Probabilités continues

Terminale S

Variable aléatoire à densité sur I

Fonction de densité sur I : fonction f continue et positive sur I telle que

$$\int_I f(t)\,\mathrm{d}t = 1$$

- $P(a \leqslant X \leqslant b) = \displaystyle\int_a^b f(t)\,\mathrm{d}t$
- $P(X = a) = 0$
- $P(a \leqslant X \leqslant b) = P(a < X \leqslant b) = P(a \leqslant X < b) = P(a < X < b)$
- $P(X \leq t) = 1 - P(X \geq t)$
- $E(X) = \displaystyle\int_I t f(t)\,\mathrm{d}t$

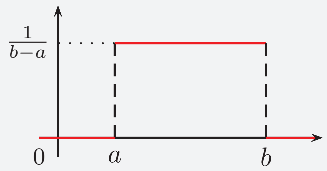
Loi uniforme sur $[\,a\,,b\,]$

Notation : $\mathcal{U}\,[\,a\,,b\,]$

$f(t) = \dfrac{1}{b-a}$

$\mathrm{P}(c \leq X \leq d) = \dfrac{d-c}{b-a}$

$E(X) = \dfrac{a+b}{2}$



Loi exponentielle sur $\mathbb{R}^+$

Notation : $\mathcal{E}(\lambda)$

$f(t) = \lambda e^{-\lambda t}$ avec $\lambda > 0$

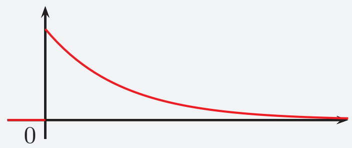
$P(a \leqslant X \leqslant b) = \mathrm{e}^{-\lambda a} - \mathrm{e}^{-\lambda b}$

$P(X \leq t) = 1 - e^{-\lambda t}$

$P(X \geq t) = e^{-\lambda t}$

$P_{X \geq t}(X \geq t+h) = P(X \geq h)$

$E(X) = \dfrac{1}{\lambda}$



Théorème de Moivre-Laplace

- $p \in\,]\,0\,;\,1\,[$ et $n \in \mathbb{N}^*$
- X_n suit la loi binomiale $\mathcal{B}(n;p)$
- $Z_n = \dfrac{X_n - np}{\sqrt{npq}}$

Pour tous réels a et b tels que $a \leq b$:

$$\lim_{n\to+\infty} P\left(a \leq Z_n \leq b\right) = \int_a^b \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\,\mathrm{d}t$$

Loi normale centrée réduite sur $\mathbb{R}$

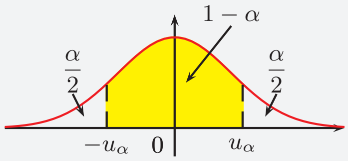
Notation : $\mathcal{N}(0;1)$

$f(t) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}$

$E(X) = 0$ et $V(X) = 1$

$\forall\,\alpha \in\,]\,0\,;\,1\,[, \exists\,!\,u_\alpha \in \mathbb{R}_*^+$ tel que

$P(-u_\alpha \leqslant X \leqslant u_\alpha) = 1 - \alpha$



$P(-1,96 \leqslant X \leqslant 1,96) = 0,95$

$P(-2,58 \leqslant X \leqslant 2,58) = 0,99$

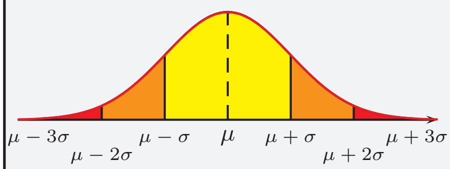
Loi normale sur $\mathbb{R}$

Notation : $\mathcal{N}(\mu;\sigma^2)$

$E(X) = \mu$ et $V(X) = \sigma^2$

X suit la loi $\mathcal{N}(\mu;\sigma^2) \iff$

$Z = \dfrac{X-\mu}{\sigma}$ suit la loi $\mathcal{N}(0\,;\,1)$



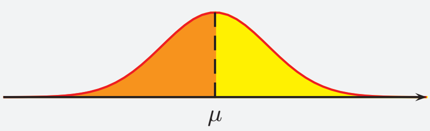
$P(\mu-\sigma \leq X \leq \mu+\sigma) \approx 0,68$

$P(\mu-2\sigma \leq X \leq \mu+2\sigma) \approx 0,96$

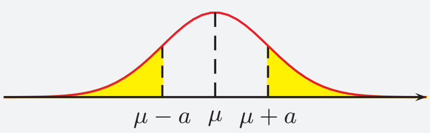
$P(\mu-3\sigma \leq X \leq \mu+3\sigma) \approx 997$

Propriétés des lois normales

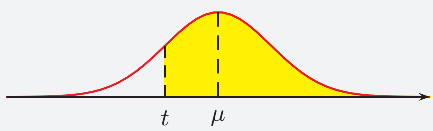
$P(X \leq \mu) = P(X \geq \mu) = 0,5$



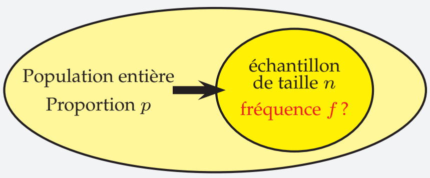
$P(X < \mu - a) = P(X > \mu + a)$



$P(X > t) = 0,5 + P(t < X < \mu)$



Intervalle de fluctuation



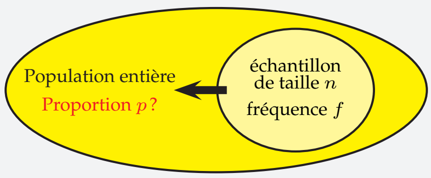
Intervalle de fluctuation asymptotique au seuil de $1-\alpha$

$$I_n = \left[\,p - u_\alpha\sqrt{\frac{p(1-p)}{n}}\,;\,p + u_\alpha\sqrt{\frac{p(1-p)}{n}}\,\right]$$

Conditions : $n \geq 30$; $nf \geq 5$; $n(1-f) \geq 5$

Seuil de 95 % : $u_{0,05} = 1,96$

Intervalle de confiance



Intervalle de confiance de p au niveau de confiance 95 %

$$\left[f - \frac{1}{\sqrt{n}}\,;\,f + \frac{1}{\sqrt{n}}\right]$$

Conditions : $n \geq 30$; $nf \geq 5$; $n(1-f) \geq 5$