答案

例1. $\vec{a}-\vec{b}+5\left(-\frac{\vec{b}}{2}+\frac{\vec{b}-3\vec{a}}{5}\right)=(1-3)\vec{a}+\left(-1-\frac{5}{2}+\frac{1}{5}\cdot 5\right)\vec{b}=-2\vec{a}-\frac{5}{2}\vec{b}$

例2：设B点坐标为(x,y,z),根据题意有

$\frac{x-2}{8}=\frac{y+1}{9}=\frac{z-7}{-12}$,且$(x-2)^2+(y+1)^2+(z-7)^2=1156$,求得

$(x,y,z)=(18,17,-17)$或者$(x,y,z)=(-14,-19,31)$

例3.

(1). $2\vec{a}+3\vec{b}=2\cdot(4,-1,3)+3\cdot(5,2,-1)=(23,4,-3)$

(2). 设B点坐标为(x,y,z),则$(x-6,y+3,z-3)=-2\cdot(4,-1,3)$,则

$(x,y,z)=(-2,-1,-3)$.

(3). 设C点坐标为$(x1,y1,0)$,则$\frac{x1-6}{4}=\frac{y1+3}{-1}=\frac{0-3}{3}$,解得$(x1,y1,0)=(2,-2,0)$

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例4. 设向量$\overrightarrow{P_1P_2}$的方向角为α,β,γ, 则$\alpha=\frac{\pi}{3},\cos\alpha=\frac{1}{2},\beta=\frac{\pi}{4},\cos\beta=\frac{\sqrt{2}}{2}$,

由于$cos^2\alpha+cos^2\beta+cos^2\gamma=1$, 所以$\cos\gamma=\pm\frac{1}{2}$. 设$P_2$的坐标为$(x,y,z)$,

则$(x-1,y,z-3)=2\cdot\left(\frac{1}{2},\frac{\sqrt{2}}{2},\pm\frac{1}{2}\right)$,求得$(x,y,z)=(2,\sqrt{2},4)$或者 $(x,y,z)=(2,\sqrt{2},2)$

例5. (1) $\vec{a}\cdot\vec{b}=1\cdot1+1\cdot(-2)+(-4)\cdot2=-9.$

(2) $\cos\theta=\frac{a_xb_x+a_yb_y+a_zb_z}{\sqrt{(a_x^2+a_y^2+a_z^2)}\sqrt{(b_x^2+b_y^2+b_z^2)}}=\frac{1}{-\sqrt{2}}$, 因此$\theta=\frac{3\pi}{4}$

(3) $\vec{a}\cdot\vec{b}=|\vec{b}|Prj_b\vec{a}$, 所以$Prj_b\vec{a}=\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=-3$

例6. 0向量是与0向量平行的.

答案

例7.解$\overrightarrow{AC}=(0,4,-3),\overrightarrow{AB}=(4,-5,0)$,三角形ABC的面积为：

$S=\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{AB}\right|=\frac{1}{2}\sqrt{15^2+12^2+16^2}=\frac{25}{2},\left|\overrightarrow{AC}\right|=\sqrt{4^2+(-3)^2}=5,S=\frac{1}{2}\left|\overrightarrow{AC}\right|\cdot\left|\overrightarrow{BD}\right|$,所以$\left|\overrightarrow{BD}\right|=5$

例8. 根据向量内积和向量的向量积的定义可知$\left(\vec{\alpha}\cdot\vec{\beta}\right)=|\vec{\alpha}|\left|\vec{\beta}\right|\cos\theta,\left(\vec{\alpha}\times\vec{\beta}\right)=|\vec{\alpha}|\left|\vec{\beta}\right|\sin\theta$,所以$(\vec{\alpha}\cdot\vec{\beta})^2+|\vec{\alpha}\times\vec{\beta}|^2=|\vec{\alpha}|^2|\vec{\beta}|^2cos^2\theta+|\vec{\alpha}|^2|\vec{\beta}|^2sin^2\theta=|\vec{\alpha}|^2|\vec{\beta}|^2$

答案

例9. 不妨设A为原点，则A与其它三个点的向量为

$\overrightarrow{AD}=(-2,0,-2),\overrightarrow{AC}=(-2,4,0),\overrightarrow{AB}=(0,4,-2)$

故四面体的体积为

$$V=\frac{1}{6}\begin{vmatrix}-2 & 0 & -2\\ -2 & 4 & 0\\ 0 & 4 & -2\end{vmatrix}=\frac{16}{3}$$

而平面BCD上有$\overrightarrow{BD}=(-2,-4,0),\overrightarrow{BC}=(-2,0,2)$,

其面积为$S=\frac{1}{2}\cdot\left|\overrightarrow{BD}\times\overrightarrow{BC}\right|=\frac{1}{2}\cdot\sqrt{8^2+4^2+8^2}$ =6，所以A到平面距离为$d=\frac{3V}{S}=\frac{8}{3}$