According to http://en.wikipedia.org/wiki/Thue-Morse_sequence the n-th Thue-Morse sequence bit is

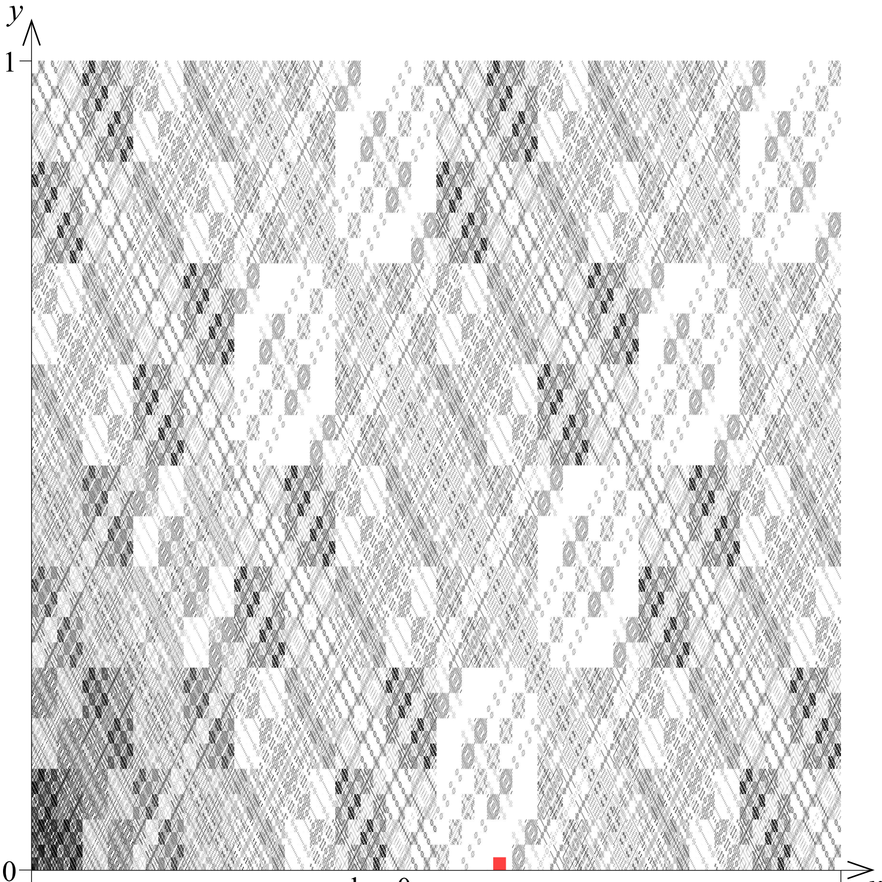
$t(n) = \left(\sum_{k=0}^{\infty} (\text{Floor}[n / 2^k] \bmod 2) \right) \bmod 2 = \text{DigitCount}[n, 2, 1] \bmod 2$

with function Floor[z] defined at http://functions.wolfram.com/IntegerFunctions/ and DigitCount at http://functions.wolfram.com/NumberTheoryFunctions/.

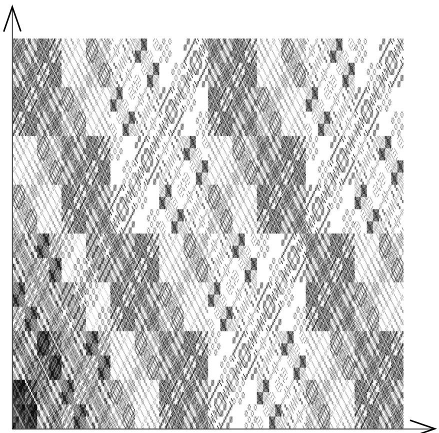
These coordinate systems show all points (X(k, n), Y(k, n)) with $n \in \mathbb{N}$,

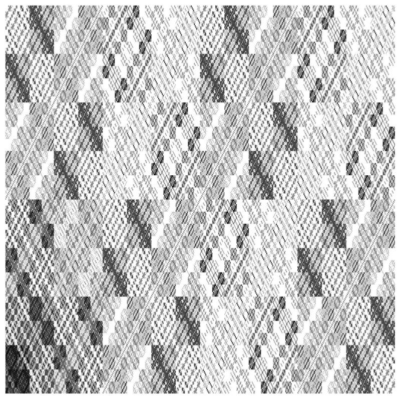
$X(k, n) = \sum_{a=n+1}^{\infty} 2^{n-a}\ t(a^2/2 + a/2 - n + k - 1)$ [when k=0 the arguments of t are: when n=0: 0, 2, 5, 9, 14, …, n=1: 1, 4, 8, 13, 19, …, n=2: 3, 7, 12, 18, 25, …] and

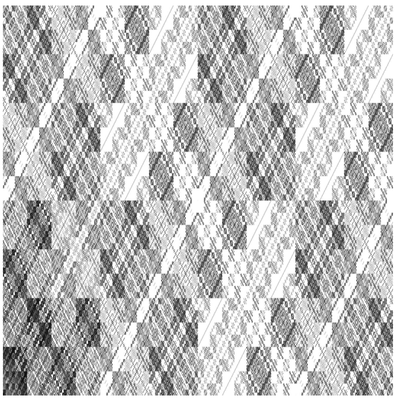
$Y(k, n) = \sum_{a=n}^{\infty} 2^{n-a-1}\ t(a^2/2 + a/2 + n + k)$ [when k=0 the arguments of t are: when n=0: 0, 1, 3, 6, 10, …, n=1: 2, 4, 7, 11, 16, …, n=2: 5, 8, 12, 17, 23, …]:

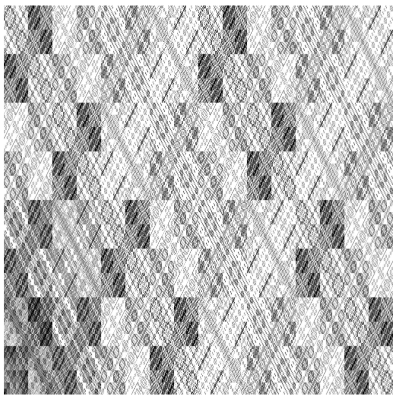


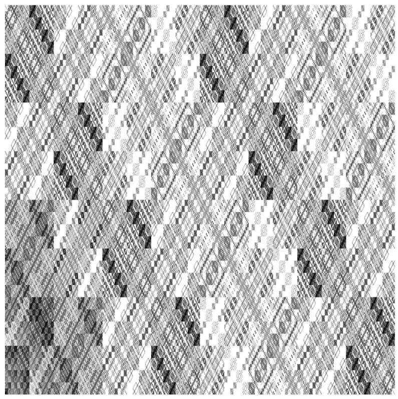
k = 0


k = 1


k = 2


k = 3


k = 4


k = 5

The visible pattern suggests to conjecture that none of these points lies in the red square ($73/128 \le x \le 75/128$ and $y \le 1/64$), so that the conjectured equality is

$$\Sigma_{n=0}^{\infty} \ (1 + \mathrm{Sign}[1/128 - \mathrm{Abs}[X(0, n) - 37/64]])\ (1 + \mathrm{Sign}[1/64 - Y(0, n)]) \ = \ 0$$

with Sign[z] and Abs[z] from http://functions.wolfram.com/ComplexComponents/.
Full assembling of the left side with function writing from there yields:

Sum[(1+Sign[1/128-Abs[Sum[2^(n-a)*Mod[DigitCount[(a^2+a)/2-n-1,2,1],2],{a,n+1,Infinity}]-37/64]])*(1+Sign[1/64-Sum[2^(n-a-1)*Mod[DigitCount[(a^2+a)/2+n,2,1],2],{a,n,Infinity}]]),{n,0,Infinity}]

http://wolframalpha.com *does not tell why* it evades an earnest processing of it at all,
while shorter command lines with these functions like
- Sum[2^-k*Mod[DigitCount[n,2,1],2],{n,1,Infinity}]
- Sum[Mod[Sum[Mod[Floor[n/2^k],2],{k,0,Infinity}],2]/2^k,{n,1,Infinity}]

at least give vague attempts to process the entire actual commands, in these 2 cases ascertaining the http://en.wikipedia.org/wiki/Thue-Morse_constant × 2.