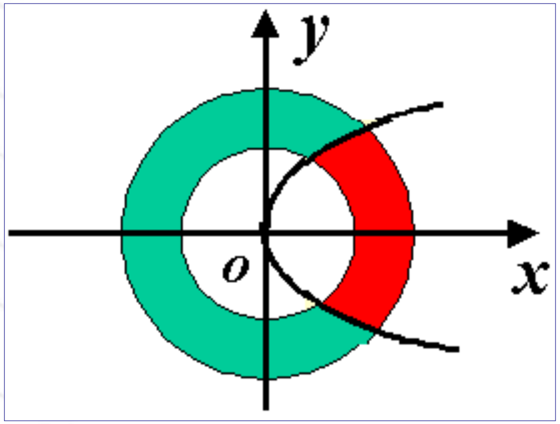
答案

例4. 解 $\begin{cases} |3-x^2-y^2| \le 1 \\ x-y^2>0 \end{cases} \Rightarrow \begin{cases} 2 \le x^2+y^2 \le 4 \\ x>y^2 \end{cases}$



所求定义域为

$D = \{(x,y)|2 \le x^2+y^2 \le 4, x>y^2\}$

例7. 跟据题意可知,

$\left|(x^2+y^2)\sin\frac{1}{x^2+y^2}\right| \le x^2+y^2$,因此

$\lim\limits_{(x,y)\to(0,0)}(x^2+y^2)\sin\frac{1}{x^2+y^2}$=0

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例8. $\lim\limits_{(x,y)\to(0,0)}\frac{\sin(x^2y)}{x^2+y^2}=\lim\limits_{(x,y)\to(0,0)}\frac{\sin(x^2y)}{x^2y}\cdot\frac{x^2y}{x^2+y^2}$

由于 $\lim\limits_{(x,y)\to(0,0)}\frac{\sin(x^2y)}{x^2y}=1,\left|\frac{x^2y}{x^2+y^2}\right|\le\frac{1}{2}|x|,\therefore\lim\limits_{(x,y)\to(0,0)}\frac{\sin(x^2y)}{x^2+y^2}=0$

例9. 取$y=kx^3$, $\lim\limits_{(x,y)\to(0,0)}\frac{x^3y}{x^6+y^2}=\lim\limits_{(x,y)\to(0,0)}\frac{x^3kx^3}{x^6+k^2x^6}=\frac{k}{1+k^2}$,

其值随k的不同而变化，故极限不存在.

例10. 取$x=\rho\cos\theta,y=\rho\sin\theta$, $|f(x,y)-f(0,0)|=|\rho(\sin^3\theta+\cos^3\theta)|<2\rho$,$\forall\varepsilon>0,\exists\delta=\frac{\varepsilon}{2}$,当$0<\sqrt{x^2+y^2}<\delta$时,

$|f(x,y)-f(0,0)|<2\rho<\varepsilon$, $\lim\limits_{(x,y)\to(0,0)}f(x,y)=f(0,0)$,故函数在(0,0)处连续.

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例11. 取$y = kx$, $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = \lim\limits_{(x,y)\to(0,kx)} \frac{kx^2}{x^2+k^2x^2} = \frac{k}{1+k^2}$,其值随$k$的不同而变化，极限不存在，故函数在(0,0)处不连续 。

例12. 原式=$\lim\limits_{(x,y)\to(0,0)} \frac{xy+1-1}{xy(\sqrt{xy+1}+1)} = \lim\limits_{(x,y)\to(0,0)} \frac{1}{\sqrt{xy+1}+1} = \frac{1}{2}$.

例13.不妨设$f(x,y) = \frac{x^3y^2}{(x^2+y^4)^2}$,取y=kx,$f(x,kx) = \frac{x^3k^2x^2}{(x^2+k^4x^4)^2} \to 0$

但是$\lim\limits_{(x,y)\to(0,0)} f(x,y)$不存在，原因若取$x = y^2, f(y^2,y) = \frac{y^6y^2}{(y^4+y^4)^2} \to \frac{1}{4}$