例1 求函数$z = xe^{2y}$在点$P(1,0)$处沿从点$P(1,0)$到点$Q(2,-1)$的方向的方向导数。这里方向$\vec{l}$即为$PQ = \{1,-1\}$,

故x轴到方向$\vec{l}$的转角$\varphi = -\frac{\pi}{4}$.

$$\because \frac{\partial z}{\partial x}|_{(1,0)} = e^{2y}|_{(1,0)} = 1; \frac{\partial z}{\partial y}|_{(1,0)} = 2xe^{2y}|_{(1,0)} = 2,$$

所求方向导数

$$\frac{\partial z}{\partial l} = \cos\left(-\frac{\pi}{4}\right) + 2\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}.$$

例2 解 由方向导数的计算公式知

$$\frac{\partial f}{\partial l}|_{(1,1)} = f_x(1,1)cos\alpha + f_y(1,1)sin\alpha$$

$$= (2x - y)|_{(1,1)}cos\alpha + (2y - x)|_{(1,1)}sin\alpha$$

$$= cos\alpha + sin\alpha = \sqrt{2}\sin(\alpha + \frac{\pi}{4})$$

（1）当$\alpha = \frac{\pi}{4}$时，方向导数达到最大值$\sqrt{2}$;

（2）当$\alpha = \frac{5\pi}{4}$时，方向导数达到最小值$-\sqrt{2}$;

（3）当$\alpha = \frac{3\pi}{4}$和$\alpha = \frac{7\pi}{4}$时，方向导数等于0.

例3. 令$f(x,y)=\frac{1}{x^2+y^2}$,则$f_x=-\frac{2x}{(x^2+y^2)^2}$, $f_y=-\frac{2y}{(x^2+y^2)^2}$,所以$\mathrm{grad}\, f=-\frac{2x}{(x^2+y^2)^2}i-\frac{2y}{(x^2+y^2)^2}j$.

例4.$f_x=2x, f_y=2y, f_z=2z$,所以$grad\, f(1,-1,2)=(2,-2,4)$

例5.根据梯度的定义可以知道$grad\,(u+v)=\left((u+v)_x,(u+v)_y,(u+v)_z\right)=\left(u_x,u_y,u_z\right)+\left(v_x,v_y,v_z\right)=grad\, u+grad\, v$

例6. 解. 根据题意得此函数在两点的梯度为 $(2,0,2),(0,2,0)$,根据夹角的定义得$\cos\alpha=\frac{0}{2\sqrt{8}}=0$,所以$\alpha=\frac{\pi}{2}$.