1. Amortization of level loan (Pt relationship):
A loan is repaid with level monthly payments based on an annual effective interest rate of 5%. The 36th monthly payment consists of $18.19 of interest and $170.01 of principal. Calculate the amount of principal paid in the seventh monthly payment.

- [ ] 150.00
- [ ] 151.10
- [ ] 153.30
- [ ] 152.20

Solution: $P_{t+s} = P_t \cdot (1+i)^s$. The relationship of Principal between different times.

So $P_{36} = P_7 \cdot (1+i)^{29}$. i is the effective interest rates of month.

$(1+i)^{12} = 1.05$.

$\Rightarrow P_7 = 151.10$.

Duration: t	Payment: R	Interest Paid: $I_t = iB_{t-1}$	Principal Repaid: $P_t = R - I_t$	Outstanding Principal: $B_t = B_{t-1} - P_t$
0				$a_{\overline{n}\mid}$
1	1	$ia_{\overline{n}\mid} = 1 - v^n$	v^n	$a_{\overline{n}\mid} - v^n = a_{\overline{n-1}\mid}$
2	1	$ia_{\overline{n-1}\mid} = 1 - v^{n-1}$	v^{n-1}	$a_{\overline{n-1}\mid} - v^{n-1} = a_{\overline{n-2}\mid}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	1	$ia_{\overline{n-t+1}\mid} = 1 - v^{n-t+1}$	v^{n-t+1}	$a_{\overline{n-t+1}\mid} - v^{n-t+1} = a_{\overline{n-t}\mid}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	1	$ia_{\overline{1}\mid} = 1 - v$	v	$a_{\overline{1}\mid} - v = 0$
Total	n	$n - a_{\overline{n}\mid}$	$a_{\overline{n}\mid}$	

This shows that if the payments are level, the amount of principal repaid are in geometric progression. with common ratio $(1+i)$.

You should know:

Principal repayments P_t:

(a). Portion of a payment which is devoted to reducing (paying off) the outstanding loan balance.

(b). $P_t = R \cdot v^{n-t+1} = R - I_t$

(c). Principal repayments increase over time, according to a geometric progression.

(d). Successive values of P_t differ by a factor of $(1+i)$. i.e. $P_{t+s} = P_t \cdot (1+i)^s$.

2. Refinancing:

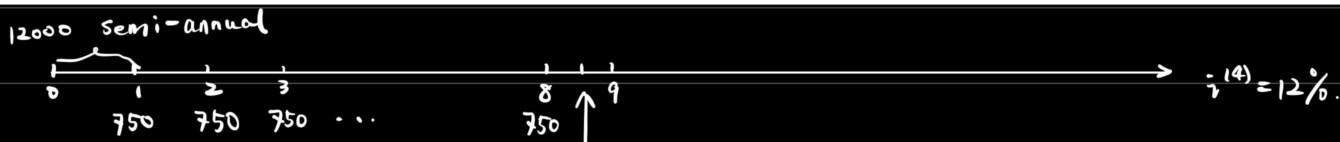
A small business takes out a loan of 12,000 at a nominal rate of 12%, compounded quarterly, to help finance its start-up costs. Payments of 750 are made at the end of every 6 months for as long as is necessary to pay back the loan. Three months before the 9th payment is due, the company refinances the loan at a nominal rate of 9%, compounded monthly. Under the refinanced loan, payments of R are to be made monthly, with the first monthly payment to be made at the same time that the 9th payment under the old loan was to be made. A total of 30 monthly payments will completely pay off the loan.

$i^{(4)} = 12\%$.

$j^{(12)} = 9\%$.

· Determine R.

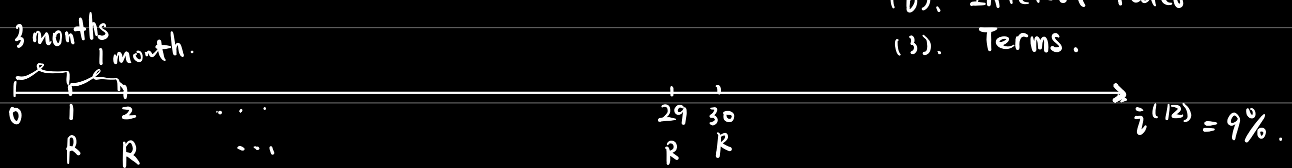
- [ ] 452
- [ ] 461
- [ ] 448
- [ ] 456



You should know: (a). Outstanding Balance.

(b). Interest rates

(3). Terms.



(a). Outstanding Balances: $B_0^{New} = B_{t=8.5}^{old} = 12000(1+i)^{8.5} - 750 \cdot s_{\overline{8}|i} \cdot (1+i)^{\frac{1}{2}} = 12163.03$.

The effective interest rate of half-year.

$(1+i)^2 = (1+\frac{i^{(4)}}{4})^4$. $\Rightarrow i = 0.0609$.

(b). Interest rates for new loan: $j = \frac{i^{(12)}}{12} = 0.0075$.

(c). Term: 30.

$\Rightarrow$ $B_0^{New} = R \cdot a_{\overline{30}|j} \underbrace{(1+j)^{-2}}_{\text{Reason: ?}}$ $\Rightarrow R = 461.13$. #.

Summary:

1. A loan could be refinanced at any time during its term due to several reasons including a change in interest rate, in payment amount, and in loan term.
2. Loan refinance questions involve computing the revised term of the loan, interest rate, payment amount, outstanding balance, amount of interest and principal repayment.
3. A general approach to solving loan refinance questions is to determine the outstanding loan balance at the time of refinancing under the original terms. Then, set this equal to the PV of the revised payments under the new terms of the loan.

3. Drop payment:
Christine takes out a 30-year, \$100,000 mortgage loan with monthly payments of \$812.54 beginning at the end of the first month, at an effective annual interest rate of 9.5%. Immediately after five years of monthly payments, she renegotiates the loan in order to pay it off more quickly. She will now make monthly payments of \$1,500, and the interest rate has dropped to an 8.0% effective annual rate. Christine will also make one final, partial payment (made one month after the last \$1,500 payment). Determine the amount of the final, partial payment.

☐ At least \$300, but less than \$600	☐ (A) Less than \$300
☐ At least \$600, but less than \$900	☐ At least \$900, but less than \$1,200

- The final payment is larger than the level payments. — Balloon payment
- The final payment is smaller than the level payments. — Drop payment.

For this question,

The first thing is outstanding balance after the 60th payment.

$$B_0^{New} = B_{60}^{old} = 812.54\, a_{\overline{300}|j} = 95962.05.$$

Then, the number of payments of 1500. $B_0^{New} = R \cdot a_{\overline{n}|k}$.

$R = 1500$. $k = (1+8\%)^{\frac{1}{12}} - 1 = 0.0064$. $\Rightarrow 82 < n < 83$.

So the drop payment will occur 83 months after the 60th month.

We should solve the equation $95962.05 = 1500\, a_{\overline{82}|k} + R \cdot v^{83}$. $R = 1047$. #

4. Convenient amortization formula for level payments:
A mortgage of $200,000 at 5.5% effective is being repaid by level annual payments at the end of each year for 30 years. Determine the interest paid in the 9th payment.

- [ ] 9620.46
- [ ] 9523.69
- [ ] 9580.72
- [ ] 9645.26

· Interest payments I_t:

(a). Portion of a payment which is devoted to covering the interest charged for the use of the borrow money.

(b). Each period, the interest incurred is equal to the interest rate multiplied by the loan balance at the begining of the period. ($I_t = i \cdot B_{t-1}$).

(c). Interest is paid as it is incurred.

(d). $I_t = R \cdot (1 - v^{n-t+1}) = iB_{t-1}$.

Method 1: Determine the outstanding Balance at the end of the 8th year using the prospective method.

$$B_8 = \frac{B_0}{a_{\overline{30}|}} \cdot a_{\overline{22}|} = 173157.98.$$

Multiply this B_8 by 0.055 and get $I_9 = iB_8 = 9523.69$. #.

Method 2: Retrospective method to get the outstand Balance at the end of the 8th year.

$$200000(1+i)^8 - \frac{B_0}{a_{\overline{30}|}} \cdot s_{\overline{8}|} = B_8 = 173157.98.$$

Multiply this B_8 by 0.055 and get $I_9 = iB_8 = 9523.69$. #.

Method 3: Use the formula for the interest paid in the t^{th} payment.

since $R = \frac{200000}{a_{\overline{30}|}} = 13761.08$.

$$I_t = R \cdot (1 - v^{n-t+1}) = 13761.08 (1 - (1.055)^{-22}) = 9523.69.$$

5. level principal payment:

Sol borrows 1200. He promises to make payments at the end of each month for 12 months, with each payment equal to the sum of (i) and (ii):

(i) is equal to 100 toward principal.

(ii) is equal to 1½% of the outstanding balance at the end of the previous month.

Alice decides to buy this series of loan payments at a price X that will give her a nominal yield rate of 24% per annum compounded monthly.

Determine X.

- [ ] 1154
- [ ] 1164
- [ ] 1184
- [ ] 1174

Sol:

End of Month	Outstanding Balance	Interest Paid	Principal Paid	Total Payment
0	1200	-	-	-
1	900	18	100	118
2	800	16.5	100	116.5
3	700	15	100	115
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
12	0	1.5	100	101.5

Alice:

$$X = 100 \cdot a_{\overline{12}|2\%} + 1.5\,(Da)_{\overline{12}|2\%}.$$

$$= 1164.379.$$

1. As an approach to paying off a loan, the equal principal repayments method involves:
   (a) Level amounts of principal
   (b) Interest on the outstanding balance
2. Over time, such an approach involves decreasing total payments
   (a) Interest payments decrease over time, as the loan balance falls due to the principal repayments

6. A loan is being repaid by a series of annual non-level payments at 5% effective. The 9-th payment is \$3500 and the principal paid from this payment is \$3000. The 10th payment is \$2500. How much principal is contained in this 10th payment?

- [ ] 3150
- [ ] 1950
- [ ] 2150
- [ ] 4150

Method 1:

$$P_{10} = (1.05)\cdot P_9 + (R_{10} - R_9) = (1.05)\cdot(3000) + (2500 - 3500) = 2150.$$

If the payments remained at a level \$3500, we know that the principal repayments would form a geometric series with comon ratio $(1+i)$. Thus, the principal in the next payment would be $(1.05)\cdot(3000)$.

But the next payment is actually 2500, or 1000 less than the 9th payment of 3500.

This entire additional ammount must go toward principal. (The interest contained in any payments is completely independent of the amount of that payment; it depends only on the previous outstanding loan balance.)

Thus, the principal repaid in the \$2500 payment is $(1.05)(3000) + (2500 - 3500)$.

Method 2:

$I_9 = B_8 \cdot (0.05) = 3500 - 3000 = 500. \Rightarrow B_8 = 10000.$

$B_9 = B_8 - R_9 = 7000.$

$I_{10} = B_9 \cdot (0.05) = 350.$

$P_{10} = R_{10} - I_{10} = 2150.$

(This method is Recurision formula). I_t, B_{t-1}, B_t. P_t. P_t.

$I_t = i \cdot B_{t-1}$. $P_t = R_t - I_t$. $B_t = B_{t-1} - P_t$.

8. Ming borrows X for 10 years at an annual effective interest rate of 8%. If he pays the principal and accumulated interest in one lump sum at the end of 10 years, he would pay 468.05 more in interest than ifhe repaid the loan with 10 level payments at the end of each year. Calculate X.

- [ ] 750
- [ ] 725
- [ ] 675
- [ ] 700
- [ ] 775

Interest:

(1). Lump Sum : $X(1.08)^{10} - X$.

(2). Amortization: $10 \cdot \frac{X}{a_{\overline{10}|}} - X$.

$$\Rightarrow \quad X(1.08)^{10} - X = 10 \cdot \frac{X}{a_{\overline{10}|}} - X + 468.05$$

$$X = 700.$$

1. ***Amortization***: paying off a loan over time with a single payment in each payment period
   (a) Individual payments have both ***principal repayment*** and ***interest*** components
   (b) Indebtedness (outstanding balance on loan) decreases over time
   (c) Portion of each payment devoted to principal repayment increases over time
   (d) Portion of each payment devoted to interest decreases over time
2. ***Outstanding loan balance***:
   (a) Balance at any point in time is equal to the total present value of all future payments on the loan
   (b) ***Prospective formula***: $B_t^p = R \cdot a_{\overline{n-t}|}$
   (c) ***Retrospective formula***: $B_t^r = R\left[a_{\overline{n}|}(1+i)^t - s_{\overline{t}|}\right]$
   (d) Original balance = amount borrowed
   (e) For level annual payments R: $B_0 = R \cdot a_{\overline{n}|}$
   (f) $R_t = P_t + I_t$
3. Principal repayments P_t:
   (a) Portion of a payment which is devoted to reducing (paying off) the outstanding loan balance
   (b) $P_t = Rv^{n-t+1} = R - I_t$
   (c) Principal repayments increase over time, according to a geometric progression
   (d) Successive values of P_t differ by a factor of $(1+i)$, i.e., $P_{t+s} = P_t \cdot (1+i)^s$
4. Interest payments I_t:
   (a) Portion of a payment which is devoted to covering the interest charged for the use of the borrowed money
   (b) Each period, the interest incurred is equal to the interest rate multiplied by the loan balance at the beginning of that period
   (c) Interest is paid as it is incurred
   (d) $I_t = R\left[1 - v^{n-t+1}\right] = iB_{t-1}$