AMMM Project

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1 Modelling

1.1 Decision vars

- $w_{n,h}(\mathbb{B})$: whether the nurse n works at the hour h
- $z_n(\mathbb{B})$: whether the nurse n works during the shift(24h) or not
  - $zn = 1 \Rightarrow$ The nurse n works at least 1 hour, $\exists h, w_{n,h} = 1$
  - $zn = 0 \Rightarrow \forall h, w_{n,h} = 0$
- $s_n(\mathbb{N})$: hour in which the nurse n starts working, such that $w_{n,s_n} = 1$ and $w_{n,s_n-i} = 0$, $\forall i : 1 \leq s_n - i < s_n$
- $e_n(\mathbb{N})$: hour in which the nurse n stops working, such that $w_{n,e_n} = 1$ and $w_{n,e_n+i} = 0, \forall i : e_n < e_n + i \leq 24$

1.2 Known instance variables

- $demand_h$
- $nNurses$
- $minHours$
- $maxHours$
- $maxConsec$
- $maxPresence$

1.3 Objective function

Min: $\sum_{n=1}^{nNurses} z_n$

1.4 Constraints

- set the zn values correctly: $\forall n : 1 \leq n \leq nNurses$,
  $24 \cdot z_n \geq \sum_{1 \leq h \leq 24} w_{n,h}$
  $z_n \leq \sum_{1 \leq h \leq 24} w_{n,h}$

- At any hour h, at least demandh nurses should be working:
  $\forall h : 1 \leq h \leq 24$,
  $\sum_{1 \leq n \leq nNurses} w_{n,h} \geq demand_h$

- Each nurse that works, should work at least minHours:
  $\forall n : 1 \leq n \leq nNurses$
  $\sum_{1 \leq h \leq 24} w_{n,h} \geq minHours \cdot z_n$

- Each nurse that works, should work at most maxHours:
  $\forall n : 1 \leq n \leq nNurses$
  $\sum_{1 \leq h \leq 24} w_{n,h} \leq maxHours \cdot z_n$

- Each nurse works at most maxConsec consecutive hours:
  $\forall n : 1 \leq n \leq nNurses$,
  $\forall h_1 : 1 \leq h_1 \leq 24 - maxConsec$,
  $\sum_{h_1 \leq h \leq h_1 + maxConsec} w_{n,h} \leq maxConsec$

- Each nurse can stay in the hospital at most maxPresence hours:
  $\forall n : 1 \leq n \leq nNurses, \forall h : 1 \leq h \leq 24, e_n \geq h \cdot w_{n,h}$
  $\forall n : 1 \leq n \leq nNurses, s_n \geq 0$
  $\forall n : 1 \leq n \leq nNurses, \forall h : 1 \leq h \leq 24, s_n \leq (h - 24) \cdot w_{n,h} + 24 \cdot z_n$
  $\forall n : 1 \leq n \leq nNurses, e_n - s_n + 1 - (2 \cdot 24) \cdot (1 - z_n) \leq maxPresence \cdot z_n$

- Each nurse can rest at most one consecutive hour (exam hint version):

  $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23 : r_{n,h} = 1 - w_{n,h}$
  $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23 : wa_{n,h} = w_{n,h+1}$
  $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23 : wb_{n,h} = w_{n,h-1}$
  $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 23, \forall M : M \geq 24$
  $M \cdot (1 - r_{n,h}) + M \cdot wb_{n,h} - 24 \cdot wa_{n,h} + 24 \cdot r_{n,h} \geq \sum_{1 \leq h_i \leq h} w_{n,h_i}$

  which is equal to :
  $2 \cdot M \cdot (1 - r_{n,h}) + M \cdot wb_{n,h} - M \cdot wa_{n,h} + M \cdot r_{n,h} \geq \sum_{1 \leq h_i \leq h} w_{n,h_i}$

- Each nurse can rest at most one consecutive hour:

  $\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 22, \forall M : M \geq 24$
  $M - M \cdot w_{n,h-1} + M \cdot w_{n,h} + M \cdot w_{n,h+1} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$

can be rewritten as :
$\forall n : 1 \leq n \leq nNurses, \forall h : 2 \leq h \leq 22, \forall M : M \geq 24$
$M - M \cdot wb_{n,h} + M \cdot w_{n,h} + M \cdot wa_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$
$M - M \cdot wb_{n,h} + M \cdot (1 - r_{n,h}) + M \cdot wa_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$
$M \cdot (2 - r_{n,h}) - M \cdot wb_{n,h} + M \cdot wa_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$
$2 \cdot M \cdot (1 - r_{n,h}) - M \cdot wb_{n,h} + M \cdot wa_{n,h} + M \cdot r_{n,h} \geq \sum_{h+1 \leq h_i \leq 24} w_{n,h_i}$
$2 \cdot M \cdot (1 - r_{n,h}) - M \cdot wb_{n,h} + M \cdot wa_{n,h} + M \cdot r_{n,h} \geq \sum_{h \leq h_i \leq 24} w_{n,h_i}$