4. 求下列函数的导数:

(1) $y = 4x^3 + 2x$.

$y' = 12x^2+2$

(2) $y = \frac{1}{x^3} + \frac{3}{x^2} + 4$.

$y = x^{-3}+3x^{-2}+4$

$y' = -3x^{-4}\cancel{+}-6x^{-3}$.

(3) $y = 2e^x + 3\tan x$. $= 2e^x+3\frac{\sin x}{\cos x}$

$y' = 2e^x+3\frac{\cos x\cos x+\sin^2 x}{\cos^2 x} = 2e^x+\frac{3}{\cos^2 x} = 2e^x+3\sec^2 x$

(4) $y = 3\ln x + 4\lg x + \ln 5$.

$y' = \frac{3}{x} + \frac{4}{x\ln 10}$

(5) $y = \sin x \ln x$.

$y' = \cos x\ln x + \frac{1}{x}\sin x$

(6) $y = x^2 e^x \cos x$.

$y' = 2x\cdot e^x\cos x + x^2\cdot e^x\cos x - x^2e^x\cdot\sin x$

$= xe^x(2\cos x + x\cos x - x\sin x)$.

(7) $y = \frac{5x^2+3x}{1+x^2}$.

$y' = \frac{(10x+3)(x^2+1)-(5x^2+3x)\cdot 2x}{(x^2+1)^2}$

$= \frac{\cancel{10x^3}+3x^2+10x+3-\cancel{10x^3}-6x^2}{x^4+2x^2+1}$

$= \frac{-3x^2+10x+3}{x^4+2x^2+1}$.

(8) $y = \frac{x^2-\ln x}{x^2+\ln x}$.

$y = \frac{x^2+\ln x-2\ln x}{x^2+\ln x} = 1-\frac{2\ln x}{x^2+\ln x}$

$y' = -\frac{\frac{2}{x}(x^2+\ln x)-2\ln x(2x+\frac{1}{x})}{(x^2+\ln x)^2} = \frac{\cancel{2x}-2x-\cancel{\frac{2}{x}\ln x}+4x\ln x+\cancel{\frac{2}{x}\ln x}}{(x^2+\ln x)^2}$

$= \frac{4x\ln x-2x}{(x^2+\ln x)^2}$.

5. 求 a 为何值时曲线 $y = \ln x$ 与曲线 $y = ax^2$ 相切.

设两曲线在 $x=x_0$ 处相切.

$$\begin{cases} \ln x_0 = a x_0^2 \\ \frac{1}{x_0} = 2a x_0 \end{cases} \quad \therefore a = \frac{1}{2e}$$

6. 求下列函数的导数:

(1) $y = (3x-2)^{10}$. $y' = 10(3x-2)^9 \cdot 3 = 30(3x-2)^9$.

(2) $y = \sin(4x+1)$. $y' = 4\cos(4x+1)$

(3) $y = e^{-x^2}$. $y' = e^{-x^2}(-2x) = -2x \cdot e^{-x^2}$.

(4) $y = \ln(3x^2+2)$. $y' = \frac{6x}{3x^2+2}$

(5) $y = \arcsin(x^2)$. $y' = \frac{2x}{\sqrt{1-x^4}}$

(6) $y = (\arcsin x)^2$. $y' = \frac{2\arcsin x}{\sqrt{1-x^2}}$.

(7) $y = \ln\sin 2x$. $y' = \frac{1}{\sin 2x}(\sin 2x)' = \frac{2\cos 2x}{\sin 2x} = \frac{2}{\tan 2x}$

(8) $y = \sqrt{a^2+x^2}\cos x$. $y' = \frac{1}{2}(a^2+x^2)^{-\frac{1}{2}}\cos x \cdot 2x - \sqrt{a^2+x^2}\sin x$

$$= \frac{x\cos x}{\sqrt{a^2+x^2}} - \sqrt{a^2+x^2}\sin x.$$

(9) $y = e^{3x}\sin(5x+1)$.

$$y' = e^{3x} \cdot 3\sin(5x+1) + 5e^{3x}\cos(5x+1)$$
$$= 3e^{3x}\sin(5x+1) + 5e^{3x}\cos(5x+1).$$

(10) $y = \arccos\sqrt{x+1}$.

$$y' = -\frac{1}{\sqrt{1-(x+1)}} \cdot \frac{1}{2}(x+1)^{-\frac{1}{2}} = -\frac{1}{\sqrt{-x}} \cdot \frac{1}{2\sqrt{x+1}}$$
$$= \frac{-1}{2\sqrt{-x^2-x}}$$

(11) $y = \ln(\sec x - \tan x)$.

$y' = \frac{1}{\sec x - \tan x}(\tan x \sec x - \sec^2 x)$

$= \frac{(\tan x - \sec x)\sec x}{\sec x - \tan x} = -\sec x$.

(12) $y = a^{a^x} + a^{x^a} + a^{a^a}$.

~~$y' = a^{a^x}(a^x)' + a^{x^a}(x^a)'$~~

~~$= a^{a^x} \cdot a^x \ln a + a^{x^a} \cdot a \cdot x^{a-1} = a^{a^x+x}\ln a + a^{x^a+1} \cdot x^{a-1}$~~

$y' = a^{a^x}\ln a \cdot a^x \ln a + a^{x^a} \cdot \ln a \cdot a x^{a-1}$

(13) $y = \arcsin\sqrt{\frac{1-x}{1+x}}$.

$y' = \frac{1}{\sqrt{1-\frac{1-x}{1+x}}} \cdot \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-\frac{1}{2}} \cdot \frac{-(1+x)-(1-x)}{(1+x)^2} = -\frac{1}{(x+1)\sqrt{2x(1-x)}}$

(14) $y = e^{\arctan\sqrt{x}}$.

$y' = e^{\arctan\sqrt{x}} \cdot \frac{1}{1+x} \cdot \frac{1}{2}x^{-\frac{1}{2}} = \frac{e^{\arctan\sqrt{x}}}{(x+1)2\sqrt{x}}$

7. (1) 设 $y = f(e^{\sin^2 2x})$, 其中 $f(x)$ 可导, 求y'.

$y' = f'(e^{\sin^2 2x}) \cdot e^{\sin^2 2x} \cdot 2\sin 2x \cdot \cos 2x \cdot 2$

$= 2e^{\sin^2 2x}\sin 4x \cdot f'(e^{\sin^2 2x})$

(2) 设函数 $F(x)$ 在 $x=0$处可导，函数 $g(x) = \begin{cases} x^2\sin\frac{1}{x}, & x \neq 0, \\ 0, & x = 0. \end{cases}$ 求复合函数 $F(g(x))$ 在 $x=0$ 处的导数.

$x \neq 0$ $\quad g'(x) = 2x\sin\frac{1}{x} + (-x^2\cos\frac{1}{x}\frac{1}{x^2})$

$= 2x\sin\frac{1}{x} - \cos\frac{1}{x}$

$x=0$ $\quad g'(x) = 0$.

~~$[F(g(x))]' = F'(g(x)) \cdot g'(x)$~~

故 $[F(g(0))]' = F'(g(0)) \cdot g'(0) = 0$

$f'(x)=f(x)\cdot[\ln f(x)]'$

8. 用对数求导法求下列函数的导数.

(1) $y=(\cos x)^{\cos x}$;

$y'=(\cos x)^{\cos x}(\cos x\ln\cos x)'$

$=(\cos x)^{\cos x}\left[-\sin x\ln\cos x+\cos x\cdot\frac{1}{\cos x}\cdot(-\sin x)\right]$

$=-(\cos x)^{\cos x}\sin x\cdot(1+\ln\cos x)$

(2) $y=\sqrt{e^{\frac{1}{x}}\sqrt{x\sqrt{\sin x}}}$;

$\ln y=\ln(e^{\frac{1}{2x}}\cdot x^{\frac{1}{4}}\cdot \sin x^{\frac{1}{8}})$

$=\ln\frac{1}{2x}+\frac{1}{4}\ln x+\frac{1}{8}\ln\sin x$

$y'=\sqrt{e^{\frac{1}{x}}\sqrt{x\sqrt{\sin x}}}\left(-\frac{1}{2x^2}+\frac{1}{4x}+\frac{\cos x}{8\sin x}\right)$

$=\left(-\frac{1}{2x^2}+\frac{1}{4x}+\frac{\cot x}{8}\right)\sqrt{e^{\frac{1}{x}}\sqrt{x\sqrt{\sin x}}}$.

(3) $y=\frac{\sqrt{x+1}(3-x)^3}{(x+2)^4}$.

$y'=\frac{\sqrt{x+1}(3-x)^3}{(x+2)^4}\left[\frac{1}{2}\ln(x+1)+3\ln(3-x)-4\ln(x+2)\right]'$

$=\frac{\sqrt{x+1}(3-x)^3}{(x+2)^4}\left[\frac{1}{2(x+1)}-\frac{3}{3-x}-\frac{4}{x+2}\right]$