

Consider the statement S : “If two integers are both even, then their product is even”.

(a) Write S in the form “for all... , if... , then...”, using only mathematical symbols and variables.

S : $\forall a, b \in \mathbb{Z}$, if a and b are both even, then ab is even.

(b) Write the negation of S using only mathematical symbols and variables.

Negation of S : $\exists a, b \in \mathbb{Z}$ such that a and b are both even, but ab is not even.

(c) Write the converse of S using only mathematical symbols and variables.

Converse of S : $\forall a, b \in \mathbb{Z}$, if ab is even, then a and b are both even.

(d) Write the contrapositive of S using only mathematical symbols and variables.

Contrapositive of S : $\forall a, b \in \mathbb{Z}$, if ab is not even, then a or b is not even.

(e) Either prove or disprove S .

To prove S , we need to show that for any two even integers a and b , their product ab is even.

Proof: Let a and b be even integers. Then, we can write $a = 2k$ and $b = 2m$, where k and m are integers. Now, the product of a and b is $ab = (2k)(2m) = 4km$. Since k and m are integers, $4km$ is also an integer. Therefore, ab is even.

This proof is now complete.