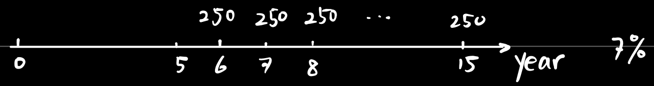
1.



$$PV = 250 \cdot {}_{5|}a_{\overline{10|}} = 250 \cdot v^5 \cdot \frac{1-v^{10}}{i} = 1251.93.$$

since $v = \frac{1}{1+i} = (1.07)^{-1}$

$v = \frac{1}{1+i} = 1-d.$

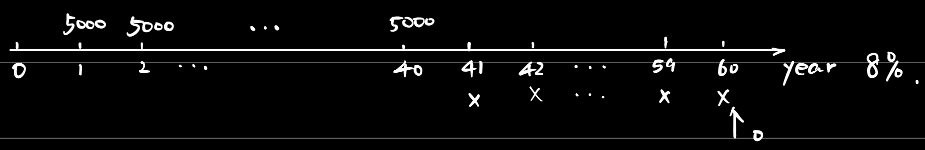
$\Rightarrow d = \frac{i}{1+i}. \quad \Rightarrow i = \frac{d}{1-d}.$

(B).

Method 2. $PV = 250 \cdot {}_{6|}\ddot{a}_{\overline{10|}} = 250 \cdot v^6 \cdot \frac{1-v^{10}}{d}.$

since $v = (1.07)^{-1}$. $d = 1 - v = \frac{i}{1+i}.$

2.



$$5000 \cdot s_{\overline{40|}} \cdot (1+i)^{20} - X \cdot s_{\overline{20|}} = 0.$$

$$1295282.59\,(1+i)^{20} = X \cdot (45.7620)$$

$$X = 131927.29.$$

(B).

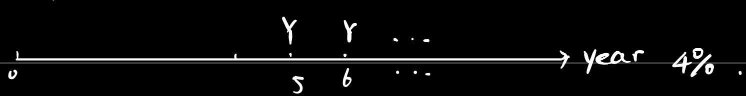
Method 2. $5000 \cdot s_{\overline{40|}} = X \cdot a_{\overline{20|}}$

Method 3. $5000 \cdot a_{\overline{40|}} = X \cdot a_{\overline{20|}} \cdot v^{40}.$

Note: You guys should notice that how to explain the reason of equation.

3.

Perpetuity A.



Perpetuity B.

$$5\, {}_{3|}\ddot{a}_{\overline{\infty|}} = Y \cdot {}_{5|}\ddot{a}_{\overline{\infty|}}.$$

$$5 \cdot v^3 \cdot \frac{1}{d} = Y \cdot v^5 \cdot \frac{1}{d}$$

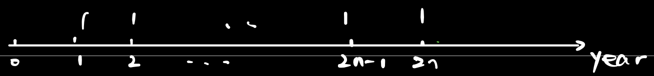
$$5 = Y \cdot v^2$$

$$Y = \frac{5}{v^2} = 5.41.$$

(A).

Method 2. $5 \cdot {}_{2|}a_{\overline{\infty|}} = Y \cdot {}_{4|}a_{\overline{\infty|}}$

Method 3. $5 \cdot {}_{2|}a_{\overline{\infty|}} = Y \cdot {}_{5|}\ddot{a}_{\overline{\infty|}}$

4. 

I: $PV = a_{\overline{2n}|}$.

II: $PV = 3 \cdot {}_{2n|}a_{\overline{\infty}|} = 3 \cdot v^{2n} \cdot \frac{1}{i}$

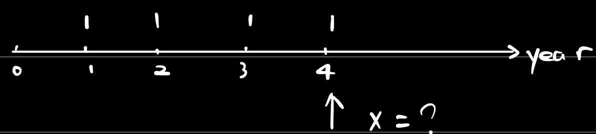
$PV|_{t=0 \text{ for I}} = PV|_{t=0 \text{ for II}}$. $\Rightarrow a_{\overline{2n}|} = \frac{3}{i} v^{2n}$.

$\Rightarrow \frac{1-v^{2n}}{i} = \frac{3v^{2n}}{i} \Rightarrow 4v^{2n} = 1. \quad v^{2n} = \frac{1}{4}$

$\Rightarrow (1+i)^{-2n} = \frac{1}{4} \Rightarrow (1+i)^{2n} = 4 \Rightarrow (1+i)^n = 2.$ (A).

Method 2. $a_{\overline{2n}|} + {}_{2n|}a_{\overline{\infty}|} = a_{\overline{\infty}|}$. (This because the relationship of annuity between different years).

Or the relationship between deferred annuity and term annuity).

5. 

Since the force of interest is related to the time t, the effective rate of interest are different in different time.

Then, we should find FV using the force of interest.

$a(t_1, t_2) = \exp(\int_{t_1}^{t_2} \delta_r \, dr)$.

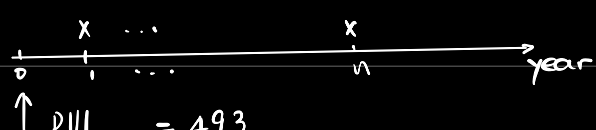
$\Rightarrow a(t, 4) = \exp(\int_t^4 \frac{1}{5-r} dr) = \exp(-\ln(5-r)|_t^4) = \exp(\ln(5-t) - \ln 1) = 5-t.$

Lastly, $X = FV|_{t=4} = s_{\overline{4}|} = 1 \cdot a(1,4) + 1 \cdot a(2,4) + 1 \cdot a(3,4) + 1 \cdot a(4,4)$

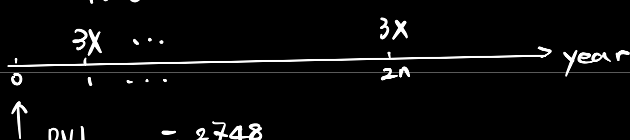
$= 1 \cdot 4 + 1 \cdot 3 + 1 \cdot 2 + 1 \cdot 1 = 10.$

Note: This Question, you should know the relationship between force of interest and accumulated value.

In the same time, you should know how to find the $PV|_{t=0}$.

6. Dottie 

$PV|_{t=0} = 493$.

Sam 

$PV|_{t=0} = 2748$

$$PV\Big|_{t=0}^{\text{Dottie}} = X \cdot \frac{1-v^n}{i} = 493. \quad (1)$$

$$PV\Big|_{t=0}^{\text{Sam}} = 3X \cdot \frac{1-v^{2n}}{i} = 2748. \quad (2).$$

(2) ÷ (1). $$\frac{2748}{493} = 3X \cdot \frac{1-v^{2n}}{i} \Big/ X \cdot \frac{1-v^n}{i}.$$

then. $$\frac{2748}{493} = \frac{3-3v^{2n}}{1-v^n}.$$

Note: Don't calculate $\frac{2748}{493}$, and let it equal to X.

Then, $X(1-v^n) = 3-3v^{2n}$.

$\Rightarrow X - Xv^n = 3-3v^{2n}$.

$3v^{2n} - Xv^n + X - 3 = 0$.

Here calculate X by calculator. $X = 5.57$.

$v^n = \frac{X \pm \sqrt{\Delta}}{6}$, $\Delta = (-X)^2 - 4 \cdot 3 \cdot (X-3) = 0.1849$.

$v^n = 0.8567$. or $v^n = 1$. Consider why $v^n \neq 1$?

Method 2: (The relationship between $a_{\overline{n}|}$ and $a_{\overline{2n}|}$)

$\frac{a_{\overline{2n}|}}{a_{\overline{n}|}} = 1+v^n$. (algebraically). $a_{\overline{2n}|} = a_{\overline{n}|} + {}_{n|}a_{\overline{n}|} = a_{\overline{n}|} + v^n \cdot a_{\overline{n}|} = a_{\overline{n}|} \cdot (1+v^n)$.

The fact that the PV of a 2n-year annuity is the sum of the PV of an n-year annuity and the PV of an n-year deferred n-year annuity.

Similarly.

$a_{\overline{3n}|} = (1+v^n+v^{2n})\,a_{\overline{n}|}$.

$PV\Big|_{t=0}^{\text{Dottie}} = X \cdot a_{\overline{n}|}$, $PV\Big|_{t=0}^{\text{Sam}} = 3X \cdot a_{\overline{2n}|}$.

$$\frac{2748}{493} = \frac{3X \cdot a_{\overline{2n}|}}{X \cdot a_{\overline{n}|}} \Rightarrow \frac{2748}{3 \cdot (493)} = \frac{a_{\overline{2n}|}}{a_{\overline{n}|}} = 1+v^n \Rightarrow v^n = 0.8567.$$

7.

Time	Payment
1 to n	98
n+1 to 3n	196

Short Way: $AV = 98 \cdot s_{\overline{3n}|} + 98 \cdot s_{\overline{2n}|} = 8000$.

$$\frac{(1+i)^{3n}-1}{i} + \frac{(1+i)^{2n}-1}{i} = \frac{8000}{98}$$

Since $(1+i)^n = 2$. then $\frac{2^3-1+2^2-1}{i} = \frac{8000}{98}$

$i = 12.25\%$.

Long way: $AV = 98(1+i)^{2n} \cdot s_{\overline{n}|} + 198 \cdot s_{\overline{2n}|}$

8. Short way: $PV|_{t=0} = 12\cdot a_{\overline{25}|} - 2\cdot a_{\overline{22}|} - 10\cdot a_{\overline{15}|} + 8\cdot a_{\overline{10}|}$

$AV|_{t=25} = 8\cdot s_{\overline{25}|} - 8\cdot s_{\overline{15}|} + 10\cdot s_{\overline{10}|} + 2\cdot s_{\overline{3}|}$.

9. I: $\frac{a_{\overline{10}|}(1+i\cdot s_{\overline{10}|})}{1+\ddot{s}_{\overline{9}|}} = \frac{a_{\overline{10}|}\left(1+i\cdot\frac{(1+i)^{10}-1}{i}\right)}{s_{\overline{10}|}} = \frac{a_{\overline{10}|}\cdot(1+(1+i)^{10}-1)}{s_{\overline{10}|}} = \frac{s_{\overline{10}|}}{s_{\overline{10}|}} = 1$.

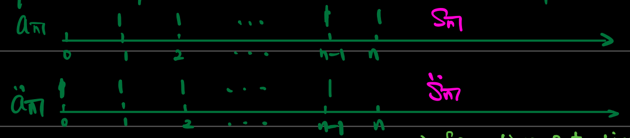
This, you should know $\ddot{s}_{\overline{k}|}+1 = s_{\overline{k+1}|}$. and $a_{\overline{10}|}(1+i)^{10} = s_{\overline{10}|}$.

II: $v^{10}\cdot\ddot{s}_{\overline{10}|} - a_{\overline{9}|} = \ddot{a}_{\overline{10}|} - a_{\overline{9}|} = a_{\overline{9}|}+1-a_{\overline{9}|} = 1$.

This, you should know $\ddot{a}_{\overline{k}|} = a_{\overline{k-1}|}+1$.

III: $(1+i)^{10}\cdot a_{\overline{10}|} - \ddot{s}_{\overline{9}|} = s_{\overline{10}|} - \ddot{s}_{\overline{9}|} = s_{\overline{10}|} - (s_{\overline{10}|}-1) = 1$.

This question you should know the relationship between annuity immediate and annuity due.



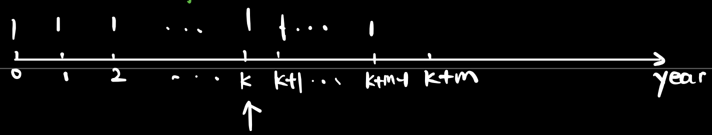
$s_{\overline{n}|} = \ddot{s}_{\overline{n-1}|} + \underbrace{1}_{\text{at time n}}$.

or $\ddot{s}_{\overline{n}|} = \underbrace{s_{\overline{n+1}|}}_{\text{from time 1 to n}} \underbrace{-1}_{\text{at time n}}$.

from time 1 to n-1

$\ddot{a}_{\overline{n}|} = \underbrace{1}_{\text{at time 0}} + \underbrace{a_{\overline{n-1}|}}_{\text{from time 1 to n-1}}$ or $a_{\overline{n}|} = \underbrace{\ddot{a}_{\overline{n+1}|}}_{\text{from time 0 to time n}} \underbrace{-1}_{\text{at time 0}}$.

10.



- Before time k, there are k times payments, at time k, there is 1 times payment, and after time k. there are m-1 times payments.

(1). $1+\ddot{s}_{\overline{k}|} + a_{\overline{m-1}|}$. (✓).

- Since there are k+m times payments at the beginning of each year,

In the time 0, $PV|_{t=0} = \ddot{a}_{\overline{k+m}|} = a_{\overline{k+m}|}(1+i)$.

and In the time k. the Value is $PV|_{t=0}\cdot(1+i)^k = a_{\overline{k+m}|}(1+i)\cdot(1+i)^k = a_{\overline{k+m}|}(1+i)^{k+1}$.

(2). $a_{\overline{k+m}|}(1+i)^{k+1}$ (✓).

- In the time k+m, $AV|_{t=k+m} = \ddot{s}_{\overline{k+m}|}$. and in the time k, the value is $AV|_{t=k+m}\cdot v^m$

$= \ddot{s}_{\overline{k+m}|}\cdot v^m$.

If you use $s_{\overline{k+m}|}$. it represent the AV from 1 to k+m, So there lack the first payment in the time 0, and more a payment in the k+m.
then it is,

$\left(s_{\overline{k+m}|} + 1\cdot(1+i)^{k+m} - 1\right)\cdot v^m$.

(3). $s_{\overline{k+m}|}\cdot v^m$ (✗).

- In the time k, if we use notation of $\ddot{s}_{\overline{k}|}$, it presents the AV from time 0 to time k-1.

and $\ddot{a}_{\overline{m}|}$ represents the PV from time k to k+m-1.

(4). $\ddot{s}_{\overline{k}|} + \ddot{a}_{\overline{m}|}$ (✓).

Since $\ddot{s}_{\overline{k}|} = s_{\overline{k+1}|} - 1$. and $\ddot{a}_{\overline{m}|} = a_{\overline{m-1}|} + 1$.

(5). $\ddot{s}_{\overline{k}|} + \ddot{a}_{\overline{m}|} = s_{\overline{k+1}|} - 1 + a_{\overline{m-1}|} + 1 = s_{\overline{k+1}|} + a_{\overline{m-1}|}$ (✓).

Note: This question, you also solve it by drawing time line with different choices.