

AMMM Project

Adrian Rodriguez Bazaga, Pau Rodriguez Esmerats

November 2017

1 Modelling

1.1 Decision vars

- $w_{n,h}(\mathbb{B})$: whether the nurse n works at the hour h
- $z_n(\mathbb{B})$: whether the nurse n works during the shift(24h) or not
 - ★ $z_n = 1 \Rightarrow$ The nurse n works at least 1 hour, $\exists h, w_{n,h} = 1$
 - ★ $z_n = 0 \Rightarrow \forall h, w_{n,h} = 0$
- $s_n(\mathbb{N})$: hour in which the nurse n starts working,
such that $w_{n,s_n} = 1$ and $w_{n,s_n-i} = 0, \forall i : 1 \leq s_n - i < s_n$
- $e_n(\mathbb{N})$: hour in which the nurse n stops working,
such that $w_{n,e_n} = 1$ and $w_{n,e_n+i} = 0, \forall i : e_n < e_n + i \leq 24$

1.2 Known instance variables

- $demand_h$
- $nNurses$
- $minHours$
- $maxHours$
- $maxConsec$
- $maxPresence$

1.3 Objective function

Min: $\sum_{n=1}^{nNurses} z_n$

1.4 Constraints

- set the z_n values correctly: $\forall n : 1 \leq n \leq nNurses$,

$$24 \cdot z_n \geq \sum_{1 \leq h \leq 24} w_{n,h}$$

$$z_n \leq \sum_{1 \leq h \leq 24} w_{n,h}$$
- At any hour h , at least $demand_h$ nurses should be working:
 $\forall h : 1 \leq h \leq 24$,

$$\sum_{1 \leq n \leq nNurses} w_{n,h} \geq demand_h$$
- Each nurse that works, should work at least $minHours$:
 $\forall n : 1 \leq n \leq nNurses$

$$\sum_{1 \leq h \leq 24} w_{n,h} \geq minHours \cdot z_n$$
- Each nurse that works, should work at most $maxHours$:
 $\forall n : 1 \leq n \leq nNurses$

$$\sum_{1 \leq h \leq 24} w_{n,h} \leq maxHours \cdot z_n$$
- Each nurse works at most $maxConsec$ consecutive hours:
 $\forall n : 1 \leq n \leq nNurses$,
 $\forall h_1 : 1 \leq h_1 \leq 24 - maxConsec$,

$$\sum_{h_1 \leq h \leq h_1 + maxConsec} w_{n,h} \leq maxConsec$$
- Each nurse can stay in the hospital at most $maxPresence$ hours:
 $\forall n : 1 \leq n \leq nNurses, e_n \leq 24 \cdot z_n$
 $\forall n : 1 \leq n \leq nNurses, \forall h : 1 \leq h \leq 24, e_n \geq h \cdot w_{n,h}$
 $\forall n : 1 \leq n \leq nNurses, s_n \geq 0$
 $\forall n : 1 \leq n \leq nNurses, \forall h : 1 \leq h \leq 24, s_n \leq (h - 24) \cdot w_{n,h} + 24 \cdot z_n$
 $\forall n : 1 \leq n \leq nNurses, e_n - s_n + 1 \leq maxPresence \cdot z_n$
- Each nurse can rest at most one consecutive hour (exam hint version):
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23 : r_{n,h} = 1 - w_{n,h}$
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23 : wa_{n,h} = w_{n,h+1}$
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23 : wb_{n,h} = w_{n,h-1}$
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23, \forall M : M \geq 24$

$$M \cdot (1 - r_{n,h}) + M \cdot wb_{n,h} - 24 \cdot wa_{n,h} + 24 \cdot r_{n,h} \geq \sum_{1 \leq h_i \leq h} w_{n,h_i}$$

which is equal to :

$$2 \cdot M \cdot (1 - r_{n,h}) + M \cdot wb_{n,h} - M \cdot wa_{n,h} + M \cdot r_{n,h} \geq \sum_{1 \leq h_i \leq h} w_{n,h_i}$$
- Each nurse can rest at most one consecutive hour:
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 22, \forall M : M \geq 24$

$$M - M \cdot w_{n,h-1} + M \cdot w_{n,h} + M \cdot w_{n,h+1} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

can be rewritten as :

$$\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 22, \forall M : M \geq 24$$

$$M - M \cdot wb_{n,h} + M \cdot w_{n,h} + M \cdot wa_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

$$M - M \cdot wb_{n,h} + M \cdot (1 - r_{n,h}) + M \cdot wa_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

$$M \cdot (2 - r_{n,h}) - M \cdot wb_{n,h} + M \cdot wa_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

$$2 \cdot M \cdot (1 - r_{n,h}) - M \cdot wb_{n,h} + M \cdot wa_{n,h} + M \cdot r_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

$$2 \cdot M \cdot (1 - r_{n,h}) - M \cdot wb_{n,h} + M \cdot wa_{n,h} + M \cdot r_{n,h} \geq \sum_{h \leq h_i \leq 24} w_{n,h_i}$$