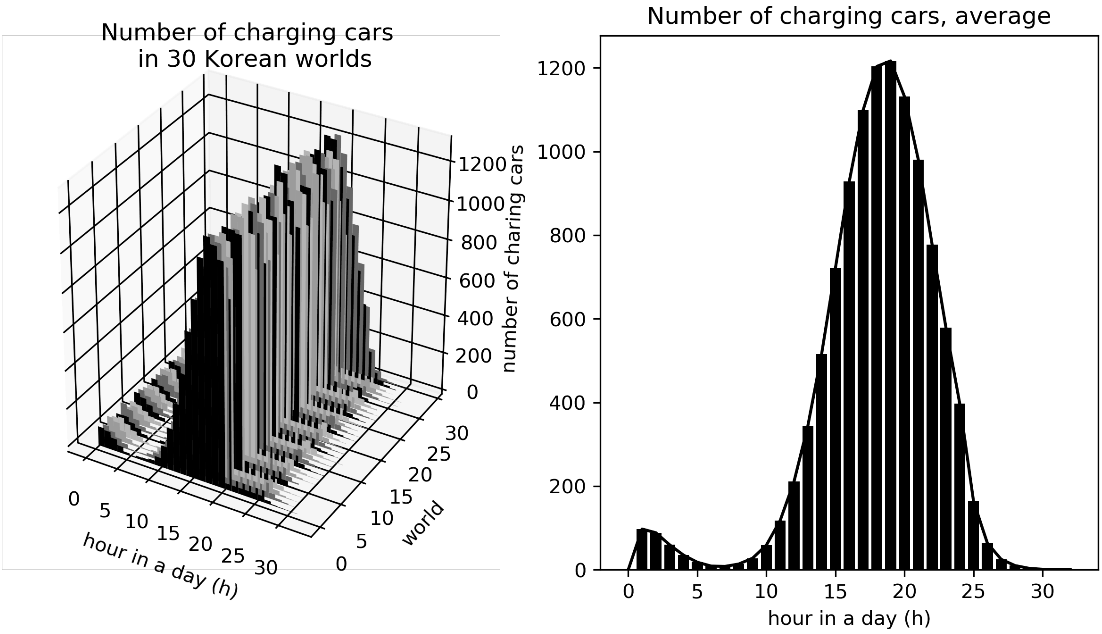

Figure 16: Monte Carlo method for Korea