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Solving Min-Max Vehicle Routing Problem

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***Abstract*—The present study is focused on the Min-Max Vehicle Routing Problem (MMVRP). According to the characteristics of model, hybrid genetic algorithm is used to get the optimization solution. First of all, use natural number coding so as to simplify the problem; apply improved insertion method so as to improve the feasibility of the solution; retain the best selection so as to guard the diversity of group. The study adopts 2- exchange mutation operator, combine hill-climbing algorithm to strengthen the partial searching ability of chromosome. Secondly, Boltzmann simulated annealing mechanism for control genetic algorithm crossover and mutation operations improve the convergence speed and search efficiency. At last, it uses simulated experiments to prove the effectiveness and feasibility of this algorithm, and provides clues for massively solving practical problems.**

***Index Terms*—MMVRP, improved insertion method, simulated annealing mechanism, hill-climbing algorithm, hybrid genetic algorithm**

I. Introduction

Vehicle Routing Problem is a typical NP problem. The research method of VRP mainly includes exactness algorithm, heuristic algorithm and intelligent optimized algorithm. Mester applied the evolution strategy with positive guidance into CVRP and reaped good optimization results [1]. Based on simple moving rules, Ergun constitutes large-scale neighboring scope search algorithm [2].

When getting the solution of problem with big scale and multi-restricted-condition, intelligent algorithm is more widely applied. Bell modified the ant colony algorithm and proposed the ant colony algorithm applicable to solve VRP [3]. Ali applied genetic algorithm to study dynamic VRP with the different capacities [4]. Bent used simulated annealing algorithm to solve VRP with time windows [5]. Fermin applied TS algorithm to solve VRPB [6].

In practice, there exists a type of problems, whose aim is not to demand the shortest distance or the cheapest expenditure of the whole route, but to demand the shortest distance or the shortest time of the longest sub route throughout the whole route, for which is called Min-Max Vehicle Routing Problem, MMVRP. For example, the arrangement of patrol route for the patrol vehicles [7], the arrangement of delivery routes for the airdropped goods in emergency [8], and the arrangement of delivery routes for the postmen [9].

Michael firstly solved the minimum boundary value of the objective function in MMVRP, and then used taboo search algorithm to get the solution [10]. Carlsson studied MMMDVRP, and used heuristic algorithm based on region division to solve the problem [11]. Esther proposed an approximate algorithm to solve MMVRP. This algorithm consists of two parts: (1) seek the shortest route in each route's distance constraints. (2) reduce the longest distance as possible as it can [12]. David applied improved branch and bound algorithm to solve MMVRP [13].

Considering the complexity of MMVRP, the essay proposed to apply hybrid genetic algorithm. Experiments proved that this algorithm can achieve not only better calculating results, but also better calculation efficiency and quicker convergence rate.

II. Mathematical Model

$$Z = Min\left\{ Max \sum_{i\in S}\sum_{j\in S}\sum_{k\in V} X_{ijk} d_{ij} \right\} \tag{1}$$

Constraints:

$$\sum_{k\in V} Y_{ik} = 1, \quad i \in H \tag{2}$$

$$\sum_{i\in H}\sum_{j\in S} q_i X_{ijk} \le W_k, \quad k \in V \tag{3}$$

$$\sum_{i\in S} X_{ijk} = Y_{ik}, \quad j \in S, \quad k \in V \tag{4}$$

$$\sum_{j\in S} X_{ijk} = Y_{ik}, \quad i \in S, \quad k \in V \tag{5}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijk} \le |m| - 1, \forall m \subseteq \{2,3...,n\}, \quad k \in V \tag{6}$$

$$\sum_{k\in V}\sum_{i\in S} X_{ijk} d_{ij} \le D_k, \quad j \in H \tag{7}$$

Decision Variable:

$X_{ijk} = 1$, if travel vehicle k moves from node i to node j, $i \ne j \in S$, $k \in V$; or $X_{ijk} = 0$.

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$Y_{ik} =$ 1, if travel vehicle k belongs to node i , $i \in H$, $k \in V$, or $Y_{ik} = 0$.

In the formula: $G\{g_r | r = 1,...,R\}$ is a series of aggregations of distribution centre in the place R (this essay only has one); $H\{h_i | i = R+1,..R+N\}$ is a series of clients' aggregations in the place N ; $S\{G\} \cup \{H\}$ is the combination of all distribution centers and clients. $V\{v_k | k = 1,..K\}$ is travel vehicle k 's aggregation; q_i is the demand amount of client $i(i \in H)$; W_k is travel vehicle k 's loading capacity; d_{ij} is the linear distance from client i to client j ; D_k is the travel vehicle k 's maximum travel mileage.

In the formula (1), the objective function is not to demand the shortest distance of routes in the whole circuit, but to demand the shortest distance of the longest route in the whole circuit; constraint (2) ensures each client to be served by only one travel vehicle of one type; constraint (3) is for travel vehicle's loading capacity, in order to ensure every vehicle in each route not to surpass its loading capacity; constraint (4) ensures every vehicle to reach the client and the distribution centre only once; constraint (5) ensures certain vehicle to be dispatched only once from certain distribution centre or certain receiving point; constraint (6) ensures the routes' connectivity, that is to say, once the vehicle arrives at a client node, it must leave from the node; constraint (7) is for travel vehicle's mileage, in order to ensure every vehicle in each route not to surpass its maximum travel mileage.

III. Parameter Design for Hybrid Genetic Algorithm

A. Genetic Coding

Adopt natural number coding and utilize one structure body to express the information of each supply point. When coding is used natural number of customer name, zero is expressed by distribution centre all along. Chromosome is just a path and the length of chromosome refers to amount of all points including starting and ending point. When the length of chromosome is ln, n-1 is the ending point.

B. The Formation of Initial Solution

Given h_k as the total number of client nodes served by vehicle k , aggregation $R_k = \{y_{ik} | 0 \le i \le h_k\}$ to correspond the client nodes served by the number k vehicle, Y_{ik} signified that vehicle k served in node i , Y_{0k} signified that the number k vehicle's beginning point was distribution centre. The procedures as such:

Step1: Order vehicles' initial remaining load capacity: $w_k^1 = w_k$, $k = 0$, $h_k = 0$, $R_k = \Phi$;

Step2: The demand amount corresponding to the i client node in a route q_i , order $k = 1$;

Step3: if $q_i \le w_k^1$, then order $w_k^1 = Min\{(w_k^1 - q_i), w_k\}$, if not turn to Step6;

Step4: if $w_k^1 - q_i \le w_k$, and $D_{i-1} + D_i \le D_k$; then $R_k = R_k \cup \{i\}$, $h_k = h_k + 1$ if not turn to Step6;

Step5: if $k > K$, then $k = K$, otherwise, $k = k$;

Step6: $k = k + 1$, turn to Step3;

Step7: $i = i + 1$, turn to Step2;

Step8: repeat Step2-7, K recorded the total used vehicles, R_k recorded a group of feasible routes.

C. Select Operator

The concrete steps are as followings.

Step1: Calculate the fitness value f_i of each individual for community. Suppose that the highest fitness value of actual group is f_{best} .

Step2: Calculate the total fitness value $\sum f_i$.

Step3: Calculate the corresponding fitness value $p_i = \frac{f_i}{\sum f_i}$ of each individual in community.

Step4: Randomly create α in [0,1]. If $p_1 + ... + p_{i-1} < \alpha < p_1 + ... + p_{i-1} + p_i$, select individual i enter into the next generation community.

D. Crossover Operator

Ordinal crossover can keep and combine different sequence ordinal structure unit. When two father individual cross, select one part of father individual 1 to keep into the corresponding sequence of father individual 2 and produce son individual.

However, if all chromosome crossover dots are not zero, good gene will be destroyed when ordinal crossover. It cannot guarantee that algorithm is convergent to the optimization. Therefore, ordinal crossover operation is improved in order to keep the part of good gene of parents and get the optimization as whole. The concrete operations can be shown as followings.

Step 1: Randomly select one part as crossover part from two father chromosome.

Step 2: If two genes at chromosome crossover dot are zero, directly have crossover operation.

Step 3: If genes at chromosome crossover dot are all zero, move the crossover dots to the left (right) till genes at left and right crossover dot are all 0. And then have ordinal operation.

For example, randomly select one part as crossover part according to the following gene representation of two father individuals.

$$p_1 : (0120|345|067890) \quad p_2 : (0341|067|205980)$$

Two crossover dots of p_1 are zero and they are directly as the father generation of ordinal crossover. These parts $|345|$ of good genes cannot be destroyed. All crossover dots of p_2 are not zero, and they can become zero after move them in the right. The crossover part of second father individual can be gotten, and good gene

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part $|672|$ can not be destroyed. The following formula can be gotten after exchange the crossover parts of p_1 and p_2.

$p'_1 : (0120|345|067890)$ $p'_2 : (03410|672|05980)$

Delete the same factors besides crossover parts in turn to p'_1 and p'_2. And supplement never appearing factors in chromosome so as to form into two individual gene strings of legal filial generation.

$o_1 : (0140|672|089350)$ $o_2 : (07210|345|06980)$

Comparing with standard ordinal crossover operator, improved crossover operator can make chromosome of filial generation inherit more gene information of father generation so as to guarantee the algorithm convergent. Therefore, ability of searching optimization solution is stronger than standard ordinal crossover operator.

E. Mutation Operator

The mutation strategy of this study is adopted 2-commutation mutation strategy, namely, randomly selecting mutated individual chromosome according to some mutation probability and two gene locations in this chromosome, exchanging gene in two places and form into new gene clusters.

If it is continuously appeared with zero code in gene clusters, exchange zero code and non-zero code in random place. Execute this step much times till that new gene cluster become the legal child generation individual.

Through validating, 2-exchanging mutation mode is in favor of jumping optimized solution as part, and it can obviously improve the efficiency of total algorithm.

For example, select the individual by probability, and randomly select two mutation gene places, for instance, the fifth place and eighth place.

Exchange the gene in two mutation gene situation and get new gene string.

$p^1 : (0120|345|067890)$

If zero code continuously appears in new gene string, exchange zero code with non zero in any situation. And execute this step many times till new gene string becomes into legal filial generation individual.

The experiment can prove that 2- exchange mutation mode is advantageous to skip partial optimization solutions and obviously improve the efficiency of total algorithm.

F. Mountain climbing operation

Add mountain climbing algorithm into genetic algorithm. Improve the solution of each generation structure so as to reduce the expense of solving route and quicken algorithm constringency speed. Mountain climbing algorithm adopts 2-opt method to take point exchanging operation.

Supposed routing line before exchanging is $s=\{...,x_i,x_{i+1},..x_j,x_{j+1},..\}$, it can get the routing line $s'=\{...,x_j,x_{i+1},...x_i,x_{j+1},..\}$ after exchanging location of two points. If the distance is equal to $\{d(x_{i+1},x_j)+d(x_i,x_{j+1})\}<\{d(x_{i+1},x_i)+d(x_j,x_{j+1})\}$, exchanging is successful and keeps exchanging result.

Otherwise, exchanging attempt is failure, cancel exchanging and furbish former routing line.

To the optimized individual of each generation group through genetic operation, it can realize mountain climbing operation through searching in neighbors.

The study adopts gene exchanging operator to realize climbing operation. The concrete steps are as followings.

Step1: Initial recycling time variable t=1, when the most optimal solution at present $s^*=s$ and its length is $l(s^*)$.

Step2: Randomly selecting two top points x_i, x_j in the most optimal route and $i<j$. x_i is not close to x_j.

Step3: Calculate saving distance,

$$\Delta c=\{d(x_{i+1},x_j)+d(x_i,x_{j+1})\}<\{d(x_{i+1},x_i)+d(x_j,x_{j+1})\} \quad (8)$$

If $\Delta c>0$, it isn't exchanged. If $t=t+1$, it shifts into step4.

Otherwise, execute exchanging. And the corresponding solution is s'. And the optimal solution is $s^*=s'$. If $t=1$, it shifts into step 2.

Step4: If $l(s^*)$ isn't reduced in the last x a circulation, this algorithm is over. Otherwise, it shifts into step 2.

Step5: Repeat step 1 to step 4 till reaching certain exchanging times.

G. Simulated annealing operation

In order to avoid falling into the region optimization process, utilize Boltzmann mechanism of simulated annealing algorithm, control the crossover and mutation operation of genetic algorithm. This hybrid algorithm can make convergence of optimal solutions jump from partial optimal so as to improve the quality and searching efficiency, make up a single optimization and realize global convergence.

Suppose that the fitness value of individual in some generation is f_1. After selection, crossover and mutation of genetic algorithm, fitness value of new individual is f_2. Suppose $\Delta f = f_2 - f_1$, the current temperature is T.

According to the mutation value of individual adaptability Δf and probability value, control individuals. The concrete steps are as followings.

Step1: If $\Delta f<0$, fitness value f_2 can be kept in the next generation. And fitness value f_1 can be removed from the overall.

Step2: If $\Delta f>0$, calculate $\exp(-\Delta f/T)$.

Step3: When take random number in $\exp(-\Delta f/T)>[0,1]$, the individual of fitness value f_1 can be kept in the next generation, and fitness value f_2 can be removed from the overall.

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Step4: When take random number in $\exp(-\Delta f / T) \le [0,1]$, the individual of fitness value f_2 can be kept in the next generation, and fitness value f_1 can be removed from the overall.

Ⅳ.EXPERIMENTAL CALCULATION AND RESULT ANALYSIS

Example one: The data originates from Document [14]. There are one depot and 20 client nodes, the coordinates and demand amount of each node is created randomly, as indicated in table 1(the depot's number is 0); give six vehicles of the same type, and the vehicle's load capacity is 8.

TABLE I.
KNOWN CONDITION OF EXAMPLES

Item	coordinate	Distribution amount
0	(52,4)	0
1	(15,49)	1.64
2	(0,61)	1.31
3	(51,15)	0.43
4	(25,71)	3.38
5	(38,62)	1.13
6	(35,45)	3.77
7	(100,4)	3.84
8	(10,52)	0.39
9	(26,79)	0.24
10	(87,7)	1.03
11	(24,89)	2.35
12	(19,25)	2.60
13	(20,99)	1.00
14	(73,91)	0.65
15	(100,95)	0.85
16	(7,73)	2.56
17	(69,86)	1.27
18	(24,3)	2.69
19	(66,14)	3.26
20	(9,30)	2.97

A. Solution of Hybrid Genetic Algorithm

Hybrid genetic algorithm adopts the following parameters as part.

The main parameters: population size of 60, the maximum number of iterations is 500; crossover 0.90, mutation operator is 0.2. Initial temperature is T_0 =250. Temperature coefficient is $\delta = 0.85$.

Randomly solve ten times and calculation results can be seen as table 2.

It can be known that new tabu search algorithm in the study all get the much higher solution during the course of ten times from table 2.

The average value of total distance is 1090.247(km) and the average using vehicles are six. The calculation result of algorithm is relatively steady. The total distance of least solution is only better 1.180 percent than the best.

TABLE III.
RESULTS OF MMVRP USING HYBRID GENETIC ALGORITHM

Calculation order	Hybrid Genetic Algorithm		
	Total distance	The longest line	Vehicle amount
1	1084.013	205.767	6
2	1095.813	205.767	6
3	1090.863	205.767	6
4	1084.013	205.767	6
5	1095.136	205.767	6
6	1087.674	205.767	6
7	1084.013	205.767	6
8	1096.809	205.767	6
9	1087.674	205.767	6
10	1096.460	205.767	6
Average value	1090.247	205.767	6
Standard deviation	5.444	0	0

Here, the longest line is 205.767 km, the corresponding optimal total length of 1084.013 km. The concrete route can be seen in table 3 and figure 1.

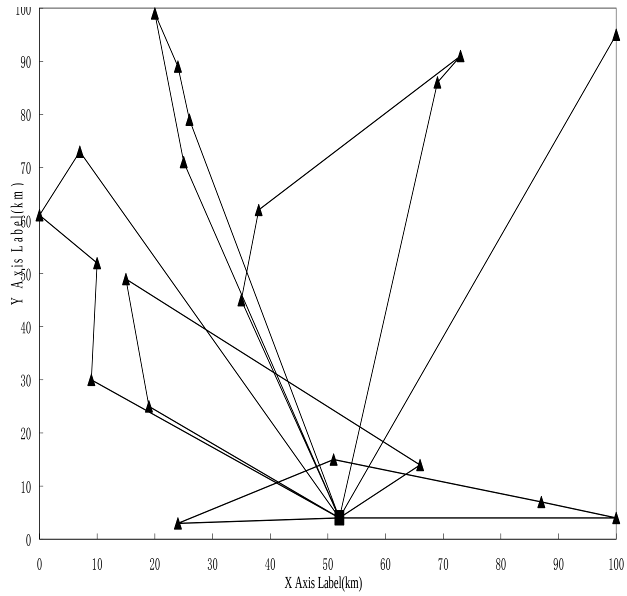

Figure 1. Optimization route of MMVRP using hybrid genetic algorithm.

TABLE II.
RESULTS BY HYBRID GENETIC ALGORITHM

Line No.	Running Path	Mileage
1	0-16-2-8-20-0	181.416
2	0-9-11-13-4-0	201.293
3	0-18-3-10-7-0	155.784
4	0-6-5-14-17-0	197.247
5	0-15-0	205.767
6	0-12-1-19-0	142.506
The Total Mileage	1084.013 km	
Average Mileage	180.669 km	
The longest line	205.767 km	

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B. Solutions by genetic algorithm

Reference [14] is adopted genetic algorithm to get the solution.

The main parameters: population size of 50, the maximum number of iterations is 400; crossover 0.80, mutation operator is 0.2. The concrete route can be seen in table 4 and figure 2.

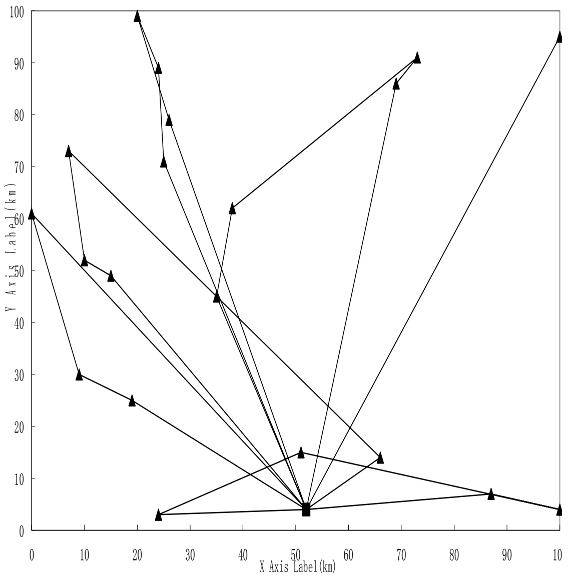

Figure 2. Optimization route of MMVRP using genetic algorithm.

TABLE VI.
OPTIMAL RESULTS BY GENETIC ALGORITHM

Line No.	Running Path	Mileage
1	0-1-8-16-19-0	185.945
2	0-9-13-11-4-0	201.293
3	0-18-3-7-10-0	156.254
4	0-6-5-14-17-0	197.247
5	0-15-0	205.767
6	0-12-20-2-0	159.731
The Total Mileage	1106.237 km	
Average Mileage	184.373 km	
The longest line	205.767 km	

C. Solutions by Tabu Search Algorithm

Reference [14] is adopted tabu search algorithm to get the solution.

The main parameters: the size of the neighborhood list of 20, the candidate set of 10, taboo list is 10, and end time is 5 seconds. The concrete route can be seen in table 5 and figure 3.

TABLE IV.
OPTIMAL RESULTS BY TABU SEARCH ALGORITHM

Line No.	Running Path	Mileage
1	0-18-10-7-0	152.486
2	0-3-9-16-2-8-20-0	199.298
3	0-15-0	205.767
4	0-4-13-11-5-0	201.529
5	0-12-1-19-0	142.506
6	0-6-17-14-0	193.550
The Total Mileage	1095.136 km	
Average Mileage	182.523 km	
The longest line	205.767 km	

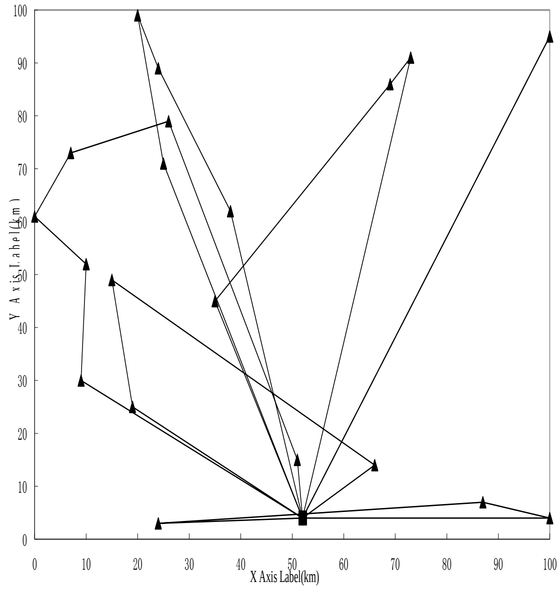

Figure 3. Optimization route of MMVRP using tabu search algorithm.

D. Analysis on Three Algorithms

Compared the optimal scheme of reference [14], the proposed hybrid genetic algorithm has a strong search capability, high computational efficiency and high quality on algorithm solving.

TABLE V.
COMPARISON AMONG GA, TS AND ALGORITHM OF THIS STUDY

	Genetic Algorithm	Tabu Search Algorithm	Algorithm of This Study
Average value	-	-	1090.247
Standard deviation	-	-	5.444
Vehicle amount	-	-	6
Optimal Solution No.	-	-	3
The Total Mileage	1106.237	1095.136	1084.013
Average Mileage	184.373	182.523	180.669
The longest line	205.767	205.767	205.767

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V. CONCLUSIONS

In general, the proposed hybrid genetic algorithm has strong searching ability, rapid convergence rate, strong ability to overcome the fall into local optimum and high solving high quality. Therefore, it is more practical significance and value so as to reduce operating cost and improve economic benefit.

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