

EXTENDING NEO DEFLECTION FORMULAE TO HIGH FLUENCES

IAA PDC Conference 2023

April 5, 2023

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Introduction

- An asteroid as large as the one in the 2023 PDC exercise (> 220 m) is difficult or impossible to deflect using kinetic impactors.
- Nuclear explosives can deliver much more energy/kg than an impactor. That energy is mostly in the form of x-rays that will ablate the asteroid's surface resulting in deflection or disruption of the asteroid.
- The detailed calculation of how the x-rays are absorbed and ablate the surface is very time consuming. Planners designing a mission to deflect an asteroid on an impact trajectory need to consider many scenarios as they plan an optimal mission.
- Therefore a simple analytic formula is needed to approximate the Δv generated by any given scenario.

History

- Several years ago Joe Wasem ran a series of calculations for several stand-off distances and several yields at a fixed stand-off. [2.5 orders of magnitude in fluence (kt/m^2)]
- These results were used to fit coefficients for an analytic formula derived using simplifying assumptions.
- This simple formula has been used in planning PDC scenarios for several years.
- The Planetary Defense group at LLNL has worked for a couple years to create an energy deposition source to use in hydrodynamics-only calculations so groups around the world can calculate the deflection of many types of asteroids without the complexity of full radiation-hydrodynamic calculations (see Mary Burkey's talk which follows).
- As we developed this source and looked at this year's scenario we discovered that the analytic formula would under-predict Δv for the high-fluence (kt/m^2) cases we needed to deflect this asteroid.

Outline

- The assumptions used in the analytic formula
- The data used to fit the coefficients
- Why this does not extrapolate to high fluences
- New calculations
- Updating the analytic formula

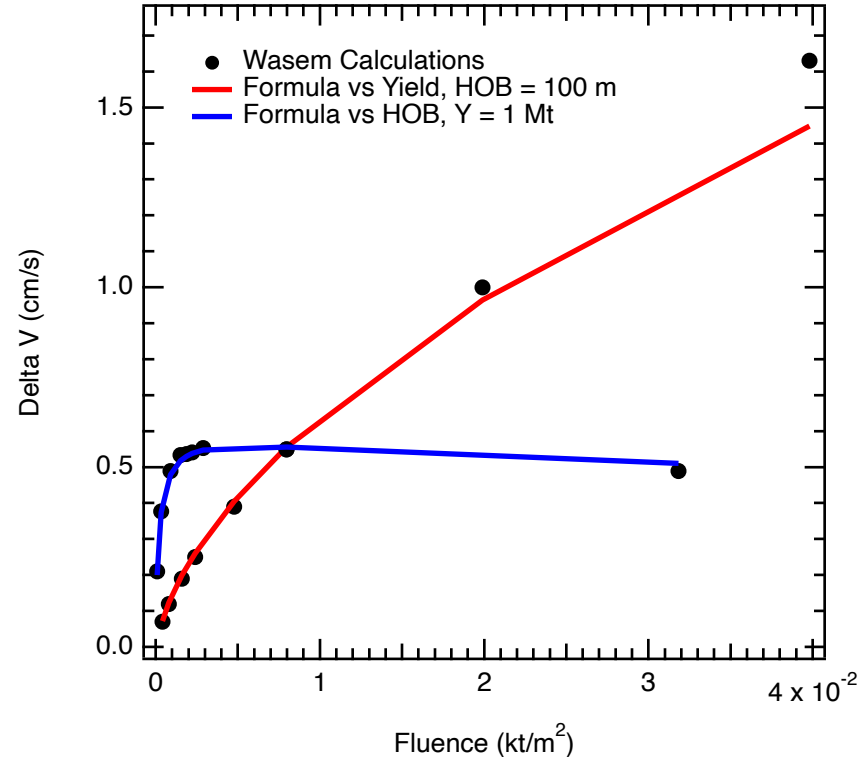
The derivation of the analytic formula

- The formula for a spherical asteroid is built from some simple assumptions:
 - The x-ray deposition profile is a single exponential, characterized by λ_d .
 - The melt energy, $\varepsilon_{\text{melt}}$, gives the depth of melted material.
 - The velocity generated is, $\Delta v \sim \sqrt{2ME}/M_{\text{ast}}$ where M is the melted mass, E is the deposited energy, and M_{ast} is the asteroid mass.
- These approximations allow you to integrate the formula in closed form
- For ease of fitting the parameters, λ_d and $\varepsilon_{\text{melt}}$, appear in the coefficients, $a/\sqrt{\rho} = \frac{3}{4}\sqrt{\lambda_d/(\pi\rho)}$ and the fluence at which melt begins, $b\rho = 4\pi\rho\lambda_d\varepsilon_{\text{melt}}$
In terms of the dimensionless variables $x = d/R$ and $y = Y/(b\rho d^2)$

$$\Delta v = \frac{a}{R^2} \sqrt{\frac{Y}{\rho}} \sqrt{\frac{x^2}{1+x} \left\{ \frac{2}{x} [1 + \ln y] - \left(1 + \frac{2}{x}\right) \ln \left(1 + \frac{2}{x}\right) \right\} \left[1 - \sqrt{1 - (1+x)^{-2}}\right]}$$

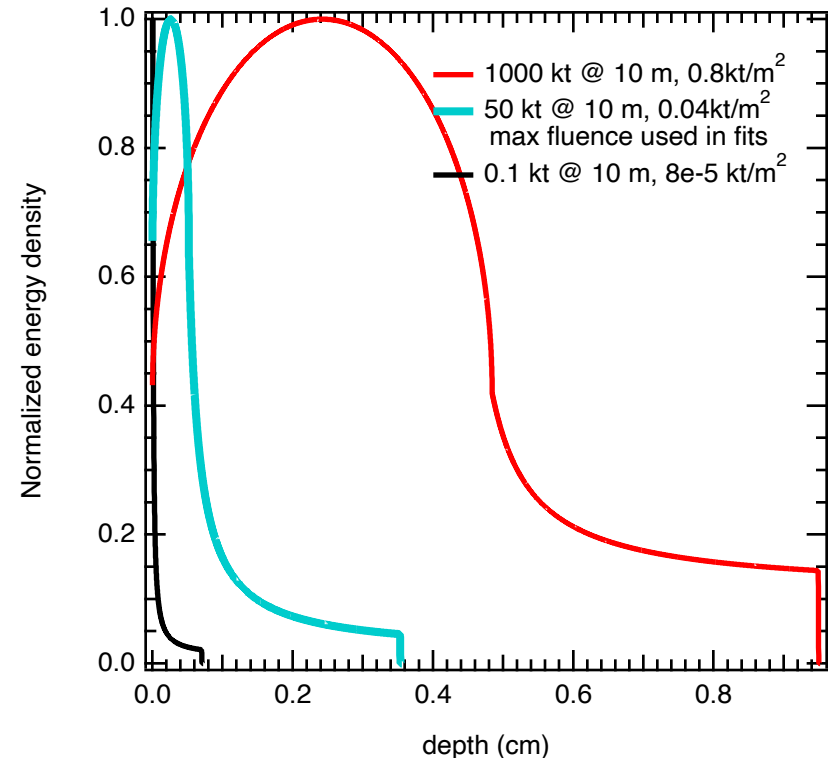
The analytic formula matches the data well except at one point

- The data from which the formula was fit go up to a fluence of 0.04 kt/m^2
- For this years scenario we looked at up to 1 Mt at 10m for a fluence of 0.8 kt/m^2 , 20 times larger than the data.
- The largest yield data point (red curve) shows the formula under-predicts Δv . It is not clear if the same result would be seen at smaller HOB's (blue curve)
 - Changing the HOB changes the geometry of the deposition as well as the fluence.



Issues at high fluence

- As the fluence increases the nature of the energy deposition changes
- At low fluence it is dominated by an exponential and shallow in depth (black curve).
- As the fluence increases the deposition depth increases and the near surface behavior transitions into a thermal wave.
- The thermal wave portion is not very pronounced in the depositions used to fit the formula (see middle curve).
- Since the deposition profile changes at high fluence I am not surprised that the formula does not work as well.

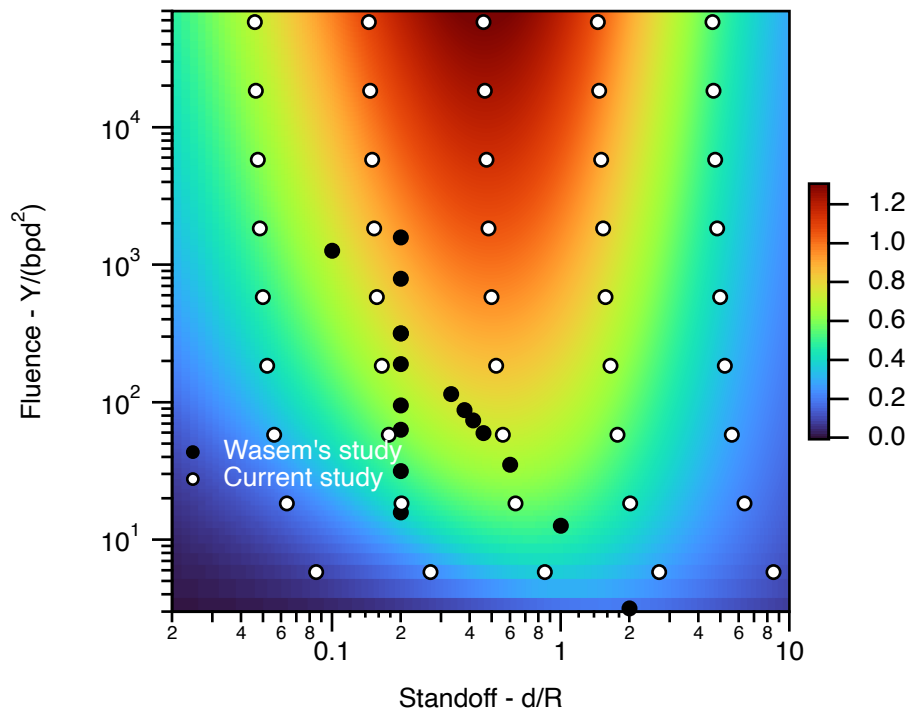


New Ares calculations

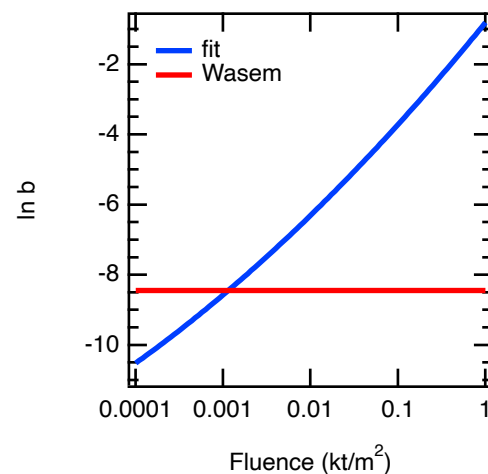
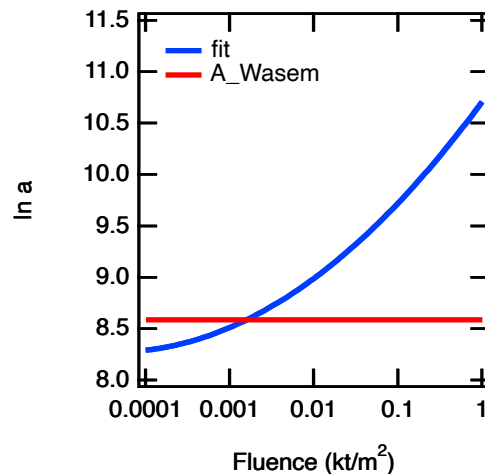
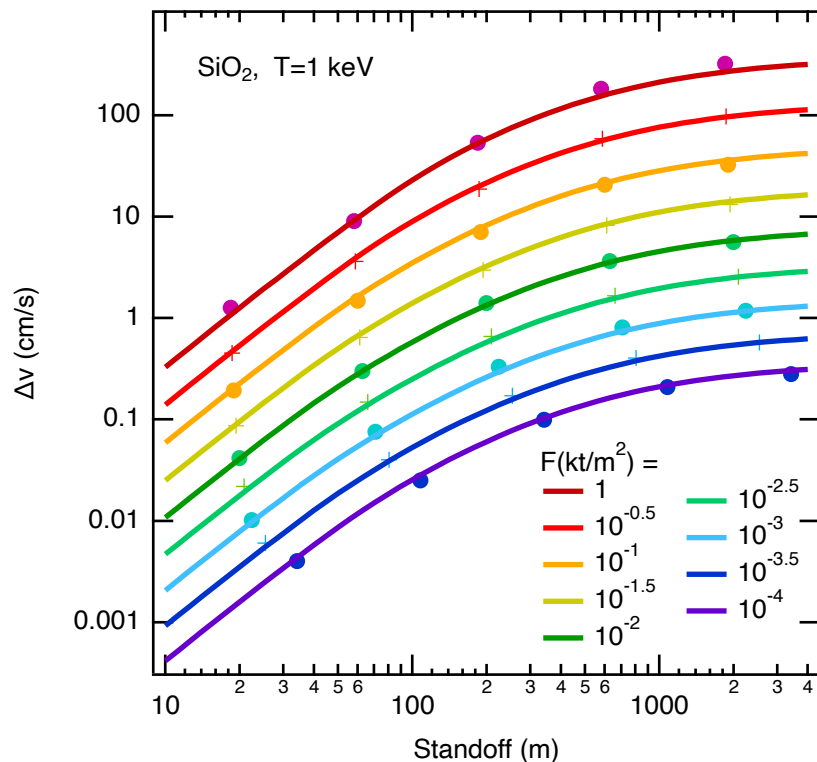
- Ares calculations have been done with Mary Burkey's energy deposition function (see the next talk), which depends on depth, angle of incidence, fluence, temperature, length of the source, material, and porosity. These are pure hydro calculations and run relatively quickly.
- The calculations were done for SiO_2 using $Y = 1 \text{ Mt}$, $\phi = 0$, $t_{\text{src}} = 50 \text{ ns}$, $T_{\text{src}} = 1 \text{ \& } 2 \text{ keV}$ and a diameter of 800 m.
- These calculations are used to fit an updated formula

Calculations to fit the formula with

- This plot shows the grid of calculations done to fully map out the nature of the formula. The calculations by Wasem are shown in black and did not fully cover the shape of the function.
- The color image shows the dimensionless part of the analytic formula.
- To get an acceptable fit the coefficients, a and b , needed to depend on the fluence, F .



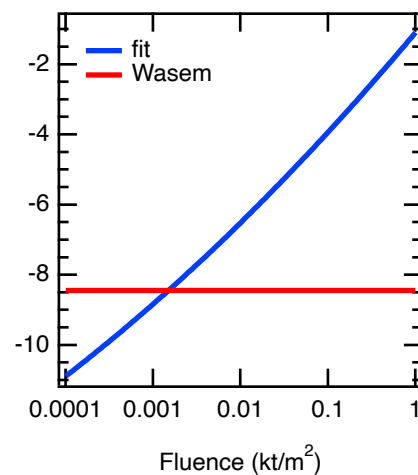
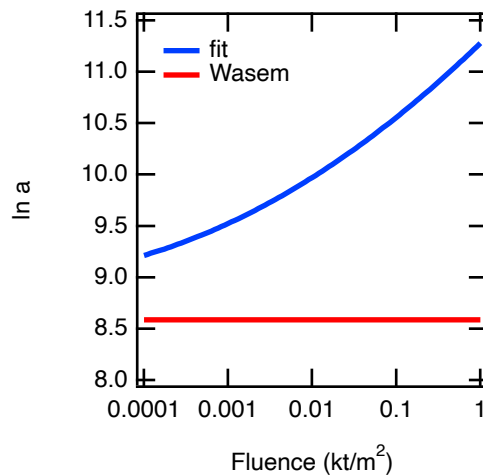
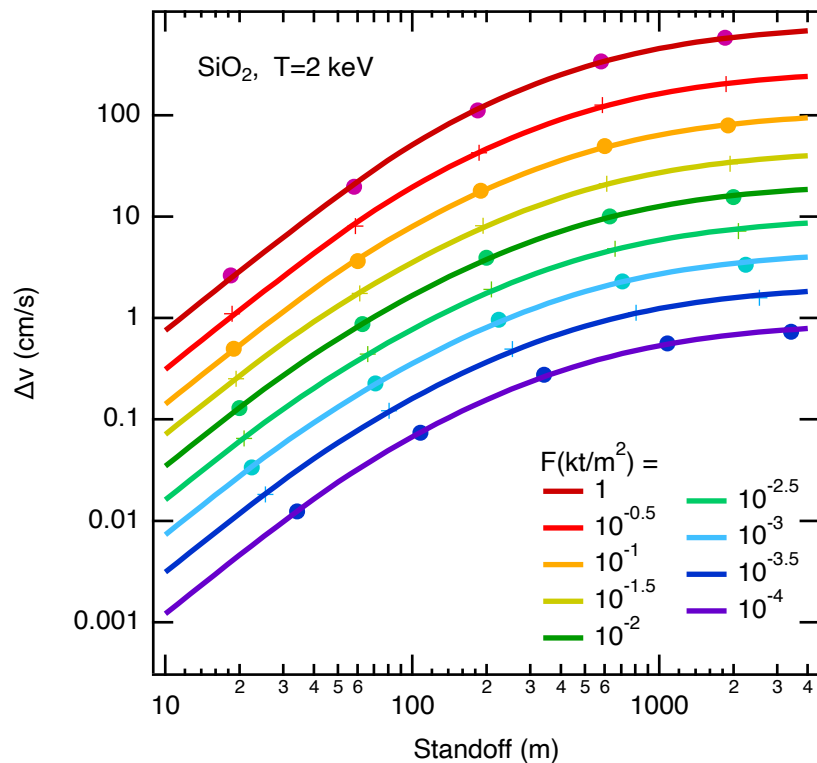
Results for 1 keV



$$\ln a = 10.709 + 1.115 \log F + 0.128(\log F)^2$$

$$\ln b = -0.808 + 3.066 \log F + 0.160(\log F)^2$$

Results for T = 2keV

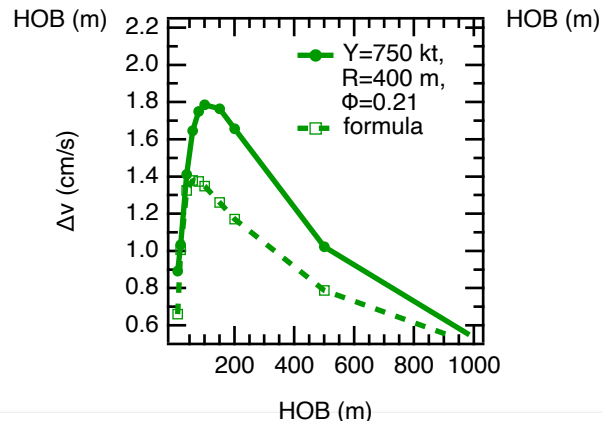
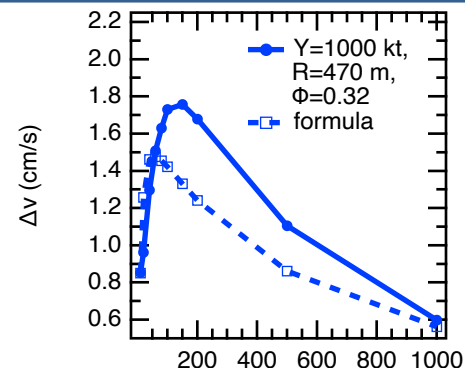
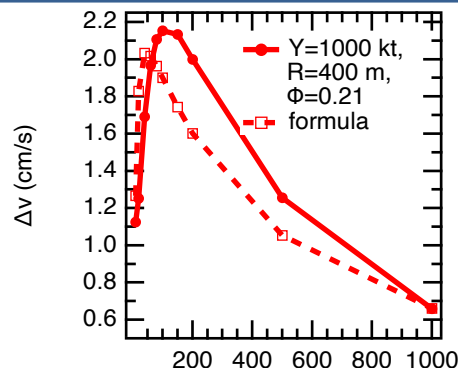


$$\ln a = 11.280 + 0.792 \log F + 0.0689(\log F)^2$$

$$\ln b = -1.094 + 2.972 \log F + 0.131(\log F)^2$$

Problems with the new formula

- The grid of calculations were done at full SiO_2 density.
- However, the PDC scenario uses a more realistic porosity. The analytic formula included a density dependence to account for this
- While this dependence improves the agreement with calculations it is not perfect.
- Here we see a comparison with calculations for an 800m diameter with $\phi = 0.21$ and with the upper bounding size of 939 m and a porosity of $\phi = 0.32$.
- The disagreement is much better than the factor of 5 the old formula was low by.



Conclusions

- A new deflection formula for nuclear payloads is available for SiO_2 .
- Mission planners have a new tool to use.
- Work is underway on other materials:
 - forsterite, iron-nickel, and ice
- Calculations in Spheral on non-spherical and/or rubble-pile asteroids are planned.

