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PLURALITY VOTING: THE STATISTICAL LAWS OF DEMOCRACY IN BRAZIL

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We explore the statistical laws behind the plurality voting system by investigating the election results for the mayor in Brazil in 2004. Our analysis indicate that the vote partition among mayor candidates of the same city tends to be "polarized" between two candidates, a phenomenon that can be closely described by means of a simple fragmentation model. Complex concepts like "government continuity" and "useful vote" can be identified and even statistically quantified through our approach.

Keywords: Election statistics; voting process; fragmentation model.

Understanding the process by which the individuals of a society make up their minds and reach opinions about different issues can be of great importance. In this context, the election is a fundamental democratic process and the vote certainly represents the most effective instrument for regular citizens to promote significant changes in their communities.

General elections in Brazil are held every four years, when citizens vote for executive as well as legislative mandates. Voting is compulsory and ballots are collected in electronic voting machines. In the case of the legislative mandates, which include elections for congressmen, state deputies and city counselors, a *proportional* voting system is used, where candidates compete for a limited number of seats and become elected in proportion to their corresponding voting fraction. In two previous studies,[1,2] the statistical analysis for the Brazilian 1998 and 2002 elections revealed that the proportional voting process for federal and state deputies displays scale invariance.[3] It has been shown that the distribution of the number of candidates N receiving a fraction of votes v follows a power-law $N(v) \sim v^{\alpha}$ for intermediate values of v only with $\alpha \approx -1$, while it deviates downwards for both large and small v.[4,5] The striking similarity in the distribution of votes in all states, regardless of large diversities in social and economical conditions in different regions of the country, has been taken as an indication of a common mechanism in the decision process. More precisely, it has been suggested that the explanation for this robust scale invariance is a multiplicative process in which the voter's choice for a candidate is governed by a product, instead of a sum of probabilities.[6]

For the selection to executive mandates (president, state governors and mayors), one of the most common election formats is the so called *plurality* voting system, where the winning candidate is only required to receive the largest number of votes in his/her favor, after which all other runners automatically and completely lose. This system is applied in 43 of the 191 countries in the United Nations, Brazil being the largest democracy in this group. Plurality voting has been studied extensively in political science[7] and effects such as the approval of previous administrations or tactical voting, including the so called "useful vote", have been discussed from psychological and sociological points of view. What is missing is a careful statistical description and a mathematical model that include these effects revealing common mechanisms in the decision process.

Here we analyze the election statistics for an executive mandate in Brazil. On 6 October 2004, there was an election in Brazil's 5562 cities in which 102 817 864 electors chose one from among up to 14 candidates for mayor. The collection of ballots was entirely electronic, thus permitting a very rapid count and publication of the results.[8] In Fig. 1 we show the distribution of the fraction of votes v for the winner (right) and for the loser (left), if only two candidates were in the race. The superposition of both sides leads to a distribution that displays a pronounced cusp

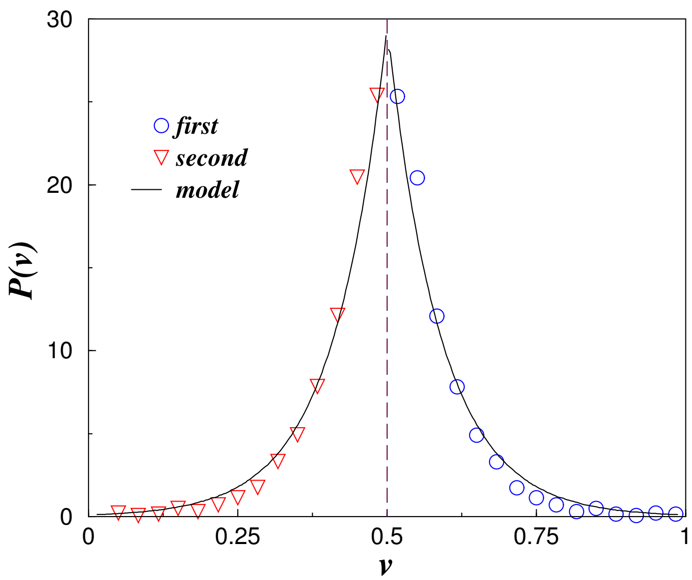

Fig. 1. Histograms of voting fraction for mayor elections of Brazilian cities in 2004. The data correspond to elections with only two candidates and all ordinates have been divided by a factor of 10^3. The circles give the frequency of fraction votes for the winner, while the downward triangles are the results for the loser. The solid lines are symmetric with respect to $v = 0.5$ and represent the best fits to the data by the exponential function, $P(v) \propto \exp(-|v - 0.5|/\lambda)$.

at $v = 0.5$ and differs strongly from a uniform distribution. As shown in Fig. 1 (solid lines), the entire data set can be well described by an exponential decay of the form,

$$P(v) \propto \exp\left(\frac{-|v - 0.5|}{\lambda}\right), \tag{1}$$

with the parameter $\lambda \approx 0.08$. The values for the left and right sides correspond to the excess and deficit of votes for the winning and losing candidates, respectively. The sharpness of the curve beautifully illustrates the effect of polarization which drives a typical ballot close to the marginal situation of a tie.[9]

In Fig. 2 we show the statistics of the winner for cities with three and four candidates. As depicted, both distributions display a cusp-shaped maximum close to the same value $v = 0.5$, and exponential tails on both sides. This behavior can be described by a generalization of the fragmentation model of Ref. 10, based on the well known fact that the approval (or disapproval) of the previous municipal administration usually decides rather early whether the acting mayor or the candidate he/she supports as follower is reelected or not. We start by dividing the electorate into two fractions, v_1 and $r_1 = 1 - v_1$. Keeping intact the fraction v_1, we divide r_1 into v_2 and $r_2 = r_1 - v_2$. At a third step, while v_2 remains undivided, the fraction r_2 is partitioned again, and so on. As opposed to Ref. 10, where the limit of an infinite number of fragments is investigated, here we consider a process in which a finite number n of fragments (fraction of the

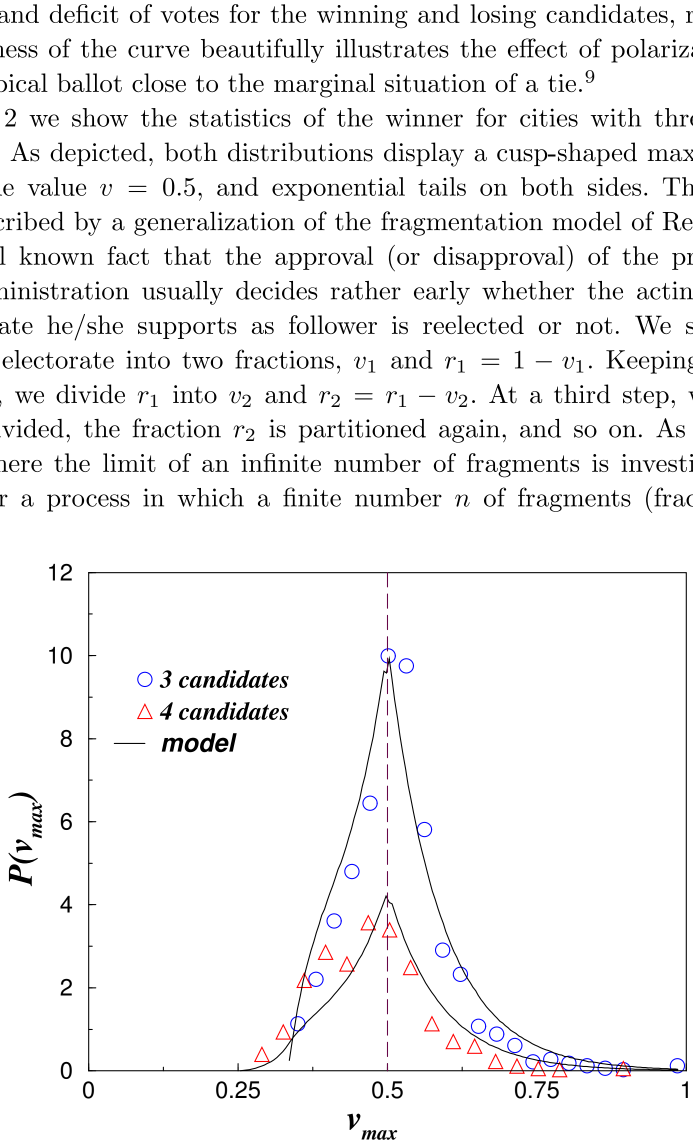

Fig. 2. Histograms of the voting fraction for the most voted candidates $v_{\max}$ in elections with three (circles) and four (upward triangles) candidates. The solid lines are the predictions of the fragmentation model.

electorate) is generated with sizes that can be written as $v_1 = x_1$, $v_2 = (1-x_1)x_2$, $v_3 = (1-x_1)(1-x_2)x_3, \ldots, v_{n-1} = (1-x_1)(1-x_2)\cdots(1-x_{n-1})x_n$, and $v_n = (1-x_1)(1-x_2)\cdots(1-x_{n-2})(1-x_{n-1})$, with $0 \leq x \leq 1$ being a random variable distributed according to the same function $\rho(x)$. This randomness excludes at this point any tactical voting strategies. We simply attribute to each candidate i a fraction of votes v_i, with $i = 1, 2, 3, \ldots, n-1, n$.This is justifiable if we assume that v_i should be closely related with the fraction of electors "decided" to vote in candidate i. In this way, it is reasonable to adopt the distribution given by Eq. (1) (see also Fig. 1) as a first approximation for $\rho(x)$. Under this framework, one can also think of x_1 as being the fraction of the electorate voting for "continuity". Following this model, for example, the fraction of votes $v_{\max}$ of the most voted among a finite number n of competing candidates is given by,

$$v_{\max} = \max[v_1, v_2, v_3, \ldots, v_{n-1}, v_n]\,. \tag{2}$$

For the numerical solution of our model, we first generate $n-1$ random numbers distributed according to Eq. (1). From these, we calculate the entire set of v_i fractions and determine the largest one, $v_{\max}$. We then repeat this process $N = 10^5$ times in order to produce a histogram for $v_{\max}$, as displayed in Fig. 2. As depicted, the agreement between the real data and the model predictions for $v_{\max}$ with $n = 2$ and 3 is very good, without the need of any adjusting parameter of the fragmentation model. This confirms the validity of our approach.

The selection of one among n candidates by the population during an electoral campaign is certainly not a static process. For example, the dynamics of a typical voting process that is studied here is shown in Fig. 3, where we present the results of a sequence of polls made before the mayor election in São Paulo during the campaign of 2004.[11] First, the time evolution of these polls illustrates well the "polarization" between the first and the second most voted candidates. Second, the growth in popularity of both candidacies is a clear consequence of the loss of votes of the two less voted candidates. This tactical transference of votes is explained as follows. Being driven by the results of election polls widely spread in the media during the campaign, the electors tend to adopt the so-called "useful vote", either to try to guarantee or to prevent the victory in the first round of the most voted candidate.

In summary, based on a Brazilian dataset for plurality voting of unseen quality, we have discovered the existence of a strongly peaked exponential distribution in the case of only two candidates, showing the effect of polarization. When more candidates are involved, a simple fragmentation model based on early decisions concerning continuity is able to explain the shape of the distribution of the winner.

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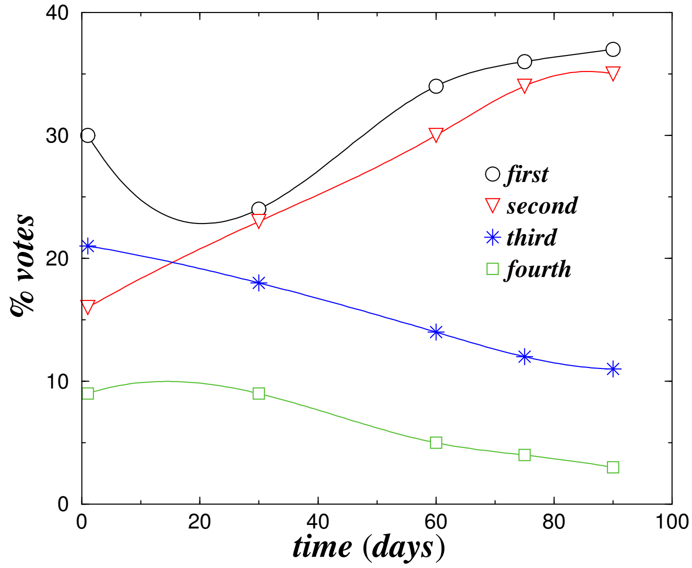

Fig. 3. Time evolution of the percentage of votes for the first four most voted candidates participating in the election for mayor in São Paulo during the campaign of 2004. These polls have been made by the Brazilian agency IBOPE.[11] The time in days is counted from the date of the first poll, namely 28 of June of 2004. The gradual approximation between the first two most voted candidates shows the polarization phenomenon, while the growth of both candidacies illustrates the "useful vote" effect. The solid lines are cubic splines drawn to facilitate the view.

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