

Prove that $(A - B) \subseteq B \iff (A - B) = \emptyset$.

Assume $(A - B) \neq \emptyset$. First we consider the forward direction:

By definition 2.1.4, $(A - B) = \{x \in A \mid x \notin B\}$, which means $(A - B)$ consists of elements in A but not in B .

Since $(A - B) \subseteq B$, every element $x \in (A - B)$ must also belong to B . This leads to a contradiction.

Thus, there can't be any element in $(A - B)$, which implies that $(A - B) = \emptyset$.

Then we consider the reverse direction:

Since $(A - B) = \emptyset$, there can't be any element in $(A - B)$. By definition, an empty set is a subset of any set.

Therefore, $(A - B) \subseteq B$.

We conclude that:

$$(A - B) \subseteq B \iff (A - B) = \emptyset$$