

FACULTAT INFORMÀTICA DE BARCELONA

ALGORITHMIC METHODS FOR MATHEMATICAL MODELING
AMMM(MIRI)

AMMM Project

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1 Introduction

Write the context of the Project: mixed integer linear programming (SIMPLEX and branch and cut) and metaheuristics algorithms to model and solve mathematical problems.

The motivation: to model, implement and test each of the models applied to a problem.

Introduce the problem (copy from assignment).

Explain the outcomes of the project: 3 models of the problem, time comparison of MILP versus Metaheuristics, Comparison of the 2 Metaheuristics.

2 Models

2.1 ILP Model

This section explains how the Mixed Integer LP model has been designed and implemented according to the constraints of the problem (see Introduction for more details). First, the mathematical constraints that allows us to build a SIMPLEX problem instance are explained. Second, the code used to implement the model in ILOG script is presented. Finally, we present a brief explanation of the tests that have been performed on the model to determine its correctness.

2.1.1 Mathematical formulation of the model

Decision vars

- $w_{n,h}(\mathbb{B})$: whether the nurse n works at the hour h
- $z_n(\mathbb{B})$: whether the nurse n works during the shift(24h) or not
 - ★ $z_n = 1 \Rightarrow$ The nurse n works at least 1 hour, $\exists h, w_{n,h} = 1$
 - ★ $z_n = 0 \Rightarrow \forall h, w_{n,h} = 0$
- $s_n(\mathbb{N})$: hour in which the nurse n starts working,
such that $w_{n,s_n} = 1$ and $w_{n,s_n-i} = 0, \forall i : 1 \leq s_n - i < s_n$
- $e_n(\mathbb{N})$: hour in which the nurse n stops working,
such that $w_{n,e_n} = 1$ and $w_{n,e_n+i} = 0, \forall i : e_n < e_n + i \leq 24$

Known instance variables

- $demand_h$
- $nNurses$
- $minHours$
- $maxHours$
- $maxConsec$
- $maxPresence$

Objective function Min: $\sum_{n=1}^{nNurses} z_n$

Constraints

- set the z_n values correctly: $\forall n : 1 \leq n \leq nNurses$,

$$24 \cdot z_n \geq \sum_{1 \leq h \leq 24} w_{n,h}$$

$$z_n \leq \sum_{1 \leq h \leq 24} w_{n,h}$$
- At any hour h , at least $demand_h$ nurses should be working:

$$\forall h : 1 \leq h \leq 24,$$

$$\sum_{1 \leq n \leq nNurses} w_{n,h} \geq demand_h$$
- Each nurse that works, should work at least $minHours$:

$$\forall n : 1 \leq n \leq nNurses$$

$$\sum_{1 \leq h \leq 24} w_{n,h} \geq minHours \cdot z_n$$
- Each nurse that works, should work at most $maxHours$:

$$\forall n : 1 \leq n \leq nNurses$$

$$\sum_{1 \leq h \leq 24} w_{n,h} \leq maxHours \cdot z_n$$

- Each nurse works at most maxConsec consecutive hours:
 $\forall n : 1 \leq n \leq nNurses,$
 $\forall h_1 : 1 \leq h_1 \leq 24 - \text{maxConsec},$

$$\sum_{h_1 \leq h \leq h_1 + \text{maxConsec}} w_{n,h} \leq \text{maxConsec}$$
- Each nurse can stay in the hospital at most maxPresence hours:
 $\forall n : 1 \leq n \leq nNurses, \forall h : 1 \leq h \leq 24, e_n \geq h \cdot w_{n,h}$
 $\forall n : 1 \leq n \leq nNurses, s_n \geq 0$
 $\forall n : 1 \leq n \leq nNurses, \forall h : 1 \leq h \leq 24, s_n \leq (h - 24) \cdot w_{n,h} + 24 \cdot z_n$
 $\forall n : 1 \leq n \leq nNurses, e_n - s_n + 1 - (2 \cdot 24) \cdot (1 - z_n) \leq \text{maxPresence} \cdot z_n$
- Each nurse can rest at most one consecutive hour (exam hint version):
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23 : r_{n,h} = 1 - w_{n,h}$
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23 : wa_{n,h} = w_{n,h+1}$
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23 : wb_{n,h} = w_{n,h-1}$
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23, \forall M : M \geq 24$

$$M \cdot (1 - r_{n,h}) + M \cdot wb_{n,h} - 24 \cdot wa_{n,h} + 24 \cdot r_{n,h} \geq \sum_{1 \leq h_i \leq h} w_{n,h_i}$$

which is equal to :

$$2 \cdot M \cdot (1 - r_{n,h}) + M \cdot wb_{n,h} - M \cdot wa_{n,h} + M \cdot r_{n,h} \geq \sum_{1 \leq h_i \leq h} w_{n,h_i}$$
- Each nurse can rest at most one consecutive hour:
 $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 22, \forall M : M \geq 24$

$$M - M \cdot w_{n,h-1} + M \cdot w_{n,h} + M \cdot w_{n,h+1} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

can be rewritten as :

$$M - M \cdot wb_{n,h} + M \cdot w_{n,h} + M \cdot wa_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

$$M - M \cdot wb_{n,h} + M \cdot (1 - r_{n,h}) + M \cdot wa_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

$$M \cdot (2 - r_{n,h}) - M \cdot wb_{n,h} + M \cdot wa_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

$$2 \cdot M \cdot (1 - r_{n,h}) - M \cdot wb_{n,h} + M \cdot wa_{n,h} + M \cdot r_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$$

$$2 \cdot M \cdot (1 - r_{n,h}) - M \cdot wb_{n,h} + M \cdot wa_{n,h} + M \cdot r_{n,h} \geq \sum_{h \leq h_i \leq 24} w_{n,h_i}$$

2.1.2 ILOG script implementation of the ILP model

2.1.3 Testing the ILP model

2.2 GRASP model

A brief context of the GRASP:

- acronym
- building blocks / components
- intensification, diversification and strategy applied

2.2.1 GRASP model pseudocode

This paragraph shows the pseudocode of our GRASP model for the problem of interest.

After the pseudocode is shown, brief comments of each line should be added.

2.2.2 GRASP model test

Tests performed to determine if the model is correct. Results obtained (very brief)

2.3 BRKGA model

A brief context of the BRKGA metaheuristic algorithm:

- acronym
- building blocks / components
- intensification, diversification and strategy applied

2.3.1 BRKGA model pseudocode

This paragraph shows the pseudocode of our BRKGA model for the problem of interest.

After the pseudocode is shown, brief comments of each line should be added.

2.3.2 BRKGA model test

Tests performed to determine if the model is correct. Results obtained (very brief)

3 Tests

3.1 Instance generation

This section shows how the instances have been generated. Possibly adding a very schematic pseudocode of our generation algorithm.

This section also explains the sets of instances that are prepared for the project:

- small-medium set: used to compare the solving time of the ILP model versus the Metaheuristics models
- large set: used to test the Metaheuristics models parameters over a large set of instances in order to choose the best performing setup for each model.

3.1.1 Instance generation

pseudocode

comments

how feasibility is assured

tests on how to increment solving time, including some graphs with the Best Integer and Best Bound evolution for different variations of the same instance

Conclusion on how to increase the "size" of the instance without computing the solving time of the ILP model.

The next figures show different executions of a small modification of a problem instance. In each figure, a parameter of the problem is modified to increase the time it takes to Cplex to solve the instance problem. This gives us an idea of what parameters increase the size of the problem instance.

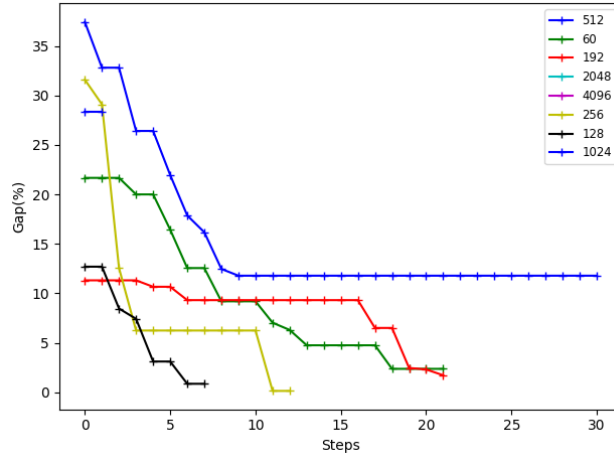


Figure 1: Evolution of Gap in similar problem instances with different number of nurses

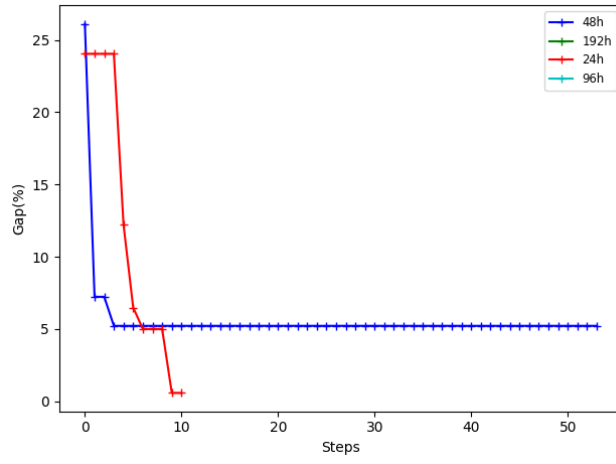


Figure 2: Evolution of Gap in similar problem instances with different number of hours

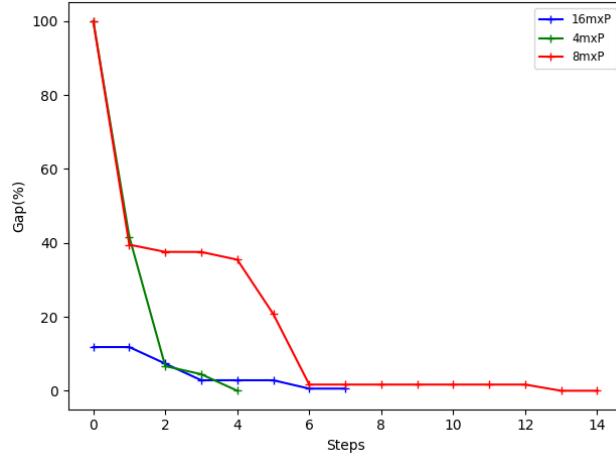


Figure 3: Evolution of Gap in similar problem instances with different values of maxPresence parameter

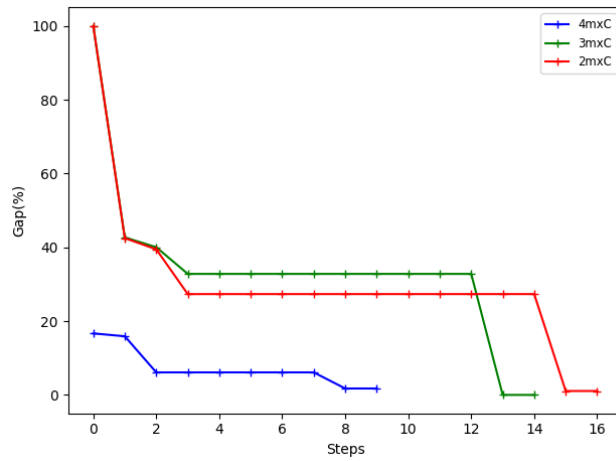


Figure 4: Evolution of Gap in similar problem instances with different values of maxConsec parameter

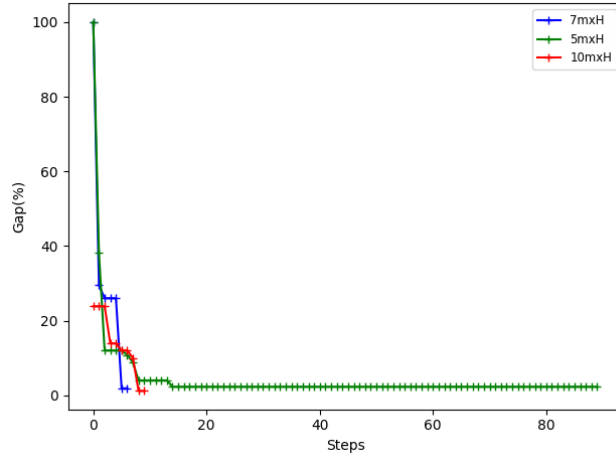


Figure 5: Evolution of Gap in similar problem instances with different values of maxHours parameter

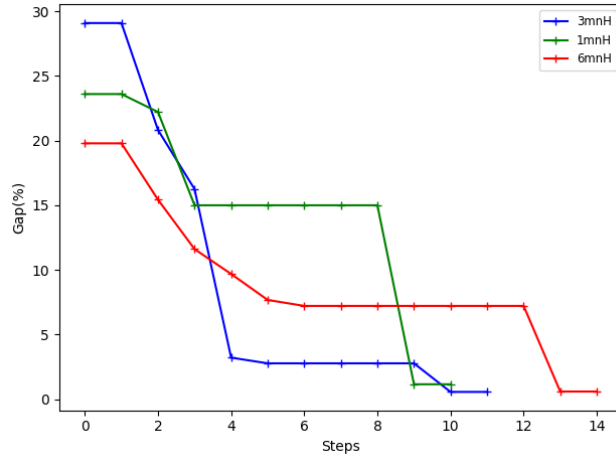


Figure 6: Evolution of Gap in similar problem instances with different values of minHours parameter

Those small observations allow us to conclude that the number of nurses and the number of hours of a schedule are the parameters of the problem that affect the most its solving time. Increasing the number of nurses or hours increases the size of the problem.

The large set of problem instances will be generated using modifications of those parameters.

3.1.2 Medium Set

Composition of the set, number of instances and solving times.

3.1.3 Large Set

Composition of the set, number of instances, solving times or another measure of the size of the problem.

3.2 Comparison of ILP and Metaheuristics models

Setup of the test:

- instances set
- execution conditions (same computer)
- models parameters
- results and times gathering procedure

Results

Chart of Solving times for all the instance set for the three models

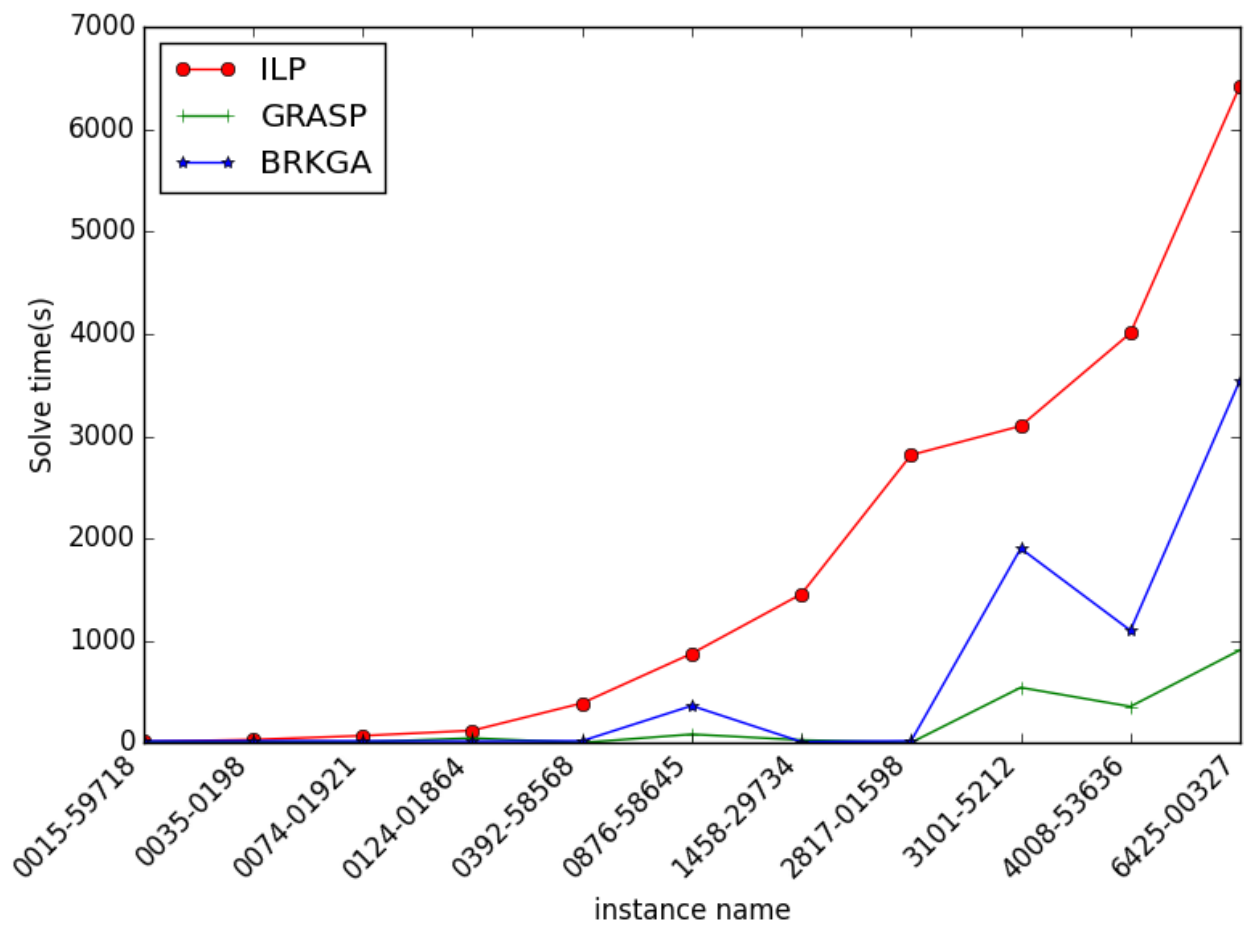
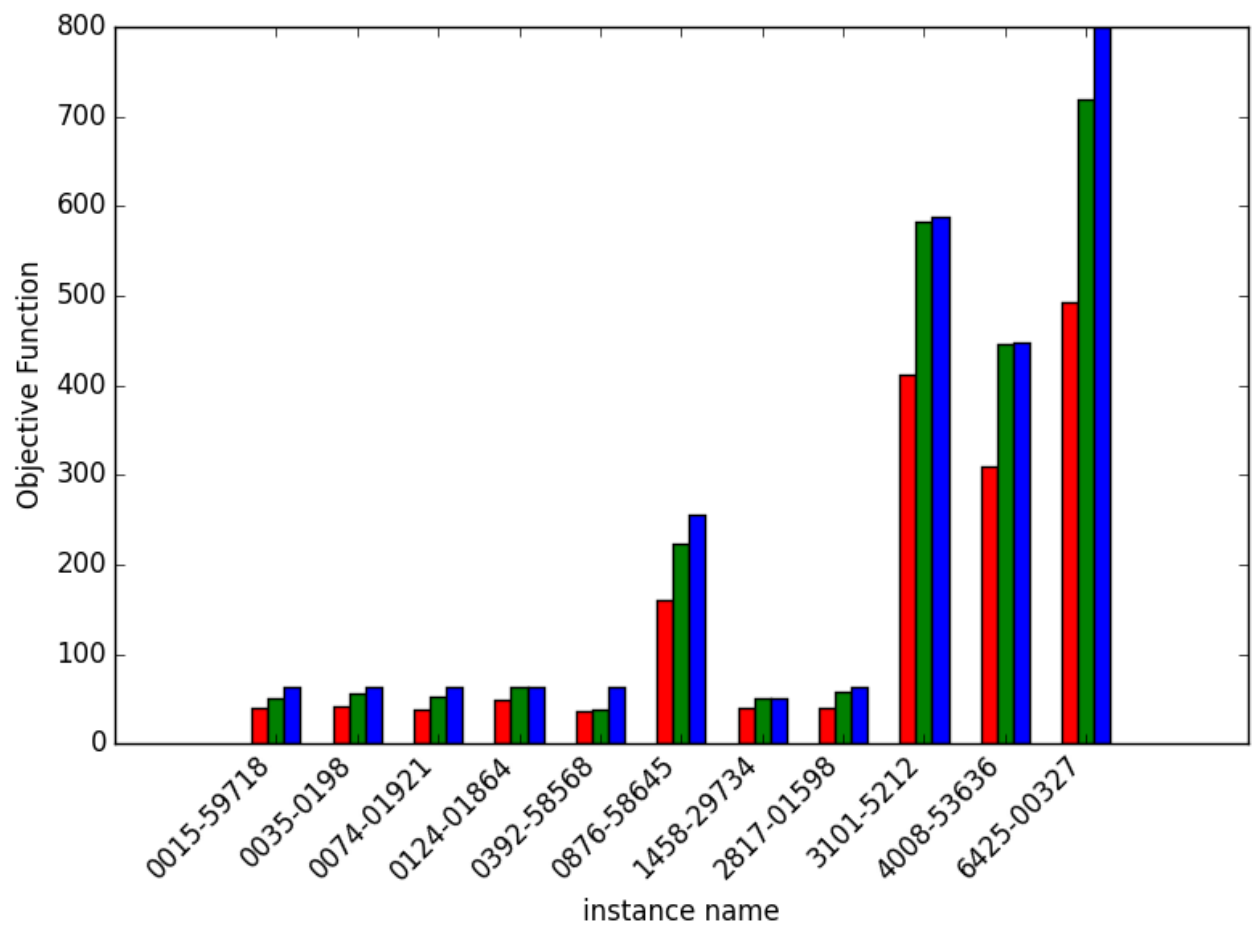


Chart of Objective function value for all the instance set for the three models



3.3 Comparison of GRASP and BRKGA models

Test of the section: parameter selection for each model, model performance comparison with best parameter setup.

3.3.1 GRASP parameter selection

Setup of the tests:

- instances set
- execution conditions (same computer)
- procedure
- results and times gathering procedure

Graph of alpha

Graph of Iterations

Graph of best improvement vs first improvement?

3.3.2 BRKGA parameter selection

Setup of the tests:

- instances set
- execution conditions (same computer)
- procedure
- results and times gathering procedure

Graph of inheritance probability

Graph of elite proportion

Graph of mutant proportion

Graph of population size

Graph of number of generations

3.3.3 Metaheuristic models comparison

Choosing the best performing parameter setup of the two models, perform a comparison of how objective function evolves in relation to time.

Draw conclusions about, why one is better than the other, why one stops sooner than the other. (think in terms of diversification and intensification and solution space exploration)

4 Conclusion

The conclusion may contain facts discovered through the comparison of solving times. Thoughts about intensification and diversification of the algorithms. Parametrizations that work and why they do, etc..