答案

例1. $\frac{\partial z}{\partial x}=2x+3y;\ \frac{\partial z}{\partial y}=3x+2y,$

$\therefore\ \frac{\partial z}{\partial x}|_{\substack{x=1\\y=2}}=2\times1+3\times2=8,\ \frac{\partial z}{\partial y}|_{\substack{x=1\\y=2}}=3\times1+2\times2=7$

例2. 证： $\frac{\partial z}{\partial x}=yx^{y-1},\frac{\partial z}{\partial y}=x^y\ln x,$

$\frac{x}{y}\frac{\partial z}{\partial x}+\frac{1}{\ln x}\frac{\partial z}{\partial y}=\frac{x}{y}yx^{y-1}+\frac{1}{\ln x}x^y\ln x=x^y+x^y=2z.$原结论成立.

例3.$\frac{\partial z}{\partial x}=\frac{1}{\sqrt{1-\frac{x^2}{x^2+y^2}}}\cdot\left(\frac{x}{\sqrt{x^2+y^2}}\right)_x'=\frac{\sqrt{x^2+y^2}}{|y|}\cdot\frac{y^2}{\sqrt{(x^2+y^2)^3}}=\frac{|y|}{x^2+y^2}$

$\frac{\partial z}{\partial y}=\frac{1}{\sqrt{1-\frac{x^2}{x^2+y^2}}}\cdot\left(\frac{x}{\sqrt{x^2+y^2}}\right)_y'=\frac{\sqrt{x^2+y^2}}{|y|}\cdot\frac{(-xy)}{\sqrt{(x^2+y^2)^3}}=-\frac{x}{x^2+y^2}sgn\,\frac{1}{y},$

$\frac{\partial z}{\partial y}|_{\substack{x\neq0\\y=0}}$不存在

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例4. 证 $p=\frac{RT}{V}\Rightarrow\frac{\partial p}{\partial V}=-\frac{RT}{V^2}$; $V=\frac{RT}{p}\Rightarrow\frac{\partial V}{\partial T}=\frac{R}{p}$;$T=\frac{pV}{R}\Rightarrow\frac{\partial T}{\partial p}=\frac{V}{R}$,

$$\frac{\partial p}{\partial V}\cdot\frac{\partial V}{\partial T}\cdot\frac{\partial T}{\partial p}=-\frac{RT}{V^2}\cdot\frac{R}{p}\cdot\frac{V}{R}=-\frac{RT}{pV}=-1$$

例5. $f_x(0,0)=\lim\limits_{x\to 0}\frac{\sqrt{|x\cdot 0|}-0}{x}=0=f_y(0,0)$

例6. $\frac{\partial z}{\partial x}=3x^2y^2-3y^3-y,\frac{\partial z}{\partial y}=2x^3y-9xy^2-x;$

$$\frac{\partial^2 z}{\partial x^2}=6xy^2,\frac{\partial^3 z}{\partial x^3}=6y^2,\frac{\partial^2 z}{\partial y^2}=2x^3-18xy;$$

$$\frac{\partial^2 z}{\partial x\partial y}=6x^2y-9y^2-1,\frac{\partial^2 z}{\partial y\partial x}=6x^2y-9y^2-1.$$

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例7.$\frac{\partial u}{\partial x} = ae^{ax}\cos by\,, \frac{\partial u}{\partial y} = -be^{ax}\sin by;$

$$\frac{\partial^2 u}{\partial x^2} = a^2e^{ax}\cos by, \frac{\partial^2 u}{\partial y^2} = -b^2e^{ax}\cos by;$$

$$\frac{\partial^2 u}{\partial x \partial y} = -abe^{ax}\sin by, \frac{\partial^2 u}{\partial y \partial x} = -abe^{ax}\sin by$$

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例8 解 当$(x,y)\neq(0,0)$时，

$$f_x(x,y)=\frac{x^4y+4x^2y^3-y^5}{(x^2+y^2)^2},f_y(x,y)=\frac{x^5-4x^3y^2-xy^4}{(x^2+y^2)^2};$$

当$(x,y)=(0,0)$时，

$$f_x(0,0)=\lim_{\Delta x\to 0}\frac{f(0+\Delta x,0)-f(0,0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{0-0}{\Delta x}=0,$$

$$f_y(0,0)=\lim_{\Delta y\to 0}\frac{f(0,0+\Delta y)-f(0,0)}{\Delta y}=\lim_{\Delta y\to 0}\frac{0-0}{\Delta y}=0.$$

再根据二阶偏导数定义，有

$$f_{xy}(0,0)=\lim_{\Delta y\to 0}\frac{f_x(0,0+\Delta y)-f_x(0,0)}{\Delta y}=\lim_{\Delta y\to 0}\frac{\frac{-\Delta y^5}{\Delta y^4}-0}{\Delta y}=-1,$$

$$f_{yx}(0,0)=\lim_{\Delta x\to 0}\frac{f_y(0+\Delta x,0)-f_y(0,0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\frac{\Delta x^5}{\Delta x^4}-0}{\Delta x}=1.$$

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例9 解 由于$ln\sqrt{x^2+y^2}=\frac{1}{2}\ln(x^2+y^2)$, $\frac{\partial u}{\partial x}=\frac{x}{x^2+y^2}$, $\frac{\partial u}{\partial y}=\frac{y}{x^2+y^2}$,

$$\frac{\partial^2 u}{\partial x^2}=\frac{x^2+y^2-2x^2}{(x^2+y^2)^2}=\frac{y^2-x^2}{(x^2+y^2)^2},$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{x^2+y^2-2y^2}{(x^2+y^2)^2}=\frac{x^2-y^2}{(x^2+y^2)^2},$$

因此

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{y^2-x^2}{(x^2+y^2)^2}+\frac{x^2-y^2}{(x^2+y^2)^2}=0.$$