Task 1: Poisson

The probability that there are k stars in a volume V is given by the Poisson distribution:

$$P(k, V) = \frac{(nV)^k e^{-nV}}{k!}$$

where n is the number density of stars. The probability that the nearest star is within a distance R is one minus the probability that there are no stars in the sphere of radius R:

$$1 - P(0, \frac{4}{3}\pi R^3) = 1 - e^{-4\pi n R^3/3}.$$

This gives the cumulative probability that the nearest star lies within distance R.

Alternatively, if ask probability density function (PDF) that the nearest star is exactly at distance R, we differentiate the cumulative probability:

$$P(R) = \frac{d}{dR}\left(1 - e^{-4\pi n R^3/3}\right) = 4\pi R^2 n e^{-4\pi n R^3/3}.$$

Task 2: Lorentzian

The driven damped harmonic oscillator is given by:

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = Fe^{i\omega_f t}.$$

Taking the Fourier transform, we obtain:

$$\tilde{x}(\omega) = \frac{F\delta(\omega - \omega_f)}{-\omega^2 + i\gamma\omega + \omega_0^2}.$$

In the time domain, the solution is:

$$x(t) = \frac{Fe^{i\omega_f t}}{-\omega_f^2 + i\gamma\omega_f + \omega_0^2}.$$

To compute the energy absorption per cycle, we integrate:

$$\int_{t=0}^{2\pi/\omega_f} \mathrm{Re}(Fe^{i\omega_f t})\,\mathrm{Re}(x(t))\,dt.$$

Expanding the real components,

$$\int_0^{2\pi/\omega_f} dt\,\mathrm{Re}(Fe^{i\omega_f t})\,\mathrm{Re}\left(\frac{i\omega_f Fe^{i\omega_f t}}{-\omega_f^2 + i\gamma\omega_f + \omega_0^2}\right).$$

Using contour integration techniques, we arrive at the final energy absorption formula:

$$E = F\pi \frac{\gamma\omega_f}{(\omega_0^2 - \omega_f^2)^2 + \gamma^2\omega_f^2}.$$