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Surface drag

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Abstract. I will deiscuss the drag of particles on granular surfaces by a fluid. I will present simulations using FLUENT of Aeolian saltation under Earth and Mars conditions obtaining among others the static and dynamic threshold shear and the height of the saltation layer as function of the wind strength.
A universal quadratic increase of the saturated flux as function of the difference from the threshold is established.

Keywords: aeolian transport | critical phenomena | Mars | saltation
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INTRODUCTION

The motion of sand grains in a sequence of ballistic trajectories close to the ground is called saltation and is a major factor for surface erosion, dune formation and triggering of dust storms on Mars. Although this mode of sand transport has been matter of research for decades both through simulations and wind tunnel experiments under Earth and Mars conditions, it has not been possible to provide accurate measurements of particle trajectories in fully developed turbulent flow. Here we calculate the motion of saltating grains by directly solving the turbulent wind field and its interaction with the particles. On the basis of our results, we arrive at general expressions which can be applied to calculate the length and height of saltation trajectories and the flux of grains in saltation under different physical conditions, when the wind velocity is close to the minimal threshold for saltation. Our calculations show that the minimal wind velocity required to sustain saltation on Mars may be surprisingly lower than the aerodynamic minimal threshold measurable in wind tunnels. Indeed, Mars grains saltate in 100 times higher and longer trajectories and reach $5-10$ times higher velocities than Earth grains do.

THE MODEL

We simulate saltation by solving, first, the turbulent wind field inside a long two-dimensional channel (wind tunnel) and, next, the air feedback with the dragged particles. The details of the calculation procedure are described elsewhere [12]. The fluid (air) is incompressible and Newtonian, the Reynolds-averaged Navier-Stokes equations with the standard $\kappa-\varepsilon$ model being employed to describe turbulence [13].

The turbulent wind is, first, calculated without particles. The wind is generated by a pressure difference imposed between the extremities of the tunnel [12] and achieves a stable profile, the wind velocity $u(z)$ increasing with the height z as $u(z) = 2.5u_* \log z/z_0$, where z_0 is the surface roughness ($\approx 10^{-4}$ m) and u_*, the wind shear velocity, quantifies the wind strength [3]. Boundary conditions are zero wind velocity at the bottom wall and zero shear ($u_* = 0$) at the upper wall (gravity g points downwards).

Once steady-state turbulent flow is produced, particles are injected from the inlet at the ejection angle of grain-bed collisions with a velocity of the order of 60 cm/s. Grain trajectories are obtained by integrating the equation of motion:

$$\frac{dv_\mathrm{p}}{dt} = F_D\big(u - v_\mathrm{p}\big) + g\big(\rho_\mathrm{p} - \rho_\mathrm{fluid}\big)/\rho_\mathrm{p}, \qquad (1)$$

where v_p is the particle velocity, ρ_fluid and ρ_p the density of the fluid and of the particles, respectively, and $F_D\big(u - v_\mathrm{p}\big)$ represents the drag force per unit particle mass, where

$$F_D = \frac{18\eta}{\rho_\mathrm{p} d^2}\frac{C_D \mathrm{Re}}{24}, \qquad (2)$$

$\mathrm{Re} = \rho_\mathrm{fluid} d|u - v_\mathrm{p}|/\eta$ is the particle Reynolds number, d is the grain diameter, η the air viscosity and the drag coefficient C_D is taken from empirical relations [14]. The feedback on the local velocity of the fluid due to its momentum exchange with the particles is accounted for by adding the momentum change of every particle as it passes through a control volume [12],

$$F = \sum_{\text{particles}} F_D\big(u - v_\mathrm{p}\big)\dot{m}_\mathrm{p}\Delta t, \qquad (3)$$

where $\dot{m}_\mathrm{p}$ is the mass flow rate of the particles and Δt the time step. Each time a particle hits the ground it loses

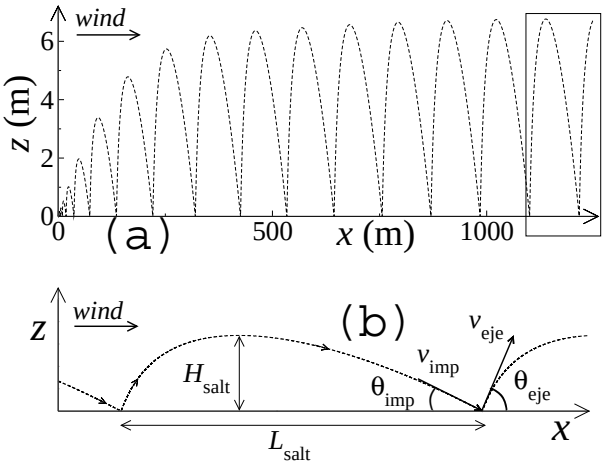

FIGURE 1. (a) The dashed line represents the trajectory of one saltating grain calculated using parameters for Mars with a wind strength of $u_* = 1.78$ m/s. Gravity points downward (negative z direction) and the particle moves downwind, i.e. from left to right. In the absence of the saltating particles, the wind strength increases logarithmically with the height. However, the wind profile is disturbed by the saltating grains as shown in ref. [12]. (b) Typical saltation trajectory obtained in the calculations — the dashed line corresponds to the saltation path enclosed by the box in (a) — showing the velocity and angle of impact ($v_{\rm imp}$, $\theta_{\rm imp}$) and of ejection ($v_{\rm eje}$, $\theta_{\rm eje}$). The length $L_{\rm salt}$ and the maximum height $H_{\rm salt}$ of a saltation trajectory are also indicated in the figure.

a fraction $r = 0.40$ of its momentum and a new stream of particles is ejected at that position with the ejection angle characteristic of saltation particles, $\theta_{\rm eje} = 36^\circ$ [5, 15]. The number of particle streams released, $n_{\rm p}$, defines the sand flux $q = \dot{m}_{\rm p} n_{\rm p}$. Particle trajectories increase in height with the downwind position, achieving a maximum after a distance ΔL, called "saturation length" [16], from the inlet. The saturated flux, $q_{\rm s}$, is identified by the maximum number of particles streams, $n_{\rm p}$, a wind of given strength u_* can transport without flux reducing due to energy loss of the trajectories [12].

In simulations for Earth, gravity $g = 9.81$ m/s^2, air viscosity $\eta = 1.78 \times 10^{-5}$ kg·m^{-1}s^{-1} and density $\rho_{\rm fluid} = 1.225$ kg/m^3 are taken, while particle diameter $d = 250\,\mu$m and density $\rho_{\rm p} = 2650$ kg/m^3 are as from sand of terrestrial dunes [3]. For Mars we use air density and viscosity $\rho_{\rm fluid} = 0.02$ kg/m^3, respectively $\eta = 1.3{\cdot}10^{-5}$ kg/m·s, while gravity is $g = 3.71$ m/s^2 and Martian particles have diameter $d = 600\mu$m and density $\rho_{\rm p} = 3200$ kg/m^3 typical of Martian dunes sand. Simulations are performed with u_* values between 0.27 m/s and 0.5 m/s for Earth and 1.12 m/s and 1.78 m/s for Mars, which means wind velocity values in the range $u = 6.21 - 11.5$ m/s (Earth) and $u = 25.8 - 41.0$ m/s (Mars) at a height of 1 m. Taking the wind velocity at this height, the fluid Reynolds number $\mathrm{Re_f} = ud\rho_{\rm fluid}/\eta$ considered in the simulations is in the range $106.8 - 197.9$ for Earth and $23.8 - 37.8$ for Mars.

While Earth trajectories achieve the maximum height after typically $\Delta L = 10$ m, Mars trajectories stabilize after $\Delta L = 1.0$ km (fig. 1a), independently of u_*. Thus, the wind tunnel used in the simulations for Mars is much larger than the one used for Earth simulations (which has length 30 m and height 1.0 m), the former being 2.0 km long with a height of 10 m. We have performed simulations using channels of larger dimensions, both for Mars and for Earth. In both cases, it was found that increasing the size of the tunnel does not change the results.

RESULTS AND DISCUSSION

In Figure 2 we show the increase of the resulting maximum flux of particles in saltation, $q_{\rm s}$, with the shear velocity u_* on Mars. For comparison, the same simulations were performed using terrestrial atmospheric conditions and grain size and density characteristic of dunes on Earth (inset of fig. 1).

The scaling of the saturated flux with the shear velocity is universal: $q_{\rm s} \propto (u_* - u_{*\rm t})^\alpha$, where $\alpha \approx 2.0$ is independent of the atmospheric conditions, and $u_{*\rm t}$, the *impact threshold* wind velocity — the minimal wind velocity required to sustain the motion of saltating grains at saturation [3] — obtained in the simulations is around 1.12 m/s and 0.26 m/s on Mars and on Earth, respectively. As shown in a previous work [12], we have found a rather good agreement between the data obtained from our simulations for Earth and those obtained from field measurements by Lettau and Lettau [10], thus confirming the validity of our simulation procedure [12].

The flux of sand depends not only on the wind velocity but also on gravity, atmospheric and sand properties, which must be included in the coefficient of the scaling between $q_{\rm s}$ and u_*. One expression for $q_{\rm s}(u_*)$ which includes gravity and atmospheric density was proposed and validated through wind tunnel experiments under Earth and Mars air pressure conditions [4]. The simulations of the present work, however, reveal that the air viscosity also plays a relevant role in saltation, and cannot be neglected. If the ratio $\rho_{\rm fluid}/g$ is encoded in the coefficient of $q_{\rm s}(u_*)$ [4], then dimensional analysis requires a further term with units of velocity to be included into the scaling relation of fig. 2.Universality is only obtained when the nondimensional quantity d/l_ν — where the length-scale $l_\nu = (\nu^2/g)^{1/3}$ defines, together with the time-scale $t_\nu = (\nu/g^2)^{1/3}$ the characteristic velocity $v_\nu = l_\nu/t_\nu$, with $\nu = \eta/\rho_{\rm fluid}$ — is incorporated into the prefactor of the sand flux equation. This quantity encodes the information of the amount of momen-

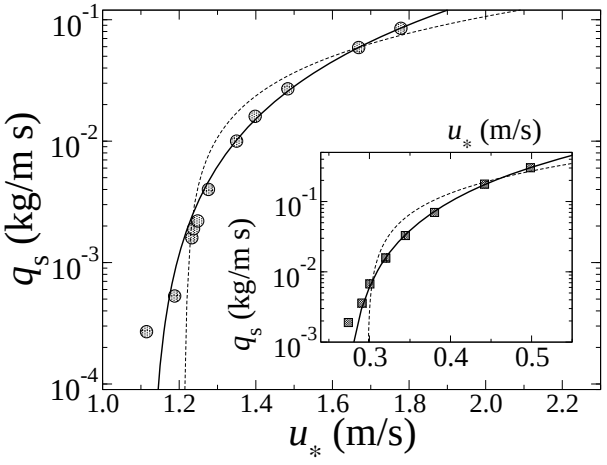

FIGURE 2. In the main plot, the circles show the saturated flux q_s calculated as function of the shear velocity u_* for Mars. The quadratic relation $q_s \propto (u_* - u_{*t})^2$, indicated by the full line, fits the data notoriously better than the classical equation for planetary sand flux [4] — whose corrected version reads [17] $q_w = C[\rho_{fluid}/g]u_*^2(1-u_{*t}/u_*)(1+u_{*t}/u_*)^2$ (dashed line) — and gives $u_{*t} = 1.12$ m/s for Mars. The range of shear velocities $u_* = 1.12 - 1.78$ m/s used for Mars corresponds to shear stress $\tau = \rho_{fluid}u_*^2$ values in the range $0.025 - 0.063$ kg m^{-1} s^{-2}. The corresponding calculations for Earth are shown in the inset. In this case, simulations were performed with shear velocities in the range $u_* = 0.27 - 0.5$ m/s.

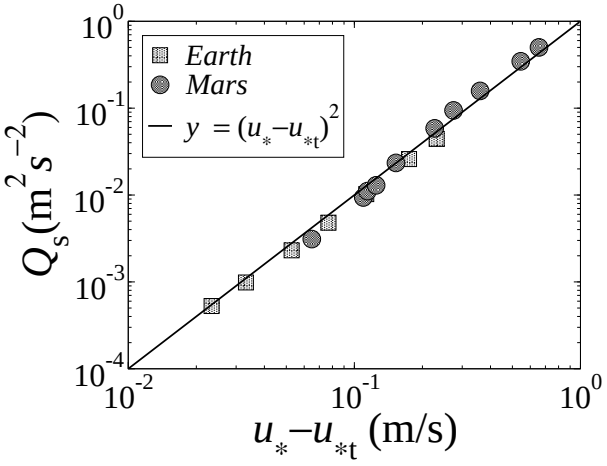

FIGURE 3. Normalized flux Q_s defined as $q_s/[227.3 \cdot u_{*t} \cdot d\rho_{fluid}^{5/3}/(\eta g)^{2/3}]$. Circles and squares correspond to calculations made for Mars and Earth, respectively. The straight line corresponds to the equation $y = (u_* - u_{*t})^2$.

tum lost due to the grain-bed collisions that has to be compensated by the drag force in the equilibrium [11]. A formula for the sand flux is then obtained, which can be applied to calculate q_s as function of u_* under arbitrary physical environments (fig. 3):

$$q_s \approx 227.3 \cdot \left[d\rho_{fluid}/(\nu g)^{2/3} \right] \cdot u_{*t} \cdot (u_* - u_{*t})^2. \quad (4)$$

Both the length and height of the saltation paths increase linearly with the shear velocity u_*, when this is close to the threshold (fig. 4). Dimensional analysis leads to a scaling of H_{salt} and L_{salt} with $t_\nu(u_* - u_{*t})$. Using our data to obtain the respective constants of proportionality, we find that universal equations for L_{salt} and H_{salt} can be obtained if the quantity $\sqrt{l_\nu/d}$ is incorporated into the coefficients of both saltation length and height. In this manner, we arrive at the following expressions, which can be used to predict the saltation trajectories under an arbitrary planetary surface, for u_* close to the threshold u_{*t}:

$$H_{salt} \approx 81.46 \cdot (\nu^2/g)^{1/3} \cdot (u_* - u_{*t})/\sqrt{gd}, \quad (5)$$

$$L_{salt} \approx 1091.5 \cdot (\nu^2/g)^{1/3} \cdot (u_* - u_{*t})/\sqrt{gd}, \quad (6)$$

where $(\nu^2/g)^{1/3}$ is the characteristic lengthscale l_ν and $\sqrt{gd}$ represents the grain velocity necessary to escape from the sand bed.

As compared to Earth trajectories, Martian saltation paths are giant. While Earth saltating particles do not exceed heights of 15 cm, maximum heights reached by particles saltating under typical sand-moving winds on Mars (fig. 1) amount to $1 - 5$ m. Moreover, Martian grains saltate in $20 - 120$ m long paths — such particles appear rather suspended when seen in a wind tunnel with length of a few meters [4]. The simulation results are in agreement with the expectation that the saltation length L_{salt} should scale with the quantity $L_{drag} = d\,\rho_p/\rho_{fluid}$, which is a characteristic length that encodes the information of the inertia of a saltating grain in the surrounding fluid [18].

DISCUSSION AND CONCLUSION

How can the simulation results presented here be helpful for the understanding of dune formation on Mars? Whether dunes could be formed at present atmospheric conditions of Mars remained in fact an open question for decades since the first images of Mars dunes taken by Mariner 9 [1, 19]. Noticeable changes on the surface of a few Martian dunes have been detected only recently (e.g. as reported in refs. [20]). These observations provided evidence that dunes can be formed and move on the present Mars. However, since the wind velocity is seldom above the threshold for saltation motion on Mars, the rate at which changes on the dunes occur is so low that no dune migration could be detected from orbiters up to now.

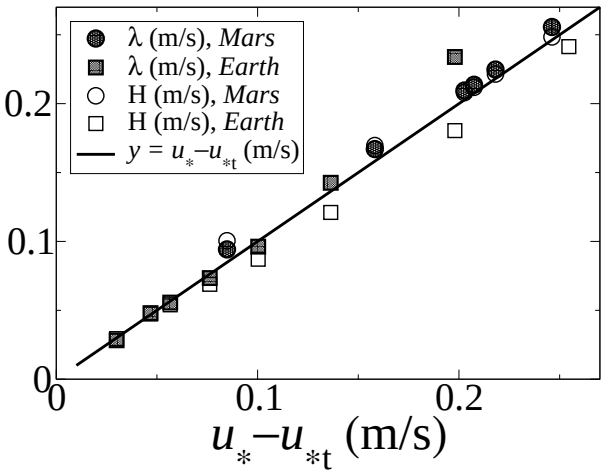

FIGURE 4. *Universal expressions for saltation transport* — The normalized length $\lambda \equiv L_{\rm salt}/[1091.5{\cdot}(l_v/\sqrt{gd})]$ and height $\mathrm{H} \equiv H_{\rm salt}/[81.46{\cdot}(l_v/\sqrt{gd})]$ as function of $u_* - u_{*\rm t}$, are shown by the full, respectively by the empty symbols, both for Mars (circles) and Earth (squares), with $u_{*\rm t} = 1.03$ m/s and 0.244 m/s for Mars and for Earth, respectively. All data seem to follow the straight line, $y = u_* - u_{*\rm t}$.

In conclusion, the present work reported the first calculation of particle trajectories in a turbulent wind. The sand flux, saltation height and length on Mars and on Earth were studied and general expressions (eqs. (4), (5) and (6)) were found which can be applied in order to calculate these quantities under different physical conditions, e.g. on the surface of Venus or Titan. The giant saltation trajectories and the high grain velocities on Mars are the missing link to understand the triggering of dust storms under typical wind velocities $u_* \approx 1.0$ m/s at present conditions of Mars [2].

The large values of impact velocities obtained in the simulations allow us to understand the high erosion of Martian soils during the rare gusts of sand transport as predicted from wind tunnel simulations of saltation under low-pressure conditions that reproduce the density of the Martian air [2] and from experiments of particle-bed collisions under typical conditions of grain velocity on Mars. It would be interesting to extend the calculations of the present work in order to study the development of saltation from the initiation of transport by aerodynamic (direct) entrainment, with inclusion of a splash function that accounts for the response of the number of ejected grains with respect to the impact velocity of saltating particles.

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