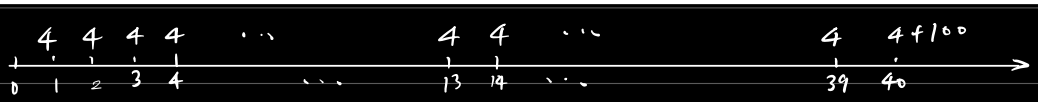
- This chapter covers:

(1). how to determine the price of a bond to earn a given yield rate.

(2). how to determine the yield rate for a bond selling at a given price.

1. A 20-year 100 par value bond with 8% semiannual coupons is purchased for 108.50. What is the book value of the bond just after the 13th coupon is paid?

A	102.24	B	106.91
C	104.89	D	103.32



$F = 100$. $r = 4\%$.

$108.50 = 4 \cdot a_{\overline{40}|i} + 100 v^{40}$. $\Rightarrow i = 3.596\%$.

$BV|_{n=13} = 4 \cdot a_{\overline{40-13}|3.596\%} + 100 v^{(40-13)} = 106.91$

1. For P = bond price and C = bond redemption value:
   (a) $P > C$ means the bond is purchased at a ***premium***
      i. Premium = $P - C$ = excess of price over redemption value
      ii. Also, $g > i$
   (b) $P < C$ means the bond is purchased at a ***discount***
      i. Discount = $C - P$ = excess of redemption value over price
      ii. Also, $i > g$
2. Bond terminology (relative to loan terminology):
   (a) Book value
      i. Used instead of "outstanding loan balance"
   (b) Amortization of premium
      i. Instead of "principal repaid"
      ii. "Writing down"
   (c) Accumulation of discount
      i. "Writing up"
   (d) Coupon
      i. Instead of "payment amount"

3. Bond amortization schedule:

(a) When bond is sold at a premium:

i. Book value is "written down" over time

ii. Interest earned = $I_t = i \cdot B_{t-1}$

iii. Amount for amortization of premium = $P_t = Coupon - I_t$

iv. Amortization reduces the book value from P to C over the life of the bond

v. Book value = $B_t = B_{t-1} - P_t$

(b) When bond is sold at a discount:

i. Book value is "written up" over time

ii. Interest earned = $I_t = i \cdot B_{t-1}$

iii. Amount for accumulation of discount = $P_t = I_t - Coupon$

iv. Accumulation of discount increases the book value from P to C over the life of the bond

v. Book value = $B_t = B_{t-1} + P_t$

2. Pt relationship:

A company buys a 10-year bond with 5% annual coupons and par value of 1,000 to yield 10% effective per annum. Let X equal the amount for "write up" in the 3rd year and let Y equal the interest earned in the 8th year. Determine X + Y.

A	115.49	B	114.65
C	113.23	D	110.90

$X = -P_3 = -(Fr - Ci)\, v^{10-3+1} = -(50 - 100)(1.1)^{-8} = 23.325.$

$Y = I_8 = C_9 - P_8 = C_9 - (Fr - Ci) \cdot v^3 = 87.566.$

$X + Y = 110.885.$

3. Differentiating bond yield and investment yield:

Dan purchases a 1000 par value 10-year bond with 9% semiannual coupons for 925. He is able to reinvest his coupon payments at a nominal rate of 7% convertible semiannually. Calculate his nominal annual yield rate convertible semiannually over the 10-year period.

A	9.2%	B	8.1%
C	9.4%	D	7.6%

$P = 925. \quad F = C = 1000. \quad Coupon = Fr = \frac{9\%}{2} \cdot (1000) = 45.$

reinvestment rate = 3.5%.

$925(1+i)^{10} = 45 \cdot s_{\overline{10}|3.5\%} + 1000$. i is the annual interest rate.

$i = 9.405\% \Rightarrow (1+i) = \left(1+\frac{i^{(2)}}{2}\right)^2 \Rightarrow i^{(2)} = 9.1937\%$.

4. price between coupon dates

A 1000 par value 5-year bond with semiannual coupons of 60 is purchased to yield 8% convertible semiannually. Two years and two months after purchase, the bond is sold at a price which maintains the same yield for the buyer. Calculate this price.

A	1089	B	1099
C	1105	D	1119

$BV|_{t=4} = 60 \cdot a_{\overline{6}|} + 1000 \cdot v^6 = 1104.84$. This the book value just after the 4th coupon payment. at the original yield rate.

The price to maintain the same yield rate $= 1104.84(1.04)^{\frac{1}{3}} = 1119.38$.

5. quoted price in financial press:

A 10,000 par value bond with 8% semiannual coupons is sold 3 years and 4 months before the bond matures. The purchase will yield 6% convertible semiannually to the buyer. The price at the most recent coupon date, immediately after the coupon payment, was 5,640. Calculate the quoted price of the bond, i.e., the price excluding accrued interest

A	5582	B	5502
C	5562	D	5542

The quoted price is equal to $B_{t+k} - kFr = B_t(1+i)^k - kFr$ with $k = \frac{2}{6} = \frac{1}{3}$.

Since the bond is sold 2 months after the last coupon payment. (k is not an integers).

The quoted price $= 5640(1.03)^{\frac{1}{3}} - \frac{1}{3} \cdot (400) = 5562.52$.

6. A 1000-par value 23-year bond with 5% semiannual coupons is callable on any coupon date in the 13th to the 23rd year.
(i) In the 13th to the 18th year, the bond is callable at a redemption value of 1260
(ii) In the 19th to the 23rd year, the bond is callable at a redemption value of 1080
Calculate the maximum purchase price for this bond that will guarantee an annual nominal yield rate of at least 4% convertible semiannually

A	(A) 1152.37	B	(C) 1170.00
C	(B) 1168.30	D	(D) 1181.63

(i). In the 13th to the 18th year. $C = 1260$. $g = \frac{25}{1260} = 0.0198 < i$.

the bond sells at a discount and the worst case for the buyer is a late call.

Assume $n = 36$. The price $P = 25\, a_{\overline{36}|2\%} + 1260 v^{36} = 1254.90$.

(ii). In the 19th to the 23rd year. $C = 1080$. $g = \frac{25}{1080} = 0.0234 > i$.

the bond sells at a premium and the worst case for the buyer is an early call.

Assume $n = 37$. The price $P = 25\, a_{\overline{37}|2\%} + 1080 v^{37} = 1168.30$.

We take the lower of the two prices: 1168.30.