

LETTER TO THE EDITOR

Precise calculation of the threshold of various directed percolation models on a square lattice

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Abstract

Using Monte Carlo simulations on different system sizes we determine with high precision the critical thresholds of two families of directed percolation models on a square lattice. The thresholds decrease exponentially with the degree of connectivity. We conjecture that p_c decays exactly as the inverse of the coordination number.

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Directed percolation (DP) describes generically the dynamics of adsorbing processes and has been applied to epidemics, forest fires, surface catalysis, etc [1–6]. DP displays a phase transition as function of the propagation probability p between an ‘adsorbing’ state and a percolating (or active) state at a critical threshold p_c . The scaling properties of DP have been known for over two decades.

The value of p_c depends on the lattice and the rule of connectivity. In 1+1 dimensions the most studied case is the tilted square lattice with nearest-neighbour connectivity (see figure 1) giving $p_c = 0.6447 \pm 0.0001$ [7]. For many applications it is of interest to consider a longer range of connectivities. In particular, also an infinite range model with a probability decaying as a powerlaw with distance has been studied [8]. Astonishingly, however, what is missing in the literature are studies of propagation probabilities of intermediate range, although these are in practice the most common situations. It is therefore the aim of the present paper to calculate the percolation thresholds of two families of models in 1+1 dimensions having finite, varying range of interactions.

Model I is defined on a tilted square lattice as shown in figure 1(a). The connectivity k is given as the number of neighbours to which a site i can be connected in the row below. In model I, the k is always even. Figure 1(a) also shows the cases $k = 2, 4$ and 6 . Model II is defined on the standard square lattice and in figure 1(b) we can see the cases $k = 1, 3$ and 5 . For this case, k is always odd.

We studied the two models up to $k = 15$ on lattices of sizes ranging from 256×20000 to 2048×20000 . We simulated for each value of k and L the model for different values of

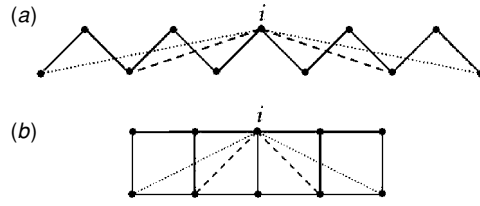


Figure 1. Schematic plot of the definitions of (a) model I and (b) model II. The full lines correspond to (a) $k = 2$ and (b) $k = 1$, adding the dashed lines one obtains (a) $k = 4$ and (b) $k = 3$ and adding the dotted lines one obtains (a) $k = 6$ and (b) $k = 5$.

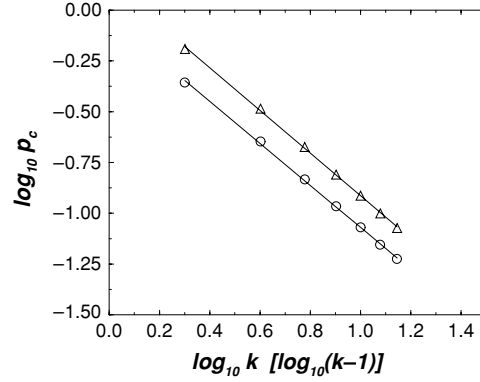


Figure 2. Double-logarithmic plot of the percolation thresholds as function of k for model I (triangles) and of $k - 1$ for model II (circles).

Table 1. Percolation thresholds of directed percolation for model I and model II.

k	I	k	II
2	0.6447 ± 0.0002	3	0.4395 ± 0.0003
4	0.3272 ± 0.0002	5	0.2249 ± 0.0003
6	0.2121 ± 0.0003	7	0.1470 ± 0.0002
8	0.1553 ± 0.0003	9	0.1081 ± 0.0002
10	0.1220 ± 0.0002	11	0.0851 ± 0.0002
12	0.0999 ± 0.0002	13	0.0701 ± 0.0002
14	0.0846 ± 0.0003	15	0.0549 ± 0.0002

p averaging over 2000 initial configurations and monitored if the system percolated or not. From the inflection point of the curve χ versus p , where χ is the fraction of realizations that percolated, we determined $p_c(L)$. Finally, we extrapolated to the thermodynamic limit p_c through the expression

$$p_c(L) = p_c(1 + aL^{-1/\nu}). \quad (1)$$

In table 1 we show the obtained thresholds with their estimated error bars, and in figure 2 we see these values in a double-logarithmic plot. Since the slope is -1.00 ± 0.05 we conclude that

$$p_c = \eta/k \quad (2a)$$

with $\eta = 1.3 \pm 0.1$ for model I and

$$p_c = \eta/(k - 1) \quad (2b)$$

with $\eta = 0.9 \pm 0.1$ for model II.

In summary, we have described in this letter numerical evidence that the percolation threshold of directed percolation decays such as η/k as a function of the coordination k . This result is consistent with the dependence of the percolation threshold on the coordination number on the Cayley tree [9] and we conjecture it to be exact. As a future work, it would be interesting to calculate the thresholds for higher-dimensional lattices.

Acknowledgments

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