1. Standard increasing annuity: Carolyn buys a 30-year increasing annuity for X. Carolyn will receive 2 at the end of the first year, 4 at the end of the second year, and for each year thereafter the payment increases by 2. The annual effective interest rate is 9%. Calculate X.

A	195.00	B	198.60
C	205.90	D	202.20

Method 1:

Let P = The first payment and Q = The common difference.

$$A = Pa_{\overline{n}|} + Q\cdot\frac{a_{\overline{n}|} - nv^n}{i}.$$

where $P=2$. $Q=2$. $X = A = 2a_{\overline{30}|} + 2\cdot\frac{a_{\overline{30}|} - 30(1.09)^{-30}}{0.09} = \underline{\underline{198.6043}}$.

$$a_{\overline{30}|} = \frac{1-(1.09)^{-30}}{0.09} = 10.2737.$$

Method 2: Since here $P=Q=2$. it is the special case. We call this an Increasing annuity.

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}.$$

$$\Rightarrow X = 2(Ia)_{\overline{n}|} = 2\cdot\frac{\ddot{a}_{\overline{30}|} - 30(1.09)^{-30}}{0.09}. \quad \text{where } \ddot{a}_{\overline{30}|} = \frac{1-(1.09)^{-30}}{\frac{0.09}{1.09}} = 11.1983.$$

$= 198.6036$.

Note: (a). An n-year annuity-immediate with payments of P, $P+Q$, $P+2Q, \cdots, P+(n-1)Q$ (i.e., payments in arithmetric progression) has the following PV:

$$PV = Pa_{\overline{n}|} + Q\cdot\frac{a_{\overline{n}|} - nv^n}{i}$$

(b). PV of an annuity due $= P\ddot{a}_{\overline{n}|} + Q\cdot\frac{a_{\overline{n}|} - nv^n}{d}$.

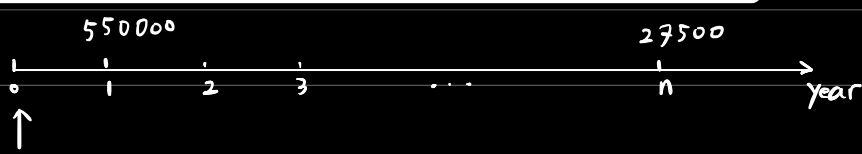
(c). AV of an annuity immediate $= PS_{\overline{n}|} + Q\frac{S_{\overline{n}|} - n}{i}$.

(d). AV of an annuity due $= P\ddot{S}_{\overline{n}|} + Q\cdot\frac{S_{\overline{n}|} - n}{d}$.

2. Standard decreasing annuity:
A company plans to depreciate a certain asset. The depreciation charge at the end of the first year will be \$550,000 and each subsequent annual depreciation charge will decrease by \$27,500 until the final depreciation charge of \$27,500.
Calculate the present value of the depredation charges assuming an annual effective interest rate 0.8%

A	3,499,000	B	3,501,024
C	3,497,988	D	3,500,012

Method 1:



$i = 0.008$.

$P = n = \frac{550}{27.5} = 20$. Q = Common difference $= 27500$.

$$PV|_{t=0} = 27500 \, \frac{n - a_{\overline{n}|}}{i} = 27500 \cdot \frac{20 - a_{\overline{20}|}}{0.08} =$$

where $a_{\overline{20}|} = \frac{1-v^{20}}{i} = 9.8181$.

Note: Determine the number of payments (n).

The formula for the n-th term of an arithmetic sequence is:

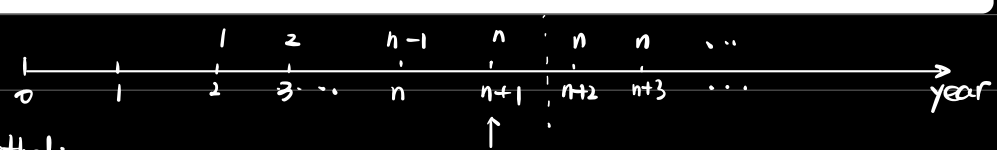
$$D_n = D_1 - (n-1)d \rightarrow \text{Decrease}.$$

final payments   first payments

3. Increasing then level annuity: A perpetuity costs 77.1 and makes annual payments at the end of the year. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, ... , n at the end of year (n + 1) . After year (n + 1), the payments remain constant at n. The annual effective interest rate is 10.5%.
Calculate n.

A 20 B 18

C 19 D 17



Method:

$$PV|_{t=0} = v\cdot\left((Ia)_{\overline{n}|} + v^n \cdot n \cdot a_{\overline{\infty}|}\right) = v\cdot\left(\frac{\ddot{a}_{\overline{n}|} - nv^n}{i} + \frac{nv^n}{i}\right) = v\cdot\frac{\ddot{a}_{\overline{n}|}}{i}.$$

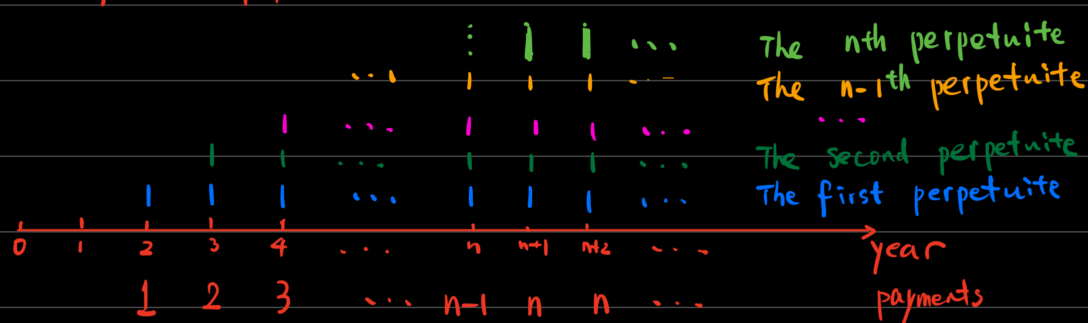
substitute: $PV|_{t=0} = 77.1$, $v = (1.105)^{-1}$, $i = 0.105$.

Sloving for n: $77.1 = (1.105)^{-1}\cdot\frac{\ddot{a}_{\overline{n}|}}{0.105} \Rightarrow \ddot{a}_{\overline{n}|} = \frac{1-(1.105)^{-n}}{0.105}\cdot(1.105) = 8.9455$.

$1-(1.105)^{-n} = 0.85 \Rightarrow (1.105)^{-n} = 0.15 \Rightarrow -n \lg 1.105 = \lg 0.15 \Rightarrow \underline{n = 19}$.

Method 2: Thinking of the perpetuity as consisting of a set of perpetuites with level payments of 1. In case, there is a finite set of such perpetuities: the first one starts at time 2. the second one starts at time 3. ..., and the nth and last one starts at time n+1.

If you add up the payments of these n perpetuites, you will find that they are 1.2.3.⋯.n. followed by level payments of n at the end of the (n+1)st and subsequent years.



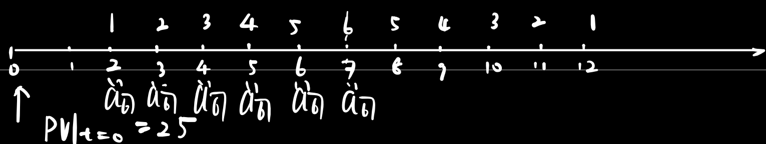
So $PV|_{t=0} = v\cdot a_{\overline{\infty}|} + v^2\cdot a_{\overline{\infty}|} + \cdots + v^n\cdot a_{\overline{\infty}|} = \frac{1}{i}\left(\frac{v(1-v^n)}{1-v}\right) = v\cdot\frac{1-v^n}{i(1-v)} = \frac{v}{i}\cdot \ddot{a}_{\overline{n}|}$.

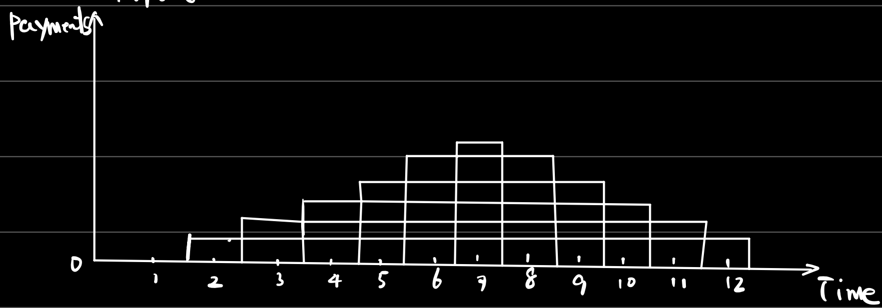
4. palindromic annuity: An 11-year annuity has a series of payments 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, with the first payment made at the end of the second year. The present value of this annuity is 25 at interest rate i.
A 12-year annuity has a series of payments 1, 2, 3, 4, 5, 6, 6, 5, 4, 3, 2, 1, with the first payment made at the end of the first year. Calculate the present value of the 12-year annuity at interest rate i.

A 29.5 B 30.5

C ✓ 30.0 D 31.0

Method: Annuity I.

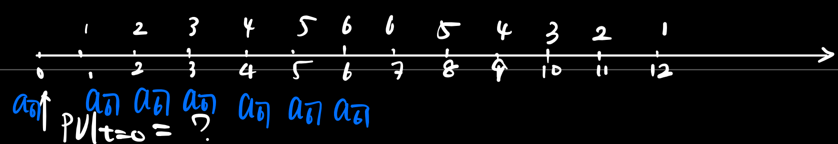




We first split the annuity into 6 level annuities, each of which pays 1 for 6 years.
The time of the first payment for these 6 annuities will vary from 1 to 6.

$I = PV|_{t=0} = \ddot{a}_{\overline{6}|}\cdot a_{\overline{6}|}\cdot v = 25 \Rightarrow a_{\overline{6}|}\cdot a_{\overline{6}|} = 25 \Rightarrow a_{\overline{6}|} = 5$.

Annuity II.



The same way. $II = PV|_{t=0} = a_{\overline{6}|}\cdot \ddot{a}_{\overline{7}|} = a_{\overline{6}|}\cdot(a_{\overline{6}|}+1) = 5\times 6 = 30$.

5. geometrically varying annuity: Stan makes deposits at the end of every year into a fund for 15 years. The first deposit at the end of first year is $5000 and each subsequent annual deposit decreases by 10% per year. The annual effective interest rate is 12%.
Calculate the accumulated value of the fund at time 20 years.

A 210,494 B 210,987

C 211,481 D 210,000

Method:

$AV|_{t=20} = PV|_{t=0}\,(1+i)^{20}$. where $i = 0.12$.

$PV|_{t=0} = 5000\,\frac{1-\left(\frac{0.9}{1.12}\right)^{15}}{0.12+0.1} = 21872.3741$.

$AV|_{t=20} = 210987.3311$

6. PQ formula: Nathan has a job that pays his income annually at the end of each year. His next paycheck will occur one year from today and it will be $50,000. Each subsequent year after the first year, he will receive a $2,000 raise. The present value of his salary over the next 20 years is $X. The annual effective interest rate is 6%. Find X

A 747,957 B 746,914

C 745,871 D 774,828

Method :

$$PV|_{t=0} = Pa_{\overline{n}|} + Q \cdot \frac{a_{\overline{n}|} - nv^n}{i}$$ same to Q1.

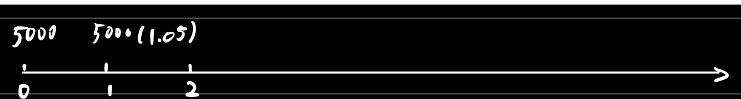
7. Geometrically increasing annuity: Stan makes deposit at the beginning of every year for 15 years. The first deposit is $5000 and subsequent deposit increase by 5% per year. Annual effective interest rate is 7%
Calculate the present value of this annuity.

A 65,984 B 61,625

C 85,874.6 D 81,785

E 170,026

Method :



$$PV = \frac{P}{1+k} \cdot a_{\overline{15}|i'} (1+i) = 65962.5247.$$

8. Find the present value of the perpetuity that pays annually. The payment starts at time 5 with RM600 and each payment thereafter increases by RM10. The interest rate is 3.5% per annum.

A 427,077 B 490,082

C 25,306 D 22,052

Method : $P = 600$. $Q = 10$.

$$PV|_{t=4} = P \cdot a_{\overline{\infty}|} + Q \cdot \frac{a_{\overline{\infty}|}}{i} = \frac{P}{i} + \frac{Q}{i^2} =$$

$$PV|_{t=0} = PV|_{t=4} \cdot v^4 =$$