Before Question 7. we review the price of Bond formulas:

(1). Basic formula: $P = Fr\,a_{\overline{n}|} + CV^n$.

(2). Premium/Discount formula: $P = C + (Fr - Ci)\,a_{\overline{n}|}$.

(3). Makeham formula: $P = K + \frac{g}{i}(C - K)$. $K = CV^n$. $g = \frac{Fr}{i}$.

Then, impact of tax on bond yield.

(1). Income Tax: For coupon payments t_1

(2). Capital gains Tax: For redemption payments t_2

$\Rightarrow$ Price = PV of "Coupon payments after tax" + PV "Redemption payments after tax".

• $i > (1-t_1)\cdot g \Rightarrow P < C$. So we should have capital gains tax.

The Price with tax:

(1). $P_2 = (1-t_1)\,Fr\,a_{\overline{n}|} + CV^n - t_2\cdot(C - P_2)\cdot V^n$.

(2). $P_2 = K + \frac{g(1-t_1)}{i}(C-K) - t_2\cdot\left(\frac{C-P_2}{C}\right)\cdot K$

This from Makeham Formula allowing tax t_1 and t_2.

If Only t_1, $P_1 = K + \frac{g(1-t_1)}{i}(C-K)$.

If t_1 and t_2, $P_2 = P_1 - t_2\cdot\frac{(C-P_2)}{C}\cdot K$.

7. An investor, who is liable to income tax at 20% but is not liable to capital gains tax, wishes to earn
a net effective rate of return of at least 5% per annum. A bond that pays coupons half-yearly in
arrear at a rate of 6.25% per annum has just been issued. The bond can be redeemed at par on a
coupon date between 10 and 15 years after the date of issue, inclusive. The date of redemption is at the option of the borrower.
Determine the maximum price that the investor is willing to pay for £100 nominal of the bond.

A 100.46	B 100.64
C 109.85	D 100.48

Net yield = 5% annually. $i = (1.05)^{\frac{1}{2}} - 1 = 2.4695\%$.

$r = \frac{6.25\%}{2} = 3.125\%$.

Since this is a callable bond, we should indentify "Tax".

Comparing $(1-t_1)\cdot g$ and i.

$(1-t_1)\cdot g = (1-0.2)\cdot(3.125\%) = 2.5\% > 2.4695\%$.

So it is premium, the yield worst at earliest date. So at 10th year.

Assume that the 19th coupon date.

The Price $P = (1-0.2)\cdot(3.125\%)\cdot(100)\cdot a_{\overline{19}|} + 100v^{19} = 100.458$ #.

8. An investor is liable to income tax at 30% and capital gains tax at 20% purchases a 5000, 8 year bond that is redeemable at par and pays coupons of 8% pa half-yearly in arrears. Calculate the price the investor should pay to obtain a net yield of 9% convertible half-yearly

A 4347.60

B 3940.31

C 4045.11

D 4010.21

$t_1 = 0.3$. $t_2 = 0.2$. $F = C = 5000$. $r = 4\%$. $n = 16$. $i = 4.5\%$.

Indentify the tax: $(1-t_1)\cdot g = 0.7\cdot(4\%) = 2.8\% < i = 4.5\%$.

So it is discount bond, and $P < C$.

So there is a capital gains tax.

$P_2 = Fr\cdot a_{\overline{n}|}\cdot(1-t_1) + Cv^n - (C-P_2)(t_2)\cdot v^n$.

$P_2(1-t_2\cdot v^n) = Fr\, a_{\overline{n}|}\cdot(1-t_1) + Cv^n - Ct_2v^n$

$= Fr\, a_{\overline{n}|}\cdot(1-t_1) + Cv^n(1-t_2)$

$$P_2 = \frac{Fr\, a_{\overline{n}|}(1-t_1) + Cv^n(1-t_2)}{1-t_2\cdot v^n} = 3940.25$$ #.

1.

Year	Spot rate
1	5%
2	6%
3	7%
4	8%

Determine price of bond using spotrates:
You are given the following spot rates:
A 3-year bond has an annual coupon rate of X and its par value of $1,000. The price of the bond is $1,191.70. Determine X .

- 14.12%
- 13.08%
- 13.46%
- 13.97%

Using Spot rates,

the Price of the bond is $P = \frac{1000X}{(1+r_1)} + \frac{1000X}{(1+r_2)^2} + \frac{1000X}{(1+r_3)^3} + \frac{1000}{(1+r_3)^3} = 1191.70$.

where $r_1 = 0.05$. $r_2 = 0.06$. $r_3 = 0.07$.

So the Coupon rate of X is, $X = \left(1191.70 - \frac{1000}{(1+r_3)^3}\right) \Big/ \left[1000\cdot\left(\frac{1}{1+r_1} + \frac{1}{(1+r_2)^2} + \frac{1}{(1+r_3)^3}\right)\right]$

$= 14.199\%$.

2. Finding forward rate with zero coupon bonds:
You are given the following information about three zero-coupon bonds, each of which matures for $1,000.
Calculate the 2-year forward rate deferred 1 year.

Maturity (years)	Price
1	$970.87
2	$873.44
3	$772.18

- [ ] 11.2%
- [ ] 12.13%
- [ ] 5%
- [ ] 25.73%

$$(1+f_{[1,3]})^2 = \frac{a(3)}{a(1)} = \frac{1000/772.18}{1000/970.87} \Rightarrow f_{[1,3]} = 12.1299\%.$$

$$a(3) = (1+r_3)^3 = \frac{1000}{772.18}. \qquad a(1) = (1+r_1) = \frac{1000}{970.87}.$$

3. identify the forward rates using coupon bonds:
You are given the following information about three bonds that pay annual coupons.
A loan arranged now. The loan commences in one year and lasts for one year. The interest rate on the loan is the one-year forward rate. Calculate the interest rate on the loan.

Maturity (years)	Coupon	Annual effective yield
1	6.0%	14.000%
2	7.0%	12.052%
3	8.0%	13.893%

- [ ] 12%
- [ ] 11%
- [ ] 10%
- [ ] 14%

We assume that $F = C = 100$.

r_t is yield of a t-year-zero coupon bond. it is spot interest.

For 1 year bond, $PV = 6 \cdot a_{\overline{1}|14\%} + 100v = \dfrac{106}{1+r_1}$. $r_1 = 14\%$.

For 2 year bond, $PV = 7 \cdot a_{\overline{2}|12.052\%} + 107 \cdot v^2 = \dfrac{7}{1+r_1} + \dfrac{107}{(1+r_2)^2} \Rightarrow r_2 = 11.98\%$.

So the interest rate on the loan, i.e. $f_{[1,2]}$.

$$1 + f_{[1,2]} = \frac{a(2)}{a(1)} = \frac{(1+r_2)^2}{1+r_1} = 1.0999 \Rightarrow f_{[1,2]} = 0.1 = 10\%$$

4. quoted rate for T-bills:
A U.S. Treasury Bill and a Government of Canada Treasury Bill have the same maturity value of $10,000, same term of 90-day, and same price of $9,873.64. Find the difference between the quoted rates of these two T-bills.

- [ ] 0.141%
- [ ] 0.152%
- [ ] 0.136%
- [ ] 0.129%

U.S. T-bills: $d = \dfrac{360}{n} \cdot \dfrac{C-P}{P} = \dfrac{360}{90} \cdot \dfrac{10000 - 9873.64}{10000} = 0.0505$

Government of Canada Treasury Bills: $i = \frac{365}{90} \cdot \frac{10000 - 9873.64}{9873.64} = 0.05190$.

So $i - d = 0.001357 = 0.136\%$.

5. Interest allowing for default risk:
Consider a 7-year loan of $6,100 that is repaid with a single payment of principal and interest at time seven. The lender requires an effective annual interest rate of 3.3% assuming no default risk and an effective annual interest rate of 3.69% if it is expected that N out of every 500 borrowers will default but with a 26% partial recovery rate. Compute N.

- [ ] 18.34
- [ ] 16.28
- [ ] 17.59
- [ ] 19.72

$$500(1.033)^7 = \underbrace{(500-N)(1.0369)^7}_{\text{The amount paid by the borrowers who don't default.}} + \underbrace{N\cdot(0.26)\cdot(1.0369)^7}_{\text{The partial recovery of 26\% from the borrowers who do default.}}$$

$N = 17.59$

Assuming that the lender gets absolutely nothing if a borrower defaults:

Let x = amount received from every borrower (assuming no defaults).

y = amount received from every borrower who does not default (assuming defaults at rate q)

q = rate of default.

The lender wants to receive at least as much as he or she would have received had there been no defaults. So the lender requires that: $x = (1-q)\cdot y$.

Thus, the minimum amount the lender requires from each borrower who does not default is:

$$y = \frac{x}{1-q}.$$

Let's introduce another factor, p, which is the fraction of the recoveries received from those who default at the end of the loan term. In this case, y is received from the fraction $(1-q)$ of the borrowers who do not default and py is received from the fraction q of the borrowers who do default. Thus, for the lender to receive the same amount with default as the lender would have received if there had been no defaults, the lender requires that:

$$(1-q)y + qpy = x \quad \text{or} \quad [1-(1-p)q]y = x$$

Solve for y: $$y = \frac{x}{[1-(1-p)\cdot q]}.$$

6. interest rate considering all factors:

The annual effective rate of interest on a loan that takes into account both default risk and inflation is 7.31 %. The annual effective interest rate that has yet to allow for default risk and inflation is 4%. The expected rate of inflation is 1.62%, and the compensation for unexpected inflation is 0.8%. Compute the credit spread or the compensation for default risk if all the rates are expressed as annual effective rates

- [ ] 0.89%
- [ ] 0.73%
- [ ] 1.04%
- [ ] 0.95%

default risk. expected inflation. unexpected inflation.

$1+R = (1+r)(1+s)(1+i_e)(1+i_u)$ with effective rates.

Thus, $1.0731 = (1.04)(1+s)(1+0.162)(1+0.008)$

$s = 0.73\%$.

Question: If the interest rates were continuous, how to calculate the credit spread?

3. Components of the interest rate: Let R be the rate on a loan with default risk, r be the rate on a loan of the same term without default risk, and s be the compensation for default risk.

(a) Default risk with no inflation: $R = r + s$ with continuous rates and $R = (1+r)(1+s) - 1$ with effective rates.

(b) Default risk with inflation present: If i is the inflation rate, $R = r + s + i$ with continuous rates and $R = (1+r)(1+s)(1+i) - 1$ with effective rates.

$s = R - r - i_e - i_u = 0.0731 - 0.04 - 0.0162 - 0.008 = 0.089 = 8.9\%$.

Q7. You are given the rate on a loan with default risk is 5.74%, the rate on a loan with the same term without default risk is 4.09%, and the inflation rate is 1.19%. Let s_1 be the compensation for default risk when all rates are compounded continuously, and s_2 be the compensation for default risk when all rates are effective annual. Compute the difference $100(s_1 - s_2)$.

(A) 3.91% (B) 4.60% (C) 5.85% (D) 6.95% (E) 7.34%

Solution:

When default risk and inflation are present, $R = r + s + i$ with continuously compounded rates and $R = (1+r)(1+s)(1+i) - 1$ with effective rates.

It is given that $R = 5.74\%$, $r = 4.09\%$, $i = 1.19\%$. Thus, $5.74 = 4.09 + s_1 + 1.19$ and $1.0574 = (1.0409)(1+s_2)(1.0119)$.

So, $s_1 = .46\%$ and $s_2 = .39052\%$. The difference: $100(s_1 - s_2) = 100(.46 - .39052) = 6.95\%$

ANS. (D)