答案

例1. 解 在点(0,0)处有$f_x(0,0)=f_y(0,0)=0$,

$\Delta z-\left(f_x(0,0)\cdot\Delta x+f_y(0,0)\cdot\Delta y\right)=\frac{\Delta x\cdot\Delta y}{\sqrt{(\Delta x)^2+(\Delta y)^2}}$,如果考虑点

$P'(\Delta x,\Delta y)$沿着直线$y=x$趋近于(0,0)则

$$\frac{\frac{\Delta x\cdot\Delta y}{\sqrt{(\Delta x)^2+(\Delta y)^2}}}{\rho}=\frac{\Delta x\cdot\Delta x}{(\Delta x)^2+(\Delta x)^2}=\frac{1}{2}$$

说明它不随着$\rho\to 0$而趋于0，当$\rho\to 0$时，

$$\Delta z-\left(f_x(0,0)\cdot\Delta x+f_y(0,0)\cdot\Delta y\right)\neq o(\rho)$$

函数在点(0,0)处不可微。

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例2. 解 在点(0,0)处有$f_x(0,0)=f_y(0,0)=0$,

$\Delta z-\left(f_x(0,0)\cdot\Delta x+f_y(0,0)\cdot\Delta y\right)=\left((\Delta x)^2+(\Delta y)^2\right)\sin\frac{1}{(\Delta x)^2+(\Delta y)^2}\leq$ $(\Delta x)^2+(\Delta y)^2=0$,所以函数在点(0,0)处可微。

例3. $\frac{\partial z}{\partial x}=ye^{xy},\frac{\partial z}{\partial y}=xe^{xy},\frac{\partial z}{\partial x}|_{(2,1)}=e^2,\frac{\partial z}{\partial y}|_{(2,1)}=2e^2$

所以全微分为$dz=e^2dx+2e^2dy$

例4. $\frac{\partial z}{\partial x}=-y\sin(x-2y),\frac{\partial z}{\partial y}=\cos(x-2y)+2y\sin(x-2y)$,

$$dz|_{(\frac{\pi}{4},\pi)}=\frac{\partial z}{\partial x}|_{(\frac{\pi}{4},\pi)}dx+\frac{\partial z}{\partial y}|_{(\frac{\pi}{4},\pi)}dy=\frac{\sqrt{2}}{8}\pi(4+7\pi)$$

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例5. 令$x=\rho\cos\theta, y=\rho\sin\theta$,

则 $\lim\limits_{(x,y)\to(0,0)} xy\sin\frac{1}{\sqrt{x^2+y^2}}=\lim\limits_{\rho\to 0}\rho^2\sin\theta\cos\theta\cdot\sin\frac{1}{\rho}=0=f(0,0)$

故函数在点(0,0)处连续，

$$f_x(0,0)=\lim_{\Delta x\to 0}\frac{f(\Delta x,0)-f(0,0)}{\Delta x}=\lim_{\Delta x\to 0}\frac{0-0}{\Delta x}=0$$

同理$f_y(0,0)=0.$

当$(x,y)\neq(0,0)$时， $f_x(x,y)=y\sin\frac{1}{\sqrt{x^2+y^2}}-\frac{x^2y}{\sqrt{(x^2+y^2)^3}}\cos\frac{1}{\sqrt{x^2+y^2}}$,

当点$P(x,y)$沿直线$y=x$趋于(0,0)时，

$\lim\limits_{(x,x)\to(0,0)} f_x(x,y)=\lim\limits_{x\to 0} x\sin\frac{1}{\sqrt{2}|x|}-\frac{x^3}{2\sqrt{2}|x|^3}\cos\frac{1}{\sqrt{2}|x|}$不存在

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所以$f_x(x,y)$在(0,0)不连续，同理可证$f_y(x,y)$在(0,0)不连续.

$\Delta f = f(\Delta x,\Delta y) - f(0,0) = \Delta x\cdot\Delta y\cdot\sin\frac{1}{\sqrt{(\Delta x)^2+(\Delta y)^2}} =$

$o(\sqrt{(\Delta x)^2+(\Delta y)^2})$故f(x,y)在点(0,0)可微，$df|_{(0,0)} = 0$

例6. 设函数$f(x,y) = x^y$.取$x = 1, y = 2, \Delta x = 0.04, \Delta y = 0.02.\because$

$f_x(x,y) = yx^{y-1}, f_y(x,y) = x^y\ln x,\ \therefore f_x(1,2) = 2, f_y(1,2) = 0$

所以$(1.04)^{2.02}\approx 1 + 2\times 0.04 + 0\times 0.02 = 1.08$