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例1. 解 $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}=e^u\sin v\cdot y+e^u\cos v\cdot 1=e^u(y\sin v+\cos v)$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}=e^u\sin v\cdot x+e^u\cos v\cdot 1=e^u(x\sin v+\cos v)$$

例2. 解 $\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}+\frac{\partial z}{\partial t}$

$$=e^t\cdot v-u\sin t+\cos t=e^t\cos t-e^t\sin t+\cos t$$

$$=e^t(\cos t-\sin t)+\cos t$$

例3. 令$u=x+y+z, v=xyz$,记$f_1'=\frac{\partial f(u,v)}{\partial u}, f_{12}''=\frac{\partial^2 f(u,v)}{\partial u\partial v}$,同理有$f_2', f_{11}'', f_{22}''$.

$$\frac{\partial w}{\partial x}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x}=f_1'+yzf_2';$$

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$$\frac{\partial^2 w}{\partial x \partial z} = \frac{\partial}{\partial z}(f_1' + yzf_2') = \frac{\partial f_1'}{\partial z} + yf_2' + yz\frac{\partial f_2'}{\partial z}$$

$$\frac{\partial f_1'}{\partial z} = \frac{\partial f_1'}{\partial u} \cdot \frac{\partial u}{\partial z} + \frac{\partial f_1'}{\partial v} \cdot \frac{\partial v}{\partial z} = f_{11}'' + xyf_{12}''$$

$$\frac{\partial f_2'}{\partial z} = \frac{\partial f_2'}{\partial u} \cdot \frac{\partial u}{\partial z} + \frac{\partial f_2'}{\partial v} \cdot \frac{\partial v}{\partial z} = f_{21}'' + xyf_{22}''$$

于是

$$\begin{aligned}\frac{\partial^2 w}{\partial x \partial z} &= f_{11}'' + xyf_{12}'' + yf_2' + yz(f_{21}'' + xyf_{22}'') \\ &= f_{11}'' + y(x+z)f_{12}'' + xy^2zf_{22}'' + yf_2'\end{aligned}$$

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例4.

$$\because d(e^{-xy} - 2z + e^{z}) = 0,$$
$$\therefore e^{-xy} d(-xy) - 2dz + e^{z}dz = 0,$$
$$e^{-xy}(xdy + ydx) = (e^{z} - 2)dz$$
$$dz = \frac{ye^{-xy}}{(e^{z} - 2)}dx + \frac{xe^{-xy}}{(e^{z} - 2)}dy$$
$$\frac{\partial z}{\partial x} = \frac{ye^{-xy}}{(e^{z}-2)}, \frac{\partial z}{\partial y} = \frac{xe^{-xy}}{(e^{z}-2)}$$

例5. $\ln z = xy\ln(x^2 + y^2)$,

$$\frac{\partial z}{\partial x} = (x^2 + y^2)^{xy} \cdot (y\ln(x^2 + y^2) + \frac{2x^2y}{x^2+y^2})$$
$$\frac{\partial z}{\partial y} = (x^2 + y^2)^{xy} \cdot (x\ln(x^2 + y^2) + \frac{2xy^2}{x^2 + y^2})$$

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$$d = (x^2+y^2)^{xy} \cdot \left(y\ln(x^2+y^2) + \frac{2x^2y}{x^2+y^2}\right)dx$$

$$+(x^2+y^2)^{xy} \cdot \left(x\ln(x^2+y^2) + \frac{2xy^2}{x^2+y^2}\right)dy$$

例6. $\frac{\partial u}{\partial x} = f_1' + f_2' \cdot \varphi_1' + f_2' \cdot \varphi_2' \cdot \psi_x$

$\frac{\partial u}{\partial z} = f_3' + f_2' \cdot \varphi_2' \cdot \psi_z$

所以$du = (f_1' + f_2' \cdot \varphi_1' + f_2' \cdot \varphi_2' \cdot \psi_x)dx + (f_3' + f_2' \cdot \varphi_2' \cdot \psi_z)dz$