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Runway Operations Optimization in the Presence of Uncertainties

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A stochastic runway planning model has been developed for scheduling of airport runway operations in the presence of uncertainty, where stochastic attributes include pushback delay, time spent on taxiway, and deviation from estimated arrival time. The runway planning problem is modeled as a process in two stages, where the first stage uses a two-stage stochastic program to find an aircraft weight-class sequence that maximizes throughput while simultaneously achieving a desirable sequence for the second planning stage without having exact information on the stochastic parameters. In the second planning stage, individual aircraft are assigned to the sequence after exact information becomes available. The computational study shows that under certain assumptions, if the schedule is dense enough, there is a potential benefit of using the stochastic runway planner over a first-come, first-served planning policy or a deterministic runway planner.

Nomenclature

$c^{\omega}_{f,i}$ = cost of assigning flight f to slot i in scenario ω
e_f = maximum time flight f can be early
F = set of flights
$F(k)$ = set of flights of type k
$h(\cdot)$ = convex function describing the cost of runway throughput
$h_f(\cdot)$ = convex function describing the cost of schedule deviation for flight f
I = set of runway slots
$I(f)$ = set of runway slots available for flight f
K = set of aircraft types
K' = set of aircraft types preceding the flight set under consideration
M = a sufficiently large constant
n = number of runway slots
n_k = number of flights of aircraft type k
Q = set of linear segments approximating cost function $h(\cdot)$
$Q(f)$ = set of linear segments approximating cost function $h_f(\cdot)$ for flight f
$S^{\min}_{k_1,k_2}$ = minimum separation requirement between leading aircraft type k_1 and trailing aircraft type k_2
T_f = realized runway arrival/departure time for flight f
T^L = latest scheduled arrival/departure time
T^{ω}_f = runway arrival/departure time for flight f in scenario ω
$\bar{t}_k$ = time for latest slot of type $k \in K'$
V = set of sequences where the triangle inequality is not satisfied
w_i = aircraft type assigned to slot i
α^q_f, β^q_f = slope and intersect of linear segment $q \in Q(f)$ for flight f
α^q, β^q = slope and intersect of linear segment $q \in Q$
λ_{ω} = probability of scenario ω
Ω = set of scenarios

I. Introduction

THE airport surface is a very complex operating environment as the entities in the system, namely the aircraft, must pass through many different subcomponents between the time they land and takeoff. The sequence of airport components that a typical aircraft operation passes through begins with the arrival runway. After landing, an aircraft exits the runway and enters the taxiway on its way to the ramp area and ultimately the gate. While at the gate, the aircraft is refueled, new passengers board and the sequence of operations is reversed. During peak hour of operations, the resources in each of these subcomponents are less than the demand and a significant amount of coordination needs to take place to ensure safe and efficient operations. In addition, there is a constant interaction between the components; e.g., delay in the ramp or taxiway area impacts the departure process on the runway, or a gate shortage can create congestion in the ramp area.

The runway subsystem has been identified as the major source of delay in the departure process [1]. At peak hours the demand is often higher than the effective capacity of the runway or runways, which generally causes aircraft to queue before departing. The capacity of the runway is determined by the fleet mix and by the wake vortex separation requirements. To ensure safe operations, the Federal Aviation Administration (FAA) has defined separation requirements that apply to different phases of both the arrival and departure procedures. The FAA formally defines three aircraft weight classes from which the requirements are defined: small (S), large (L), and heavy (H) [2]. In their documentation, the FAA frequently considers Boeing 757 (B757 or 7) as an exception, which, in practice, makes it four weight classes. In this paper, four weight classes are considered and the separation requirements are calculated by using representable landing speeds and runway occupancy times for the different weight classes. Given that the runways are considered the bottleneck in the departure process, procedures and models have

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been developed with the goal to increase departure throughput [3–5]. Most previous methods to plan or optimize the departure operations assume that the runway departure times are known with certainty at the time of planning. However, in real operations unanticipated delay in the pushback process or taxiway operations may lead to disruptions in the departure schedule, and the benefit provided by the planning or optimization can be lost.

Significant research has been performed on the arrival side of the process as well [6–9]. It has been recognized that the first-come, first-served (FCFS) method of sequencing arrival aircraft, the method most widely used in practice, is very robust in practice. Advanced sequencing methods improve landing rates over FCFS only when the arrival rate is greater than the maximum runway landing capacity and the aircraft weight classes are sufficiently mixed [10]. Furthermore, in the same study it is shown that optimal deterministic sequences may be suboptimal in a dynamic environment.

The deterministic version of the runway planning problem has been well studied in the literature. Bianco et al. [6] noted that the problem is NP-complete and presents integer programming based formulations. Before that, Dear [7] described a computationally more tractable version of the arrival sequencing problem where the number of shifts from a nominal first-come, first-served policy is constrained. Various dynamic-programming based algorithms [8,9,11] extend the constrained position shifts idea. For the departure-sequencing problem, Anagnostakis and Clarke [3] suggested a two-stage approach, where the first stage is used to find a departure sequence based on aircraft weight classes only. In the second stage individual flights are assigned to positions in the departure sequence. Other papers on departure sequencing discuss constrained number of position shifts [4] and departure queue positions [5].

Most literature on runway scheduling treats the departure planning problem separately from the arrival planning. This is a valid assumption in most cases, since the departure and arrival processes can work independently from one another. Many larger airports, especially airports serving as hubs, have configurations with close parallel runways,[¶] where one runway is a dedicated arrival runway and the other runway is dedicated to departures. Under instrument flight rules (IFR) these runways can no longer be treated as independent and for efficient usage of the close parallel runways the arrival and departure planning process should be merged.

The contributions in this paper include a novel two-stage stochastic runway planning model. To the best of our knowledge, it is the first stochastic planning model for runway operations. The objective of the two-stage model is to increase throughput simultaneously minimize the deviation from scheduled departure or arrival time for individual aircraft. This is done by generating an aircraft weight-class sequence that is used in a later planning stage. The structure of the two-stage runway planning model allows for efficient computations using a Benders decomposition scheme. The concept of virtual runways is introduced to allow modeling of a general airport configuration including both arrival and departure runways. Furthermore, some stochastic characteristics of the departure and arrival processes are presented and included in the formulation. The model is designed for traffic scenarios with high demand for arrivals and departures, and in situations where runway dependencies have an impact on the throughput of the dependent sets of runways, such as airport designs with close parallel runways.

The paper is organized as follows: in the next section a two-stage planning process is presented together with the mathematical programs used in each planning stage. Various aspects of the model such as cost functions and distributions for the stochastic modeling are discussed. The paper ends with a description and results from the computer simulations that were performed.

II. Problem Formulation

The stochastic runway planning problem is modeled as a planning process in two stages. The first planning stage seeks to find a weight-class sequence that maximize throughput while simultaneously achieve a desirable sequence for the second stage. In the second planning stage, uncertain parameters are realized and individual aircraft are assigned to positions in the weight-class sequence to minimize the cost of fuel consumption, emission, and crew and passengers delays.

The motivation behind the two-stage planning process is the observation that the throughput on a set of runways depends only on the aircraft weight-class sequence and not on the characteristics of individual aircraft. This planning approach was first introduced by Anagnostakis and Clarke [3] as a computationally efficient way to find good departure schedules, both with respect to runway throughput and the deviation from scheduled departure time for individual aircraft (where the scheduled departure times are deterministic). In addition, the two-stage planning process is appropriate for a stochastic version of the runway planning problem, since the information required in the first planning stage (namely, the pool of aircraft weight classes) is more robust than the scheduled time at runway for individual aircraft, which is the information needed in the second planning stage [12]. To exemplify, an air traffic controller with a 1 h look-ahead window is likely to predict the aircraft weight-class mix better than the arrival and departure times for individual aircraft.

The first planning stage is solved using a two-stage stochastic program. The objective of the two-stage program is to find an aircraft weight-class sequence that maximizes runway throughput and at the same time is "good" for the second planning stage with respect to the anticipated sequence of individual aircraft. Since the departure and arrival times are considered to be stochastic at this phase of the planning process, the two-stage mathematical program consists of a set of scenarios of possible runway departure and arrival times for the individual aircraft. The output from the first planning stage, i.e., the two-stage stochastic program, is an aircraft weight-class sequence. Hence, the two-stage stochastic program is referred to as the *sequence optimizer* (see Fig. 1).

In the second planning stage, runway departure and arrival times are assumed to be realized. The objective in this planning stage is to assign the aircraft under consideration to positions in the weight-class sequence obtained in the first planning stage. The complexity of the problem is decreased, since the aircraft weight-class sequence, with corresponding separation requirements, is fixed. Given the nature of this planning stage, the mathematical program used here is referred to as the *assignment optimizer*. The assignment optimizer is a mixed-integer program (MIP) that takes the aircraft weight-class sequence as input and assigns individual aircraft to the available slots. The MIP also updates the scheduled time of operation so that all the separation requirements are met. Although formulated as a MIP, the assignment optimization is easy to solve in practice, since the aircraft weight-class sequence has been specified; therefore, the MIP resembles a traditional assignment problem with some additional constraints.

Although the overall planning process requires both the sequence optimizer and the assignment optimizer, the emphasis in this paper is on the sequence optimizer, i.e., the two-stage stochastic program finding the aircraft weight-class sequence. The assignment optimizer is important for generating the final runway schedule and maintain separation requirements between aircraft, but from a computational perspective, this is a relatively simple deterministic model that is not hard to solve.

To facilitate the development of a model that is general enough to be used at any airport, the concept of virtual runways is introduced. Using virtual runways, all interactions between operations on the physical runways (not including taxiways) can be defined by the user. With this ability the planner can be used for different configurations, and even different airports, without changing the underlying model. The definitions of the virtual runways are given as user input to the model.

The runway planning problem does not always have a clearly defined objective. Maximizing runway throughput and minimizing flight delays seem to be the most commonly used objectives in the literature. In this application, a global cost function is developed that

[¶]Defined as parallel runways whose extended centerlines are separated by less than 4300 ft.

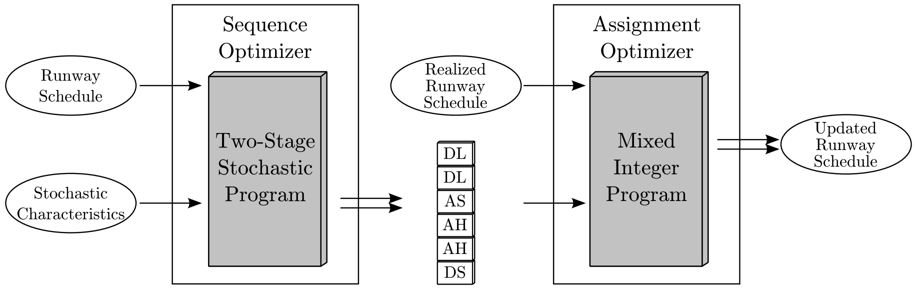


Fig. 1 Overall planning process is performed in two stages. In each stage a mathematical program is used.

captures relevant cost such as fuel, emissions, crew and passengers. By putting a dollar value on both the runway throughput and the individual flight delays, the two objectives can be used simultaneously.

There are many factors that can impact the scheduled runway departure time or the scheduled runway arrival time. In the stochastic version of the problem, the arrival and departure times are not known with certainty when the first-stage planning problem is solved; e.g., weather or surrounding traffic can delay arriving flights. Similarly, heavy traffic on the surface or a pushback delay can cause a departing flight to miss the scheduled runway departure time. Based on the characteristics of these sources of uncertainty, a number of outcomes for the time a flight reaches the runway are generated. For a sequence of flights, the combination of outcomes form a number of runway time scenarios. Since the total number of scenarios grow exponentially with the number of flights, only a subset of the scenarios are considered and sampling techniques are used to generate good solutions.

A. Sequence Optimizer

The objective of the stochastic program used in the first planning stage is to generate an aircraft weight-class sequence that is optimal with respect to runway utilization and with respect to the expected cost of assigning aircraft to positions in this sequence, which is done in the second planning stage. To find such a sequence, multiple scenarios are generated where each scenario lists a possible realization of runway departure and arrival times for the aircraft under consideration. The stochastic program is decomposed into two stages where runway utilization is handled in the first stage and flight assignment is considered in the second stage.

Using the standard four weight classes, the first-stage problem seeks to find an aircraft weight-class sequence that maximizes throughput in the presence of uncertainty about the estimated runway departure and arrival times. Maximization of the throughput is achieved by minimizing the time of the last operation scheduled on a runway, or set of runways. The significance of the sequence relates to the fact that the mandated separation requirements are asymmetrical; e.g., an arrival of a heavy aircraft followed by a small aircraft requires a larger separation than the reversed sequence.

Given a set of flights with attributes such as weight class, runway assignment, and scheduled runway departure/arrival time, it is implicitly defined how many slots that need to be considered and how many slots on each runway that need to be designated for a specific weight class. In this context, the combination of runway and aircraft weight class is defined as an aircraft type; e.g., AH (arrival runway and heavy), DL (departure runway and large), and AS (arrival runway and small) are three members of the set of aircraft types. With this in mind the first-stage model is formulated below.

1. *First-Stage Model*

The decision variables are as follows: t_i is the time of arrival/departure for runway slot $i \in I$, $x_{i,k}$ is 1 if runway slot $i \in I$ is used for aircraft type $k \in K$ and 0 otherwise, and r is the cost of exceeding latest scheduled time

The mathematical formulation is

$$\min r \tag{1}$$

$$\text{subject to } \sum_{i \in I} x_{i,k} = n_k \quad \forall\ k \in K \tag{2}$$

$$\sum_{k \in K} x_{i,k} = 1 \quad \forall\ i \in I \tag{3}$$

$$t_{i+1} - t_i \geq S^{\min}_{k_i,k_{i+1}}(x_{i,k_i} + x_{i+1,k_{i+1}} - 1) \quad \forall\ i \in I \quad \forall k_i, \quad k_{i+1} \in K \times K \tag{4}$$

$$t_0 - \bar{t}_k \geq S^{\min}_{k,k_0} x_{0,k_0} \quad \forall\ k_0 \in K, \quad k \in K' \tag{5}$$

$$r \geq (t_n - T^L)\alpha_q + \beta_q \quad \forall\ q \in Q \quad t_i \geq 0 \quad \forall\ i \in I \quad x_{i,k} \in \{0,1\} \quad \forall\ i \in I, \quad k \in K \quad r \geq 0 \tag{6}$$

In the first-stage model the binary variables $x_{i,k}$ define the sequence of aircraft weight classes. To determine the cost of runway utilization and simultaneously maintain separation requirements, each slot has a time, t_i, associated with it. The actual cost of the runway utilization is captured in the variable r.

The objective (1) is to minimize the cost of violating the last scheduled arrival/departure time: i.e., minimize the cost of runway utilization. The first set of constraints (2) states that the number of slots for each aircraft type must match the number of aircraft of each type in the data set. Furthermore, only one aircraft type can be assigned to each slot, per constraint (3). The next set of constraints (4) define the separation requirements for consecutive positions in the sequence; e.g., if aircraft type AH is assigned to slot 1 and aircraft type DS is assigned to slot 2, then the separation between these operations must be at least $S^{\min}_{\text{AH,DS}}$ time units. Similarly, constraint (5) defines the separation requirements between slots preceding the data set under consideration and the first slot that is being scheduled. The convex cost function $h(\cdot)$ describing the cost of runway throughput (see Sec. II.D) is approximated by a piecewise-linear function. Since the piecewise-linear approximation is convex, standard techniques can be used to model the objective function. More specifically, each linear segment is modeled in constraint set (6), where $(t_n - T^L)$ expresses the delay from the last scheduled operation.

2. *Second-Stage Model*

In the second stage of the sequencing model, individual aircraft are assigned to the sequence obtained in the first stage. Since the arrival and departure times are not known with certainty, a number of scenarios are created. A scenario is a sequence of flights where the runway departure/arrival time has been realized through sampling from the distribution describing the time of arrival/departure. The cost of assigning an aircraft to a certain position in the sequence is given by $c^{\omega}_{f,i} = h_f(\bar{t}_i - T^{\omega}_f)$. Here, the indices ω, f, i denote the scenario, flight and position in the sequence, respectively. T^{ω}_f is the realized runway departure/arrival time for flight f in scenario ω and $\bar{t}_i$ is the estimated time for position i in the sequence. The estimation is based on a first-come, first-served schedule, which is likely to be close to the optimal sequence.

The input to the model is the aircraft type sequence obtained in the first stage and the realized runway departure/arrival time for each flight in the given scenario. Since the assignment in one scenario is independent from the assignment in other scenarios, one second-stage model is defined for each scenario and solved independently. The output from the second-stage model is the flight-to-slot assignment in each scenario. In addition, each subproblem generate dual information that is used in the decomposition framework. For each scenario, the second-stage subproblem is modeled as follows:

The parameters from the first stage are as follows: $\bar{x}_{i,k}$ is 1 if runway slot $i \in I$ is used for aircraft type $k \in K$ and 0 otherwise. The decision variables are as follows: $y^{\omega}_{f,i}$ is 1 if flight $f \in F$ is assigned to runway slot $i \in I$ in scenario $\omega \in \Omega$ and 0 otherwise.

The mathematical formulation is as follows. For each $\omega \in \Omega$,

$$\min \sum_{f\in F}\sum_{i\in I} c^{\omega}_{f,i} y^{\omega}_{f,i} \tag{7}$$

$$\text{subject to } \sum_{i\in I} y^{\omega}_{f,i} = 1 \quad \forall\ f \in F \tag{8}$$

$$\sum_{f\in F(k)} y^{\omega}_{f,i} = \bar{x}_{i,k} \quad \forall\ i \in I, \qquad k \in K \tag{9}$$

$$y^{\omega}_{f,i} \in \{0,1\} \quad \forall\ f \in F, \qquad i \in I \tag{10}$$

The objective (7) in the second-stage model is to minimize the total cost of assigning individual flights to positions in the sequence. This is a pure assignment problem where each flight must be assigned to exactly one slot, depicted in constraint set (8). Furthermore, only aircraft of the appropriate weight class and runway assignment can be assigned to a specific position in the sequence, illustrated in constraint set (9). Note that the weight-class assignment is given as input to the problem from the first-stage solution. With the assignment structure of the problem, the integrality constraint on the decision variables can be relaxed, which allows for fast solution times for the subproblems [13]. Hence, the integrality constraint (10) can be replaced with

$$y^{\omega}_{f,i} \ge 0 \quad \forall\ f \in F, \qquad i \in I \tag{11}$$

3. *Benders's Decomposition*

Benders's decomposition is a decomposition method suitable for mathematical programs where a limited number of the variables are linking variables [14]. A common area of application is stochastic two-stage programs where the second stage can be modeled as a linear program. If the linking variables are excluded from the constraint matrix the remaining variables form a block-diagonal structure. Each block can then be solved independently from the other blocks or, in other words, the model decomposes into one or several subproblems that can be solved with much less effort than solving the complete program. Within the context of two-stage stochastic programs, each scenario form one subproblem. Information from the subproblems are passed back to the problem containing the linking variables, called the master problem. The iterative process is continued until the optimal solution to the full problem is obtained.

The structure of the sequence optimizer allows for efficient computations using Benders's decomposition, where the first-stage model is considered to be the master problem. The binary $x_{i,k}$ variables defining the sequence are the linking variables. The master problem is solved as mixed-integer program. Each scenario forms a linear subproblem that can be solved independently from all other subproblems, given the sequence from the master problem. More specifically, let $u^{\omega,p}_{f}$ and $v^{\omega,p}_{i,k}$ be the dual variables for constraints in constraint sets (8) and (9), respectively. P denotes the set of iterations before the current iteration. The dual objective in subproblem $\omega \in \Omega$ in iteration $p \in P$ is shown in Eq. (12):

$$\max \sum_{f\in F} u^{\omega,p}_{f} + \sum_{i\in I}\sum_{k\in K} v^{\omega,p}_{i,k} x_{i,k} \tag{12}$$

Since the subproblems in the decomposition scheme are always feasible, only Benders's optimality cuts need to be considered. Based on computational experiments it turns out that for this specific problem, the aggregated Benders's cut outperforms the traditional approach, where one cut is added for each subproblem. To the first-stage model, also referred to as the master problem, the unrestricted variable z is added together with the set of constraints forming the cuts:

$$\sum_{\omega\in\Omega} \lambda_{\omega}\left(\sum_{f\in F} u^{\omega,p}_{f} + \sum_{i\in I}\sum_{k\in K} v^{\omega,p}_{i,k} x_{i,k}\right) \le z \quad \forall\ p \in P \tag{13}$$

B. Assignment Optimizer

As time progresses, the uncertainty in the runway arrival/departure time for individual aircraft is reduced and the scheduling of the operations on the runways can take place. This is considered to be the second planning stage in the overall runway planning procedure. At this stage the runway departure and arrival times are treated as deterministic, where they have been realized from the probability distributions used in the first planning stage. Given the aircraft weight-class sequence from the first planning stage, the objective of the assignment optimizer is to assign the aircraft to positions in this sequence. In addition, the runway arrival and departure times are included in the model to ensure that the separation requirements between aircraft are maintained. It is worth pointing out that the realized scenario may not have been included as one of the scenarios in the sequence optimizer. As noted in Sec. II.E, due to computational limitations it is not possible to include all possible scenarios. Hence, the assignment of aircraft to positions in the sequence must be explicitly modeled in the assignment optimizer. The output from the assignment optimization provides an updated runway schedule for all arrivals and departures in the flight set.

The decision variables are as follows: t_i is the time of arrival/departure for runway slot $i \in I$, r_f is the cost of schedule deviation for flight $f \in F$, $y_{f,i}$ is 1 if flight $f \in F$ is assigned to runway slot $i \in I$ and 0 otherwise, and r is the cost of exceeding latest scheduled time.

The mathematical formulation is

$$\min \sum_{f\in F} r_f + r \tag{14}$$

$$\text{subject to } \sum_{i\in I(f)} y_{f,i} = 1 \quad \forall\ f \in F \tag{15}$$

$$\sum_{f\in F(w_i)} y_{f,i} = 1 \quad \forall\ i \in I \tag{16}$$

$$t_j - t_i \ge S^{\min}_{w_i,w_j} \quad \forall\ i,j \in I, \quad \text{and} \quad i < j \tag{17}$$

$$t_i \ge T_f - e_f - M(1 - y_{f,i}) \quad \forall\ i \in I, \qquad f \in F \tag{18}$$

$$r_f \ge (t_i - T_f)\alpha^q_f + \beta^q_f - M(1 - y_{f,i}) \quad \forall\ i \in I, \quad f \in F, \qquad q \in Q(f) \tag{19}$$

$$r \ge (t_n - T^L)\alpha_q + \beta_q \quad \forall\ q \in Q \tag{20}$$

$$t_i \ge 0 \qquad r_f \ge 0 \qquad r \ge 0 \quad \forall\ i \in I \quad \forall\ f \in F$$
$$y_{f,i} \in \{0,1\} \quad \forall\ f \in F, \qquad i \in I$$

The binary variables $y_{i,f}$ used in the assignment optimizer has the same purpose as the variables used in the second stage of the sequence optimizer: namely, to assign individual aircraft to slots in the sequence. In addition, a time for each slot, t_i, is used to capture the runway utilization and deviations from the realized runway schedule for the individual aircraft. The cost of runway utilization and aircraft deviations are captured in variables r and r_f, respectively.

The objective (14) in the assignment optimizer is to minimize the combined cost of deviations for individual aircraft and the cost of runway utilization. Aircraft types for the different slots are decided in the sequence optimization and aircraft can only be assigned to slots of the corresponding aircraft type, as given by Eq. (15). Similarly, per Eq. (16), a slot can only accommodate aircraft of the corresponding type. Since the sequence of aircraft types is known, the separation requirements can easily be modeled using Eq. (17). Many constraints in Eq. (17) are dominated by other constraints in the same set and can be eliminated in the preprocessing stage. The time assigned to each slot is determined by spacing requirements and the scheduled runway departure/arrival time for the aircraft assigned to the slot. By assumption, no departure can take place before the scheduled departure time, modeled in constraint set (18) by setting $e_f = 0$. The earliest possible arrival time is determined by the distance from the airport and the speed increase that is possible. Constraint sets (19) and (20) model the piecewise-linear functions for aircraft deviation cost and runway throughput cost, respectively.

C. Virtual Runways

The runway planner with its subcomponents, the sequence optimizer and the assignment optimizer, is designed to be general so that configurations at different airports can be modeled. In addition, it is desirable that the configuration can be given as user input to the model without having to change the optimization model. To achieve this, the concept of virtual runways is developed. A virtual runway is defined to be either a single runway or a set of physical runways where every runway in the set interacts with at least one other runway in the set. The interaction refers to the operations on the runway and does not include taxiway crossings. Airports can consist of multiple virtual runways, which decompose the overall runway scheduling problem into smaller subproblems that can be solved independently of the operations on other virtual runways.

For example, consider an airport with two close parallel runways where one runway is dedicated for arrivals and the other runway for departures. Under IFR these two runways are not independent, hence the two physical runways form a virtual runway. The separation requirements used for two close parallel runways are shown in Table 1.

It must be noted that the separation requirements do not satisfy the triangle inequality, which may cause a problem in the mathematical formulation. As an example, consider the sequence AH-DL-AS, where the arrival of the heavy aircraft is the first flight in the sequence and the arrival of the small aircraft is the last. The separation requirement between AH-DL is 15 s (between arrival touchdown and departure start of takeoff roll) and the requirement for DL-AS is 80 s (see Fig. 2). Since the first-stage model in its current state only consider consecutive positions in the sequence, the separation between AH-AS would be 95 s. This violates the separation requirement of 240 s.

To overcome the violation of the triangular inequality, the first-stage model needs to be extended and the cases where the triangle

Table 1 Separation requirements, in seconds, for two close parallel runways

	Trailing operation							
Leading operation	AH	A7	AL	AS	DH	D7	DL	DS
AH	96	138	138	240	15	15	15	15
A7	96	108	108	198	15	15	15	15
AL	60	72	72	162	15	15	15	15
AS	60	72	72	102	15	15	15	15
DH	48	56	56	80	90	90	120	120
D7	48	56	56	80	90	90	120	120
DL	48	56	56	80	60	60	60	60
DS	48	56	56	80	60	60	60	60

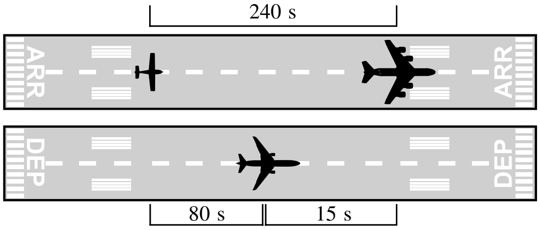


Fig. 2 Illustration of a case where the triangular inequality does not hold. The required separation requirements, in seconds, for the sequence AH-DL-AS are shown.

inequality is not satisfied must be explicitly modeled. Letting V be the set of sequences in which the triangle inequality is not satisfied, the aircraft types defining the sequences can be denoted k_0, k_r and R for the first type, the last type and the ordered set R containing the intermediate types. The length of the sequence is $r + 1$.

The additional constraints are modeled in constraint set (21) and added to the first-stage model:

$$t_{i+r} - t_i \geq S^{\min}_{k_0,k_r}\left(x_{i,k_0} + \sum_{r' \in R} x_{i+r',k_r} + x_{i+r,k_r} - (\|R\| + 1)\right)$$
$$\forall\ i \in I \quad \forall\ (r, k_0, kr, R)V \tag{21}$$

Similar to constraint set (5) modeling the initial separation to preceding slots, the explicit constraints for the triangle inequality must be included in a similar manner. This is done in constraint set (22), which is also added to the first-stage model:

$$t_r - \bar{t}_{k_0} \geq S^{\min}_{k_0,k_r}\left(\sum_{r' \in R} x_{r'-1,k_{r'}} + x_{r,k_r} - \|R\|\right)$$
$$\forall\ (r, k_0, k_r, R) \in V \quad \text{and} \quad k_0 \in K' \tag{22}$$

D. Global Cost Function

One of the key attributes for each flight is the scheduled runway departure or arrival time. In this application the runway departure time refers to the start of the takeoff roll and the runway arrival time refers to time of touchdown. With these definitions the scheduled runway time** can be used when modeling the separation requirements, as the requirements are based on the time for the start of takeoff roll and time for touchdown, respectively. In an ideal case, where the runway capacity is not a limiting factor, a flight would depart on the scheduled runway time. However, since aircraft have to be separated due to physical reasons and mandated separation requirements, there will be deviations from the scheduled runway time. The term *deviation* is used, as an arriving aircraft can reach the runway earlier through a speed increase. For departures, no flights are allowed to be rescheduled to a time before the scheduled departure time.

As noted in earlier sections, the objective of the mathematical model is twofold: to maximize runway throughput and minimize the deviation from scheduled runway time. Both of these objectives make use of a global cost function capturing all relevant costs. By putting a monetary value on deviation from scheduled runway time, the two objectives can be combined and used simultaneously in the model. The cost function developed by Solveling et al. [15] includes fuel cost; cost of emissions of CO_2, CO, NO_x, SO_2, and HC; noise cost; and cost related to crew and passengers. With the assumption that arriving aircraft initially fly according to their fuel- and emission-optimal speed, the cost functions are convex and can therefore be approximated with a piecewise-linear convex function in the formulation. To give some intuition to the convexity of the cost function, consider an arriving aircraft that touches down on time. From the assumption of optimal flight speed, the cost will be 0. If the

**The term runway time is introduced as a generalization of runway departure time and runway arrival time when the direction is irrelevant for the discussion.

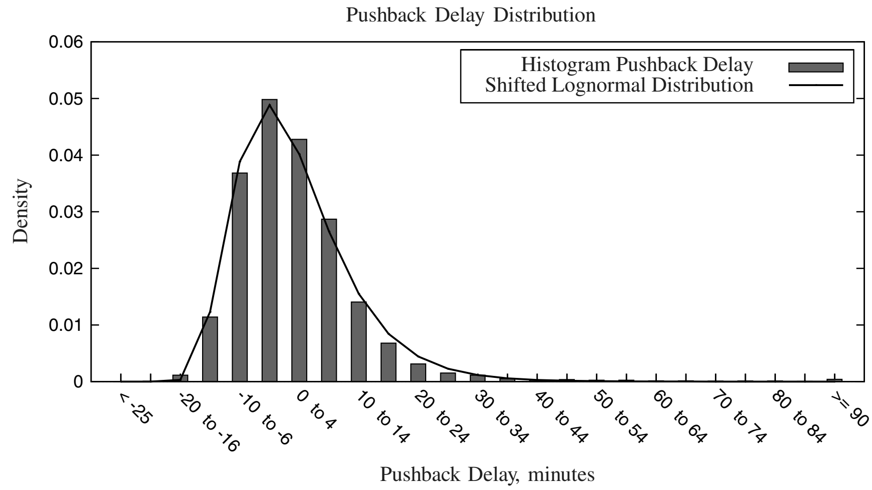


Fig. 3 Histogram of pushback delays is approximated with a shifted lognormal distribution starting at −25 with mean 26.1 and standard deviation 9.55.

flight is delayed by a short period of time, a small cost is incurred, mainly because of additional fuel burn. If the aircraft experience a longer delay, the cost will not only include fuel burn, but also missed passenger connections and additional crew cost. Similarly, if the flight touches down before scheduled time, a cost is incurred due to increased fuel burn.

With a global cost function defined for each flight in the data set, a cost function for runway throughput is developed. Let F denote the flights included in the optimization model and let F' be the flights succeeding the flights in F. For both sets F and F', the flights in the sets are ordered by scheduled runway time. For the purpose of constructing the cost function, the last flight in F is assumed to be optimized so that the updated time is the same as the scheduled runway time, $\Delta_0 = 0$. With this assumption, the flights in F' are scheduled according to a FCFS policy until the FCFS policy gives a zero delay for a flight or all the flights in F' are scheduled. The cost of the delay for each flight in F' is calculated, and adding these costs together the cost for violating the last scheduled runway time in F, T^L, by Δ_0 minutes is obtained. Repeating this with the assumption that the last flight in F' is delayed by $\Delta_1, \Delta_2, \ldots, \Delta_\eta$ minutes, a cost function for violating T^L can be constructed. The parameters η and Δ_i, $i = 0, \ldots, \eta$, can be chosen by the user to achieve the desired level of detail in the piecewise-linear function that is created. By the properties of convex functions, this function is also convex.

E. Stochastic Characteristics and Sampling

As mentioned earlier, each flight has a scheduled departure/arrival time. However, at the time of execution of the first-stage runway planner, the actual operation times are not known with certainty. As one of many reasons for this, consider a departing flight whose pushback is delayed due to problems in the boarding process. Another example could be a departing flight that has pushed back, but due to surrounding traffic, the ramp area is congested and taxiing is delayed. To simplify matters, two sources of uncertainties for the scheduled departure time from the runway are considered: pushback delay and delay along the taxi path from the gate to the runway.

An analysis of airport turn operations has been performed and an approximation of the pushback delay distribution has been obtained. The analysis was performed with 20,234 data points from the Bureau of Transportation Statistics data on on-time performance for Detroit Metropolitan Wayne County Airport (DTW) for the period between January and November 2006. The pushback delay in Fig. 3 is calculated as the difference between the actual turn time and the scheduled turn time for those flights where there is no arrival delay. Although in real-world operations there can be many sources for pushback delays, e.g., delays in the boarding process, late incoming crew or delays as a consequence of runway scheduling, the pushback delay in the stochastic model is treated as an exogenous parameter. The analysis on stochastic characteristics for the taxiing process was obtained from the literature [16].

An analysis similar to the one presented in Fig. 3 was performed for arriving flights. By studying data from FAA's Enhanced Traffic Management System estimates on the uncertainty of arrival prediction were obtained. More than 300,000 data points were studied and the sample means and sample standard deviations for the different time horizons can be seen in Fig. 4. For each time interval, a beta distribution was fitted and used to draw samples from.

Given these stochastic characteristics, the distribution around the estimated departure or arrival time is discretized to three levels. For the computational study, the mean and ± 1 standard deviation were used as the three levels. With three outcomes per flight, a sequence of n flights would generate 3^n scenarios if all scenarios are generated, which is a relatively large number even for small n. Although efficient, the two-stage model is not capable of handling all scenarios, even for n as small as 8. Instead, the sample average approximation [17] (SAA) method is used. The SAA method is a simulation-based approach to stochastic optimization problems where the objective function is approximated by repeatedly solving smaller instances of the problem. The main idea in the SAA method is that it is significantly faster to evaluate the expected value function given a prespecified solution than it is to find a solution. In other words, a solution is generated using a small number of scenarios, and the expected value of that solution is evaluated using a larger set of scenarios. This process is then repeated, and the best solution at termination is the solution the algorithm returns.

The SAA method used for the sequence optimizer is summarized in the following steps:

1) Generate N identically distributed scenarios based on the three outcomes for each flight.

2) Solve the two-stage model for the generated scenarios. Store the sequence corresponding to the optimal solution.

3) Repeat steps 1 and 2 for M replications. A list of M sequences (one sequence may be in the list more than once) is now obtained.

4) Collapse all identical sequences in the list to one sequence. A sequence needs to be listed only once.

5) For sequences in the list, fix the solution in the first-stage problem according to the sequence. Solve the second stage for all 3^n scenarios and store the objective-function value.

6) The solution to the complete procedure is obtained by selecting the sequence with the minimal objective-function value in step 5.

F. Implementation

The mathematical models are implemented in C++ using Cplex[††] 12.1 as the optimization solver, with Concert as the modeling layer. As in most stochastic programming formulations, the problem size

[††]Data available online at www.cplex.com.

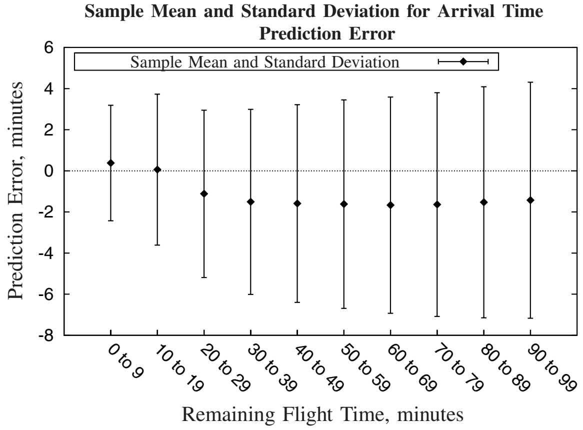


Fig. 4 Sample mean and sample standard deviation for arrival prediction observations. The prediction error was calculated as the actual arrival time minus the estimated arrival time. The prediction error depends on remaining flight time before reaching the runway.

grows fast. To be able to use the model in an environment where the data come from a simulation model, a rolling-horizon scheme is developed that allows larger data sets to be processed. After the sequencing problem is solved for a subset of m flights, the first $m/2$ positions in the sequence are fixed and the horizon is shifted by $m/2$ positions. For the results presented in Sec. III, $m = 8$ was used.

With regard to a potential practical application and for use in a simulation environment, the time horizons considered are of great importance. For the evaluation of the runway planner a time horizon of 60 min is used for the sequence optimization. All flights with a scheduled runway time that is within 60 min of the current simulation time are included to determine the sequence. Those slots that are scheduled for the next 30 min are kept fixed, and the flights scheduled for the next 15 min are assigned to slots fixed in previous iterations.

G. Limitations

Although the suggested two-stage planning model can add value to a stochastic planning framework, there are several aspects of the process that needs to be further investigated. These include some shortcomings of the proposed set of algorithms that have to be addressed before a practical application can be developed. An important factor that is not discussed here or in earlier work is the timing of decisions in a two-stage approach: in particular, on the arrival side. When should decisions be made about a desired sequence and when should this be communicated to individual aircraft? What flexibilities/limitations exist for this time frame? Currently, no such limitations, e.g., overtaking or fairness considerations, are included in the runway planner.

Given the two-stage planning process, the main shortcoming in the *sequence optimizer* is the simplified second-stage model. The assignments of individual aircraft to positions in the sequence are done using a fixed cost based on the first-come, first-served sequence. Ideally, the timing of each aircraft would be included in the second-stage model, providing the exact cost of assigning aircraft to positions in the sequence. Including this information into the second-stage model increases the complexity of the problem significantly, as the second stage must be modeled as a mixed-integer program and the current decomposition structure can no longer be used. The *assignment optimizer* uses the weight-class sequence as input when performing the flight-to-slot assignment after the runway times are realized. However, in the current version of the assignment optimizer, the sequence cannot be violated; i.e., if the realized outcome is significantly different from the input sequence, the optimization model has no way to detect this and allow the assignment to violate the weight-class sequence. Research is ongoing to address both these issues.

A more general shortcoming in stochastic optimization is the discretization of continuous probability functions. Continuous functions or discrete functions with a large number of possible realizations must be discretized to an appropriate level before they can be used for computations. In this application, the probability distribution for deviation from expected runway departure/arrival time has been discretized.

III. Computational Study

To evaluate the performance and impact of the runway planner, a simulation environment was developed. In addition to the optimization components, a module to progress the simulation and to generate realized departure and arrival times was developed. For reference purposes, this module is called simulation progression. A set of Python[‡‡] scripts tied the runway planner and the simulation module together. The simulation procedure is described below.

1) Start simulation 15 min before the first scheduled flight.
2) Run the runway planner.
   a) Perform sequence optimization for all flights with scheduled runway time in the next 60 min.
   b) Fix the sequence for the slots in the next 30 min.
   c) Assign flights with realized runway time within the next 15 min to the available slots.
3) Run the SP to increase the simulation time by 15 min.
   a) Remove all flights before the new current time.
   b) Realize runway time for flights scheduled in the next 15 min.
4) If there are no more flights to optimize, stop.
5) Go to step 2.

The data set used in the simulation was taken from a realistic flight schedule for DTW. Flights from a 2 h peak period were extracted, and the runway assignments were updated to achieve a balanced load. The airport configuration considered in the study, southwest-bound flow at DTW, has two pairs of close parallel runways, but only one pair was considered in the simulation. For runway planning models in general, the fleet mix has an impact on the results due to the asymmetric separation requirements. A second fleet mix with different characteristics was obtained [18], and simulations were performed on the updated fleet mix. The fleet mix in the original data set is referred to as fleet mix 1, and the second fleet mix is referred to as fleet mix 2. The characteristics of the data set can be seen in Table 2.

As the density of the schedule has a large impact of the performance of the stochastic runway planner, four copies of the initial schedule were created in which the span of the schedule has been compressed by 15, 30, 45, and 60 min. The resulting rates are presented in Table 3. To exemplify how the copies were created, consider a 2 h schedule compressed by an hour. If the beginning of the first hour is considered to be time zero and the runway times are expressed as units after time zero, the runway times are simply divided by two.

To evaluate the benefits of a stochastic runway planner over a FCFS policy and a deterministic policy, three different planning procedures were used. The *FCFS* procedure uses a one-stage first-come, first-served assignment. The first planning stage is not used and no aircraft weight-class sequence is defined. The scheduling, i.e., only enforcing separation requirements, is done for flights within 15 min of the current simulation time. The second planning procedure uses the two stages in the planning process, but does not use the two-stage stochastic program in the first planning stage. Instead, the aircraft weight-class sequence is generated by performing an optimization where the runway departure/arrival times are treated as deterministic. This is referred to as the *deterministic* procedure. The third planning procedure, the *stochastic* procedure, is the procedure presented in this paper. In the stochastic procedure, 30 scenarios and five samples were used in the experiments reported. Note that the deterministic procedure is equivalent to the stochastic procedure with one simple scenario where the runway departure times in the scenario are the scheduled runway departure/arrival times.

In the study, 100 Monte Carlo replications were performed for each of the five schedules with independent realizations of aircraft

[‡‡]Data available online at www.python.org.

Table 2 Characteristics for the data set containing flights scheduled for a 2 h window on two close parallel runways

Rates, flights/h	Fleet mix 1	Fleet mix 2
Arrival rate: 21.0	Heavy: 2%	Heavy: 34%
Departure rate: 25.5	B757: 5%	B757: 8%
Total rate (two runways): 46.5	Large: 85%, small: 8%	Large: 19%, small: 39%

Table 3 Resulting rates after compressing the initial schedule

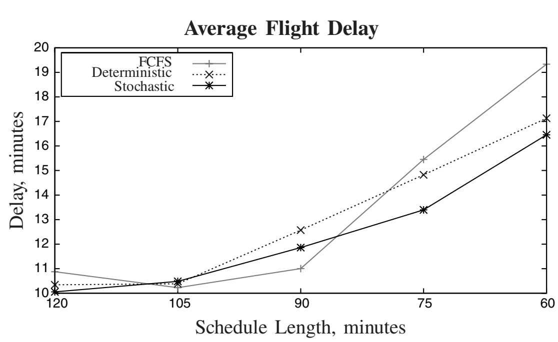
Schedule length, min	Arrival rate, flights/h	Departure rate, flights/h	Total rate (two runways), flights/h
120	21.0	25.5	46.5
105	24.0	29.1	53.1
90	28.0	34.0	62.0
75	33.6	40.8	74.4
60	42.0	51.0	93.0

runway departure/arrival times between replications. In each replication the realized runway times were generated using the stochastic characteristics presented earlier. To achieve comparable results, the same 100 replications were applied for each of the three planning procedures; i.e., the realizations of the aircraft runway times were identical for all three experiments.

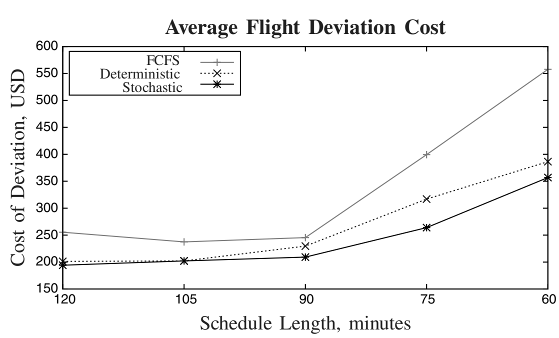
The result is shown in Figs. 5 and 6. Figures 5a and 6a show the average delay per flight with the initial schedules on the x-axis and the three models represented by the three data series. Figures 5b and 6b show the resulting average delay cost and Figs. 5c and 6c show the delay on the runway. The runway delay is defined as the difference between the time for the last scheduled flight and the latest realized flight time.

In Figs. 5 and 6 it can be seen that the stochastic runway planning procedure is able to reduce the cost of the combined arrival and departure operations on the runway, given that the schedule is dense enough. As the length of the schedule is reduced and the arrival and departure rates increase, the delay and the resulting cost of the delay both increase. For both fleet mixes under consideration, the FCFS policy results in less delay than the optimized schedules when the combined arrival and departure rates are less than 74 flights/h: i.e., when the length of the schedule is more than 75 min. With rates higher than or equal to 74 flights/h, the stochastic procedure reduces delay both with respect to the FCFS policy and the deterministic planning procedure.

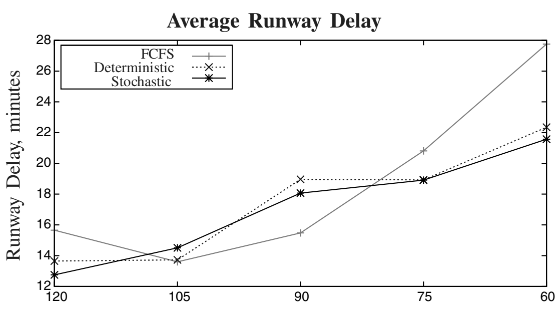
It is important to emphasize the difference in the planning logic between the FCFS policy and the optimization based planners. As described in the simulation procedure in the beginning of this section, the optimized planners commit to an aircraft weight-class sequence based on estimated runway arrival/departure times. At a later stage, when the realized times are known, individual aircraft are assigned to positions in the sequence. In the FCFS policy, the estimated runway arrival and departure times are not considered. Instead, the FCFS sequence is based on the realized times at the runway. Hence, the FCFS policy can result in less delay than the optimized schedules, which is the case when the combined arrival and departure rates are



a) The average delay per flight for the different initial schedules

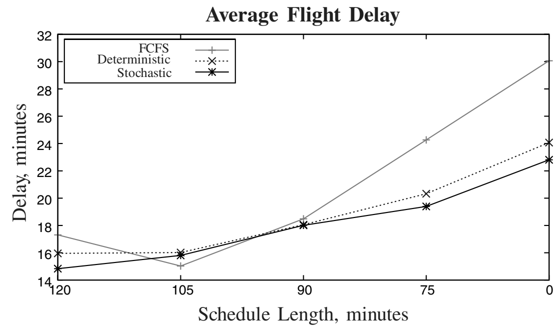


b) The average delay cost per flight, in USD, for the different initial schedules.

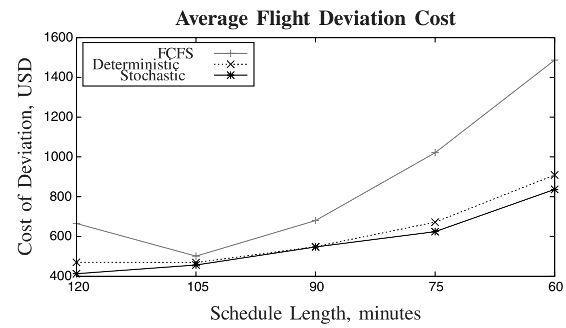


c) The average delay on the runway per replication

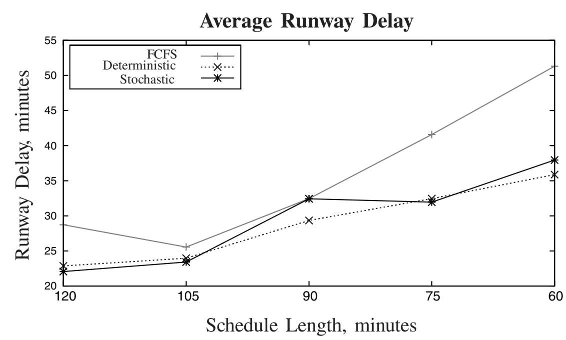
Fig. 5 Simulation results for fleet mix 1.



a) The average delay per flight for the different initial schedules



b) The average delay cost per flight, in USD, for the different initial schedules



c) The average delay on the runway per replication

Fig. 6 Simulation results for fleet mix 2.

Table 4 Average run time, in minutes, to simulate 93 flights

	Fleet mix 1			Fleet mix 2		
Schedule compression	FCFS model	Deterministic model	Stochastic model	FCFS model	Deterministic model	Stochastic model
0	0.1	7.9	48.6	0.1	23.7	158.8
15	0.1	6.5	40.6	0.1	27.9	164.6
30	0.1	7.7	48.6	0.1	26.8	184.7
45	0.1	6.6	37.8	0.1	25.0	205.9
60	0.1	5.0	28.4	0.1	17.4	112.1

less than 74 flights/h. In terms of runtime, the stochastic two-stage model is efficient. However, improved performance of the planner would allow for larger data sets in the rolling-horizon implementation, which could potentially increase the quality of the solution.

The runtimes for the simulations can be seen in Table 4, where the time refers to the time it takes to complete one simulation with 93 flights. The optimization component is executed every 15 min of simulation time.

IV. Conclusions

In this paper a two-stage stochastic runway planning model was developed and embedded in a two-stage planning procedure. To the best of our knowledge this is the first stochastic optimization model for runway scheduling. The core model is formulated as a two-stage stochastic program, where the first stage seeks to find an aircraft weight-class sequence such that runway throughput is maximized. After the uncertain parameters are assumed realized, individual aircraft are assigned to positions in the sequence such that the deviations from the scheduled arrival and departure times are minimized. The runway planner is designed for conditions when the arrival/departure rate is higher than the runway capacity, which is often the case when visibility is limited and IFR apply. In such conditions, close parallel runways cannot be operated independently and all dependent runways must be considered when planning runway operations. To accommodate multiple runways, the concept of virtual runways is introduced and used in the model.

The computational results shows that, under certain assumptions, there is value in using the stochastic runway planner over a FCFS policy when the combined arrival and departure rates are high compared with runway capacity. In this study it is assumed that the probability distributions and realizations of runway departure/arrival times are independent; i.e., there is no correlation between aircraft. In actual operations when the arrival and departure rates are high, deviations from scheduled runway arrival/departure will most likely be correlated between aircraft. Adjusting the planning framework to include correlation and evaluate the benefits of such framework is suggested as a future research direction.

The stochastic model results in less average delay compared with a deterministic counterpart, which indicates that deterministic methods of determining runway schedules are not necessarily optimal in a dynamic environment. Further research is required to find the specific arrival/departure rate, specifying when it is beneficial to use a stochastic model over a FCFS policy, which may have to be characterized separately for each major airport.

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