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# A heuristic approach for solving a rich min-max vehicle routing problem with mixed fleet and mixed demand

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ABSTRACT

We propose a new heuristic approach for a new variant of vehicle routing problem with a min-max objective function and mixed types of service demands satisfied by a mixed fleet. It is assumed that vehicles with unlimited service capacity, which have different service and transfer speeds, operate on demand points grouped in regions with boundaries that cannot be passed. The proposed solution method is based on swarm intelligence. A numerical study is carried out to evaluate the performance of the proposed method.



## 1. Introduction

The research on Vehicle Routing Problem (VRP) started with the pioneering work of Dantzig and Ramser (1959). They analyzed dispatching of trucks to optimize routing of a gasoline truck fleet between a bulk terminal and service stations. Since then, a large literature is created on VRP and its variants. Survey papers (e.g. Fisher (1995); Laporte, Gendreau, Potvin, & Semet (2000); Toth & Vigo (2001); Marinakis & Migdalas (2002); Laporte (2009)), books (e.g. Golden, Raghavan, & Wasil (2008); Crainic & Laporte (2012)) and bibliographies (e.g. Laporte & Osman (1995); Marinakis & Migdalas (2007)) written on VRPs and their solution methods comprise a substantial reference for researchers. Since the problem is computationally hard, besides exact methods, heuristic and metaheuristic methods are proposed. VRP has several variants on the problem environments of depots, vehicles, delivery methods and customer preferences: VRPs with multiple depots, with multiple use of vehicles, with capacitated/uncapacitated vehicles, with time windows, with split delivery, with backhauls, with pickup and deliveries, etc. Differently, some researchers focus not just on the minimization of total cost or total travel time, but also on different objectives like the minimization of maximum cost or maximum travel time and the minimization of the latest or average arrival time at a demand point. Especially, the former kind of objective functions are encountered mostly in the non-profit fields like humanitarian logistics, disaster relief and other services provided by governmental or non-profit organizations (Duran, Gutierrez, & Keskinocak, 2011; Renkli & Duran, 2015).

As stated in the review paper of Lahyani, Khemakhem, and Semet (2015), in recent years, due to the progress in methodology and computing technology, new realistic VRP variants with complex objective functions and constraints attract attention of researchers. These extended problems with various real-life attributes are often called Rich VRPs. Since our problem is more real-life oriented in the dimensions of requests (multiple demand types), fleet (multiple platform types) and route structure (open and balanced routes in different regions), we describe it as a Rich VRP.

Our main motivation is the naval anti-mine warfare operations such as mine sweeping or mine hunting in areas contaminated or suspected to be contaminated by anchored or buried sea mines laid close to our harbors and straits by the enemy. This kind of operations have certain limitations according to type of mines, type of search that can be conducted by platforms, transition and operation speeds of platforms and engagement risk on or between the regions where the operations take place. Although it is not a must, we expect a mine counter-measure operation to employ various types of ships and to deal with various types of enemy mines. This is by the nature of increasingly growing research and development in both mine and mine counter-measure technologies, while the life-cycles of a modern mine and the platforms used in mine counter-measure operations are very long. Therefore, in order to have a more general and practical model, we assume mixed fleet and mixed demand which reflect various types of platforms and various types of mines, respectively. On the other hand, the

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min-max objective function reflects the fact that mine sweeping and hunting operation should be completed as soon as possible in order to have full capability at open sea concurrently in the shortest time.

With a fleet of mixed types of platforms and when the enemy's mine inventory is known, a decision maker should come up with a good solution in a short time and execute it before the battle fleet should start to operate at sea. Similar problems can also be encountered in civilian environments such as humanitarian logistics or disaster relief efforts where heterogeneous vehicles or teams are on duty in disconnected regions and regional safety problems are expected.

In our problem, we assume that we have a mixed fleet of limited number of vehicles. While some of the vehicles conduct one type of service, the others may be capable of providing multiple types of service. Moreover, the vehicles may have varying transition and operation speeds. Since the vehicles are considered to have sufficient fuel capacity to conduct all of the assigned service duties, the problem is assumed be uncapacitated. We also assume that, after the assignment to regions, the vehicles cannot change their regions due to the limitations related to safety or geographic characteristics of the area on which the operation takes place.

A similar problem, where the service delivery at each service point is allowed to be split by different vehicles, is analyzed in the study of Yakıcı and Karasakal (2013). The authors proposed a classical heuristic solution method. A set of routes is chosen randomly by using normal distribution from a larger set built up by a deterministic approach and a Set Covering Problem (SCP) is solved to find a set of routes for each of the vehicles which collectively cover all of the demand points. Since split delivery is not practical in service operations in many cases, we consider the case where splitting of the service in a demand point is not allowed. In our solution method, we try to explore the solution space effectively and exploit the promising regions with a tailored swarm optimization metaheuristic approach.

There are some studies handling min-max routing problems with metaheuristic approaches. França, Gendreau, Laporte, and Müller (1995) applies a tabu search method for min-max $m$-TSP problem which consists of $m$ identical vehicles without capacity limit and only one region. Golden, Laporte, and Taillard (1997) also uses tabu search for solving min-max capacitated VRP and its version with multiple use of vehicles. Ren (2011) solves min-max VRP using a hybrid genetic algorithm.

There are also different approaches other than metaheuristics. Applegate, Cook, Dash, and Rohe (2002) proposes a branch and bound algorithm for a specific min-max VRP instance and Carlsson, Ge, Subramaniam, Wu, and Ye (2009) applies a linear programming based approach and region partitioning for min-max multiple depot VRP.

Related works are reported by Campbell, Vandenbussche, and Hermann (2008) and Xu, Xu, and Li (2010). Campbell et al. (2008) studies minimization of latest arrival time and average arrival time for TSP and VRP, while Xu et al. (2010) works on minimizing the latest service completion time in an uncapacitated path cover problem with multiple depots, homogeneous demand type and homogeneous fleet.

Swarm intelligence with ants, called ant colony optimization (ACO), was first applied to Traveling Salesman Problem (TSP), by the work reported in Dorigo and Gambardella (1997). Later, this approach has been applied to VRP and its extensions (Bell & McMullen, 2004; Doerner, Hartl, & Lucka, 2005; Li & Tian, 2006; Mazzeo & Loiseau, 2004; Reimann, Doerner, & Hartl, 2004). However, to the best of our knowledge, this technique has not been applied to a min-max VRP with mixed fleet and mixed demand before.

In the following sections of this article, the model is introduced along with its mathematical formulation, the proposed heuristic approach is explained, and the design and the comparative results of numerical experiments are reported. Finally, we present our concluding remarks and mention future research directions in the last section.

## 2. Model definition

In this section, we present the required notation and define the Min-max VRP with Mixed Fleet and Mixed Service Demand (MFMDVRP). The time spent by a vehicle while making transfers from one demand point to another and the time spent while providing service for a demand point are called transition time and service time, respectively. The total time spent by a vehicle until the service completion of the last visited demand point is called the travel time of that vehicle. The terms demand point, node and vertex are used interchangeably throughout this paper. In the definition of the problem the following sets and parameters are used:

$G = (\{V \cup (c)\}, E)$: complete digraph with set of vertices $(1, \ldots, |V|)$ and central vertex $c$. $E$ denotes the set of edges $(i,j)$ where $i$ and $j \in \{V \cup (c)\}$,

$K = (1, \ldots, |K|)$: set of service vehicles,
$D = (1, \ldots, |D|)$: set of service types,
$K_d$: set of service vehicles with type $d$ ($j \in D$) service capability,
$V_d$: set of vertices with type $d$ ($j \in D$) service demand,
$P = (p_{ij})$: distance matrix for graph $G$,
$S = (s^k)$: vector of transfer speed for the vehicles in $K$,
$Q = (q_i)$: vector of service demand for the vertices in $V$,
$R = (r_d^k)$: service rate matrix for the vehicles in $K$ and the service types $D$.

A solution to MFMDVRP requires $K$ routes starting at central vertex $c$, and satisfaction of all service requirements. The decision variables used in the problem are as follows:

$x_{ij}^k \in \mathbb{B}$ takes value 1 if edge $(i,j)$ belongs to the $k^{th}$ route; 0 otherwise,
$y \in \mathbb{R}$ denotes the maximum travel time,
$w_d^k \in \mathbb{R}$ denotes the type $d$ service time by vehicle $k$ $(= \frac{q_j}{r_d^k} x_{ij}^k)$,
$I_i^k \in \mathbb{R}$ is a dummy variable used for subtour elimination.

$$(\textbf{MFMDVRP}) \quad z_{MFMDVRP} = \min y \tag{1}$$

subject to

$$\sum_{d \in D} w_d^k + \sum_{i \in \{V \cup (c)\}, j \in \{V \setminus (i)\}} \frac{p_{ij}}{s^k} x_{ij}^k \leqslant y \quad \forall k \in K \tag{2}$$

$$\sum_{k \in K_d,\, i \in \{V \cup (c)\}} x_{ij}^k = 1 \quad \forall j \in \{V_d \setminus i)\}, \forall d \in D \tag{3}$$

$$\sum_{i \in V} x_{ci}^k \leqslant 1 \quad \forall k \in K \tag{4}$$

$$\sum_{i \in \{V \cup (c)\}} x_{ij}^k = \sum_{i \in \{V \cup (c)\}} x_{ji}^k \quad \forall k \in K, \forall j \in \{V \setminus i)\} \tag{5}$$

$$I_i^k - I_j^k + (M+1) x_{ij}^k \leqslant M \quad \forall k \in K, \forall i \in V, \forall j \in V \tag{6}$$

$$x_{ij}^k \in \{0, 1\} \quad \forall i \in \{V \cup (c)\}, \forall k \in K, \forall j \in V \tag{7}$$

$$y \geqslant 0 \tag{8}$$

$$w_d^k \geqslant 0 \quad \forall d \in D, \forall k \in K \tag{9}$$

$$I_i^k \geqslant 0 \quad \forall i \in V, \forall k \in K \tag{10}$$

The objective function (1) represents the minimization of $y$ which is given as an upper bound on travel times of $|K|$ vehicles in Constraint Set (2). Constraint Set (3) ensures that each demand point should be visited once by a vehicle matching its service demand type. Constraint Set (4) ensures that each vehicle exits from central vertex once at most. Constraint Sets (5) and (6) prevent multiple visits to the nodes and prevent subtours. Constraint Sets (7)–(10) define the feasible decision space for the problem.

Note that still we can reduce the size of the feasible space defined by Constraint Sets (2)–(10). Although we assign big numbers for the distances between the nodes of different regions and we know that the optimal solution or the solutions close to optimal do not contain cross-regional tours, Constraint Sets (11) and (12) which eliminate all of the routes that cross the regional boundaries are expected to decrease the elapsed time to reach the optimal solution. Moreover, demand type cuts which prevent vehicles from visiting demand points that they cannot serve are defined as additional constraints by Constraint Set (13).

$$\sum_{g\epsilon G} A_g^k \leqslant 1 \quad \forall k\epsilon K \tag{11}$$

$$\sum_{i\epsilon V} x_{in^g}^k \leqslant A_g^k \quad \forall k\epsilon K, \forall n^g \epsilon V^{(g)}, \forall g\epsilon G \tag{12}$$

$$\sum_{k\epsilon\{K\setminus K_d\}, i\epsilon\{V\cup(c)\}} x_{ij}^k = 0 \quad \forall j\epsilon V_d, \forall d\epsilon D \tag{13}$$

In the Constraint Sets (11) and (12), $G$ and $A_g^k$ respectively represent the set of regions and a binary decision variable which is equal to 1 when vehicle $k$ is assigned to region $g$. $n^g$ is a member of $V^{(g)}$ which is the set of vertices in region $g$.

It is easy to observe that the proposed problem MFMDVRP with Constraints (1)–(10) is NP-Hard. Consider that we have only one region, one demand type, and one vehicle type. If we assume that all demands and distances from demand points to depot are equal to zero, then MFMDVRP converts to $m$-TSP with a min-max objective function, which minimizes the maximum of the tours of $m$ uncapacitated identical vehicles, starting and ending at the depot.

## 3. Heuristic

The first Ant Colony Optimization algorithm, called AS (Ant System), is proposed by Dorigo, Maniezzo, and Colorni (1996). The approach is inspired by the foraging behavior of ants and their depositing pheromone on the routes between nest and food sources. Their direction selection is correlated to the strength of pheromone on that direction. Within this framework, some extensions like ACS (Ant Colony System) and MMAS (MAX-MIN Ant System) are proposed to improve the performance of AS (Dorigo & Gambardella, 1997; Stützle & Hoos, 2000). ACS has differences like deamon action (addition of extra pheromone on the components of the best solution), pseudo-random-proportional rule (choosing some movements in a greedy manner) and step-by-step pheromone updates (updating pheromone trails of partial solutions). Another extension MMAS employs deamon actions and restricts pheromone values to a specified interval.

In the original Ant System, at each iteration each ant constructs a solution. Differently in our proposed algorithm, waiving this population based characteristic of the ACO algorithm, we consider the vehicles as ants. Therefore, the collection of the routes constructed by the vehicles at each iteration makes up one feasible solution. We have chosen the delayed pheromone update approach. The employed quality function, which affects the level of pheromone addition at each iteration, is made equal to $\frac{z^b}{z_l}$ where $z^b$ and $z_l$ denote the objective function values of *best* solution (obtained until the corresponding time) and the *last* iteration solution, respectively. A common approach of pheromone evaporation is applied for avoiding a rapid convergence towards a local optimal solution.

The distinguishing parts of our approach can be listed as the assignment of vehicles to regions and the usage of both node and edge pheromone. The reason for using node and edge pheromone separately is the independence of service time on nodes and transit time between nodes. The details about the assignment of vehicles using *vehicle assignment pheromone trail*, the construction of routes using *edge and node pheromone trails* and the pheromone update procedures are given in the explanations of each procedure introduced in this section.

In order to give a formal definition of the heuristic, we will refer to the following notations;

$T$: set of vehicle types,
$t$: index for vehicle types,
$T_t$: set of vehicles of type $t$,
$G$: set of regions,
$g$: index for regions,
$\mu_{tg}$: number of vehicles of type $t$ assigned to region $g$.

The main steps of the algorithm is given in Fig. 1. The procedures are explained in detail in the order of their appearance in the algorithm.

**assignVehicles**

In this procedure, there are two methods of assigning a vehicle to a region: *Method 1* and *Method 2*. In *Method 1*, vehicles are distributed to regions randomly. While doing these assignments, we consider the types of services that each vehicle can supply and each region demands. Therefore, we keep two sets for each service type: one keeps the unassigned vehicles having the capability of providing the corresponding service type and the other keeps the regions which require the corresponding service type but not yet assigned a capable vehicle. Whenever the numbers of vehicles and regions in these two sets get equal, then the vehicles in the vehicle set are randomly assigned to the regions in the region set at the same time. In *Method 2*, the assignment of vehicles to regions is executed w.r.t. ACO approach. For applying *Method 2*, it is required to wait until a number of iterations pass in order the *pheromone trails related to vehicle assignment to regions* to accumulate. Therefore, before applying *Method 2*, *Method 1* is applied first. Although these methods can be employed alternatively throughout the heuristic algorithm, in our experiments, *Method 1* is only applied in the first $k$ iterations in order to accumulate the pheromone trail related to vehicle assignment to regions. The number $k$ is set to 100 in our experiments within 1000 iterations. This allows that in the first 10% of iterations, solutions are constructed randomly.

In *Method 2*, in order to make the assignment, we need to construct a probability distribution. Let $ph(dist)_{tmg}$ denotes the pheromone corresponding to the assignment of $m$ vehicle of type $t$ to region $g$. The assignment probability of $m$ vehicle of type $t$ to region $g$ ($p(assign)_{tmg}$), and the assignment probability of type $t$ vehicle to region $g$ ($p(typeassign)_{tg}$), are given in Eqs. (14) and (15), respectively. This pheromone trail is defined by recursive Eq. (20) explained in the **updatePheromone** procedure.

$$p(assign)_{tmg} = \frac{ph(dist)_{tmg}}{\sum_{g\epsilon G,\, t\epsilon T,\, m\leqslant|T_t|} ph(dist)_{tmg}} \tag{14}$$

$$p(typeassign)_{tg} = \sum_{m\leqslant|T_t|} p(assign)_{tmg} \tag{15}$$

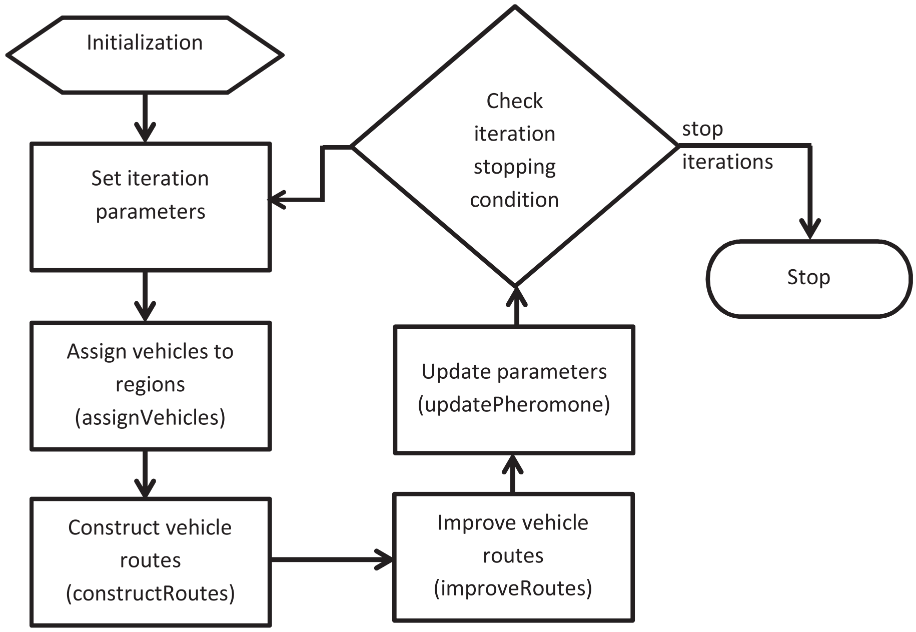


**Fig. 1.** Main steps of the heuristic algorithm.

Steps of the procedure **assignVehicles** are given as follows:

- **Step 1.** If accumulation of $ph(dist)_{tmg}$ is realized, go to Step 2, o.w. assign vehicles w.r.t. *Method 1* and STOP.
- **Step 2.** Set $p(typeassign)_{tg}$ for all $t\epsilon T$ and $g\epsilon G$ w.r.t. the current $ph(dist)_{tmg}$ and go to Step 3.
- **Step 3.** Randomly choose a $(t,g)$ w.r.t. $F(typeassign)$ which is cdf (cumulative distribution function) for $p(typeassign)_{tg}$. If assigning a vehicle of type $t$ to region $g$ does not create a result such that remaining nodes cannot be covered with the remaining unassigned vehicles, update $\mu_{tg}$ and go to Step 4; otherwise repeat this step.
- **Step 4.** Set $p(typeassign)_{tg}$ to $p(typeassign)_{tg} - p(assign)_{t\mu_{tg}g}$ and go to Step 5.
- **Step 5.** If all vehicles are assigned to a region STOP; o.w. normalize the values $p(typeassign)_{tg}$ such that they sum up to 1, update $F(typeassign)$, go to Step 3.

**constructRoutes**

With the application of this procedure, a solution to the problem (MFMDVRP) is constructed with the ACO approach. Before defining the procedure formally, we need to explain the additional required parameters:

$t(real)_k$: realized travel time by vehicle $k$.
$t(prospective)_{kj}$: travel time by vehicle $k$, if it visits and serves (non-visited) demand point $j$, which is equal to $t(real)_k + \frac{p_{ij}}{s^k} + \frac{q_j}{r_d^k}$.
$myopic_{kj}$: local desirability (visibility) of serving the demand point $j$ by vehicle $k$, which is equal to the reciprocal of $t(prospective)_{kj}$.
$ph(node)_{kj}$: Pheromone concentration on demand point $j$ for vehicle $k$.
$ph(edge)_{kj}$: Pheromone concentration on the edge exiting from current position of vehicle $k$ and entering to demand point $j$ for vehicle $k$.

We define two different transition probabilities: $p(myopic)_{kj}$ and $p(global)_{kj}$, given in Eqs. (16) and (17), respectively. They are employed in constructing probability distribution which represent vehicle $k$'s visiting (and serving) probability of demand point $j$. $p(myopic)_{kj}$ is used until $ph(node)_{kj}$ and $ph(edge)_{kj}$ accumulate in the same manner as it is in the accumulation of $ph(dist)_{tmg}$ explained in the procedure **assignVehicles**. Then $p(global)_{kj}$ is used for determining transition of a vehicle to a demand point.

$$p(myopic)_{kj} = \frac{myopic_{kj}}{\sum_{k\epsilon K\, j\epsilon V} myopic_{kj}} \tag{16}$$

$$p(global)_{kj} = \frac{ph(node)_{kj} * \alpha + ph(edge)_{kj} * \beta + myopic_{kj} * \gamma}{\sum_{k\epsilon K\, j\epsilon V} ph(node)_{kj} * \alpha + ph(edge)_{kj} * \beta + myopic_{kj} * \gamma} \tag{17}$$

Note that $p(myopic)_{kj}$ represents only visibility, while $p(global)_{kj}$ represents both visibility and learnt knowledge. The weight parameters of $p(global)_{kj}$; $\alpha$, $\beta$ and $\gamma$ represent the biases for pheromone trails on nodes and on edges, and visibility, respectively. The steps of the procedure **constructSolution** is given as follows:

- **Step 1.** If accumulation of $ph(edge)_{kj}$ and $ph(node_{kj})$ is realized, set $p(global)_{kj}$ for all $k\epsilon K$ and $j\epsilon V$ and go to Step 3, o.w. set $p(myopic)_{kj}$ for all $k\epsilon K$ and $j\epsilon V$, go to Step 2.
- **Step 2.** Randomly choose a $(k,j)$ w.r.t. $F(myopic)$ which is cdf for $p(myopic)_{kj}$, assign demand point $j$ as the next node on the path of vehicle $k$, go to Step 4.
- **Step 3.** Randomly choose a $(k,j)$ w.r.t. $F(global)$ which is cdf for $p(global)_{kj}$, assign demand point $j$ as the next node on the path of vehicle $k$, go to Step 4.
- **Step 4.** If all demand points are visited (and served) by the vehicles STOP, o.w. update $p(myopic)_{kj}$ if accumulation of $ph(edge)_{kj}$ and $ph(node)_{kj}$ is realized and go to Step 3, o.w. go to Step 2.

**improveRoutes**

An improvement heuristic is applied to the solution obtained by **constructSolution**. In our experiments we applied 3-opt method in order to shorten the travel times of $k$ vehicles. Another approach may be exact optimization of visit sequence when the amount of visited nodes are below a specific level for a quick solution.

**updatePheromone**

The pheromone update rules are similar for all of the three pheromone trails used in the model. Let $z^b$ and $z_l$ denote the objective function values of the best solution (obtained until the corresponding time) and the solution found in the last iteration. $\Delta ph$ is equal to $\frac{z^b}{z_l}$ for the visited nodes, the traversed edges and the vehicle-region assignments used in the last iteration, while it is equal to zero for the remaining unused nodes, edges and vehicle-region assignments. The update rules are given in the following formulas:

$$ph(node)_{kj} := eva(node) * ph(node)_{kj} + \Delta ph \quad (18)$$

$$ph(edge)_{kj} := eva(edge) * ph(edge)_{kj} + \Delta ph \quad (19)$$

$$ph(dist)_{tmg} := eva(dist) * ph(dist)_{tmg} + \Delta ph \quad (20)$$

Note that $\Delta ph$ is common for all of the update functions, while the evaporation parameters $eva(node)$, $eva(edge)$ and $eva(dist)$ are defined separately to allow more flexibility.

## 4. Experiments

Since there is no benchmark data, due to similarity of the problems MFMDVRP and VRPSDHD (Vehicle Routing Problem with Split Delivery and Heterogeneous Demand), the experiments have been conducted with the same instances used in the experiments of VRPSDHD reported in the study of Yakıcı and Karasakal (2013). Note that the heuristic solution values for VRPSDHD serves as a reference for the performance of the heuristic approach suggested in this study. Since the optimal solution of VRPSDHD (which allows splitting demand) is a lower bound for the optimal solution of MFMDVRP, finding close solutions to VRPSDHD solutions (or better solutions) proves the performance of the suggested heuristic. The test problems are taken from the library of CVRP (available in http://neo.lcc.uma.es/radi-aeb/WebVRP/).

The speed and the service rates for each vehicle type is given in Table 1. The data required for the test instance with 31 nodes (coordinates and regional membership of demand points, required quantity and type of demand for each demand point) are given in Table 2. The required data for the remaining instances are given in the paper of Yakıcı and Karasakal (2013).

In the test instances, two types of demand (*Type 1 Demand* and *Type 2 Demand*) and three regions are utilized. Two of four vehicle types satisfy only *Type 1 Demand* and one type meets only *Type 2 Demand*. Remaining type of vehicle can serve both of the service demands.

Before the numerical experiments, we conduct an analysis on the factors of evaporation rate and weight parameters: $eva(edge)$, $eva(node)$, $eva(dist)$, $alpha$, $beta$ and $gamma$. The test instance with 31 nodes (see Table 2) is used in this analysis. Each parameter is given four levels (0.1, 0.4, 0.7, 1), and for each combination of these levels, the heuristic is run for 5 times. Average of the heuristic solutions for each combination is taken as response. Several models are evaluated w.r.t. the sum of squares due to error. The model with maximum $R^2$ has nonlinear regressors (like $alpha * beta$, $beta/gamma$, etc.) and $R^2$ value of 58%. Since adding nonlinear regressors have resulted in insignificant increase in $R^2$, the best linear model having the regressors $eva(edge)$, $eva(dist)$, $alpha$, $beta$ and $gamma$ is considered as the reference model ($R^2 = 57\%$). In the experiments, the parameter values are set to their minimum level according to the negative coefficients of this regression model.

**Table 1**
Data of vehicles in test instances.

| Vehicle type | Transfer speed | Service rate (*Type 1 Demand*) | Service rate (*Type 2 Demand*) |
|---|---|---|---|
| A | 7 | 3.5 | 0 |
| B | 6 | 3 | 0 |
| C | 10 | 0 | 5 |
| D | 8 | 4 | 4.5 |

**Table 2**
Data of vertices in test instance E031 (Generated using E031-09h, Hadjiconstantinou et al., 1995).

| Vertex | Coordinates | Region | Demand type | Demand |
|---|---|---|---|---|
| 0 | 40 40 | – | – | – |
| 1 | 22 22 | 2 | 2 | 18 |
| 2 | 36 26 | 1 | 2 | 26 |
| 3 | 21 45 | 2 | 2 | 11 |
| 4 | 45 35 | 1 | 2 | 30 |
| 5 | 55 20 | 2 | 1 | 21 |
| 6 | 33 34 | 1 | 1 | 19 |
| 7 | 50 50 | 1 | 2 | 15 |
| 8 | 55 45 | 2 | 2 | 16 |
| 9 | 26 59 | 2 | 1 | 29 |
| 10 | 40 66 | 2 | 2 | 26 |
| 11 | 55 65 | 2 | 1 | 37 |
| 12 | 35 51 | 1 | 1 | 16 |
| 13 | 62 35 | 3 | 2 | 12 |
| 14 | 62 57 | 3 | 2 | 31 |
| 15 | 62 24 | 3 | 1 | 8 |
| 16 | 21 36 | 2 | 1 | 19 |
| 17 | 33 44 | 1 | 2 | 20 |
| 18 | 9 56 | 3 | 2 | 13 |
| 19 | 62 48 | 3 | 1 | 15 |
| 20 | 66 14 | 3 | 2 | 22 |
| 21 | 44 13 | 2 | 2 | 28 |
| 22 | 26 13 | 3 | 1 | 12 |
| 23 | 11 28 | 3 | 2 | 6 |
| 24 | 7 43 | 3 | 1 | 27 |
| 25 | 17 64 | 3 | 1 | 14 |
| 26 | 41 46 | 1 | 1 | 18 |
| 27 | 55 34 | 1 | 2 | 17 |
| 28 | 35 16 | 2 | 1 | 29 |
| 29 | 52 26 | 1 | 1 | 13 |
| 30 | 43 26 | 1 | 1 | 22 |

The heuristic algorithm is run for 10 times for each test instance. A limit of 10 min and 1000 iterations, whichever comes first, is applied to each run of the heuristic. After the heuristic algorithm is applied, a Set Covering Problem (SCP) is solved with the routes generated in the iterations of the heuristic run. In order to avoid misvaluation, the routes of the heuristic run which has the closest solution value to the average of 10 solutions are, taken. In order to avoid redundancy, the routes longer than the heuristic solution of corresponding run are eliminated in creating the data for SCP. A time limit of 2 min is used, and the best solution found in this period is reported as the solution of SCP.

Application of the exact solution method to MFMDVRP is limited to 1 h. All experiments are conducted with a personal computer with 2.6 GHz CPU. The exact solution method for solving MFMDVRP and SCP is applied with MIP solver of CPLEX in GAMS 24.2.2.

The results are reported in Table 3, where the instance code is given in the first column. The first section denotes the group and node quantity as in original notation in the library of CVRP. The quantities of each vehicle type are shown after the symbol $k$ respectively. $x$ indicates that the demand originally defined in the library is multiplied by the number coming after $x$. Although multiplying demand does not add a practical use for our problem, we did the multiplications to have the same test instances presented in the study of Yakıcı and Karasakal (2013) in order to compare the results. In the columns named as MIP and MIP+, we have reported the lower bound and the incumbent solution of the exact method application after one hour is elapsed. MIP refers to the

**Table 3**
Experiment results.

| Problem ID | MIP1 | | MIP+ | | MFMDVRP Heuristic | | | | VRPSDHD |
|---|---|---|---|---|---|---|---|---|---|
| | LB[a] | Sol'n[b] | LB [a] | Sol'n [b] | Ave. Sol'n[c] | Best Sol'n[d] | SCP Sol'n[e] | SCP Time[f] | Heur.Sol'n[g] |
| E031-k2/2/2/2 | 23.90 | 29.77 | 28.16 | 28.88 | 33.29 | 31.47 | 32.38 | 11 | 26.69 |
| E-n41-k3/3/3/3:*x*5 | 87.95 | 110.34 | 88.26 | 106.74 | 126.00 | 123.96 | 101.66 | 63 | 97.85 |
| A-n45-k2/2/4/6:*x*5 | 56.88 | 95.04 | 57.33 | 86.90 | 100.29 | 92.99 | 72.98 | 115 | 70.46 |
| P-n51-k2/1/3/4 | 24.16 | | 24.14 | 34.82 | 40.41 | 38.45 | 32.84 | 22 | 26.73 |
| P-n51-k2/2/4/6 | 17.24 | 36.84 | 17.28 | 26.91 | 34.47 | 33.05 | *23.43* | 65 | 27.09 |
| E-n76-k1/1/2/4 | 49.41 | | 49.81 | 71.31 | 79.64 | *73.35* | | 5 | 77.75 |
| E-n76-k2/2/4/6 | 28.16 | | 28.22 | 65.78 | 54.23 | 51.04 | *45.06* | 120 | 46.04 |
| E-n101-k1/1/2/4 | 52.80 | | 52.96 | | *83.52* | *82.29* | *79.95* | 18 | 83.68 |
| E-n101-k2/2/4/6 | 29.61 | | 30.16 | | 59.09 | 58.69 | 57.74 | 46 | 50.35 |
| M-n121-k1/1/2/4 | 45.64 | | 45.58 | | 73.29 | *69.38* | *71.43* | 96 | 71.58 |
| M-n121-k2/2/4/6 | 25.61 | | 25.68 | | 52.65 | 50.71 | 45.12 | 120 | 44.45 |
| M-n151-k1/1/2/4 | 78.09 | | 78.09 | | 128.51 | 124.31 | *119.21* | 12 | 119.59 |
| M-n151-k2/3/4/6 | 41.78 | | 42.16 | | 82.36 | 79.10 | *74.84* | 52 | 74.96 |

[a] Lower bound found in one hour.
[b] Best solution found in one hour.
[c] Average solution of 10 heuristic runs. Each run is limited to 1000 iterations and 10 min.
[d] Best solution in 10 heuristic runs.
[e] SCP is solved with the routes generated by the heuristic. The run of which solution is closest to corresponding average solution is referenced. The routes longer than the solution of the corresponding run are eliminated before SCP application.
[f] The time elapsed in seconds when the reported SCP solution is found. The time given to SCP is limited to 2 min.
[g] The heuristic solution values for the VRPSDHD instances reported in Yakıcı and Karasakal (2013) serves as a reference for the performance of the applied heuristic in this study. Finding close or even better solutions without splitting demands proves good performance of the proposed method.

model defined in (1)–(12). In MIP+, the additional cut defined in (13) is added in order to see its contribution to the solution process. In the columns under the title MFMDVRP Heuristic; the average solution of 10 heuristic runs (limited to 1000 iterations and 10 min), the best solution in 10 heuristic runs, the objective function value of SCP solution using the routes generated by heuristic and the time elapsed in seconds when the reported SCP solution is found are given. Since we use heuristic solution values for VRPSDHD reported in the study of Yakıcı and Karasakal (2013) as a supporting reference for the performance of the applied heuristic in this study, it is reported in the last column. The solutions under the MFMDVRP Heuristic column (the average solution, the best solution and the SCP solution) are typed in italic characters, if they are less than the corresponding VRPSDHD solution.

The effect of cut (13) is observed when the solutions of MIP and MIP+ are compared. However, as the problem size increases, the application of exact method on both of the formulations fail to find a feasible solution.

Note that, an MFMDVRP heuristic solution worse than corresponding VRPSDHD heuristic solution does not imply inferiority of the proposed method, while the opposite case shows that the MFMDVRP heuristic method works better. It is clear that, when the problem size increases, using proposed method applying SCP mostly results in better solutions than VRPSDHD heuristic solution. This result also shows that, application of the proposed technique will improve solutions for the problem where the split delivery of service is allowed.

## 5. Conclusion and future research

A rich VRP with mixed fleet, mixed demand and regional constraints is introduced and solved using an algorithm based on the Ant Colony Optimization techniques. The problem takes its root from optimization requirement of a generic operation type conducted by a naval mine sweeping/hunting fleet. The platforms with varying sweeping/hunting capabilities and varying operation and transition speed operate on several channels or areas to neutralize sea mines of different types. Similar problems may also emerge in humanitarian logistics or disaster relief efforts.

In the experiments, it is observed that when a Set Covering Problem with the input composed of the routes generated in iterations of the proposed algorithm is solved, better solutions can be obtained in a very limited time period. Therefore, a decision maker should better use the proposed algorithm along with the application of SCP improvement with the obtained routes.

There may be improvement opportunities for the proposed algorithm. For example, several techniques used with ACO algorithms, like deamon action and application of restrictions on the maximum and minimum pheromone values, may be applied to see whether they lead to a significant improvement on the solution or not. Also, hybridization of metaheuristic approaches may give promising results.

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