

Introduction to Bayesian statistics

Part 1 — Concepts

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🔗 <https://github.com/jorgetendeiro/GSMS-2020>

Bayes rule

- ▶ \mathcal{D} = data
- ▶ θ = unknown parameter

$$p(\theta|\mathcal{D}) = \frac{p(\theta)p(\mathcal{D}|\theta)}{p(\mathcal{D})}$$

In words,

$$\text{posterior} = \frac{\text{prior} \times \text{likelihood}}{\text{evidence}}$$

The *evidence* does not depend on θ ; let's hide it:

$$\text{posterior} \propto \text{prior} \times \text{likelihood}$$

The symbol \propto means “proportional to”.

Bayes rule

$$\text{posterior} \propto \text{prior} \times \text{likelihood}$$

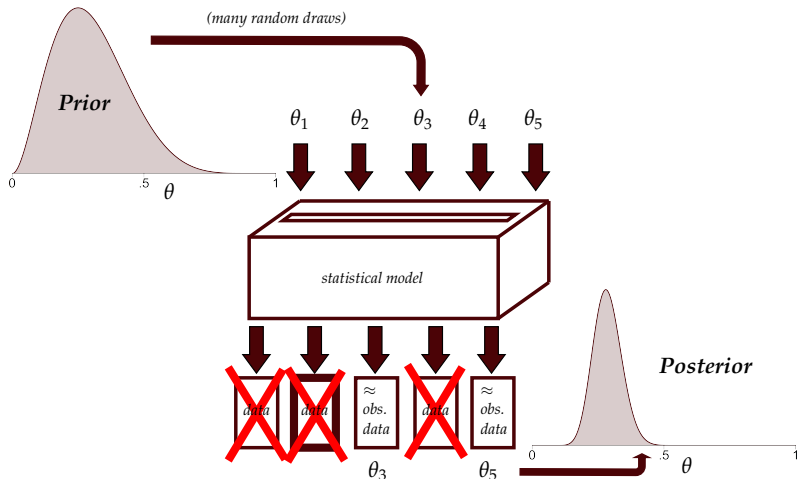
- ▶ *Prior*: Belief about the 'true' value of θ , *before looking at the data*.
- ▶ *Likelihood*: The statistical model, linking θ to data.
- ▶ *Posterior*: Updated knowledge about θ , in light of the observed data.

Bayes rule

One useful way to think about the Bayes rule is by considering *Approximate Bayesian Computation* (ABC; see [Wiki](#)).

- ▶ ABC is actually computationally *very* inefficient.
- ▶ But, it is *conceptually* very clear!

Bayes rule



Bayes rule

The Bayes rule from the ABC perspective:

Find the values of θ that allow the model to predict data pretty much like our observed data.

Humm. . .

MLE, anyone?

Bayesian inference can be thought of as an extension of MLE!

Bayes rule – Summary

$$\text{posterior} \propto \text{prior} \times \text{likelihood}$$

Bayesian modelling requires three ingredients:

- ▶ Data.
- ▶ Priors, reflecting our subjective belief about the parameters.
- ▶ A statistical model, relating parameters to data.

Bayes rule is a mathematically rigorous means to combine prior information on *parameters* with the *data*, using the *statistical model* as the bridge between both.

Bayes rule – Example

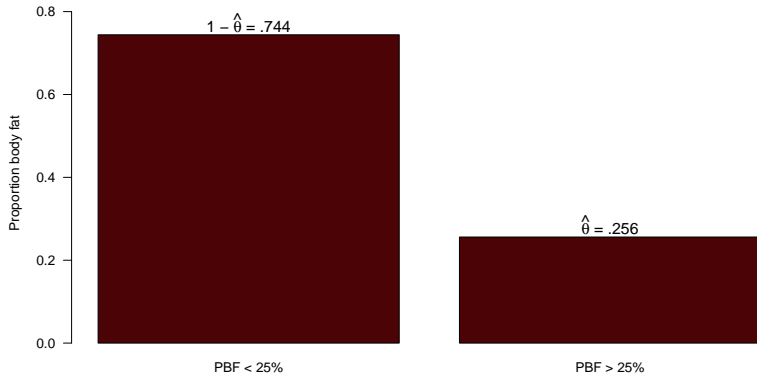
Data here:

<https://das1.datadescription.com/datafile/bodyfat/>.

- ▶ Various measurements of 250 men.
- ▶ To keep it simple: I dichotomize the percentage of body fat (PBF).
- ▶ 0 = PBF lower than 25%;
1 = PBF larger than 25%.
- ▶ *Goal*: Infer the proportion of obese men in the population.

Let's denote the population proportion by θ .

Bayes rule – Example



Bayes rule – Example

Let's use the Bayesian machinery.

Recall that we need three ingredients:

- ▶ Data.
- ▶ Prior.
- ▶ Model.

Bayes rule – Example

- *Data.* For now, let's only use the first 10 scores.

0	0	1	0	1	0	0	0	0	0
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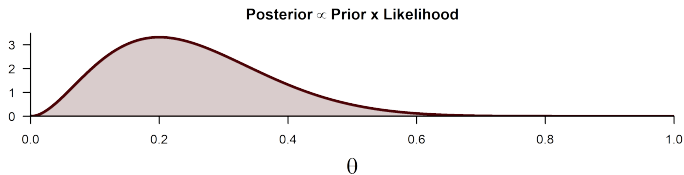
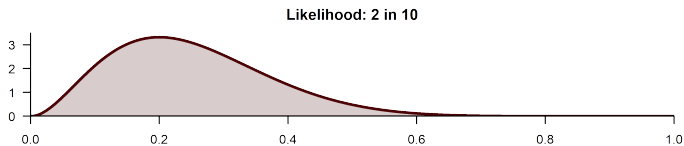
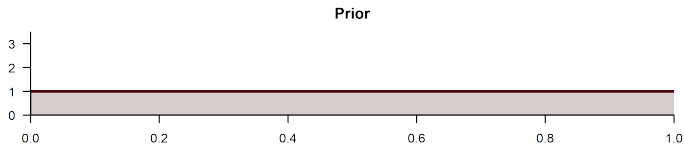
Bayes rule – Example

- *Model.* We'll use the binomial model. Assumptions:
 - ✓ Independence between measurements.
 - ✓ One population with underlying rate θ .
 - ✓ Random sample.

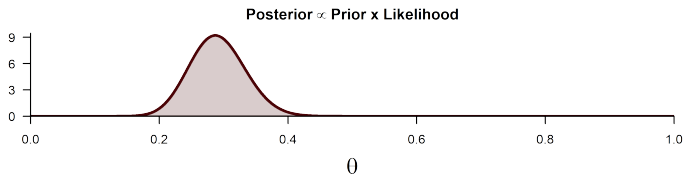
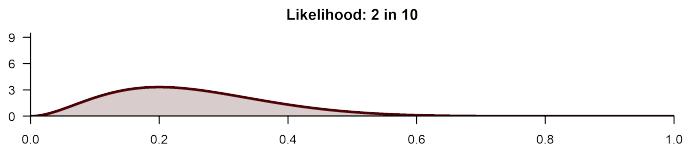
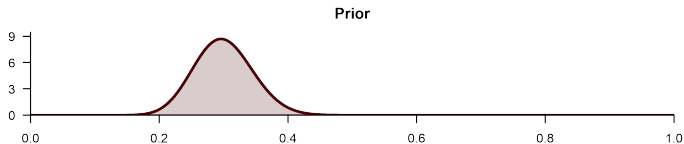
Bayes rule – Example

- ▶ *Prior.* We'll try several.

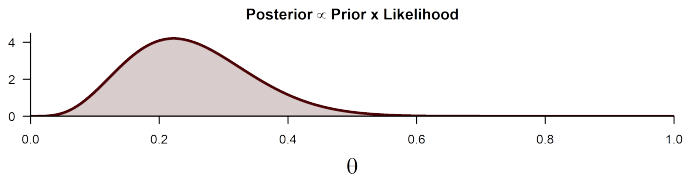
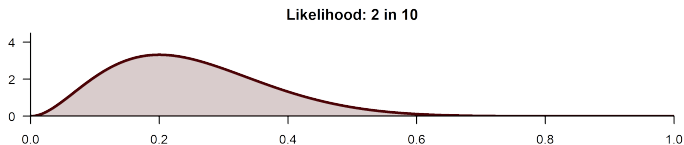
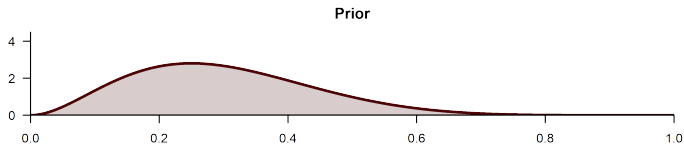
Bayes rule – Example



Bayes rule – Example



Bayes rule – Example



References

Forder, L., & Lupyan, G. (2019). Hearing words changes color perception: Facilitation of color discrimination by verbal and visual cues. *Journal of Experimental Psychology: General*, 148(7), 1105. doi: [10.1037/xge0000560](https://doi.org/10.1037/xge0000560)