9. 求下列函数的二阶导数.

(1) $y = x^3 + \cos x$.

$y' = 3x^2 - \sin x$

$y'' = 6x - \cos x$

(2) $y = (1 + x^2)\arctan x$.

$y' = 2x \cdot \arctan x + (1+x^2) \cdot \frac{1}{1+x^2}$

$= 2x \cdot \arctan x + 1$

$y'' = 2\arctan x + 2x \cdot \frac{1}{1+x^2}$

$= 2\arctan x + \frac{2x}{1+x^2}$

(3) $y = xe^{-x^2}$;

$y' = e^{-x^2} + x \cdot e^{-x^2} \cdot (-2x)$

$= e^{-x^2}(1 - 2x^2)$

$y'' = e^{-x^2} \cdot (-2x)(1-2x^2) + e^{-x^2} \cdot (-4x)$

$= e^{-x^2}(-6x + 4x^3)$

(4) $y = \ln\sqrt{\frac{1-x}{1+x}}$.

$y = \frac{1}{2}\ln\frac{1-x}{1+x} = \frac{1}{2}[\ln(1-x) - \ln(1+x)]$

$y' = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$

$y'' = \frac{1}{2}\left[\frac{1}{(x-1)^2} - \frac{1}{(x+1)^2}\right]$

$= -\frac{1}{2}\left[\frac{1}{(x-1)^2} + \frac{1}{(x+1)^2}\right]$

10. 求下列函数的 n 阶导数的一般表达式.

(1) $y = \frac{1}{x^2+4x-12}$. $= \frac{1}{(x+6)(x-2)} = \frac{1}{8}\left(\frac{1}{x-2} - \frac{1}{x+6}\right)$

$y^n = \frac{1}{8}\left[\left(\frac{1}{x-2}\right)^{(n)} - \left(\frac{1}{x+6}\right)^{(n)}\right]$

$= \frac{1}{8}\cdot(-1)^n\cdot n!\cdot\left[\frac{1}{(x-2)^{n+1}} - \frac{1}{(x+6)^{n+1}}\right]$

(2) $y = \cos^4 x$.

$\cos^4 x = \left(\frac{1+\cos 2x}{2}\right)^2 = \frac{1}{4}\left(1+2\cos 2x + \frac{1+\cos 4x}{2}\right) = \frac{1}{8}(3+4\cos 2x+\cos 4x)$

$y^{(n)} = \frac{1}{8}\left[0+4\cos\left(2x+\frac{n\pi}{2}\right)\cdot 2^n + \cos\left(4x+\frac{n\pi}{2}\right)\cdot 4^n\right]$

$= 2^{n-1}\cos\left(2x+\frac{n\pi}{2}\right) + 2^{2n-3}\cdot\cos\left(4x+\frac{n\pi}{2}\right)$

(3) $y = x^2e^{2x}$.

$\because y' = e^{2x}(2x^2+2x) = 2^0\cdot e^{2x}\left(2x^2+2\cdot x+\frac{1^2-1}{2}\right)$

$y'' = 2e^{2x}(2x^2+4x+1) = 2^1\cdot e^{2x}\left(2x^2+2\cdot 2x+\frac{2^2-2}{2}\right)$

$y^{(3)} = 4e^{2x}(2x^2+6x+3) = 2^2\cdot e^{2x}\left(2x^2+2\cdot 3x+\frac{3^2-3}{2}\right)$

$\therefore y^{(n)} = 2^{n-1}e^{2x}\left(2x^2+2nx+\frac{n^2-n}{2}\right)$

(4) $y = \frac{x}{\sqrt[3]{x+1}}$.

$y^{(n)} = \sum_{i=0}^{n} C_n^i (x)^{(i)}\cdot\left(\frac{1}{\sqrt[3]{x+1}}\right)^{(n-i)} = x\cdot\left(\frac{1}{\sqrt[3]{x+1}}\right)^{(n)} + n\cdot\left(\frac{1}{\sqrt[3]{x+1}}\right)^{(n-1)}$

$= x\cdot(-1)^n\cdot\left(n-\frac{2}{3}\right)!\cdot(x+1)^{-(n+\frac{1}{3})} + n\cdot(-1)^{n-1}\left(n-\frac{5}{3}\right)!\cdot(x+1)^{-(n-\frac{2}{3})}$

11. 求由下列方程所确定的隐函数 y 的导数 $\frac{dy}{dx}$:

(1) $\ln(x^2+y) = x^3y+\sin x$;

$\frac{2x+y'}{x^2+y} = 3x^2y + x^3\cdot y' + \cos x$

$2x+y' = (x^2+y)(3x^2y+\cos x) + x^5y' + x^3y\cdot y'$

$(x^5+x^3y-1)y' = 2x-(x^2+y)(3x^2y+\cos x)$

$\frac{dy}{dx} = y' = \frac{2x-(x^2+y)(3x^2y+\cos x)}{x^5+x^3y-1}$

(2) $e^{xy} + y^2 = \cos x$.

$e^{xy}\cdot(y+xy') + 2y\cdot y' = -\sin x$

$y'(xe^{xy}+2y) = -\sin x - ye^{xy}$

$y' = -\dfrac{\sin x + y\cdot e^{xy}}{xe^{xy}+2y}$

12. 求由下列方程所确定的隐函数 $y = y(x)$ 的二阶导数 $\frac{d^2y}{dx^2}$：

(1) $y = x + \arctan y$.

两边求导：$y' = 1 + \dfrac{1}{1+y^2}\cdot y'$

$(1+y^2)y' = 1+y^2+y'$

$y^2y' = 1+y^2$

$y' = \dfrac{1+y^2}{y^2}$

$y^2y' = 1+y^2$ 两边求导得

$2y\cdot(y')^2 + y^2\cdot y'' = 2y\cdot y'$

$y'' = \dfrac{2yy'(1-y')}{y^2} = \dfrac{2y'(1-y')}{y}$

$= -\dfrac{2(1+y^2)}{y^5}$

(2) $y = 1 + xe^y$.

两边求导：$y' = e^y + xe^y\cdot y'$

$y' = \dfrac{e^y}{1-xe^y}$

$y' = e^y + xe^y\cdot y'$ 两边求得

$y'' = e^y\cdot y' + (e^y + x\cdot e^y\cdot y')y' + xe^y\cdot y''$

$= 2e^y\cdot y' + xe^y(y')^2 + xe^y\cdot y''$

$\therefore y'' = \dfrac{2e^y\cdot y' + x\cdot e^y(y')^2}{1-xe^y}$

$= \dfrac{(2-xe^y)\cdot e^{2y}}{(1-xe^y)^3}$

13. 求下列参数方程所确定的函数的导数$\frac{dy}{dx}$：

(1) $\begin{cases} x = 2t - \cos t, \\ y = 1 - \sin t. \end{cases}$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{-\cos t}{2+\sin t}$$

(2) $\begin{cases} x = \arctan t, \\ 2y - ty^2 + e^t = 5. \end{cases}$

$2y-ty^2+e^t=5$ 两边求导： $2y' - y^2 - 2ty \cdot y' + e^t = 0$

$$y' = \frac{y^2-e^t}{2-2ty} \quad \therefore \frac{dy}{dt} = \frac{y^2-e^t}{2-2ty}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y^2-e^t}{(2-2ty)\cdot\frac{1}{t^2+1}}$$

$$= \frac{(y^2-e^t)(t^2+1)}{2-2ty}$$

14. 求下列参数方程所确定的函数的二阶导数$\frac{d^2y}{dx^2}$：

(1) $\begin{cases} x = \sin t - t, \\ y = 1 - \cos t. \end{cases}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin t}{\cos t - 1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$= \frac{\frac{\cos t(\cos t-1) - \sin t(-\sin t)}{(\cos t-1)^2}}{\cos t - 1}$$

$$= \frac{1-\cos t}{(\cos t-1)^3}$$

$$= -\frac{1}{(\cos t-1)^2}$$

(2) $\begin{cases} x = \ln(1+t^2), \\ y = \arctan t. \end{cases}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{t^2+1}}{\frac{1}{t^2+1}\cdot 2t} = \frac{1}{2t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{-\frac{1}{2t^2}}{\frac{1}{t^2+1}\cdot 2t} = -\frac{t^2+1}{4t^3}$$

15. 求曲线 $\begin{cases} x = e^t \sin 2t, \\ y = e^t \cos t \end{cases}$ 在点$(0,1)$处的切线方程和法线方程.

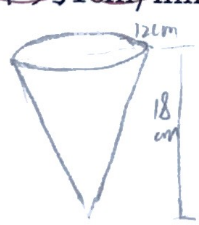
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t(\cos t - \sin t)}{e^t(\sin 2t + 2\cos 2t)} = \frac{\cos t - \sin t}{\sin 2t + 2\cos 2t}$$

$x=0$时, $y' = \left.\frac{dy}{dx}\right|_{t=0} = \frac{1}{2}$

∴切线: $y-1 = \frac{1}{2}x \Rightarrow x-2y+2=0$

法线: $y-1 = -2x \Rightarrow 2x+y-1=0$

16. 溶液自深18cm顶直径12cm的正圆锥形漏斗中漏入一直径为10cm的圆柱形筒中.开始时漏斗中盛满了溶液.已知当溶液在漏斗中深为12cm时, 其表面下降的速率为1cm/min, 问此时圆柱形筒中溶液表面上升的速率为多少?



设圆锥液面高度为h时

圆柱液面高度为y

则圆锥内液面半径为 $\frac{1}{3}h$

$$\frac{1}{3}\pi\cdot 6^2\cdot 18 - \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \pi\cdot 5^2\cdot y$$

$\Rightarrow$

$$216\pi - \frac{1}{27}\pi h^3 = 25\pi y$$

$$216 - \frac{1}{27}h^3 = 25y$$

两边求导 $-\frac{1}{9}h^2\cdot\frac{dh}{dt} = 25\frac{dy}{dt}$

∴当 $h=12$ $\frac{dh}{dt} = 1$ 时 -1

$$\frac{dy}{dt} = +\frac{16}{25}$$

17. 求下列函数的微分:

(1) $y=(x+1)^x+\arctan\ln x$.

$y=e^{x\ln(x+1)}+\arctan\ln x$

$y'=e^{x\ln(x+1)}\cdot\left[\ln(x+1)+\frac{x}{x+1}\right]+\frac{1}{1+(\ln x)^2}\cdot\frac{1}{x}$

$=(x+1)^x\left[\ln(x+1)+\frac{x}{1+x}\right]+\frac{1}{x(1+\ln^2 x)}$

$dy=\left\{(x+1)^x\left[\ln(x+1)+\frac{x}{1+x}\right]+\frac{1}{x(1+\ln^2 x)}\right\}dx$

(2) $y=\arctan\sqrt{2-x}$.

$y'=\frac{1}{1+2-x}\cdot\frac{1}{2}\frac{1}{\sqrt{2-x}}\cdot(-1)$

$=\frac{1}{2(x-3)\sqrt{2-x}}$

$dy=\frac{1}{2(x-3)\sqrt{2-x}}dx$

(3) $y=[g(x)]^{x+1}$, ($g(x)$有一阶导数, $g(x)>0$).

$\ln y=(x+1)\ln[g(x)]$

$y'=y\cdot(\ln y)'=[g(x)]^{x+1}\cdot\left\{\ln[g(x)]+(x+1)\cdot\frac{1}{g(x)}\cdot g'(x)\right\}$

$dy=[g(x)]^{x+1}\left[\ln[g(x)]+(x+1)\cdot\frac{g'(x)}{g(x)}\right]dx$

(4) $y=2^{-\frac{1}{\cos x}}$.

$y'=2^{-\frac{1}{\cos x}}\cdot\ln 2\cdot\left(-\frac{\sin x}{\cos^2 x}\right)$

$dy=2^{-\frac{1}{\cos x}}\cdot\ln 2\left(-\frac{\sin x}{\cos^2 x}\right)dx.$