Directions: Let $f : A \to B$ be a function, A_1 and A_2 be subsets of A, and B_1 and B_2 be subsets of B.

1. Prove that $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$

   Proof. Since f is a function, it is injective (one-to-one) and surjective (onto). $\square$

2. Prove that $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$ but $f(A_1) \cap f(A_2) \subseteq f(A_1 \cap A_2)$ doesn't necessarily hold.

   Proof. Let $f : A \to B$ be a function such that $f(A_1) = B_1$ and $f(A_2) = B_2$. $\square$

3. Explain what about the definition of functions enables the statement in (a) to be true, but precludes $f(A_1) \cap f(A_2) \subseteq f(A_1 \cap A_2)$. (Hint: Think about what all counterexamples to this statement have in common.)

   Proof. Since f is a function, it is injective (one-to-one) and surjective (onto). $\square$