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Optimal Solution of a Vehicle-routeing Problem: Transporting Mentally Handicapped Adults to an Adult Training Centre

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Many organizations must devise tours for vehicles to collect from or deliver to a given set of destinations—the vehicle-routeing problem. This paper presents the optimal solution to a real-world problem with 38 destinations and four vehicles, and is thought to be the largest such problem that has been solved optimally. The organization concerned is a local authority (Berkshire County Council) which has aims other than profit maximization. Therefore explicit consideration is given to the multi-objective nature of the problem. Attempts to solve the problem using a commercial mathematical-programming package failed, but a specially written computer program was run on a mini-computer. In the optimal solution, total travel time is reduced by 15.7% and total distance by 11.5%, while the number of empty seats in each vehicle is more equally distributed.

Key words: computers, integer, multi-objective, scheduling, transport, travelling salesman

INTRODUCTION

The problem of devising tours for vehicles, which has received considerable attention in the literature over the past 25 years, has been extensively reviewed.[1] A vehicle-routeing problem (VRP) is solely concerned with devising tours to visit a specified set of destinations, and the order and timing of these visits is irrelevant.

This is a pilot study to investigate the ease with which the VRPs facing the management of a local authority can be solved and the benefits that follow. Explicit consideration is given to the multi-objective nature of the VRP. A specially written computer program is used to find an optimal solution for a VRP with 38 destinations. This is thought to be one of the largest real-world applications for which optimal answers have been obtained.

The next section sets out the problem, which is followed by a description of the data. The multiple objectives and the constraints are then presented. After a consideration of solution procedures, the empirical results are discussed. The final section contains some general conclusions.

THE PROBLEM

Berkshire Social Services (BSS) run an adult training centre (ATC) at Bennet Road in South Reading. This day centre provides education, training and employment for mentally handicapped adults (trainees), who have moved on to an ATC after attending a special school. The trainees are over 18 years of age with an IQ of under 70. The catchment area for Bennet Road is the area covered by Reading Social Services Division (i.e. Reading Borough) and the North Newbury Division of Newbury Social Services (a largely rural area to the west of Reading). A few trainees from Wokingham district also attend the centre.

There is a legal duty on Social Services to assist trainees with transport to ATCs, and it is the policy of BSS to provide free transport to all trainees who require it. Each weekday in April 1984, 132 trainees attended the centre between the hours of 9 a.m. and 4 p.m. Of these, 85 travelled by public transport, and three were transported by relatives, leaving 44 trainees to be transported by BSS. For this purpose the centre had the use of three large, identical transit minibuses and one Mercedes ambulance with a tail-lift. Each morning the four vehicles left the centre to collect the trainees from their homes, and in the evening, the same vehicle returned them home. In addition to his other duties, the manager of the Bennet Road ATC devises the vehicle tours. The aim of this study is to develop an appropriate computer-based model that will solve his VRP.

This paper presents the same case study as a paper by Okonjo-Adigwe published in this journal in July 1989. His paper relies upon the work presented here.

The only previous study of which we are aware that has devised tours for transporting trainees to an ATC is by Sonnenberg,[2] who investigated transport to ATCs in Hampshire. His approach was largely manual, although he did use linear programming in studying the trade-off between costs and travel times implicit in specified solutions which used fewer vehicles than the existing solution.

THE DATA

BSS supplied a list of the addresses of the 44 trainees who were transported each day to Bennet Road in April 1984. Since two trainees lived in the same house, there were only 43 separate calls to be made. To simplify the problem, the trainees living in two small villages to the east of Reading—Beenham (three trainees) and Burghfield Common (four trainees)—were treated as just two destinations. This resulted in there being 38 destinations to be visited by the four vehicles each morning and evening. Denoting Bennet Road as destination 1 gives a total of 39 destinations.

(i) *Distances*

Using a street map of Reading and an ordnance survey map of the rural area to the west, the road distances were computed between every pair of destinations. Many previous studies of VRPs have used 'crow-flies' rather than road distances, despite the obvious disadvantages of the former. Foulds *et al.*[3] used both road distances and 'crow-flies' distances in their study. They found that the saving in total road distance travelled when 'crow-flies' distances were used was only 55% of the saving when road distances were used. For some routes in Reading, the choice of which sequence of roads to use involved an implicit trade-off between minimizing travel time and minimizing distance. The route used in such cases was that which an experienced local driver might choose. As the centre of Reading contains a large one-way system, some non-symmetries in the routes through the central area were expected, but none were found. Hence, the distance matrix is symmetric.

(ii) *Travel Times*

Since there are 741 ($39 \times 38/2$) pairs of destinations and only 42 of the routes between these destinations are actually used, travel times had to be estimated. Even for the routes that are currently used, a single journey time would have been unsuitable. This is because, unlike distances, travel times are stochastic, and many observations would have been necessary to establish an estimate of the average travel time. For this reason, and to ensure consistency between all the travel times, it was decided to generate estimates of the travel times using the distance matrix. The resulting numbers can be regarded as estimates of the average travel times and, in consequence, the time objectives presented below must be interpreted in terms of expected values.

To convert distances into times, estimates of vehicle speeds are necessary. Some previous empirical studies have made explicit estimates of vehicle speed, and these have ranged from 9 m.p.h. to 50 m.p.h. for 'crow-flies' distances.[4–11] It should be noted that where the distances are measured 'as the crow-flies', the corresponding speeds should be lower than the actual road speed to produce the actual travel times.

Most previous studies have used the same speed on all routes (and for all vehicles). For the Bennet Road study this would have been inappropriate. The routes involve a complete range of road types, e.g. motorway, rural lanes, suburban roads and urban streets, and there is considerable variation in driving speeds. For this reason, the catchment area of Bennet Road was divided into four zones. These zones are: (1) Central Reading—the area within a one-mile radius of the centre of Reading, plus the first mile of the Oxford Road; (2) Outer Reading—the area covered by street maps of Reading, excluding central Reading and the motorway; (3) Motorway—the M4 between junctions 11 and 12; and (4) Rural—the remaining area, which is the rural area to the west and south of Reading.

Each element of the distance matrix was disaggregated into the distance travelled in each of these four zones. The speeds used to convert these distances into travel times were: Central Reading 10 m.p.h., Outer Reading 20 m.p.h., Rural 30 m.p.h., and Motorway 50 m.p.h. The same speed was used for each of the four vehicles because three of them are identical and the fourth travels at the same speed as the others.

In addition to allowing for changes in driving speed on different types of road, four road junctions were identified where motorists experience considerable delays. If a route passed through one or more of these junctions, the appropriate delays were added to the travel time for that route. As a check on the accuracy of the calculation of total travel times, the actual times for the tours used by BSS were compared with those calculated using the techniques described above. The two sets of times were reasonably similar, supporting the vehicle speeds and junction delays used. Like the distance matrix, the travel-time matrix is symmetric.

(iii) *Capacity*

The capacities of the four vehicles, expressed in terms of the number of ambulant trainees who can be carried, are 13, 14 and 13 for the three minibuses and 15 for the ambulance. The second minibus can carry one more trainee because, unlike the other two, it does not have an escort. The capacity used on the vehicles is one place for each ambulant trainee and one and a half places for each trainee who remains in their wheelchair on board the vehicle. The three trainees living at destinations 37, 38 and 39 were in wheelchairs, could not be lifted into a minibus and so had to be transported by the ambulance with the tail-lift. Hence they each used one and a half places on the ambulance.

Two trainees (at destinations 6 and 17) were lifted out of their wheelchairs into a minibus, or *vice versa*, four times per day. If these two trainees were transported by the ambulance with a tail-lift, they could remain in their wheelchairs and remove the need for the driver to lift them. This possibility led to an unusual complication. Each of these trainees takes up only one place if they are lifted out of their wheelchair, but will occupy one and a half places if they remain in their wheelchair. Thus the 'size' of the items to be carried depends upon which vehicle is involved.

(iv) *Trip Times*

To compute the total trip times, it is necessary to add in the time taken by the trainees to board the vehicle from their homes in the morning or to leave the vehicles in the evening. The times taken by trainees to board the vehicles in the morning are considerably longer than those taken to leave the vehicles in the evening. As a result, the evening VRP differs very slightly from the morning VRP. It was decided to use boarding times, and hence explicitly to solve the morning VRP. For an ambulant trainee, the boarding time is 2 minutes, and for trainees in wheelchairs, a time of 5 minutes was used. Whether trainees 6 and 17 are lifted out of their wheelchairs into a minibus or use the tail-lift on the ambulance, the time taken is 5 minutes.

OBJECTIVES

To devise a preferred set of tours it is necessary to specify some criterion by which alternative tours can be judged. Previous empirical studies have used four main criteria: (1) cost (either variable or total), (2) distance, (3) time (either travel or total) and (4) number of tours or vehicles required. Variable cost, distance and time all tend to be positively correlated, so that minimizing any one of them will also tend to minimize the other two. If a single criterion is used, then, unless total cost is to be minimized, a choice must be made between minimizing variable cost (or some correlated variable) and the number of vehicles (or some correlated variable). Second, a choice must be made as to which specific variable to minimize.

For Bennet Road it was not possible to increase the size of the vehicle fleet, and a reduction would have made the problem infeasible. Nor was it possible to alter the mix of vehicles used. Since the vehicle fleet was exogenous, using the number of vehicles as an objective was ruled out, and attention focused on the criteria in the variable cost group. While there may be strong correlations between variable cost, distance and time, these correlations will not be perfect, and so the choice between them is of importance.

In a commercial situation the minimization of cost may be a suitable single objective, but when providing a service to mentally handicapped adults, other factors may also be of importance. Five objectives were identified: (1) minimization of the total distance travelled by all the vehicles, (2) minimization of the total time (boarding and travel) taken by all the vehicles, (3) equalization of the trip times of the individual vehicles, (4) equalization of the number of unused places in each vehicle and (5) the use of the ambulance (which has a tail-lift) to carry trainees 6 and 17 in wheelchairs.

The inclusion of distance and time was partly as a proxy for variable cost, on which data was unavailable, but it was also because keeping down the total time and distance travelled are themselves worthwhile objectives. Thus, it was thought desirable to minimize the time which trainees have to spend each morning and evening sitting in a vehicle. The time objective (trip time) is the sum of the travel times and the time taken for the trainees to board the vehicle from their homes in the morning. Each tour is deemed to start or end with the vehicle fully loaded at Bennet Road, and so the times to board or leave the vehicle at the centre are irrelevant.

The objective of equalizing the total time taken by each vehicle was included to ensure that total time was not reduced by having some vehicles making very long trips while other vehicles made only short trips. Such a situation would mean that some trainees would be subject to long travel times, and there would be an unequal balance in the workload between drivers.

Leaving the same number of unused places in each vehicle would make it easier to assign a new trainee to a vehicle without the necessity of recomputing the new optimal solution. Of course, this decision involves a trade-off between the inefficiencies introduced by insisting on the equalization of spare capacity and the subsequent manual allocation of new trainees to routes, and the costs of reoptimization. Roughly half a dozen changes each year occur in the identity (and location) of the trainees who are transported, and the recomputation of the optimal solution was thought to be desirable once or twice a year.

In discussions with BSS it emerged that total time was the most important objective; total distance and equal unused places were objectives of intermediate importance; and equal time was of least importance. Therefore, the objective of the mathematical programming problem was specified to be the minimization of the total time taken by all the vehicles, subject to constraints on the maximum and minimum trip time for each vehicle and the maximum and minimum number of seats occupied in each vehicle. The relationship between time and distance was relied upon to reduce the total distance.

CONSTRAINTS

First, no vehicle is allowed to leave Bennet Road more than once. This means it is infeasible for a vehicle to make two tours. For the problem under consideration this condition is reasonable. Second, every destination, except Bennet Road, has to be visited exactly once. This guarantees that all trainees are transported. The last three destinations must be serviced by the ambulance, as the trainees concerned are in wheelchairs and must be lifted in and out of the vehicle by a tail-lift. Third, the tours must be continuous; i.e. if a vehicle arrives at a destination, the same vehicle must also leave from that destination. Fourth, the formation of subtours, i.e. continuous tours that neither leave from nor return to the origin, must be ruled out. Fifth, the capacity of each of the vehicles must not be exceeded. Sixth, maximum and minimum trip times for each vehicle must be observed; and last, upper and lower limits on the number of seats occupied on each vehicle are specified.

SOLUTION PROCEDURES

The solution to a VRP consists of a tour (or set of tours) specifying the sequence in which the destinations are to be visited. This can be viewed as a zero-one problem; either a route between two destinations is used or it is not. Algorithms exist for finding solutions to reasonably large travelling-salesman problems (TSP), but when constraints, e.g. on capacity, are added to turn the TSP into a VRP, these algorithms are no longer directly applicable.

Almost all previous models of real-world VRPs have used heuristic procedures, and these techniques may not find the optimal solution. Heuristics are used because it has been impossible to find optimal solutions to problems with a realistic number of destinations. However, in recent years two developments have facilitated finding optimal solutions for small real-world problems. First, there have been theoretical developments in the mathematical techniques for solving large integer-programming problems, and second, the speed and storage capacity of computers have increased.

The size of problem that can be solved optimally is still severely limited. Laporte et al.[12,13] used a computer program specially written to solve a hypothetical VRP. The largest number of destinations for which they were able to find an optimal solution was 50 for Euclidean distances, and 60 for non-Euclidean distances. Since a real-world VRP has Euclidean distances, the relevant figure is 50. Christofides[14] reported the optimal solution of a VRP with 53 destinations. These are thought to be the largest problems for which optimal solutions have been obtained. Only two previous papers have reported the optimal solution of real-world VRPs: (a) the 15-destination study by Balinski and Quandt,[15] and (b) the heavily simplified problem of Yano et al.,[16] who used a microcomputer to solve their problem.

Since the Bennet Road problem involves only 44 trainees, it was decided to attempt to find an optimal solution. Whilst previous optimal solutions to VRPs have been obtained using specially written computer programs, an attempt was made to solve the problem using a general-purpose mathematical programming package, SCICONIC. This is a commercially available package that can be implemented on a variety of mainframes and uses the branch-and-bound technique for solving integer problems, such as the VRP. The use of such software would considerably facilitate the routine solution of the model by BSS, and of similar problems by other local authorities. However, despite considerable effort, even the relatively small VRP considered in this paper proved too large for optimal solution by such a package. Subsequently Waters[17] reported an investigation of the feasibility of using SCICONIC for solving VRPs. Simple extrapolation of his results for 3–10 destinations suggests SCICONIC might take 120,000 years of VAX 8600 CPU time to solve a VRP with 38 destinations; i.e. the use of SCICONIC is currently an infeasible approach to solving VRPs of this size. Therefore, a purpose-designed program was written to solve the problem optimally.[18,19]

RESULTS

The actual solution used by BSS in April 1984 is set out in Table 1. This solution differs from that in Okonjo-Adigwe,[18,19] which is incorrect. The distances and times used to evaluate both the BSS and the optimal solutions are those generated for this study. While the distances are reasonably objective, the times are more questionable, and BSS may have been optimizing with respect to a different time matrix. Therefore, the comparison of alternative solutions is conditional on the acceptance of the time and distance matrices of this study. This may bias the comparisons against the BSS solution.

TABLE 1. *BSS solution*

Vehicle	Total time (minutes)	Distance (miles)	Seats used	Deviations from average time	Deviations from average spare capacity
1	105	28.4	12	−1.8	−1.4
2	104	17.7	13	−2.8	−1.4
3	115	21.9	11	+8.3	−0.4
Ambulance	103	21.0	9.5	−3.7	+3.2
Totals	427	89.0	45.5	|16.6|	|6.4|

The VRP was solved using a specially written FORTRAN computer program, which was run on a VAX 11/780 computer and took over 5 hours of CPU time to produce the solution set out in Table 2.[18,19] Access to a machine with the power of this mini-computer should be available to most local authorities.

TABLE 2. *Optimal solution*

Vehicle	Total time (minutes)	Distance (miles)	Seats used	Deviations from average time	Deviations from average spare capacity
1	80	11.4	11	−14	−0.4
2	106	28.7	12	+12	−0.4
3	87	18.6	12	−7	−1.4
Ambulance	103	20.1	10.5	+9	+2.1
Totals	376	78.8	45.5	|42|	|4.3|

The objective was to minimize the total trip time, and a reduction on the BSS solution of 11.5% was achieved. (This differs from the 14.5% in Okanjo-Adigwe[18,19] owing to the use of a different time for the BSS solution.) Whatever the tours, the same trainees had to board and leave the vehicles, and since these times were independent of the vehicle involved, the total boarding and leaving time is a constant and could not be reduced by devising a superior set of tours. The total boarding time for all trainees was 103 minutes, and the optimal solution achieved a reduction of 15.7% in the total travel time. As a by-product of reducing the total trip time, there was a reduction of 11.5% in the total mileage. (This differs from the 6% in Okonjo-Adigwe.[18,19]) Such reductions in time and distance may not lead to reductions of the same magnitude in the variable costs. For example, although petrol costs may be proportional to mileage, the wages of the driver may not. Therefore caution must be used in converting the time and distance savings into cost savings.

To equalize the trip times of the four vehicles, the lower bound on the individual trip time was varied from 65 minutes to 80 minutes, while the upper bound was fixed at 120 minutes. This variation in minimum trip time had no effect on the optimal solution. As can be seen from Table 2, when the minimum trip time was raised to 80 minutes, it became binding on vehicle 1. When the lower bound was raised above 80 minutes, it was not possible to find a feasible solution. This implies that, if a feasible solution is desired, the opportunity cost of imposing the time-equalization constraints is zero. For all of these runs, the upper and lower bounds on the number of occupied seats were 6 and 13 (14 for vehicle 2) respectively.

Despite the attempts to equalize trip times, the optimal solution has considerably less equality of trip times than the BSS solution. This is because the optimal solution has little effect on the trip times of two vehicles, but causes a considerable reduction in the times of the other two vehicles. However, the optimal solution does have a more equal distribution of unused seats. Therefore, the optimal solution performs well on the three criteria of most importance to BSS: total time, total distance and equal unused seats.

Another objective, although of low importance, was for trainees 6 and 17 to be carried by the ambulance, but in none of the previous solutions was this the case. So the VRP was resolved with this additional constraint. The minimum trip time was set at 70 minutes, the maximum trip time at 160 minutes, the minimum number of occupied seats at six and the maximum at 13 (14 for vehicle 2). The resulting solution required a total trip time of 454 minutes, i.e. 22% worse than the optimal value without this restriction, and involved a total distance of 97.5 miles, i.e. 24% worse. Therefore, owing to the unfortunate geographical distribution of the trainees wishing to be collected by the ambulance, a high price had to be paid for ensuring that trainees 6 and 17 remained in their wheelchairs.

CONCLUSIONS

It has been found that, even though the performance of commercially available integer programming packages is improving, it is not yet possible to use them to solve small real-world VRPs optimally. However, it has been shown that specially written programs are capable of producing optimal answers to such problems. It appears that it is possible to generate optimal solutions to realistic problems with up to 40–50 destinations.

It was found that the achievement of an optimal solution to a problem of this size involves a very considerable investment of time in the development of an appropriate computer program, in addition to the use of a mini-computer. For the Bennet Road problem, the 3-opt heuristic found the optimal solution both when trainees 6 and 17 were free to travel on any vehicle and when they were not. The subsequent large computing effort just verified the optimality of these solutions. Although it is necessary to compute the optimal solution before the quality of a heuristic can be evaluated, and the sub-optimality of a heuristic may vary from problem to problem, these results provide real-world support for the view that using heuristics can be a fruitful approach to solving real-world VRPs.

While some previous studies have had multiple objectives, their use of heuristics has clouded any analysis of the underlying trade-offs because of possible variations in the degree of sub-optimality. The calculation of optimal solutions for the Bennet Road study means that quantification of the various trade-offs is not confused by such effects.

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