1. [An investment requires an initial payment of 10,000 and annual payments of 1,000 at the end of each of the first 10 years.] [Starting at the end of the eleventh year, the investment returns five equal annual payments of X.] this is return.
Determine X to yield an annual effective rate of 10% over the 15-year period

This is a Project

- [x] 11,050
- [ ] 10,900
- [ ] 10,750
- [ ] 11,200

Outflow: 10000 1000 1000 . . . 1000

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 year

Inflow: X X X X X

Method 1:

$$10000(1+i)^{15} + 1000 \cdot s_{\overline{10|}} \cdot (1+i)^5 = X \cdot s_{\overline{5|}}$$

Method 2:

$$10000 + 1000 \cdot a_{\overline{10|}} = X \cdot a_{\overline{5|}} \cdot v^{10}.$$

$$X = 11046.$$

There is an effective rate of interest that exists such that the PV of inflows will yield the PV of outflows. This interest rate is called a yield rate or an internal rate of return (IRR) as it indicates the rate of return that the investor can expect to earn on their investment.

In other words, there exists a certain interest rate where the net present value is equal to 0.

$$NPV = \sum_{t=0}^{n} \frac{\text{net } CF_t}{(1+IRR)^t} = 0.$$

Under the cash flow approach, $PV|_{t=0} = \sum_{t=0}^{n} CF_t^{in} \cdot v_i^t - \sum_{t=0}^{n} CF_t^{out} \cdot v_i^t = 0.$

$$\Leftrightarrow \sum_{t=0}^{n} CF_t^{in} \cdot v_i^t = \sum_{t=0}^{n} CF_t^{out} \cdot v_i^t.$$

2. Reinvested interest (standard increasing annuity):
Jasmine invests $500 at the beginning of each year for 20 years at an annual effective interest rate of 5%. The interest credited at the end of each year is reinvested at an annual effective rate of 6%.
Determine the accumulated value at the end of 20 years.

- [ ] 17,653.36
- [x] 17,913.64
- [ ] 17,740.32
- [ ] 17,827.28

0 1 2 3 . . . 19 20

payments 500 500 500 500 . . . 500 0

Interest 500i 1000i 1500i . . . 9500i 10000i

$5\% = i$

$6\% = j$

$$AV|_{t=20} = 500 \times 20 + 500i \cdot (Is)_{\overline{20|}j} = 10000 + 25 \cdot \left(\frac{\ddot{s}_{\overline{20|}j} - n}{j}\right) = 17913.$$

Reinvestment Rates.

- the yield rate that is calculated assumes that the positive returns (or cash outflows) will be reinvested at the same yield rate.
- the actual rate of return can be higher or lower than the calculated yield rate depending on the reinvestment rates.

Case 1: An investment of 1 is invested for n years and earns an annual effective rate of i.

The interest payments are reinvested in an account that credits an annual effective rate of j.

$$FV|_{t=n} = (i \cdot 1)\, s_{\overline{n}|j} + 1.$$

If $i=j$, $\quad FV|_{t=n} = (1+i)^n$.

Case 2: For Qestion Q: $\quad FV|_{t=n} = i \cdot (Is)_{\overline{n-1}|j} + n \times 1$.

If $i=j$, $\quad FV|_{t=n} = s_{\overline{n}|i}$.

Note: Notice that the payment time is start or ending of one year.

3. On January 1, 2007, you initiate an investment account, with the value and deposit/withdrawal activity during the year is as per the table given. (The "account values" represent the amount in the account immediately before the deposit or withdrawal activity on that date.) The time-weighted and dollar-weighted rates of return on the account during 2007 are equal. Find the non-zero value of X-both its magnitude, and whether it's a deposit or a withdrawal. (For the dollar-weighted rate of return, assume simple interest from the date of each deposit)

Date (2007)	Account Value	Activity
January 1	—	10,000 deposit
June 30	12,000	X
December 31	10,000	—

- [x] 4000 withdrawal
- [ ] Cannot be determined from the given information
- [ ] 2000 withdrawal
- [ ] 4000 deposit
- [ ] 2000 deposit

For the dollar-weighted Rate of Return.

Using Units of 1000: $\quad$ Interest earned: $10 - X - 10 - 0 = -X$.

$$i = \frac{-X}{0 + 10 + X(0.5)}$$

For the time-weighted Rate of Return.

$$1+i = \frac{12}{10} \cdot \frac{10}{12+X}.$$

$$\frac{12}{12+X} - 1 = \frac{-X}{12+X} = \frac{-X}{0+10+X(0.5)}. \quad X = 4. \text{ It represents withdrawl.}$$

- Dollar-Weighted Interest Rates (MWRR).

It is determined by solving the equation of value for i, based on the assumption that Simple Interest applies from the date of each deposit or withdrawal to the end of the year.

$$i = \frac{\text{Interest}}{\text{Weight-Interest}}$$

Interest — Final Value − Deposit / withdrawal Value − Initial Value

Weight-Interest — Deposit / withdrawal Future Value. (Use Assumption of Simple Interest).

Or you can build an equation to express the investment with the Return / or Final Value.

$$F_0 (1+i)^T + \sum_{s=1}^{n} C_s (1+i)^{T-t_s} = F_T.$$

Note: $(1+i)^n = (1+ni)$

F_0: The Value of the fund at time 0.

C_s: The deposits or withdrawals of the fund

F_T: The Value of the fund at time T.

- Time-Weighted Interest Rates

It is determined by assuming that the single amount is invested at the begining of the year (with no further deposits or withdrawal).

$$(1+i)^T = \frac{F_1}{F_0} \cdot \frac{F_2}{F_1+C_1} \cdot \frac{F_3}{F_2+C_2} \cdot \ldots \cdot \frac{F_T}{F_n+C_n}$$

4. Yield allowing for reinvestment:
An investment opportunity has the following characteristics: payments of \$500 will be made to you and invested into an account at the end of each year, for the next 20 years. These payments will earn an effective annual interest rate of 8%, and the interest from this account (paid at the end of each year) can be reinvested at an effective annual rate of 5%. Find the purchase price of this investment opportunity assuming an effective annual yield of 7% over the 20-year life of the investment.

- [ ] 4,885
- [ ] 5,085
- [ ] 5,285
- [ ] 4,985
- [ ] 5,185

year	0	1	2	3	4	…	19	20
payment:		500	500	500	500	…	500	500
Interest:			500i	1000i	1500i	…	9000i	9500i

$i = 8\%$.
$j = 5\%$

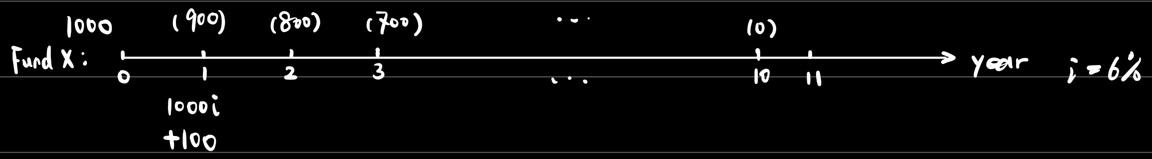
Let the purchase price is X.

$$X \cdot (1.07)^{20} = 10000 + 500i\,(Is)_{\overline{19|}j}$$

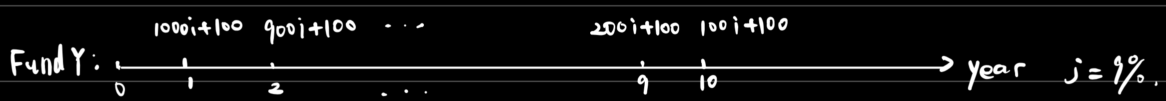
$$X = 5285.$$

5. Reinvestment with withdrawals:
1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted. The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%. Determine the accumulated value of Fund Y at the end of year 10

- [x] 2085
- [ ] 2431
- [ ] 1819
- [ ] 1519
- [ ] 2273



	Time 0	Time 1	Time 2	...	Time 10	Time 11
Account befor	0	$1000(1+i)$	$900(1+i)$	...	$100(1+i)$	0
Activity	+1000	$-100-1000i$	$-100-900i$	...	$-100-100i$	0
Balance	1000	900	800	...	0	0



$$FV\Big|^{Y}_{t=10} = 100\cdot s_{\overline{10}|j} + 100i(Ds)_{\overline{10}|j} = 2084.67.$$

6. Bill purchases an annuity at a price of 10,000. The annuity makes payments of 500 at the beginning of every 6 months for 20 years. The payments are reinvested in a fund which earns interest at an annual effective rate i. Interest payments are received every 6 months and reinvested at a nominal rate of 6% convertible semiannually. Bill realizes an overall effective annual yield of 7% on his original investment over the 20-year period. Calculate i.

- [ ] 6.2%
- [ ] 6.5%
- [ ] 6.35%
- [x] 6.05%
- [ ] 5.9%

Interests $500j$ $1000j$ $1500j$ $2000j$ $\cdots$ $19500j$ $20000j$

where j is the effective interest rate for semiannual in principle payment.

$$10000(1.07)^{20} = 20000 + 500j(Is)_{\overline{20}|m}.$$

m is the effective interest rate for semiannual in interest payment. $m = \frac{6\%}{2} = 3\%$.

$$j = 2.9785\%.$$

$$(1+j)^2 = 1+i. \qquad i = 6.0458\%.$$

7. A project requires an initial capital outlay of 30,000 and 1,000 at the end of first year will
return the following amounts (paid at the ends of year 2-6 years):
14,000, 12,000, 10,000, 8,000, 6,000.
Solve for the Net Present Value using the cost of borrowing of 6%

- [ ] RM 10,432
- [ ] RM9,170.52
- [ ] RM43,104.00
- [x] RM9,720.75

30000	1000						
0	1	2	3	4	5	6	year
		14000	12000	10000	8000	6000.	

$$PV(\text{outflows}) = 30000 + 1000\cdot(1.06)^{-1}.$$

$$PV(\text{inflows}) = \left[14000\cdot a_{\overline{5}|} - 2000\left(\frac{\ddot{a}_{\overline{5}|} - 5v^5}{i}\right)\right]\cdot v$$

$$NPV = PV(\text{inflows}) - PV(\text{outflows}) = 9720.75.$$

8. A project requires an initial capital outlay of 30,000 and 1,000 at the end of first year will
return the following amounts (paid at the ends of year 2-6 years):
14,000, 12,000, 10,000, 8,000, 6,000.
Solve for the Modified internal rate of return assuming a reinvestment rate of 3% and cost of borrowing of 6%

- [ ] 9.648%
- [x] 9.629%
- [ ] 10.921%
- [ ] 10.938%

30000	1000						
0	1	2	3	4	5	6	year
		14000	12000	10000	8000	6000.	

$$PV(\text{outflows}) = 30000 + 1000(1.06)^{-1} = 30943.4$$

$$PV(\text{outflows}) \cdot [1 + MIRR] = AV(\text{inflows})$$

$$30943.4(1 + MIRR) = (1.03)^5 \left[14000 \cdot a_{\overline{5}|} - 2000 \left(\frac{a_{\overline{5}|} - 5v^5}{2} \right) \right]$$

$$MIRR = 9.629\%.$$

Internal Rate of Return. It makes the NPV is 0.

$$NPV = \sum_{t=0}^{n} \frac{\text{net } CF_t}{(1+IRR)^t} = 0.$$

Modified - Internal Rate of Return.

AV_T of PV of outflows, with MIRR = AV_T of inflows, with j.