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Optimization of stowage plans for RoRo ships

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ABSTRACT

International vehicle transportation is primarily conducted using Roll-on/Roll-off (RoRo) ships, which are specialized to transport cargo on wheels such as cars, farming equipment, and military equipment. RoRo ships operate by going between different regions of the world according to predefined plans. In this work we focus on operational decisions that are required when operating a fleet of RoRo ships: given a ship set to travel according to a given route, we consider decisions such as which cargoes to carry, how many vehicles to carry from each cargo, and how to stow the vehicles carried during the voyage. A mathematical model is made describing the problem, and both a standard MIP solver and a specially designed heuristic method are used to solve the problem. Computational tests are conducted to analyze the difficulty of solving several variations of the problem. For certain types of instances the MIP solver works well, while for other types the heuristic is very fast and more accurate than the MIP solver.

1. Introduction

Transport by sea is the major transport mode in international trade today. According to UNCTAD [1], trade by sea in terms of weight has doubled from 1980 to 2006. The world fleet by deadweight tons has increased by 41% in the same period. Shipping of cars and trucks by Roll-on/Roll-off (RoRo) ships is classified as general cargo shipping UNCTAD [1]. The fleet size within this category decreased by 17% from 1980 to 2006, largely due to the flourishing containerized trade which has increased tenfold in the same period. Even though some shipping of cars is done in containers, the vast majority is done by RoRo ships. According to MDS Transmodal [2], intercontinental trade of vehicles was 17.1 million vehicles in 2004, growing 5% annually, and regional trade was 26.6 million vehicles, growing 3–4% annually. The capacity of RoRo ships is measured using CEUs (Car Equivalent Units). The deep sea fleet for vehicle transport, defined as ships taking more than 3000 CEUs, consisted of 355 ships in 2006 and the regional fleet (less than 3000 CEUs per ship) consisted of 152 ships.

Compared to the container shipping fleet, the intercontinental RoRo fleet is small. Herein lies one of the threats for the RoRo industry: Due to its sheer size, container shipping can obtain more efficient short sea feeder traffic in and out of main ports. The implication is that the RoRo industry must continually improve to maintain its position as the dominating transport mode for vehicles by sea.

Planning of operations in the maritime transport industry can mainly be divided into three categories, by the length of the time horizon for which the planning is done: Strategic, tactical, and operational planning. Strategic planning is concerned with a time horizon of several years, and typically involves decisions such as determining the fleet size and mix. Tactical planning usually has a time horizon in the order of months, and a typical problem could be to determine which ship should serve which route. Operational planning is concerned with short term decisions, often related to a given voyage, such as speed selection, weather routing, or stowage. When performing long-term planning, a freighter lays a strategic plan for how many and which ships to own and operate. This strategic plan is used as input for tactical planning, such as fleet deployment. When the freighter has decided which ship should sail which route, this is used as input in operational planning. In the case of stowage planning, the planner typically has been given a specific ship for a specific time interval, during which the ship should carry specific cargoes or sail a specific route. An example of a route in RoRo shipping is given in Fig. 1.

When creating these plans, the planner must balance at all times the scope of the plan and the tractability of the problem. Increased scope, that is, planning for longer time periods and more ships simultaneously, gives the planner more flexibility. This enables him/her to find solutions that exploits synergy better than if the problem was partitioned into smaller problems and solved sequentially. However, solving a planning problem of large scope is more difficult than solving the subproblems of which it is

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Fig. 1. Example of a route in RoRo shipping.

composed. In addition, problems of larger scope typically require more information, which may not be readily available.

Taking into account the presence of complex decisions regarding fleet size and mix, fleet deployment, or ship stowage in the RoRo industry, the numerous factors influencing these decisions and the limited processing capacity of the human brain, the use of operational research (OR) can be warranted. Yet, the absence of publications concerned with planning problems in this industry suggests that advanced OR is not in extensive use today. Considering both internal competition among RoRo providers and the industry's increasing competition with container shipping, RoRo providers may investigate the possibility of using more advanced planning tools. Generic tools for the tactical planning of ship routing and scheduling already exists today [3,4], but on the operational level of stowage planning the RoRo industry needs specialized tools.

Research on maritime stowage problems has mainly been considered in the context of container ships. Container ships are stowed with stacks of containers, which implies that sometimes containers must be shifted, that is, moved from one stack to another. Often, the objective is to minimize the number of shifts needed along the ship route. Surveys of container ship stowage include [5,6]. An early publication on container ship stowage is the paper by Martin et al. [7], which describes many of the considerations to be taken when stowing container ships. Avriel et al. [8] give a mathematical model for the container ship stowage problem and also provide a heuristic algorithm for solving it. Wilson and Roach [9] study the possibility of using metaheuristics for solving the problem, and specifically employ a tabu search. Kang and Kim [10] present a mathematical model with ship stability constraints and a heuristic solution method. Li et al. [11] use a Branch-and-Cut procedure, obtaining very good results with respect to the number of shifts needed.

Recently, research on stowage of bulk ships has been conducted. Bulk ships carry their cargo in tanks or compartments, and the problem is to allocate the cargo to these tanks, subject to capacity, stability, and hazardous material constraints. Pintens [12] gives a mathematical model for this problem, where the objective is to minimize a weighted sum of cost and structural stress. Hvattum et al. [13] consider several variations of the tank allocation problem, and show that it is $\mathcal{NP}$-hard even in its simplest form.

Similar research on stowage for RoRo ships seems to be missing. Most research focuses on the design of ships with emphasis on safety and avoiding roll motion, for example [14]. Kreuzer et al. [15] present a simulation tool for calculating how strongly trailers need to be secured to the deck, depending on the expected weather. The goal is to improve the loading and unloading procedure. From another perspective, Mattfeld [16] investigates transhipment terminals and explores how the terminal side should handle the transportation of vehicles, whereas Mangan et al. [17] present a methodology for port/ferry choice from a shipper's point of view.

This paper focuses on stowage planning for the deep sea fleet of RoRo ships, with the remainder of the paper structured as follows. Section 2 describes the problem of creating stowage plans for RoRo ships on an operational level, and includes a mathematical programming model. Potential solution methods are discussed in Section 3. A computational study is described in Section 4, followed by concluding remarks in Section 5.

2. Problem formulation

In the following we assume that decisions regarding the route of a ship have been fixed. A planner then has the ship available for a predefined number of days, and the ship will go along a predefined route making several port calls. At each port it may pick up one or several cargoes to be transported to a later destination on its predefined route. It may also discharge cargoes that have been picked up earlier on its route. The route is typically from a supply region, where a number of cargoes are picked up, to a demand region, where the cargoes are delivered.

A cargo consists of a fixed number of vehicles or other rolling equipment. Each vehicle has a known height, width, length, and weight, and all vehicles belonging to a given cargo are identical. Normally, specifications will not vary greatly from one cargo to another, but sometimes the ship will carry trucks, farming equipment, or military equipment, which are labeled as High-and-Heavy, and whose specifications differ accordingly. When a cargo is picked up, it must be stowed on board the ship. The ship has a specified number of decks, and each deck has a certain length and width. Some of the decks may be adjusted up and down within certain limits, so the height of each deck is not predefined. The configuration of the ship must normally be decided when the ship is empty.

Each supply port holds a known number of cargoes that can be picked up in this port, and be delivered to a port later on the route. Some cargoes are considered mandatory to carry. This corresponds to a situation where the freighter has entered into long-term contracts. Cargoes that are not mandatory to carry may be available, and we refer to these as optional cargoes. If these are carried, additional revenue is generated for the freighter. The number of vehicles to carry from each cargo is considered to be fixed.

When a cargo is picked up, it must be rolled on board the ship on a ramp and be stowed on board one or several of the ship's decks. When a cargo is delivered, it must be withdrawn from its deck and rolled off the ship's ramp. Although each deck is a continuous space, we divide a deck into several logical lanes, and say that each item in a cargo must be assigned to one lane. The only loading ramp is typically located astern, so that each lane can take cargo in a last-in first-out fashion. In a basic case we say that each lane can only hold vehicles from one cargo, but in the full model this is relaxed. Then, however, the planner will have to take into consideration that if vehicles from a cargo in the front is discharged before vehicles from another cargo in the back, the back vehicles will have to be moved out of the way. To discourage such moving, the planner may consider a small fictitious cost to be incurred for each movement.

When a ship is stowed, there are certain stability constraints that need to be taken into consideration. If too much cargo is placed on one side of the ship, the ship will lean to this side. The angle with which it leans is called the roll angle. If too much cargo is placed either in the front or the back of the ship, it will lean forward or backward. The angle with which it leans is called the trim angle. In addition, if too much cargo is placed on higher decks, the ship becomes less tolerant to sidewise instability. Roll and trim angles are illustrated in Fig. 2.

The *RoRo ship stowage problem* (RSSP), as formally defined below, is to decide upon a deck configuration with respect to height, to decide which optional cargoes to carry, and to decide how to stow all cargoes on board the ship, so that all long-term contracts are fulfilled. Example of a solution to an instance of the RSSP is given in Fig. 3.

2.1. Mathematical model

The mathematical model for the RSSP is now presented. First, parameters and variables are given. Then, a mathematical program is presented, followed by a short description of each constraint.

Indices

c	cargo
d	deck
l	lane
p	port

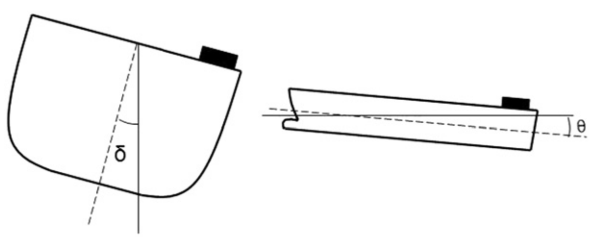

Fig. 2. Roll (δ) and trim (θ) angles.

Sets

$\mathcal{C}$	set of all cargoes
$\mathcal{C}_M$	set of all mandatory cargoes
$\mathcal{C}_O$	set of all optional cargoes
$\mathcal{D}$	set of all decks. Decks are numbered from the bottom in increasing order
$\mathcal{D}^R$	set of all decks plus the roof of the top deck
$\mathcal{L}$	set of all potential lanes on each deck. Lanes are numbered from port side to starboard side in increasing order
$\mathcal{P}$	set of all ports, except the last port on the route
$\mathcal{P}_c$	set of ports from loading port of cargo c to the port before the unloading port of cargo c, $\mathcal{P}_c = \{P^L_c, \ldots, P^U_c - 1\}$
$\mathcal{C}_c$	set of cargoes c' such that $P^L_{c'} < P^L_c < P^U_{c'} < P^U_c$. That is, if first loading vehicles from c' and later from c using the same lane, then the vehicles from c must be moved when unloading c'

Parameters

W_d	width of deck d
L_d	length of deck d
C^L_c	length of one vehicle in cargo c
C^W_c	width of one vehicle in cargo c
C^H_c	height of one vehicle in cargo c
P^L_c	loading port of cargo c
P^U_c	unloading port of cargo c, $P^U_c > P^L_c$
D^L_d	lower bound for where deck d can be placed
D^U_d	upper bound for where deck d can be placed
R^F_c	revenue for transporting optional cargo c
N_c	the number of vehicles in cargo c
C^M_c	cost incurred if cargo c needs to be moved
T^{MAX}	maximum allowable torque on the ship from the cargo around an axis going through the stern and the bow
Y^{MAX}	highest allowable center of gravity of the laden ship
C^V_c	weight of one vehicle from cargo c
W^S	lightweight of the ship

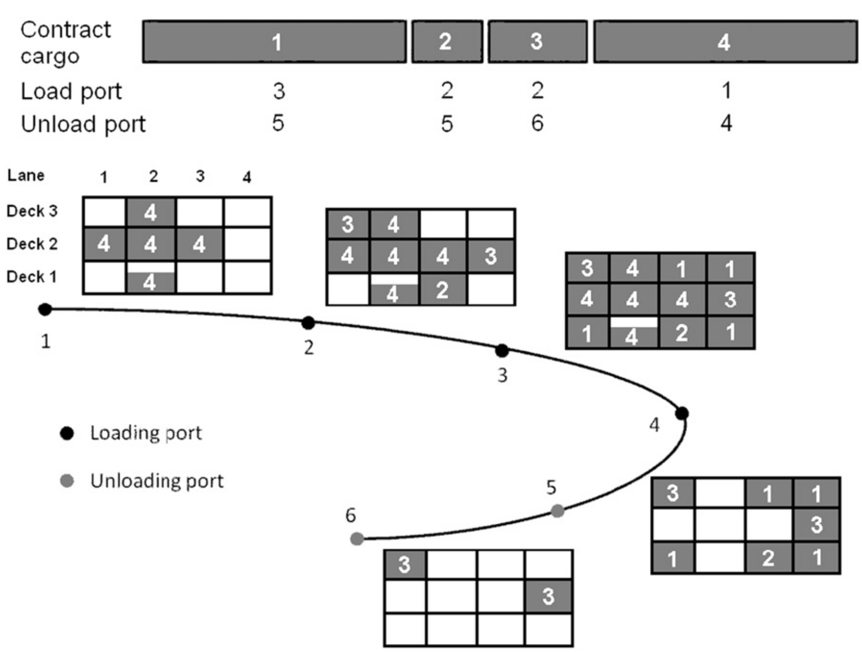

Fig. 3. Example of a solution to an instance of the RSSP, showing the stowage after departure from each port. Each box represents a lane on a deck, and the level of fill represents the quantity of vehicles stowed on the lane.

Y^S vertical distance from the ship's bottom deck to its center of gravity when empty

$\overline{X}_{dl}$ approximated horizontal distance of lane l on deck d from the ship's center of gravity, $\overline{X}_{dl} = W_d/2 - (l - \frac{1}{2}) \cdot W_d/|\mathcal{L}|$

$\overline{Y}_d$ approximated vertical distance of deck d from the ship's bottom deck, $\overline{Y}_d = (D_d^U + D_d^L)/2$

Decision variables

h_d height of deck d from the bottom deck

v_{dl} width of lane l on deck d

$w_{dlcc'} = 1$ if cargo c' is loaded in front of cargo c in lane l on deck d in port $P_{c'}^L$, /=0 otherwise

$x_{dlpc} = 1$ if lane l on deck d is used from port p to $p+1$ by cargo c, /=0 otherwise

y_{dlpc} number of vehicles from cargo c on lane l on deck d when the ship leaves port p

$z_c = 1$ if optional cargo c is taken /=0 otherwise

Objective function

$$\max \sum_{c \in \mathcal{C}_O} R_c^F z_c - \sum_{c \in \mathcal{C}} \sum_{c' \in \mathcal{C}_c} \sum_{d \in \mathcal{D}} \sum_{l \in \mathcal{L}} C_c^M w_{dlcc'} \tag{1}$$

Constraints

$$C_c^L \cdot y_{dlpc} - L_d \cdot x_{dlpc} \le 0, \quad d \in \mathcal{D},\ l \in \mathcal{L},\ c \in \mathcal{C},\ p \in \mathcal{P}_c \tag{2}$$

$$C_c^H \cdot x_{dlpc} - (h_{d+1} - h_d) \le 0, \quad d \in \mathcal{D},\ l \in \mathcal{L},\ c \in \mathcal{C},\ p \in \mathcal{P}_c \tag{3}$$

$$C_c^W \cdot x_{dlpc} - v_{dl} \le 0, \quad d \in \mathcal{D},\ l \in \mathcal{L},\ p \in \mathcal{P}_c,\ c \in \mathcal{C} \tag{4}$$

$$y_{d,l,p+1,c} - y_{dlpc} = 0, \quad d \in \mathcal{D}, l \in \mathcal{L}, c \in \mathcal{C}, \quad p \in \mathcal{P}_c \backslash \{P_c^U - 1\}, |\mathcal{P}_c| \ge 2 \tag{5}$$

$$\sum_{l \in \mathcal{L}} v_{dl} = W_d, \quad d \in \mathcal{D} \tag{6}$$

$$\sum_{d \in \mathcal{D}} \sum_{l \in \mathcal{L}} y_{dlpc} = N_c, \quad c \in \mathcal{C}_M,\ p \in \mathcal{P}_c \tag{7}$$

$$\sum_{d \in \mathcal{D}} \sum_{l \in \mathcal{L}} y_{dlpc} = N_c z_c, \quad c \in \mathcal{C}_O,\ p \in \mathcal{P}_c \tag{8}$$

$$\sum_{c \in \mathcal{C}} C_c^L \cdot y_{dlpc} \le L_d, \quad d \in \mathcal{D},\ l \in \mathcal{L},\ p \in \mathcal{P} \tag{9}$$

$$-T^{\text{MAX}} \le \sum_{c \in \mathcal{C}} \sum_{d \in \mathcal{D}} \sum_{l \in \mathcal{L}} C_c^V \cdot y_{dlpc} \cdot \overline{X}_{dl} \le T^{\text{MAX}}, \quad p \in \mathcal{P} \tag{10}$$

$$W^S Y^S + \sum_{c \in \mathcal{C}} \sum_{d \in \mathcal{D}} \sum_{l \in \mathcal{L}} C_c^V y_{dlpc} \overline{Y}_d \le \left(W^S + \sum_{c \in \mathcal{C}} \sum_{d \in \mathcal{D}} \sum_{l \in \mathcal{L}} C_c^V y_{dlpc} \right) Y^{\text{MAX}}, \quad p \in \mathcal{P} \tag{11}$$

$$x_{dl,P_c^L,c} + x_{dl,P_c^L,c'} \le 1 + w_{dlcc'}, \quad d \in \mathcal{D},\ l \in \mathcal{L},\ c \in \mathcal{C},\ c' \in \mathcal{C}_c, \tag{12}$$

$$D_d^L \le h_d \le D_d^U, \quad i \in \mathcal{D}^R \tag{13}$$

$$v_{dl} \ge 0, \quad d \in \mathcal{D},\ l \in \mathcal{L} \tag{14}$$

$$w_{dlcc'} \in \{0,1\}, \quad d \in \mathcal{D},\ l \in \mathcal{L},\ c \in \mathcal{C},\ c' \in \mathcal{C}_c \tag{15}$$

$$x_{dlpc} \in \{0,1\}, \quad d \in \mathcal{D},\ l \in \mathcal{L},\ c \in \mathcal{C},\ p \in \mathcal{P}_c \tag{16}$$

$$z_c \in \{0,1\}, \quad c \in \mathcal{C} \tag{17}$$

$$y_{dlpc} \ge 0 \text{ and integer}, \quad d \in \mathcal{D},\ l \in \mathcal{L},\ c \in \mathcal{C},\ p \in \mathcal{P}_c \tag{18}$$

The objective (1) is to maximize the sum of revenue from optional cargoes minus penalty costs incurred when having to move cargoes. Constraints (2) link the binary indicator variables x_{dlpc} for if lane l on deck d is used from port p to $p+1$ by cargo c, to the integer variables y_{dlpc} for how many vehicles from cargo c that are stowed in lane l on deck d from port p to $p+1$. The constraints also provide a bound for the number of vehicles possible to carry in a lane. Constraints (3) ensure that there is enough vertical space on the deck where the cargoes are placed. The sufficient width of the lanes is imposed by constraints (4). Constraints (5) make sure that once a cargo has been placed, it remains unmoved until it is unloaded. The partitions of decks into lanes are restricted by constraints (6). Constraints (7) and (8) link the integer variables y_{dlpc} for how many vehicles from cargo c that is stowed in lane l on deck d from port p to $p+1$ to the number of vehicles from cargo c that is carried, for respectively mandatory and optional cargoes. Constraints (9) ensure that the length of a lane is not violated by the vehicles stowed in that lane.

The stability constraints are (10) and (11). Ship stability calculations are quite complex, and involves non-linear equations. In an operational research setting, exact stability calculations may be undesirable, because the tractability of a mathematical model is greatly reduced when non-linear relationships are introduced. Therefore, linear approximations are customarily made that balance reality representation and model tractability [10,12,13]. Constraint s (10) impose that the torque from the cargo on the ship should be within the allowable limit, to avoid roll. The constants $\overline{X}_{dl}$ are approximations of the horizontal distance of a lane to the center of the ship, with negative values indicating a possible tilt to the port side and positive values indicating a tilt to the starboard side. To keep stability, the sum of the effects should be close to zero, and at least within $[-T^{MAX}, T^{MAX}]$. Trim can be controlled by a similar constraint, although this was not considered necessary for the model as presented. Constraints (11) ensure that the maximum allowable vertical distance Y^{MAX} from the ship's bottom deck to the ship's center of gravity when loaded is not exceeded. The constants $\overline{Y}_d$ and $\overline{X}_{ij}$ must be approximated from the ship characteristics, although they in reality depend on the actual stowage. The same holds for the parameters T^{MAX} and Y^{MAX} that control the tightness of the stability constraints.

When vehicles from cargo c are loaded in front of vehicles from cargo c', and cargo c' is unloaded before cargo c, there is an inconvenience as vehicles from cargo c must be moved out of the way. Constraints (12) make sure that a corresponding penalty is added to the objective function.

Upper and lower bounds on the deck heights are provided by (13). The non-negativity of lane width is ensured by constraints (14): the width of a lane may be zero if it is not being used. To make sure that the indicator variables $w_{dlcc'}$, x_{dlpc}, and z_c take binary values, restrictions (15), (18), and (17) are imposed, respectively. Finally, constraints (18) impose non-negativity and integrality on the number of vehicles carried in each lane. For an instance with 10 ports, 20 cargoes, 10 decks, and 10 lanes on each deck, the model would have up to 18,000 integer variables, 18,000 binary variables, and possibly over 50,000 constraints.

2.2. Problem complexity

The RSSP as described above is an $\mathcal{NP}$-hard optimization problem. The proof given below is based on a reduction from the *knapsack problem* (KP), which can be stated as follows [18]: We are given a finite set U, a size $s(u) \in Z^+$, and a value $v(u) \in Z^+$ for each $u \in U$. Also given are two values $B \in Z^+$ and $K \in Z^+$. The KP is to determine if there is a subset $U' \subseteq U$ such that $\sum_{u \in U'} s(u) \le B$ and $\sum_{u \in U'} v(u) \ge K$.

Theorem 1. *The RSSP is $\mathcal{NP}$-hard.*

Proof. We start with the KP, proved to be $\mathcal{NP}$-complete by Karp [19]. We will show that for every instance of the KP there is an instance of the RSSP that has an objective function value of at least K if and only if there is a feasible solution to the KP instance. We construct an RSSP instance with no mandatory cargoes and with one optional cargo for each $u \in U$. Each of the optional cargoes consists of only one vehicle. We let the ship have only one deck, and all cargoes are compatible with this deck with respect to the length and height of the vehicles. The instance has only two ports, a loading port and an unloading port. Stability constraints are wide enough to be redundant. Now, picking up an optional cargo u means using $C_u^W = s(u)$ of the deck's total width of $W = B$, and receiving $R_c = v(u)$.

If we consider a feasible instance of the KP, there must be a feasible solution of the corresponding RSSP instance that has an objective value of at least K, obtained by selecting those optional cargoes corresponding to U'. Correspondingly, if we have a solution to the RSSP with an objective value of at least K, there is a feasible solution of the KP obtained by choosing as U' those cargoes included in the RSSP solution. □

A general RSSP instance has several attributes that makes it more complex than the KP. First, the ship may have more than one deck. Second, cargoes may consist of more than one vehicle, meaning that they may have to be partitioned and placed on several decks. Third, the instance may have mandatory cargoes that must be carried without compensation. Fourth, vertical space may be a binding constraint, enforcing a trade-off in the configuration of the deck heights. Fifth, a voyage may consist of more than two ports, and sixth, there may be stability constraints that complicate the problem even further. Although these features do not affect the computational intractability from a theoretical perspective, they are likely to affect the level of difficulty of solving RSSP instances in practice.

3. Solution methods

The RSSP is relevant once the routing decisions of the ship has been fixed, in which case quite large computational times may be used to obtain its solution. However, the problem may also arise as a subproblem when routing decisions, cargo selection decisions, and stowage decisions are made simultaneously, and in this case the computational effort must be held to a minimum. When computational effort is not critical, the MIP formulation in Section 2.1 can be solved directly using standard commercial MIP solvers, and here the Xpress solver is used to this end. However, the MIP formulation puts few restrictions on the structure of the resulting solution, which may be an ambivalent aspect in practice: As will be discussed in Section 4.5 and illustrated in Fig. 6, although an optimal solution to the MIP will maximize the revenue from optional cargoes, the suggested stowage plan may be undesirable from a practical perspective. Mainly, vehicles from the same cargo may be spread across many decks and many lanes, and vehicles from many different cargoes may be stowed in the same lane. Although this allows a solution that exploits the entire ship capacity, it does not yield a stowage plan that can easily be applied in reality.

The second solution method evaluated in Section 4 is a heuristic that is described in the following. The heuristic has three parts: (1) selection of optional cargoes, (2) construction of stowage plan, and (3) improvement of stowage plan. Part 1 is only used when the problem instance includes optional cargoes and part 3 is only used when the stability restrictions (10)–(11) are active. The heuristic is designed to allow fast execution times, to cater for the possibility that the RSSP could be solved as a subproblem when planning the ship's route. Also, as opposed to the MIP formulation, the heuristic is designed to create stowage plans that are easily implemented in practice.

To solve the RSSP one has to decide both which cargoes to carry and how to place the transported vehicles on the ship. The first part of the heuristic involves selecting optional cargoes, while the two next parts involve the placement of these cargoes onboard the ship. If there are no optional cargoes, the cargo selection part of the heuristic is not invoked, and otherwise it is conducted as follows. Each optional cargo c is associated with a pseudorevenue which is initially set to R_c^F. The heuristic starts by a solution containing all the mandatory cargoes and creates a stowage plan using parts 2 and 3 as described below. It will then try to add optional cargoes greedily based on their pseudorevenue: taking each optional cargo in turn, try to create a stowage plan with the cargo added. If a stowage plan is successfully created the cargo is kept in the solution, and otherwise it is discarded. This is repeated for each optional cargo, until all have been examined. After completing this procedure, the pseudorevenue is increased for unselected cargoes by multiplying with a factor slightly larger than one and decreased for selected cargoes by dividing by the same factor. This enables the cargo selection to be repeated several times resulting in different cargo selections. The overall search terminates when a predefined number of iterations of selecting cargoes have been reached.

Having fixed the cargoes to be transported, the creation of a stowage plan is done by a constructive heuristic guided by a squeaky wheel optimization (SWO) metaheuristic [20]. This is a metaheuristic method used to guide constructive heuristics by changing the priority of decisions through an analysis of the solution that the construction gives. For the RSSP, the SWO takes as input a route and a set of quantities. The quantities are the number of vehicles of each cargo that should be transported. The heuristic makes an initial priority sequence with all the cargoes by sorting them according to increasing vehicle height. The rationale is to insert the tallest vehicles first to facilitate correct deck height adjustments.

The heuristic then proceeds by inserting each cargo in turn according to the priority sequence. For each cargo the heuristic first examines the bottom deck: if there is sufficient vertical space and width to insert a lane of vehicles the search will add vehicles from the cargo to this new lane. The number of vehicles inserted into the lane is equal to the minimum of the remaining vehicles in this cargo that have not been stowed already and the number of vehicles the length of the deck allows. This is repeated until no more lanes can be accommodated on the current deck, at which point the heuristic moves on to the next deck. When all vehicles from the current cargo have been placed, the heuristic considers the next cargo, going again back to the bottom deck to check for insertions. When the construction heuristic has considered all cargoes and all decks, it will terminate if all cargoes have been placed. If not, it will examine the ship again, lane by lane, and test whether it is possible to insert more vehicles by stowing in front of or behind the vehicles that are already placed here from other cargoes.

When the construction is completed, some cargoes may remain wholly or partially unplaced. In this case, the priority of the first unplaced cargo is increased, moving the cargo forward a random number of places in the priority sequence. Thus, between two consecutive constructions the priority sequences differ only by having one cargo that has moved forward in the sequence. With a different priority sequence the heuristic is likely to make different decisions during the next construction, and the process may be repeated until a feasible solution is found or until a maximum number of constructions have been performed.

The SWO construction does not take into account any stability restrictions, and even if the resulting stowage plan is feasible in any other respect, it may still represent an unbalanced stowage.

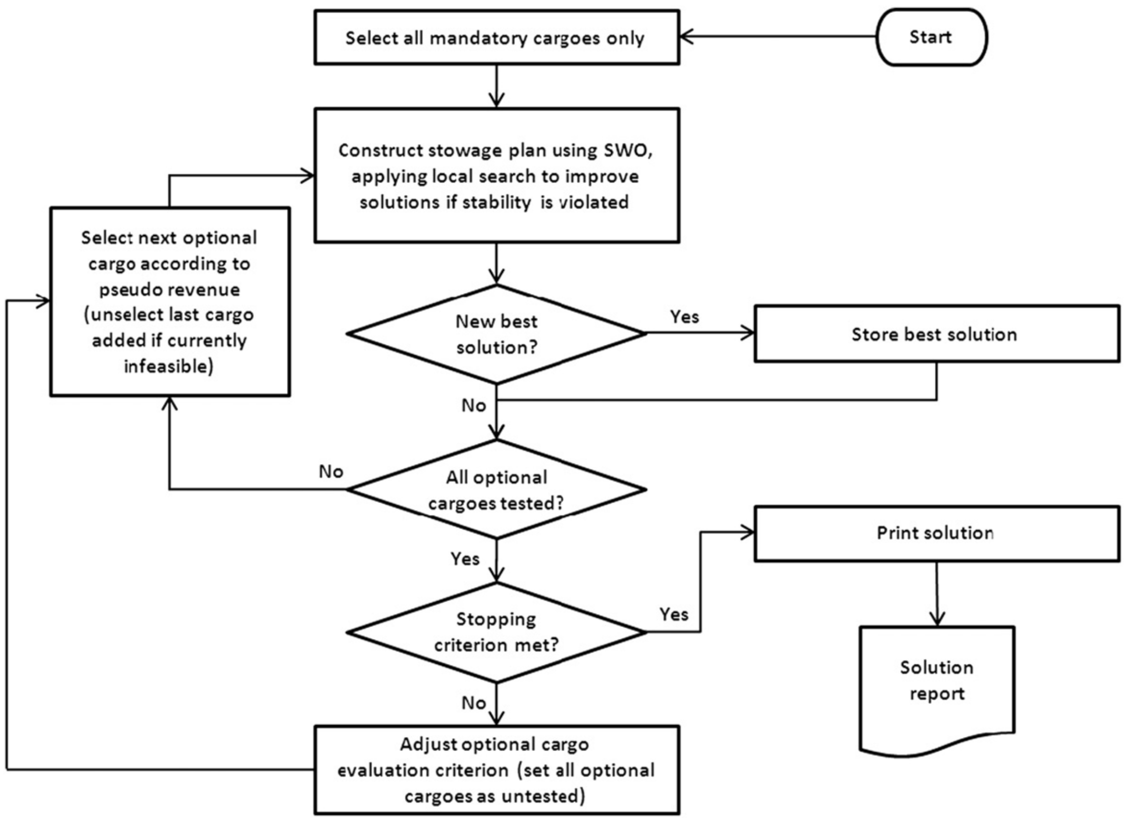

Fig. 4. Flow chart of the heuristic method for solving the RSSP.

A local search heuristic is therefore designed to repair those plans that do not fulfill the stability constraints. The move operator will try to move all vehicles from a specific cargo in a specific lane to a different lane with sufficient space. The move evaluation is a weighted sum of the improvement of the two stability constraints. If only one stability dimension is violated, improvements in the other dimension do not contribute to the move evaluation. A move is executed whenever an improving move is found, following the first improvement principle. The search continues until either the stability constraints are not violated, no further improvements can be made, or a maximum number of moves have been executed. If the local search fails to find a feasible solution, the priority sequence used by the construction heuristic will be updated as with unplaced cargoes, except that it is the first cargo to break the stability requirements that is moved forward. Fig. 4 shows how the SWO and the local search are incorporated with the selection of optional cargoes.

4. Computational study

The computational study aims to study the effectiveness of two different solution methods: Xpress version 19 employed on the mathematical formulation from Section 2.1 and the three-part heuristic described in Section 3. Simultaneously, we try to identify which properties of the RSSP make the problem more difficult to solve, and to this end we look at some simplified versions of the RSSP. All tests of the RSSP heuristic have been conducted on a computer with an Intel Xeon (2×3.06 GHz) CPU and 3.75 GB RAM. All tests with Xpress have been run on a computer with an Intel Core 2 Duo (2×2.66 GHz) CPU and 4 GB RAM.

In Section 4.1 the process of generating test instances is described. We then continue with a description of tests to calibrate the heuristic solution method in Section 4.2. A basic version of the RSSP, with no stability constraints, is studied in Section 4.3. Then, Section 4.4 describes tests showing the effect of enforcing the stability constraints. Finally, the computational testing is summarized in Section 4.5.

4.1. Test instance generation

To test the solution methods, different sets of test instances have been generated. Attempts have been made to make the test instances as realistic as possible, using publicly available data on ship and cargo attributes. Each instance is generated randomly after specifying the ship capacity in CEUs, the number of decks on the ship, the number of mandatory cargoes to carry, the number of optional cargoes available, and the number of ports to visit.

Based on the inputs, the remaining characteristics of the ship are determined, first by deciding on the length of the ship and second by deciding the number of lanes on each deck. The length of the ship is derived from a representative selection of ships from the car carrier ship database of Clarkson [21] using power law regression, as illustrated in Fig. 5. Knowing the capacity in CEU, the number of decks, and the length of the ship, the capacity in CEU per lane is calculated using the length of a standard car, 4.5 m, which in turn is used to determine the required number of lanes per deck to reach the specified total ship capacity. The width of the ship is set to be 1.8 m times the number of lanes. The final characteristic of the ship is the set of adjustable decks. Randomization is used, so that about 20% of the decks are adjustable to fit High-and-Heavy cargo. Decks for High-and-Heavy cargo should be between 3 and 5 m high, while regular car decks are between 1.5 and 2.5 m high.

Cargo characteristics are drawn randomly from uniform discrete distributions. The mandatory cargoes are set to fill between 60% and

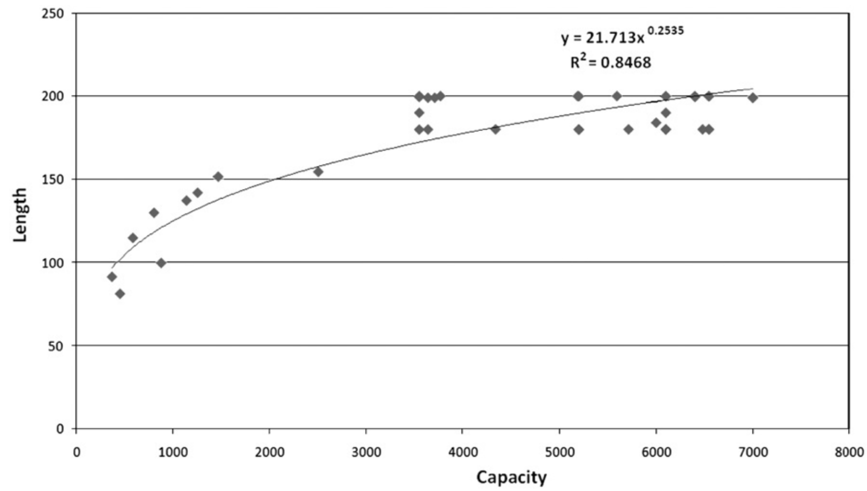

Fig. 5. Ship length as a function of capacity in CEU, based on power law regression.

90% of the ship, which gives an average number of vehicles per cargo. For each cargo, the exact number of vehicles is found by multiplying the average number by a factor between 0.2 and 1.8. In addition, 20% of all cargoes are randomly drawn to be High-and-Heavy, in which case the number of vehicles is adjusted down to compensate for the fact that these cargoes consist of larger vehicles. Using different uniform discrete distributions for normal cargo and High-and-Heavy cargo, each of the cargoes is assigned a random height, weight, width, and length. As an example, the length of vehicles from a normal cargo is drawn between 4.0 and 6.0 m, while from a High-and-Heavy cargo the length is between 8.0 and 20.0 m.

Deep sea RoRo-trade operates with loading and unloading regions. That is, a route goes from several loading ports on one continent to several unloading ports on another continent. To represent this, a random number is drawn uniformly from $\{2, \ldots, |\mathcal{P}|\}$ to represent the first unloading port. All ports before this port constitute the set of loading ports, while this and all subsequent ports constitute the set of unloading ports. Then, for each cargo, a loading and an unloading port are chosen randomly from these sets.

Stability restrictions are determined as follows. The maximum allowable momentum is equal to the momentum if all cargoes were to be evenly distributed on one side of the ship times a factor drawn randomly between 0.10 and 0.20. Then, the maximum allowable vertical distance from the ship's bottom deck to the center of gravity of the cargo is decided. First, this distance is calculated in the case that all cargoes are evenly distributed on all decks and each deck is vertically placed in the middle of its lower and upper bound. The maximum allowable distance is then set to equal this distance times a factor drawn randomly between 0.90 and 0.95.

Instances are generated separately for the various parts of the computational study, partly to reduce the risk of preserving throughout the study any bias due to random effects that may be introduced in any single instance. In addition, each test is run on sets of instances where each set consists of five instances. Table 1 summarizes the instance sets used in the computational study. Additional instances were generated to perform two full factorial experiments in Section 4.3.

Table 1
Table of RSSP instance sets used in the computational study, indicating ship size, the number of decks, the number of mandatory cargoes, the number of optional cargoes, the number of ports visited, and the number of instances in each set.

Set	Ship size (CEU)	Decks	Mand. c.	Opt. c.	Ports	No. of instances per set
1–5	500	3	2–6	2–6	3	5
6–17	250–10,000	6	3	3	3	5
18	1000	5	5	0	5	5
19	1000	5	10	0	10	5
20	2000	5	5	0	5	5
21	2000	5	10	0	10	5
22	4000	5	5	0	5	5
23	4000	5	10	0	10	5
24	6000	5	5	0	5	5
25	6000	5	10	0	10	5
26	8000	5	5	0	5	5
27	8000	5	10	0	10	5
28	500	3	3	3	3	5
29	500	6	6	6	6	5
30	1000	3	3	3	3	5
31	1000	6	6	6	6	5
32	1000	5	10	10	5	5

4.2. Calibration tests

The heuristic solution method proposed in Section 3 has some parameters that must be set. Tests showed that a promising trade-off between computational effort and the quality of the solutions obtained could be reached by using 100 iterations in the squeaky wheel optimization (part 2 of the heuristic), and using 10 iterations of cargo selection. We also limit the local search by terminating after going through the whole neighborhood at most 10 times, though the number of moves can be much higher due to using a first improvement strategy. Finally, we mentioned in Section 3 (see Fig. 4) that the evaluation criterion used when selecting optimal cargoes is adjusted between each iteration of cargo selection. This is done by adjusting the pseudorevenue associated to each optional cargo, which is divided by 1.1 if the cargo was selected, and multiplied by 1.1 if the cargo was not selected.

4.3. Testing the basic RSSP

In the following we consider instances of the RSSP with no active stability restrictions, assuming Y^{MAX} and X^{MAX} to be large constants. This is to establish whether the solution methods are able to handle realistically sized instances. We first consider instance sets with 2–6 mandatory cargoes and 2–6 optional cargoes. Each of the five sets contain five instances, with a ship size of 500 CEU, three decks, and three ports. A maximum running time of 300 s was set for Xpress. Table 2 lists average gap, average running time, and the number of instances where no feasible solution could be found for the instance sets with different number of cargoes.

The results are quite unambiguous for Xpress: an increased number of cargoes makes it more difficult to find good solution to instances of the RSSP. For the heuristic, this effect seems to be weaker. First, feasible solutions could be found for all instances with the heuristic. Second, since the gap is calculated using the best upper bound from Xpress, it is possible that the effect that can be seen for the heuristic really stems from Xpress being unable to obtain strong bounds when the number of cargoes increases.

While Table 2 shows the effect of an increased number of cargoes for a fixed ship size, the next test shows the effect of varying the ship size for a fixed number of cargoes (the size of each cargo is varied together with the ship size). Table 3 gives results for instances with six decks, three optional cargoes, three mandatory cargoes, and three ports. Ship size is varied from 250 to 10,000 CEU, giving 12 groups of five instances each. For Xpress, a time limit of 300 s for the LP-relaxation and another 300 s for the Branch-and-Cut was set. There is no clear connection between ship size and the difficulty of finding good solutions to the RSSP, neither for Xpress nor the RSSP heuristic. For the heuristic, the running time increases with ship size, because the stowage requires more processing for a larger ship, but the results with respect to optimality gap do not worsen significantly. The results also show that the heuristic performs better than Xpress, especially for instances with larger ships. At the same time we note that Xpress is able to prove optimality for several of the instances, including all five with 7000 CEU.

For these instances, without stability constraints, results indicate that the heuristic is able to find good solutions very quickly even for instances of realistic sizes. Xpress can handle most of the instances well when the number of cargoes is small, but appears to produce relatively worse solutions when the number of cargoes is increased. To further compare the two solution methods on simple instances we conducted two full factorial experiments [22, Chapter 15]. The first experiment considers the time taken to find a feasible solution, and four factors are considered: ship size (2000 or 4000 CEU), the number of decks (5 or 10), the number of mandatory cargoes (5 or 10), and the number of ports visited (5 or 10). For each of the 2^4 combinations of factors we solve five instances, and by analyzing the results we find some differences between the two solution methods.

For the heuristic, the only main effect that was significant at 5% significance level was the effect of ship size. Ship size showed a positive effect, meaning that increased ship size increases solving time. For Xpress only the number of cargoes showed a significant effect on the solution time. The interpretation of this is that the time to reach a feasible solution for the heuristic increases when the ship size increases, but remains unaffected when the number of cargoes increases. For Xpress it is the other way around, with the time increasing with the number of cargoes. For the heuristic, the stowage requires more processing when more vehicles need to be stowed, regardless of the number of cargoes into which the total number of vehicles is split. In the MIP formulation, however, the number of variables increases primarily with the number of cargoes.

The second full factorial experiment measured the gap from the best upper bound given by Xpress to the obtained solutions, and considered five factors: ship size (500 or 1000 CEU), the number of decks (3 or 6), the number of mandatory cargoes (3 or 6), the number of optional cargoes (3 or 6), and the number of ports (3 or 6). We found that both the number of mandatory cargoes and the number of optional cargoes gave significant effects in terms of the gap for Xpress, whereas only mandatory cargoes gave a similarly significant effect on the gap for the heuristic. That is, for Xpress neither the ship size, the number of decks, nor the number of ports significantly affect the gap to the best solution found, and for the heuristic neither does the number of optional cargoes.

Table 2
Average gap from the best upper bound of Xpress to the best found solution (G), average running time (T), and the number of instances where no feasible solution could be found (I), for Xpress and the RSSP heuristic for instances with different number of cargoes.

Set	$\lvert\mathcal{N}_M\rvert = \lvert\mathcal{N}_O\rvert$	Xpress			Heuristic		
		G (%)	T	I	G (%)	T	I
1	2	0	1	0	4	0.2	0
2	3	8	61	0	20	0.2	0
3	4	32	185	0	27	0.4	0
4	5	43	185	0	30	0.5	0
5	6	68	308	2	53	0.6	0
Avg.		30	148	0.4	27	0.4	0

Table 3
Average gap from Xpress upper bound to the best found solution (G), average running time in seconds (T), and the number of instances where no feasible solution could be found (I), for Xpress and the RSSP heuristic for instances with different ship size.

Set	Ship size	Xpress			Heuristic		
		G (%)	T	I	G (%)	T	I
6	250	12	222	0	12	0.3	0
7	500	20	70	0	22	0.4	0
8	1000	20	64	0	40	0.3	0
9	2000	53	183	0	43	0.4	0
10	3000	27	122	0	21	0.6	0
11	4000	50	184	0	50	0.6	0
12	5000	80	244	2	70	0.6	0
13	6000	20	65	1	13	0.7	0
14	7000	0	4	0	0	0.8	0
15	8000	52	185	0	23	0.8	0
16	9000	55	186	2	44	0.9	0
17	10,000	40	127	1	32	0.9	0
Avg.		36	138	0.5	31	0.6	0

4.4. Testing the effects of stability

The next tests aim to investigate the effect of adding stability requirements. Although RoRo ships mostly carry relatively light material, the stability of the ship can still be a very important safety issue. Tests with stability requirements are included in Table 4, reporting results for both Xpress and the heuristic on sets of instances with no optional cargoes. When stability requirements are not activated, both methods find feasible solutions to all instances, with running times being lower for the heuristic method. Adding stability requirements, in this case with Y^{MAX} and T^{MAX} equal to 20% of the maximum possible torque, gives another picture. While Xpress is still able to find feasible solutions to all but two instances, the heuristic fails to find feasible solutions to many instances. Even though the local search manages to improve the stability, the majority of the instances remain unsolved by the heuristic. The reason is presumably that the construction heuristic tends to stow all vehicles in a cargo together instead of splitting cargoes over many decks and lanes. This makes the ship more vulnerable to stability issues, especially when going half-full between ports while performing deliveries: when cargoes are stored

Table 4
Average running time in seconds (T) and number of instances where no feasible solution could be found (I) for Xpress and the RSSP heuristic for instances with and without stability constraints. The last two columns are for the heuristic with the local search for stability improvement. Instances in sets 11–20 do not include optional cargoes.

Set	Xpress				RSSP heuristic					
	No st.		St.		No st.		St.		St. & LS	
	T	I	T	I	T	I	T	I	T	I
18	1.0	0	0.7	0	0.1	0	0.1	5	0.7	3
19	3.9	0	23.3	1	0.1	0	0.1	2	0.1	0
20	0.9	0	1.3	0	0.2	0	0.2	5	2.0	5
21	2.0	0	16.5	1	0.1	0	0.2	3	1.2	1
22	1.2	0	1.8	0	0.2	0	0.2	5	3.4	4
23	4.5	0	12.1	0	0.2	0	0.2	5	7.1	5
24	2.9	0	10.7	0	0.3	0	0.3	5	6.3	5
25	6.2	0	9.3	0	0.3	0	0.3	5	7.7	5
26	3.3	0	5.4	0	0.4	0	0.3	5	10.9	5
27	12.6	0	15.5	0	0.3	0	0.5	5	8.8	5
Avg.	3.9	0	9.7	0.2	0.2	0	0.2	4.5	4.8	3.8

Table 5
Average gap from Xpress upper bound to the best found solution (G), average running time (T), and number of instances where no feasible solution could be found (I) for instances with no stability constraints and stability constraints with allowable torque of 20% and 10% of the maximum possible torque, for Xpress and the RSSP heuristic. Instances in sets 6b–10b include optional cargoes.

Set	No stability (100%)			20% torque			10% torque		
	G (%)	T	I	G (%)	T	I	G (%)	T	I
	Xpress								
28	46	184	0	33	62	0	20	63	0
29	58	244	1	79	304	1	76	305	1
30	0	11	0	0	24	0	22	123	0
31	14	248	0	20	304	0	30	305	0
32	59	306	2	72	307	3	71	307	3
Avg.	35	199	0.6	37	200	0.8	44	220	0.8
	RSSP heuristic								
28	55	0.4	1	53	2.1	1	33	0.5	1
29	76	1.6	0	76	2.7	0	81	5.5	3
30	20	0.4	0	20	1.2	0	62	9.4	1
31	29	1.7	0	28	15.8	0	59	138.9	0
32	63	4.2	0	60	4.2	0	85	43.5	2
Avg.	49	1.6	0.2	47	5.2	0.2	70	39.6	1.4

contiguously, they are more likely in this situation to be stored entirely to one side or too high.

Table 5 shows more detailed results on another set of smaller instances, which includes optional cargoes, where two different levels of stability requirements are considered. For Xpress, introducing stability constraints increased the average gap, and there was a quite large increase when going from 20% to 10% torque. Average running time also increased slightly when stability constraints were included, and there was one more instance where no feasible solution could be found within the time limit. The running time of the heuristic also increased when adding stability constraints, since the local search for stability improvement had to be used. The results indicate that the local search worked well for these instances when the stability restrictions are at 20% of the maximum. When going from 20% to 10% allowed torque, average gap, average running time, and the number of instances where no feasible solution could be found all increased quite dramatically. The gap increased because poorer solutions were found while the upper bound did not go down much, and because on seven instances no feasible solution could be found, compared to one for 20% torque.

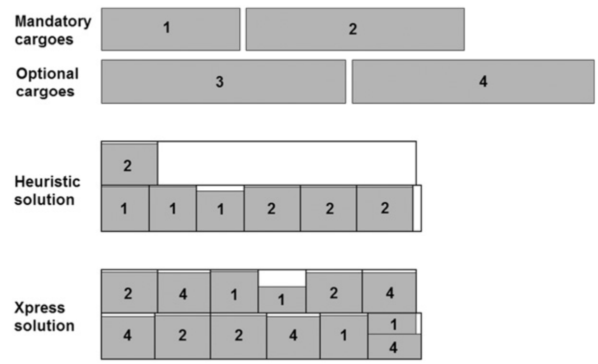

Fig. 6. Stowage plans by the heuristic and by Xpress for an instance with a 200 CEU ship, two mandatory cargoes, two optional cargoes, two decks, and six lanes. Each row represents a deck, and each rectangle represents a lane. The fill of the lane represents how much of the lane length that is used.

4.5. Summary of tests

Instances of realistic size can be solved to optimality by Xpress, even with active stability restrictions, as long as there are no optional cargoes (Table 4). For instances without stability restrictions the heuristic is very fast and produces solutions of higher quality than Xpress (Tables 2 and 3). Xpress is less effective when there are many cargoes, especially with many optional cargoes (Table 2). When introducing stability restrictions, the heuristic works badly on large instances with no optional cargoes (Table 4). However, in the presence of optional cargoes, the heuristic performs much better and can compete with Xpress despite much lower computational times (Table 5).

Another aspect is that Xpress and the heuristic will produce solutions that look very different. While the heuristic produces solutions where cargoes are grouped together and which may look appealing from a planner's perspective, the MIP solver may produce solutions where each cargo is scattered throughout the ship, even when this is not required due to stability. A small example is given in Fig. 6, which also illustrates the heuristic's inability to exploit the full deck areas and inferiority to Xpress for small instances: Xpress is able to take an optional cargo in addition to the mandatory cargoes. The heuristic is unable to do this, because the width of the lower deck is not exploited fully. This illustrates the weakness of the stowage heuristic, even though this is an extreme case, and particularly differentiating between Xpress and the heuristic since the instance is small and the relative loss of the heuristic thus is higher.

5. Concluding remarks

This paper has introduced the *RoRo ship stowage problem* (RSSP). In the RSSP one has to decide which cargoes to carry, how much of each cargo to carry, and how to stow each cargo onboard a RoRo ship during the ship's voyage. The problem is $\mathcal{NP}$-hard, and a heuristic solution procedure was designed and compared with the use of Xpress on a mathematical programming formulation.

We showed that the two solution methods have different strengths and weaknesses: when there are optional cargoes the heuristic performs better than Xpress with much lower computational times, but for tight problems with only mandatory cargoes Xpress can solve realistically sized instances quickly.

Although the stowage problem was shown to be challenging even for a given route with pick-ups and deliveries, the problem becomes even more interesting if one can incorporate more elements from the real planning situation. Planning is traditionally performed in a sequence: first determining sailing patterns, then assigning ships to voyages, and finally assigning cargoes to voyages and creating a stowage plan. When planning for several levels simultaneously, it is possible to achieve benefits that are impossible to obtain when looking at the problem in a disaggregated fashion. In future work we will add more elements to the problem currently examined, by taking into account routing decisions: taking a detour to pick up additional optional cargoes may give increased income, but at the expense of increased sailing costs, port costs, and increased time use. Another extension is to allow for flexible cargo quantities. This may occur due to contractual agreements saying that the transported volume should be flexible within specified limits, or due to the planner agreeing informally that the freighter can transport less than the agreed volume subject to some compensation. When considering routing and stowage simultaneously the RSSP will occur as a subproblem, in which case it must be solved quickly and the heuristic developed here will be valuable.

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References

[1] UNCTAD. Review of maritime transportation. Technical Report, UNCTAD; 2007.
[2] MDS Transmodal. Roro freight market overview, 〈http://www.mdst.co.uk/205089presentation.ppt〉; 2006 [accessed 10 September 2008].
[3] Fagerholt K. A computer-based decision support system for vessel fleet scheduling-experience and future research. Decision Support Systems 2004;37(1):35–47.
[4] Fagerholt K, Lindstad H. TurboRouter: an interactive optimisation-based decision support system for ship routing and scheduling. Maritime Economics and Logistics 2007;9(3):214–33.
[5] Steenken D, Voß S, Stahlbock R. Container terminal operation and operations research – a classification and literature review. OR Spectrum 2004;26(1):3–49.
[6] Stahlbock R, Voß S. Operations research at container terminals: a literature update. OR Spectrum 2008;30(1):1–52.
[7] Martin GL, Randhawa SU, McDowell ED. Computerized container–ship load planning: a methodology and evaluation. Computers and Industrial Engineering 1988;14(4):429–40.
[8] Avriel M, Penn M, Shpirer N, Witteboon S. Stowage planning for container ships to reduce the number of shifts. Annals of Operations Research 1998;76(1):55–71.
[9] Wilson ID, Roach PA. Principles of combinatorial optimization applied to container–ship stowage planning. Journal of Heuristics 1999;5:403–18.
[10] Kang J-G, Kim Y-D. Stowage planning in maritime container transportation. Journal of the Operational Research Society 2002;53(4):415–26.
[11] Li F, Tian C, Cao R, Ding W. An integer linear programming for container stowage problem. In: ICCS '08: proceedings of the 8th international conference on computational science, part I. Berlin, Heidelberg: Springer; 2008. p. 853–62.
[12] Pintens T. Geoptimaliseerde laadplanning aan boord van tankers en bulkschepen. Master's thesis, Antwerp Maritime Academy, Antwerp, Belgium; 2008.
[13] Hvattum LM, Fagerholt K, Armentano VA. Tank allocation problems in maritime bulk shipping. Computers and Operations Research 2009;36:3051–60.
[14] Vassalos D, Konovessis D. The thematic network SAFER EURORO: an integrated approach to safe European RoRo ferry design. Marine Technology 2008;45(1):1–8.
[15] Kreuzer E, Schlegel V, Stache F. Multibody simulation tool for the calculation of lashing loads on RoRo ships. Multibody System Dynamics 2007;18(1):73–80.
[16] Mattfeld D. The management of transshipment terminals: decision support for terminal operations in finished vehicle supply chains. Springer; 2006.
[17] Mangan J, Lalwani C, Gardner B. Modelling port/ferry choice in RoRo freight transportation. International Journal of Transport Management 2002;1:15–28.
[18] Garey MR, Johnson DS. Computers and intractability: a guide to the theory of np-completeness. San Francisco: Freeman; 1979.
[19] Karp RM. Reducibility of combinatorial problems. In: Miller RE, Thatcher JW, editors. Complexity of computer computations. New York: Plenum; 1972. p. 85–103.
[20] Joslin DE, Clements DP. "Squeaky wheel" optimization. Journal of Artificial Intelligence Research 1999;10:353–73.
[21] Clarkson RSL. Car Carrier Trade & Transport, 2008.
[22] Walpole R, Myers RH, Myers SL, Ye K. Probability and statistics for engineers and scientists. 7th ed. Prentice-Hall International; 2002.