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HCMUS-HLD

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Contest (1)

1	.bashrc	2 lines
2	alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \\\n-fsanitize=undefined,address'	
2	hash.sh	3 lines
2	# Hashes a file, ignoring all whitespace and comments. Use for	
3	# verifying that code was correctly typed.	
3	cpp -dD -P -fpreprocessed tr -d '[[:space:]]' md5sum cut -c-6	
3	note.txt	1 lines
3	#define rep(i, a, b) for(int i = a; i < (b); ++i)	

Mathematics (2)**2.1 Equations**

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by = e &\Rightarrow x = \frac{ed - bf}{ad - bc} \\ cx + dy = f &\Rightarrow y = \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where A'_i is A with the i 'th column replaced by b .

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \dots + c_k a_{n-k}$, and r_1, \dots, r_k are distinct roots of $x^k - c_1 x^{k-1} - \dots - c_k$, there are d_1, \dots, d_k s.t.

$$a_n = d_1 r_1^n + \dots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g.
 $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$

$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$

$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W) \tan(v-w)/2 = (V-W) \tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w .

$$a \cos x + b \sin x = r \cos(x - \phi)$$

$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry**2.4.1 Triangles**

Side lengths: a, b, c

$$\text{Semiperimeter: } p = \frac{a+b+c}{2}$$

$$\text{Area: } A = \sqrt{p(p-a)(p-b)(p-c)}$$

$$\text{Circumradius: } R = \frac{abc}{4A}$$

$$\text{Inradius: } r = \frac{A}{p}$$

Length of median (divides triangle into two equal-area triangles):

$$m_a = \frac{1}{2} \sqrt{2b^2 + 2c^2 - a^2}$$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[1 - \left(\frac{a}{b+c} \right)^2 \right]}$$

$$\text{Law of sines: } \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$$

$$\text{Law of cosines: } a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$\text{Law of tangents: } \frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$$

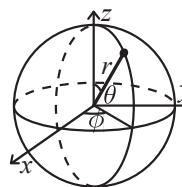
2.4.2 Quadrilaterals

With side lengths a, b, c, d , diagonals e, f , diagonals angle θ , area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180° , $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$\begin{aligned}x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\y &= r \sin \theta \sin \phi & \theta &= \arccos(z/\sqrt{x^2 + y^2 + z^2}) \\z &= r \cos \theta & \phi &= \text{atan2}(y, x)\end{aligned}$$

2.5 Derivatives/Integrals

$$\begin{aligned}\frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1+x^2} \\ \int \tan ax \, dx &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax \, dx &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} \, dx &= \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) & \int xe^{ax} \, dx &= \frac{e^{ax}}{a^2} (ax - 1)\end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x) \, dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x) \, dx$$

2.6 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\begin{aligned}\sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)\end{aligned}$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x . It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y ,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n, p)$, $n = 1, 2, \dots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$ is approximately $\text{Po}(np)$ for small p .

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\text{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\text{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let X_1, X_2, \dots be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i . π_j/π_i is the expected number of visits in state j between two visits in state i .

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i 's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k \rightarrow \infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets **A** and **G**, such that all states in **A** are absorbing ($p_{ii} = 1$), and all states in **G** leads to an absorbing state in **A**. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j , is $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is i , is $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ki} t_k$.

2.10 Bézout's identity

For $a \neq 0, b \neq 0$, then $d = \gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a, b)}, y - \frac{ka}{\gcd(a, b)} \right), \quad k \in \mathbb{Z}$$

2.11 Möbius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Möbius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d)g(n/d)$$

Other useful formulas/forms:

$$\sum_{d|n} \mu(d) = [n = 1] \text{ (very useful)}$$

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d/n)g(d)$$

$$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m)g(\lfloor \frac{n}{m} \rfloor)$$

InverseModulo.h

1491f8, 11 lines

```
template <typename T>
T inverse_modulo(T a, T m) {
    T u = 0, v = 1;
    while (a > 0) {
        T t = m / a;
        m -= t * a; std::swap(a, m);
        u -= t * v; std::swap(u, v);
    }
    assert(m == 1);
    return u;
}
```

PolyInterpolate.h

Description: Given n points $(x[i], y[i])$, computes an $n-1$ -degree polynomial p that passes through them: $p(x) = a[0] * x^0 + \dots + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi)$, $k = 0 \dots n-1$.

Time: $\mathcal{O}(n^2)$

08bf48, 13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    rep(k, 0, n-1) rep(i, k+1, n)
```

```
y[i] = (y[i] - y[k]) / (x[i] - x[k]);
double last = 0; temp[0] = 1;
rep(k, 0, n) rep(i, 0, n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
}
return res;
}
```

BerlekampMassey.h

Description: Recovers any n -order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.

Usage: berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}

Time: $\mathcal{O}(N^2)$

96548b, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
    int n = sz(s), L = 0, m = 0;
    vector<ll> C(n), B(n), T;
    C[0] = B[0] = 1;

    ll b = 1;
    rep(i, 0, n) { ++m;
        ll d = s[i] % mod;
        rep(j, 1, L+1) d = (d + C[j] * s[i - j]) % mod;
        if (!d) continue;
        T = C; ll coef = d * modpow(b, mod-2) % mod;
        rep(j, m, n) C[j] = (C[j] - coef * B[j - m]) % mod;
        if (2 * L > i) continue;
        L = i + 1 - L; B = T; b = d; m = 0;
    }

    C.resize(L + 1); C.erase(C.begin());
    for (ll& x : C) x = (mod - x) % mod;
    return C;
}
```

LinearRecurrence.h

Description: Generates the k 'th term of an n -order linear recurrence $S[i] = \sum_j S[i - j - 1]tr[j]$, given $S[0 \dots \geq n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.

Usage: linearRec({0, 1}, {1, 1}, k) // k 'th Fibonacci number

Time: $\mathcal{O}(n^2 \log k)$

f4e444, 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
    int n = sz(tr);

    auto combine = [&](Poly a, Poly b) {
        Poly res(n * 2 + 1);
        rep(i, 0, n+1) rep(j, 0, n+1)
            res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
        for (int i = 2 * n; i > n; --i) rep(j, 0, n)
            res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
        res.resize(n + 1);
        return res;
    };

    Poly pol(n + 1), e(pol);
    pol[0] = e[1] = 1;

    for (++k; k; k /= 2) {
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    }

    ll res = 0;
```

```
rep(i, 0, n) res = (res + pol[i + 1] * S[i]) % mod;
return res;
}
```

GaussianElimination.h

Description: Gaussian Elimination for solving systems of linear equations. **Usage:** gauss({{1, 2, 3}, {4, 5, 6}}); // returns {1, -2} can be use for modulo arithmetic, but be careful with division replace Z with the desired type

Time: $\mathcal{O}(N^3)$

5ba24b, 44 lines

```
using Z = double;

std::vector <Z>* gauss(std::vector <std::vector <Z>> a) {
#define ABS(x) ((x) < 0 ? -(x) : (x))
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    std::vector <int> pivot(m, -1);

    for (int col = 0, row = 0; col < m and row < n; col++) {
        int cur = row;
        for (int i = row; i < n; i++)
            if (ABS(a[i][col]) > ABS(a[cur][col]))
                cur = i;
        if (a[cur][col] == 0)
            continue;
        for (int i = col; i <= m; i++)
            swap(a[cur][i], a[row][i]);
        pivot[col] = row;

        for (int i = 0; i < n; i++) if (i != row) {
            if (a[i][col] == 0)
                continue;
            Z c = a[i][col] / a[row][col];
            for (int j = col; j <= m; j++)
                a[i][j] -= a[row][j] * c;
        }
        row++;
    }

    std::vector <Z> *ans = new std::vector <Z> (m, 0);
    for (int i = 0; i < m; i++) if (pivot[i] != -1)
        (*ans)[i] = a[pivot[i]][m] / a[pivot[i]][i];
    for (int i = 0; i < n; i++) {
        Z s = a[i][m];
        for (int j = 0; j < m; j++)
            s -= (*ans)[j] * a[i][j];
        if (s)
            return nullptr;
    }

    return ans;
#undef ABS
}
```

ModInt.h

Description: Operators for modular arithmetic.

Usage: using Z = Mint<MOD>;
Z inverse = CInv<42, MOD>;

249ad1, 97 lines

```
using i64 = long long;

template <class T>
constexpr T power(T a, i64 b) {
    T res = 1;
    for (; b; b /= 2, a *= a) {
        if (b % 2)
            pol = combine(pol, e);
        e = combine(e, e);
    }

    ll res = 0;
```

```

    }
    return res;
}

template<int P>
struct MInt {
    int x;
    constexpr MInt() : x{} {}
    constexpr MInt(i64 x) : x{norm(x % P)} {}

    constexpr int norm(int x) const {
        if (x < 0) {
            x += P;
        }
        if (x >= P) {
            x -= P;
        }
        return x;
    }
    constexpr int val() const {
        return x;
    }
    explicit constexpr operator int() const {
        return x;
    }
    constexpr MInt operator-() const {
        MInt res;
        res.x = norm(P - x);
        return res;
    }
    constexpr MInt inv() const {
        assert(x != 0);
        return power(*this, P - 2);
    }
    constexpr MInt &operator*=(MInt rhs) {
        x = 1LL * x * rhs.x % P;
        return *this;
    }
    constexpr MInt &operator+=(MInt rhs) {
        x = norm(x + rhs.x);
        return *this;
    }
    constexpr MInt &operator-=(MInt rhs) {
        x = norm(x - rhs.x);
        return *this;
    }
    constexpr MInt &operator/=(MInt rhs) {
        return *this *= rhs.inv();
    }
    friend constexpr MInt operator*(MInt lhs, MInt rhs) {
        MInt res = lhs;
        res *= rhs;
        return res;
    }
    friend constexpr MInt operator+(MInt lhs, MInt rhs) {
        MInt res = lhs;
        res += rhs;
        return res;
    }
    friend constexpr MInt operator-(MInt lhs, MInt rhs) {
        MInt res = lhs;
        res -= rhs;
        return res;
    }
    friend constexpr MInt operator/(MInt lhs, MInt rhs) {
        MInt res = lhs;
        res /= rhs;
        return res;
    }
}

```

```

    }
    friend constexpr std::istream &operator>>(std::istream &is,
                                                MInt &a) {
        i64 v;
        is >> v;
        a = MInt(v);
        return is;
    }
    friend constexpr std::ostream &operator<<(std::ostream &os,
                                                const MInt &a) {
        return os << a.val();
    }
    friend constexpr bool operator==(MInt lhs, MInt rhs) {
        return lhs.val() == rhs.val();
    }
    friend constexpr bool operator!=(MInt lhs, MInt rhs) {
        return lhs.val() != rhs.val();
    }
};

template<int V, int P>
constexpr MInt<P> CInv = MInt<P>(V).inv();

Lagrange.h

```

c3e7c1, 22 lines

```

z lagrange(const std::vector<z> &p, int x) {
    if (x < (int) p.size())
        return p[x];
    z ans = 0, prod = 1;

    for (int i = 1; i < (int) p.size(); i++) {
        prod *= x - i;
        prod /= -i;
    }

    for (int i = 0; i < (int) p.size(); i++) {
        ans += prod * p[i];
        if (i + 1 == (int) p.size())
            break;
        prod *= x - i;
        prod /= x - (i + 1);
        prod *= i - (int) p.size() + 1;
        prod /= i + 1;
    }

    return ans;
}

```

Combinatorial (3)

3.1 Permutations

3.1.1 Cycles

Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

3.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

3.1.3 Burnside's lemma

Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts “configurations” (of some sort) of length n , we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

3.2 Partitions and subsets

3.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2e5$	$\sim 2e8$

3.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + \dots + n_1 p + n_0$ and $m = m_k p^k + \dots + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$.

3.3 General purpose numbers

3.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).

$$B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$$

Sums of powers:

$$\sum_{i=1}^n n^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^{\infty} f(i) &= \int_m^{\infty} f(x) dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^{\infty} f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

3.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1$$

$$\sum_{k=0}^n c(n, k)x^k = x(x+1)\dots(x+n-1)$$

$$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$$

$$c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$$

3.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n, k) = (n - k)E(n - 1, k - 1) + (k + 1)E(n - 1, k)$$

$$E(n, 0) = E(n, n - 1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

3.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n - 1, k - 1) + kS(n - 1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

3.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n + 1) \pmod{p}$$

3.3.6 Labeled unrooted trees

on n vertices: n^{n-2}

on k existing trees of size n_i : $n_1 n_2 \cdots n_k n^{k-2}$

with degrees d_i : $(n - 2)! / ((d_1 - 1)! \cdots (d_n - 1)!)$

3.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, C_{n+1} = \frac{2(2n+1)}{n+2} C_n, C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Data structure (4)

LiChaoTreeArray LiChaoTreePtr DynamicConvexHull

LiChaoTreeArray.h

3f26e6, 41 lines

```
template <int MAX>
class LiChao {
private:
    struct Line {
        lli A, B;
        Line(lli A = 0, lli B = 0): A(A), B(B) {}
        inline lli operator () (lli X) const { return A * X + B; }
    } st[4 * MAX + 7];
public:
    void add_line(int id, int l, int r, const Line &vars) {
        if (l > r) return;
        Line cur = st[id], L = vars;
        if (cur(l) < L(l)) swap(cur, L);
        if (cur(r) >= L(r)) st[id] = L;
        else {
            int mid = (l + r) >> 1;
            if (cur(mid) > L(mid)) st[id] = L, add_line(id << 1 | 1, mid + 1, r, cur);
            else st[id] = cur, add_line(id << 1, l, mid, L);
        }
    }
    lli query(int id, int l, int r, lli X) const {
        if (l > r || X < l || X > r) return LINF;
        lli res = st[id](X);
        if (l == r) return res;
        int mid = (l + r) >> 1;
        res = min(res, query(id << 1, l, mid, X));
        res = min(res, query(id << 1 | 1, mid + 1, r, X));
        return res;
    }
};
```

LiChaoTreePtr.h

c18633, 64 lines

```
const int64_t INF = 1e18 + 7;
struct Line {
    int64_t a, b;
    Line(int64_t a = 0, int64_t b = -INF) : a(a), b(b) {}
    inline int64_t operator () (int64_t x) const { return a * x + b; }
};
struct LiChao {
    Line value;
    LiChao* lef;
    LiChao* rig;
    LiChao(void) : value(Line()), lef(nullptr), rig(nullptr) {}
    void update(int l, int r, int u, int v, const Line& LINE) {
        if (l > r or u > v or u > r or l > v)
            return;
        if (l == r) return;
        if (u > v) swap(u, v);
        if (u > r) lef = new LiChao();
        if (v < l) rig = new LiChao();
        if (u <= l & v >= r) value = LINE;
        if (u <= l & v >= r) return;
        if (u <= l) lef->update(l, u, v, LINE);
        if (v >= r) rig->update(v, r, u, LINE);
    }
};
```

```
if (u <= l and r <= v) {
    Line current = value, other = LINE;
    if (current(l) > other(l))
        swap(current, other);

    if (current(r) <= other(r))
        value = other;
    else {
        if (l == r)
            return;
        int m = (l + r) >> 1;
        if (current(m) > other(m)) {
            value = current;
            lef = lef ? lef : new LiChao();
            lef->update(l, m, u, v, other);
        } else {
            value = other;
            rig = rig ? rig : new LiChao();
            rig->update(m + 1, r, u, v, current);
        }
    }
    return;
}
```

```
int m = (l + r) >> 1;
lef = lef ? lef : new LiChao();
rig = rig ? rig : new LiChao();

lef->update(l, m, u, v, LINE);
rig->update(m + 1, r, u, v, LINE);
}

int64_t query(int l, int r, int x) {
    if (l > r or x > r or l > x)
        return -INF;
    int64_t ans = value(x);

    int m = (l + r) >> 1;
    ans = max(ans, lef ? lef->query(l, m, x) : -INF);
    ans = max(ans, rig ? rig->query(m + 1, r, x) : -INF);
    return ans;
}
```

DynamicConvexHull.h

Description: Dynamic Convex Hull find Min

Usage: For doubles, use inf = 1/.0, div(a,b) = a/b 70d773, 44 lines

```
struct Line {
    ll k, m;
    mutable ll p;
    bool operator < (const Line& o) const {
        return k < o.k;
    }
    bool operator < (const ll &x) const {
        return p < x;
    }
};

struct DynamicHull : multiset<Line, less<>> {
    const ll inf = LLONG_MAX;

    ll div(ll a, ll b) {
        return a / b - ((a ^ b) < 0 && a % b);
    }

    bool bad(iterator x, iterator y) {
        if(y == end()) {
            x->p = inf;
        }
    }
};
```

```

        return false;
    }

    if(x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);

    return x->p >= y->p;
}

void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (bad(y, z)) z = erase(z);

    if(x != begin() && bad(--x, y)) bad(x, y = erase(y));
    while((y = x) != begin() && (--x)->p >= y->p) bad(x,
        erase(y));
}

ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
}
};
```

SplayTree.h

9b5e5e, 69 lines

```

const int MAXN = 100005;

struct TNode {
    int sz, p, L, R;
    TNode() { sz = p = L = R = 0; }
};

node[MAXN];

int root, nArr;

inline void update(const int &x) {
    node[x].sz = 1 + node[node[x].L].sz + node[node[x].R].sz;
}

int buildSplay(int l, int r, int pa = 0) {
    if(l > r) return 0;
    int mid = (l + r) >> 1;
    node[mid].p = pa;
    node[mid].L = buildSplay(l, mid - 1, mid);
    node[mid].R = buildSplay(mid + 1, r, mid);
    update(mid); return mid;
}

inline void setChild(int pa, int child, bool isRight) {
    node[child].p = pa;
    if(isRight) { node[pa].R = child; } else { node[pa].L =
        child; }
}

inline void upTree(int x) {
    int y = node[x].p, z = node[y].p;
    if(x == node[y].R) {
        int b = node[x].L; setChild(y, b, 1); setChild(x, y, 0)
        ;
    } else {
        int b = node[x].R; setChild(y, b, 0); setChild(x, y, 1)
        ;
    }
    setChild(z, x, (node[z].R == y)); update(y); update(x);
}

void splay(int x) {
```

SplayTree OrderTree HashMap TwoSat TwoSatPtr

```

    while(1) {
        if(node[x].p == 0) break;
        int y = node[x].p, z = node[y].p;
        if(z > 0)
            if((y == node[z].R) == (x == node[y].R)) { upTree(y)
                ; } else { upTree(x); }
        upTree(x);
    }

    int locate(int root, int c) {
        int x(root);
        while(1) {
            int s = node[node[x].L].sz;
            if(s + 1 == c) return x;
            if(s + 1 > c) { x = node[x].L; } else { c -= s + 1; x =
                node[x].R; }
        }
    }

    void split(int T, int &A, int &B, int c) {
        if(c == 0) { A = 0, B = T; return; }
        int x = locate(T, c); splay(x);
        A = x, B = node[A].R; node[A].R = node[B].p = 0; update(A);
    }

    int join(int A, int B) {
        if(A == 0) return B;
        while(node[A].R != 0) A = node[A].R;
        splay(A); setChild(A, B, 1); update(A);
        return A;
    }
}
```

OrderTree.h

819d08, 13 lines

```

<<bits/extc++.h>>
using namespace __gnu_pbds;
template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
void example() {
    Tree<int> t, t2; t.insert(8);
    auto it = t.insert(10).first;
    assert(it == t.lower_bound(9));
    assert(t.order_of_key(10) == 1);
    assert(t.order_of_key(11) == 2);
    assert(*t.find_by_order(0) == 8);
    t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

HashMap.h

0c30ed, 6 lines

```

// To use most bits rather than just the lowest ones :
struct chash { // large odd number for C
    const uint64_t C = 11(4e18 * acos(0)) | 71;
    ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{{},{}}, {1<<16});
```

Graph (5)

TwoSat.h

Usage: add_disjunction(u, v) mean (u or v)
-u is not u

```

class TwoSat {
private:
    int n, no;
    int* comp;
    bool* was;
    std::vector<int>* g;
    std::vector<int>* g_t;
    std::vector<int> topo;
```

```

    int n;
    vector<vector<int>> forward_edge, back_edge;
    vector<bool> used;
    vector<int> order, comp;

    void dfs_first(int u) {
        used[u] = true;
        for (auto v : forward_edge[u])
            if (not used[v]) dfs_first(v);
        order.push_back(u);
    }

    void dfs_second(int u, int turn) {
        comp[u] = turn;
        for (auto v : back_edge[u])
            if (comp[v] == -1) dfs_second(v, turn);
    }

public:
    TWO_SAT(int n = 0): n(n) {
        used.assign(2 * n + 7, false);
        comp.assign(2 * n + 7, -1);
        forward_edge.resize(2 * n + 7);
        back_edge.resize(2 * n + 7);
    }

    void add_disjunction(int a, bool na, int b, bool nb) {
        a = (a << 1) ^ na;
        b = (b << 1) ^ nb;
        int neg_a = a ^ 1;
        int neg_b = b ^ 1;
        forward_edge[neg_a].push_back(b);
        forward_edge[neg_b].push_back(a);
        back_edge[a].push_back(neg_b);
        back_edge[b].push_back(neg_a);
    }

    vector<bool>* find_solution(void) {
        vector<bool> *assignment = new vector<bool> (2 * n + 7);

        for (int i = 2; i <= 2 * n + 1; i++)
            if (not used[i]) dfs_first(i);

        for (int i = 1, turn = 0; i <= 2 * n; i++) {
            int u = order[2 * n - i];
            if (comp[u] == -1) dfs_second(u, ++turn);
        }

        for (int i = 2; i <= 2 * n; i += 2) {
            if (comp[i] == comp[i + 1])
                return nullptr;
            (*assignment)[i / 2] = comp[i] > comp[i + 1];
        }

        return assignment;
    }
};
```

TwoSatPtr.h

Usage: add_disjunction(u, v) mean (u or v)
-u is not u

5ff89d, 71 lines

HCMUS-HLD

```

void add_edge(int u, int v) {
    g[u].push_back(v);
    g_t[v].push_back(u);
}

void dfs_topo(int u) {
    was[u] = 1;
    for (int v : g[u])
        if (not was[v])
            dfs_topo(v);
    topo.push_back(u);
}

void dfs_scc(int u) {
    for (int v : g_t[u]) if (not comp[v]) {
        comp[v] = comp[u];
        dfs_scc(v);
    }
}

public:
TwoSat(int _n = 0) : n(_n), no(0) {
    topo.reserve(2 * n);
    comp = new int [2 * n + 1];
    g = new std::vector <int> [2 * n + 1];
    g_t = new std::vector <int> [2 * n + 1];
    was = new bool [2 * n + 1];

    comp += n;
    g += n;
    g_t += n;
    was += n;
}

void add_disjunction(int u, int v) {
    add_edge(~u, v);
    add_edge(~v, u);
}

std::vector <int>* solve(void) {
    for (int i = 1; i <= n; i += 1) {
        if (not was[i])
            dfs_topo(i);
        if (not was[~i])
            dfs_topo(~i);
    }
    std::reverse(topo.begin(), topo.end());
    for (int u : topo) {
        if (not comp[u]) {
            comp[u] = ++no;
            dfs_scc(u);
        }
    }
    std::vector <int>* ans = new std::vector <int> (n + 1);
    for (int i = 1; i <= n; i += 1) {
        int x = comp[i], y = comp[~i];
        if (x == y)
            return nullptr;
        (*ans)[i] = x > y;
    }
    return ans;
}

```

HLD.h

2a16b9, 116 lines

```

constexpr int max_log = 18;

struct Tree {

```

HLD DinicFlow

```

int n, T;
std::vector <int> heavy, head, st, en, lvl, sz, pv;
mutable std::vector <int> visited;
std::vector <std::vector <int>> g, up;

Tree(int _n = 0) : n(_n) {
    heavy.assign(n, -1);
    head.assign(n, 0);
    st.assign(n, 0);
    en.assign(n, 0);
    lvl.assign(n, 0);
    sz.assign(n, 0);
    pv.assign(n, 0);
    g.assign(n, {});
    up.assign(max_log, {});
    visited.assign(n, 0);
}

void add_edge(int u, int v) {
    g[u].push_back(v);
    g[v].push_back(u);
}

int dfs(int u) {
    sz[u] = 1;
    heavy[u] = -1;
    int max_size = 0;
    for (int v : g[u]) if (v ^ pv[u]) {
        pv[v] = u;
        lvl[v] = lvl[u] + 1;
        dfs(v);
        if (max_size < sz[v]) {
            max_size = sz[v];
            heavy[u] = v;
        }
        sz[u] += sz[v];
    }
    return sz[u];
}

void decompose(int u, int h) {
    st[u] = T++;
    head[u] = h;
    if (heavy[u] != -1)
        decompose(heavy[u], h);
    for (int v : g[u]) if (v != pv[u] and v != heavy[u])
        decompose(v, v);
    en[u] = T;
}

void work(int x = 0) {
    T = 0;
    pv[x] = x;
    lvl[x] = 0;
    dfs(x);
    decompose(x, x);
    up[0] = pv;
    for (int j = 1; j < max_log; j += 1) {
        up[j].resize(n);
        for (int i = 0; i < n; i += 1)
            up[j][i] = up[j - 1][up[j - 1][i]];
    }
}

bool IS_ANCESTOR(int u, int v) const {
    return st[u] <= st[v] and en[u] >= en[v];
}

int LCA(int u, int v) const {

```

```

if (IS_ANCESTOR(u, v))
    return u;
if (IS_ANCESTOR(v, u))
    return v;
for (int j = max_log - 1; j >= 0; j -= 1) {
    if (not IS_ANCESTOR(up[j][u], v))
        u = up[j][u];
}
return pv[u];
}

void apply_on_path(int x, int y, const std::function <void (
    int, int, bool)>& f) const {
    int z = LCA(x, y);
    {
        int v = x;
        while (v != z) {
            if (lvl[head[v]] <= lvl[z]) {
                f(st[z] + 1, st[v], true);
                break;
            }
            f(st[head[v]], st[v], true);
            v = pv[head[v]];
        }
    }
    f(st[z], st[z], false);
    int cnt_visited = 0;
    {
        int v = y;
        int cnt_visited = 0;
        while (v != z) {
            if (lvl[head[v]] <= lvl[z]) {
                f(st[z] + 1, st[v], false);
                break;
            }
            visited[cnt_visited++] = v;
            v = pv[head[v]];
        }
    }
    for (int at = cnt_visited - 1; at >= 0; at--) {
        v = visited[at];
        f(st[head[v]], st[v], false);
    }
}

```

DinicFlow.h

c4ca54, 69 lines

```

struct DinicFlow {
    static const long long INF = (long long) 1e18 + 7;
    int numNode, numEdge;
    vector<int> dist, head, work;
    vector<int> point, next;
    vector<long long> flow, capa;

    DinicFlow(int n = 0) {
        numNode = n;
        numEdge = 0;
        dist.assign(n + 7, 0);
        head.assign(n + 7, -1);
        work.assign(n + 7, 0);
    }

    int addEdge(int u, int v, long long c1, long long c2 = 0) {
        int ret = numEdge;
        point.push_back(v); capa.push_back(c1); flow.push_back
        (0); next.push_back(head[u]); head[u] = numEdge++;
        point.push_back(u); capa.push_back(c2); flow.push_back
        (0); next.push_back(head[v]); head[v] = numEdge++;
    }

```

```

        return ret;
    }

bool bfs(int s, int t) {
    queue<int> q;
    for (int i = 1; i <= numNode; i++) dist[i] = -1;
    dist[s] = 0; q.push(s);
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int i = head[u]; i >= 0; i = next[i]) if (flow[i] < capa[i] && dist[point[i]] < 0) {
            dist[point[i]] = dist[u] + 1;
            q.push(point[i]);
        }
    }
    return dist[t] >= 0;
}

long long dfs(int s, int t, long long f) {
    if (s == t) return f;
    for (int &i = work[s]; i >= 0; i = next[i]) if (flow[i] < capa[i] && dist[point[i]] == dist[s] + 1) {
        long long d = dfs(point[i], t, min(f, capa[i] - flow[i]));
        if (d > 0) {
            flow[i] += d;
            flow[i ^ 1] -= d;
            return d;
        }
    }
    return 0;
}

long long maxFlow(int s, int t) {
    long long totFlow = 0;
    while (bfs(s, t)) {
        for (int i = 1; i <= numNode; i++) work[i] = head[i];
        while (true) {
            int d = dfs(s, t, INF);
            if (d == 0) break;
            totFlow += d;
        }
    }
    return totFlow;
}

int getFlow(int id) const {
    return flow[id];
}

bool visited(int node) const {
    return dist[node] >= 0;
}
};

Centroid.h

```

727de0, 6 lines

```

int centroid(int u, int parent, int n) {
    for (int v : adj[u])
        if (v != parent && child[v] > n/2 && !del[v])
            return centroid(v, u, n);
    return u;
}

```

FastGraphMatching.h

b4a69e, 47 lines

```

const int N = 50003;
vector<int> adj[N];
int dist[N], matx[N], maty[N], m, n, p;

```

```

    bool vi[N];

    // use Fast Matching
    bool bfs(void) {
        queue<int> qu;
        for (int i = 1; i <= m; ++i)
            if (!matx[i]) { dist[i] = 0; qu.push(i); } else { dist[i] = 1e9+7; }
        dist[0] = 1e9+7;
        while (!qu.empty()) {
            int u(qu.front()); qu.pop();
            if (dist[u] < dist[0]) {
                for (int &v : adj[u]) {
                    if (dist[maty[v]] >= 1e9+7) {
                        dist[maty[v]] = dist[u] + 1; qu.push(maty[v]);
                    }
                }
            }
        }
        return (dist[0] < 1e9+7);
    }

    bool dfs(const int &u) {
        if (!u) return true;
        for (int &v : adj[u]) {
            if (dist[maty[v]] == dist[u] + 1) {
                if (dfs(maty[v])) {
                    matx[u] = v, maty[v] = u;
                    return true;
                }
            }
        }
        dist[u] = 1e9+7;
        return false;
    }

    void process() {
        cin >> m >> n >> p;
        for (int i = 0; i < p; ++i) {
            int u, v; cin >> u >> v; adj[u].push_back(v);
        }
        int res(0);
        while (bfs())
            for (int i = 1; i <= m; ++i) if (!matx[i]) res += dfs(i);
    }
}

```

BlockCutTree.h

648ea9, 22 lines

```

void tarjan(int u) {
    low[u] = num[u] = ++num[0];
    for (int it = 0; it < int(adj[u].size()); ++it) {
        int v(adj[u][it]);
        if (!num[v]) {
            st.push(u); tarjan(v);
            low[u] = min(low[u], low[v]);
            if (low[v] == num[u]) {
                lastComp[u] = ++numBCC;
                adjp[u].push_back(numNode + numBCC);
                do {
                    v = st.top(); st.pop();
                    if (lastComp[v] != numBCC) {
                        lastComp[v] = numBCC;
                        adjp[numNode + numBCC].push_back(v);
                    }
                } while (v != u);
            }
        } else { low[u] = min(low[u], num[v]); }
    }
}

```

```

    }
    st.push(u);
}

```

OnlineBridgeCounting.h

e7c1a5, 109 lines

```

vector<int> par, dsu_2ecc, dsu_cc, dsu_cc_size;
int bridges;
int lca_iteration;
vector<int> last_visit;

void init(int n) {
    par.resize(n);
    dsu_2ecc.resize(n);
    dsu_cc.resize(n);
    dsu_cc_size.resize(n);
    lca_iteration = 0;
    last_visit.assign(n, 0);
    for (int i=0; i<n; ++i) {
        dsu_2ecc[i] = i;
        dsu_cc[i] = i;
        dsu_cc_size[i] = 1;
        par[i] = -1;
    }
    bridges = 0;
}

int find_2ecc(int v) {
    if (v == -1)
        return -1;
    return dsu_2ecc[v] == v ? v : dsu_2ecc[v] = find_2ecc(dsu_2ecc[v]);
}

int find_cc(int v) {
    v = find_2ecc(v);
    return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(dsu_cc[v]);
}

void make_root(int v) {
    int root = v;
    int child = -1;
    while (v != -1) {
        int p = find_2ecc(par[v]);
        par[v] = child;
        dsu_cc[v] = root;
        child = v;
        v = p;
    }
    dsu_cc_size[root] = dsu_cc_size[child];
}

void merge_path (int a, int b) {
    ++lca_iteration;
    vector<int> path_a, path_b;
    int lca = -1;
    while (lca == -1) {
        if (a != -1) {
            a = find_2ecc(a);
            path_a.push_back(a);
            if (last_visit[a] == lca_iteration) {
                lca = a;
                break;
            }
            last_visit[a] = lca_iteration;
            a = par[a];
        }
        if (b != -1) {
            b = find_2ecc(b);
            path_b.push_back(b);
            if (last_visit[b] == lca_iteration) {
                lca = b;
                break;
            }
            last_visit[b] = lca_iteration;
            b = par[b];
        }
    }
}

```

```

        path_b.push_back(b);
        if (last_visit[b] == lca_iteration)
            lca = b;
        break;
    }
    last_visit[b] = lca_iteration;
    b = par[b];
}

for (int v : path_a) {
    dsu_2ecc[v] = lca;
    if (v == lca)
        break;
    --bridges;
}
for (int v : path_b) {
    dsu_2ecc[v] = lca;
    if (v == lca)
        break;
    --bridges;
}
}

void add_edge(int a, int b) {
    a = find_2ecc(a);
    b = find_2ecc(b);
    if (a == b)
        return;

    int ca = find_cc(a);
    int cb = find_cc(b);

    if (ca != cb) {
        ++bridges;
        if (dsu_cc_size[ca] > dsu_cc_size[cb])
            swap(a, b);
        swap(ca, cb);
    }
    make_root(a);
    par[a] = dsu_cc[a] = b;
    dsu_cc_size[cb] += dsu_cc_size[a];
} else {
    merge_path(a, b);
}
}

```

Various (6)

MISC (7)

```
yCombinator.h c67583, 15 line
template <class Fun> class y_combinator_result {
    Fun fun_;

public:
    template <class T>
    explicit y_combinator_result(T &&fun) : fun_(std::forward<T>(fun)) {}

    template <class... Args> decltype(auto) operator()(Args &&...
        args) {
        return fun_(std::ref(*this), std::forward<Args>(args)...);
    }
};
```

```
template <class Fun> decltype(auto) y_combinator(Fun &&fun) {
    return y_combinator_result<std::decay_t<Fun>>(std::forward<
        Fun>(fun));
}

BigNum.h
52825f, 147 lines (�)

struct Bignum {
    static const int MAX_DIGIT = 1000;
    static const int BASE = (int) 1e9;
    int digits[MAX_DIGIT], numDigit;

    Bignum(ll x = 0) {
        numDigit = 0;
        memset(digits, 0, sizeof digits);

        if (!x) numDigit = 1;

        while (x > 0) {
            digits[numDigit++] = x % BASE;
            x /= BASE;
        }
    }

    Bignum(string s) {
        numDigit = 0;
        memset(digits, 0, sizeof digits);

        ll x(0); int i(s.length() - 1), l(i + 1);
        for (int i = l - 1; i >= 0; i -= 9) digits[numDigit++] =
            stoll(s.substr(i - 8, 9));
        if(l % 9) digits[numDigit++] = stoll(s.substr(0, l % 9));
    }
}

Bignum& operator += (const Bignum &x) {
    int carry(0);
    numDigit = max(numDigit, x.numDigit);
    for (int i = 0; i < numDigit; ++i) {
        digits[i] += x.digits[i] + carry;
        if (digits[i] >= BASE) {
            digits[i] -= BASE; carry = 1;
        } else { carry = 0; }
    }
    if (carry) digits[numDigit++] = carry;
    return (*this);
}

Bignum operator + (const Bignum &x) const {
    Bignum res(*this); return (res += x);
}

Bignum& operator -= (const Bignum &x) {
    int carry(0);
    for (int i = 0; i < numDigit; ++i) {
        digits[i] -= x.digits[i] + carry;
        if (digits[i] < 0) {
            digits[i] += BASE; carry = 1;
        } else { carry = 0; }
    }
    while (numDigit > 1 && !digits[numDigit - 1]) --numDigit;
}
return (*this);
}

Bignum operator - (const Bignum &x) const {
    Bignum res(*this); return (res -= x);
}
```

```

    if (digits[i] != x.digits[i]) return COMPARE(digits
        [i], x.digits[i]);
    return 0;
}

#define DEF_OPER(o) bool operator o (const Bignum &x) const
{ return compare(x) o 0; }
DEF_OPER(<) DEF_OPER(>) DEF_OPER(>=) DEF_OPER(<=) DEF_OPER
(==) DEF_OPER(!=)
#undef DEF_OPER

string toString(void) const {
    string res;
    for (int i = 0; i < numDigit; ++i) {
        int tmp = digits[i];
        for (int j = 0; j < 9; ++j) {
            res.push_back('0' + tmp % 10);
            tmp /= 10;
        }
    }
    while(res.size() > 1 && res.back() == '0')
        res.pop_back();
    reverse(res.begin(), res.end());
    return (res);
}

```

7.1 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); })`; converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29)`; kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

7.2 Optimization tricks

`__builtin_ia32_ldmxcsr(40896)`; disables denormals (which make floats 20x slower near their minimum value).

7.2.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x;) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b, 0, K) rep(i, 0, (1 << K))`
`if (i & 1 << b) D[i] += D[i^(1 << b)];`
computes all sums of subsets.

7.2.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.

FastMod FastInput

- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to $a \pmod{b}$ in the range $[0, 2b]$. 751a02, 8 lines

```

typedef unsigned long long ull;
struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m(-1ULL / b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t(m) * a) >> 64) * b;
    }
};

```

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.

Usage: `./a.out < input.txt`

Time: About 5x as fast as `cin/scanf`. 7b3c70, 17 lines

```

inline char gc() { // like getchar()
    static char buf[1 << 16];
    static size_t bc, be;
    if (bc >= be) {
        buf[0] = 0, bc = 0;
        be = fread(buf, 1, sizeof(buf), stdin);
    }
    return buf[bc++]; // returns 0 on EOF
}

int readInt() {
    int a, c;
    while ((a = gc()) < 40);
    if (a == '-') return -readInt();
    while ((c = gc()) >= 48) a = a * 10 + c - 480;
    return a - 48;
}

```