Consider the statement S: "If two integers are both even, then their product is even".

(a) Write S in the form "for all..., if..., then...", using only mathematical symbols and variables.

$$S : \forall a, b \in \mathbb{Z}, a \equiv 0 \pmod 2 \wedge b \equiv 0 \pmod 2 \implies ab \equiv 0 \pmod 2$$

(b) Write the negation of S using only mathematical symbols and variables.

$$\neg S : \exists a, b \in \mathbb{Z}, a \equiv 0 \pmod 2 \wedge b \equiv 0 \pmod 2 \wedge ab \not\equiv 0 \pmod 2$$

(c) Write the converse of S using only mathematical symbols and variables.

$$\text{Converse of } S : \forall a, b \in \mathbb{Z}, ab \equiv 0 \pmod 2 \implies a \equiv 0 \pmod 2 \wedge b \equiv 0 \pmod 2$$

(d) Write the contrapositive of S using only mathematical symbols and variables.

$$\text{Contrapositive of } S : \forall a, b \in \mathbb{Z}, ab \not\equiv 0 \pmod 2 \implies a \not\equiv 0 \pmod 2 \vee b \not\equiv 0 \pmod 2$$

(e) Either prove or disprove S.

To prove S, we need to show that for any two even integers a and b, their product ab is even.

Proof: Let a and b be even integers. Then, we can write $a = 2k$ and $b = 2m$, where k and m are integers. Now, the product of a and b is $ab = (2k)(2m) = 4km$. Since k and m are integers, $4km$ is also an integer. Therefore, ab is even.

This proof is now complete.