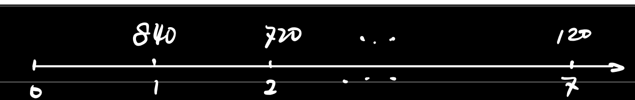
1. Two annuities have the same present value.
The first annuity is a decreasing annual annuity. The first payment is $840 due one year from today. Subsequent annual payments decrease by $120 per year. The interest rate is 4%.
The second annuity provides payments of $K per month for 7 years. The first payment is due one month from today. The interest date as well. What is K?

A. Less than 36
B. At least 40, but less than 42 ✓
C. At least 36, but less than 38
D. At least 42
E. At least 38, but less than 40

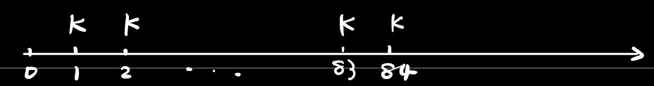


I. $PV|_{t=0} = P a_{\overline{7|}} - Q\left(\frac{a_{\overline{7|}} - 7v^7}{i}\right)$

where $P = 840$. $Q = 120$. $i = 0.04$.

$PV|_{t=0} = 2993.84$.

II.



$1+i = (1+j)^{12}$. $j = (1.04)^{\frac{1}{12}} - 1 = 0.0033$.

$PV|_{t=0} = K \cdot a_{\overline{84|}j}$ $K = 40.82$

2. Adjust the interest for perpetuity:
A perpetuity paying $10 at the start of each month has the same present value as a perpetuity paying $123 at the end of each year, when they are evaluated at the same annual effective rate of interest i . Find i .

A. 4.7% ✓
B. 3.8%
C. 4.3%
D. 4.8%

$||$

since PV is same, the payment of 123 at 1 year is equal to the future value of 12 months of 10.

i.e. $123 = 10\ddot{s}_{\overline{12|}j}$. j is the effective interest rate of month.

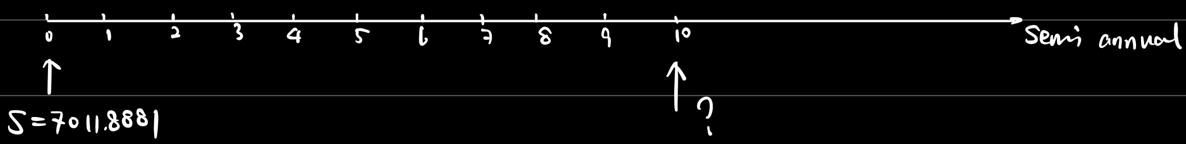
$(1+j)^{12} = 1+i$. $i = 4.047\%$.

3. adjust interest rates for level mthly annuity:
A saver pays $100 at the start of each month into a 5-year savings account that earns an annual interest rate of 6% convertible monthly. At the end of the 5 years the proceeds are transferred to another account that earns an annual rate of 4 % convertible semi-annually. The saver makes no other payments into this second account. What will the accumulated value in the second account be 5 years after the money was transferred to the second account?

A	8,547	B	8,505
C	8,561	D	6,977

Notice that 6% is nominal interest rate, $j = \frac{6\%}{12} = 0.5\%$ is effective interest rate for a month.

So at the end of the 5 years. $AV|_{t=60} = 100 \cdot \ddot{s}_{\overline{60|}j} = 7011.8881$.



Since 4% is nominal interest rate, $m = \frac{4\%}{2} = 2\%$ is effective interest rate for a half year.

$AV|_{t=10} = S \cdot (1+m)^{10} = 8547$.

4. level annuity with continuous payment:
Starting today, Percy invests continuously at a rate of 100 per annum for 40 years in a fund earning 5% per annum compounded continuously. At the end of 60 years, X is in the fund. There are no withdrawals. Determine X.

A	25395	B	12778
C	48832	D	34735

$X = AV|_{t=60} = AV|_{t=40} \cdot (1+i)^{20}$.

$AV|_{t=40} = 100 \cdot \bar{s}_{\overline{40|}} = 100 \cdot \frac{(1+i)^{40}-1}{\delta}$.

Since $\delta = 0.05$. $(1+i) = e^{\delta}$. $AV|_{t=40} = 100 \cdot \frac{(e^{0.05})^{40}-1}{0.05} =$

$X = AV|_{t=40} \cdot e^{20\delta} = 34721.14$.

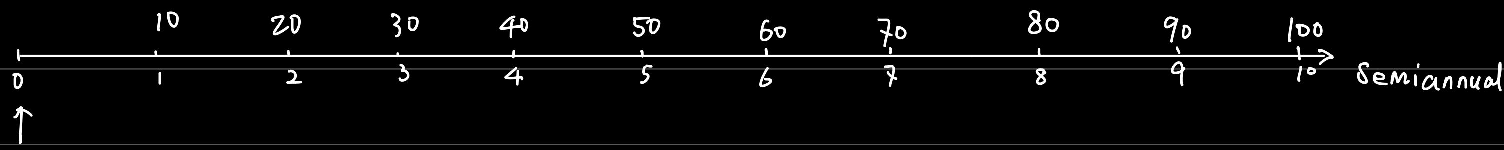
5. increasing mthly standard annuity:
Semiannual payments are to be received at the end of each 6-month period over 5 years. The first payment of \$10 occurs in 6 months, and each subsequent semiannual payment increases by \$10.The nominal interest rate convertible quarterly is 20%. Determine the present value of these payments at time 0

A 278.54

B 286.18 ✓

C 282.36

D 274.72



Since $i^{(4)} = 20\%$. $\frac{i^{(4)}}{4} = 5\%$. it is the effective interest rate of quarterly.

$\Rightarrow (1+j) = \left(1+\frac{i^{(4)}}{4}\right)^2 \Rightarrow j = (1.05)^2 - 1 = 0.1025$. It is the effective interest rate of semiannual.

$$PV|_{t=0} = 10(Ia)_{\overline{10|}j} = 10\left(\frac{\ddot{a}_{\overline{10|}} - 10v^{10}}{j}\right) = 286.18$$

6. varying stream of payment:
You continuously receive payments, at a continuously-varying annual rate of $e^{0.5t}$, for twenty years, i.e., from t = 0 to t = 20. You have a 9% annual effective interest rate. Which of the following would you solve to determine the present value, at t=0, of these payments?

A $\int_0^{20} \left(\frac{e^{0.5}}{1.09}\right)^t dt$

B $\int_0^{20} \left(\frac{e^{0.5}}{1.09}\right)^{20-t} dt$

C $\int_0^{20} \left(\frac{e^{0.9}}{1.05}\right)^t dt$

D $\int_0^{20} e^{0.5t}(1.09)^{20-t} dt$

$p(t) = e^{0.5t}$. $a(t) = (1+0.09)^t = (1.09)^t$.

$$PV|_{t=0} = \int_0^{20} p(t)\cdot a(t)\,dt = \int_0^{20} \left(\frac{e^{0.5}}{1.09}\right)^t dt$$

*. General Formulas for Continuous Paying Annuities

Assume an n-year annuity makes continuous payments at a rate of $p(t)$ at time t. Assume that interest is accumulated at a force of interest δ_t which results in an accumulation function $a(t)$.

- The PV of this annuity at $t=0$ is given by $PV = \int_0^n \frac{p(t)}{a(t)} dt$.
- The AV of this annuity at $t=n$ is given by $AV = a(n)\cdot PV$.

7\. leveraging on standard formula instead of integration by parts.

A company is introducing a new product that they think will have a 10-year life cycle, with sales increasing steadily for 5 years, after which sales will decline steadily. The company feels that the product will be so successful that they will make sales every day of the year. As a result, they model future sales by assuming net cash flows are received continuously over the 10-year horizon at the following payment rates, $\rho(t)$ given:

$$\rho(t) = \begin{cases} 100t & 0 \le t \le 5 \\ 100(10-t) & 5 < t \le 10 \end{cases}$$

The company requires an effective annual rate of return on any investment of 12.75%. What is PV of the income from this investment

- A 1331
- B 1606
- C ✓ 1414
- D 1252

$PV = 100(\bar{I}\bar{a})_{\overline{5}|} + 100 \cdot v^5 \cdot (\bar{D}\bar{a})_{\overline{5}|}$ at $i = 12.75\%$.

$= 100\left(\frac{\bar{a}_{\overline{5}|} - 5v^5}{\delta}\right) + 100 \cdot v^5 \cdot \frac{5 - \bar{a}_{\overline{5}|}}{\delta}$.

Since $\bar{a}_{\overline{n}|} = \frac{1-v^n}{\delta} = \frac{i}{\delta} \cdot a_{\overline{n}|} \Rightarrow \bar{a}_{\overline{5}|} = 3.7599$.

$PV = 1413.47$.

8\. PQ formula, change annually, paid monthly.

Monthly payments are received over 5 years. Determine the present value of payments of \$10 at the start of every month in the first year, \$20 at the start of every month in the second year, \$30 at the start of every month in the third year, and so on. The monthly effective discount rate is 1 %.

- A B 1,260.99
- B ✓ 1,249.98
- C D 1,283.01
- D C 1,272.00

10 10 ... 10 20 20 20 ... 30 30 30 ... 40 ... 50 ... 50

0 1 2 3 ... 11 12 13 14 ... 24 25 26 ... 36 48 59 60 Month.

$PV = 10(I\ddot{a})^{(12)}_{\overline{5}|} = 10 \cdot \frac{\ddot{a}_{\overline{5}|} - 5v^5}{d^{(12)}}$.

$= 1250$.

$d^{(12)} = 12\%$.

$1 - \frac{d^{(12)}}{12} = v^{\frac{1}{12}} \Rightarrow v = 0.886. \Rightarrow d = 0.1136$.

$1+i = (0.886)^{-1}$. $i = 0.12818$.

9. geometric mthly annuity, changes annually:

A senior executive is offered a buyout package by his company that will pay him a monthly benefit for the next 20 years. Monthly benefits will remain constant within each of the 20 years. At the end of each 12-month period, the monthly benefits will be adjusted upwards to reflect the percentage increase in the CPI. You are given:

(i) The first monthly benefit is R and will be paid one month from today.

(ii) The CPI increases 3.2% per year forever.

At an annual effective interest rate of 6%, the buyout package has a value of 100,000. Calculate R.

A	540	B ✓	548
C	517	D	538

The effective interest rate for month is $j = (1.06)^{\frac{1}{12}} - 1 = 0.4868\%$.

The PV of the equivalent payments at the end of years $1, 2, \cdots, 20$ is:

$$PV = R \cdot s_{\overline{12}|j} \left[1.06^{-1} + 1.032(1.06)^{-2} + \cdots + (1.032)^{19}(1.06)^{-20} \right]$$

$$= \frac{R \cdot s_{\overline{12}|j}}{1.06} \left[1 + \frac{1.032}{1.06} + \cdots + \frac{(1.032)^{19}}{(1.06)^{19}} \right]$$

$$= \frac{R \cdot s_{\overline{12}|j}}{1.06} \cdot \left[\frac{1 - \left(\frac{1.032}{1.06}\right)^{20}}{1 - \left(\frac{1.032}{1.06}\right)} \right] = R \cdot (11.6288)(15.6944) = 100000.$$

$R = 547.93$.

Method 2: $PV = T.PMT \cdot \frac{i}{i^{(m)}} \cdot a_{\overline{n}|}$.