Generalities on experimental and monitoring measures

Physics of the Hydrosphere and the Cryosphere

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Concepts related to the measurement quality

Accuracy: the difference between the measured value and the real value.
It depends both on the instrument's quality and the correctness of measurement procedures.

Precision: the, possibly random, spread of measured values around the average measured value.
Usually quantified by the standard deviation of repeatedly measured values of the same physical quantity.
Precision depends mostly on the measuring instrument and should be as small as possible.

Resolution: the smallest to be distinguished magnitude from the measured value.

Sensitivity: the ratio between the variation of the quantity which is directly measured by the instrument and the variation of the quantity to be measured.

Measurement units

The definitions and the formats to be followed in order to be consistent with the International System of Units (SI) is managed by the Bureau International des Poid et Misures (BIPM): `https://www.bipm.org/`.

The ninth edition of the SI Brochure, which has been published in 2019, can be downloaded from the following URL: `https://www.bipm.org/en/publications/si-brochure/`.

Experimental versus monitoring measurements

Experimental measurements	are performed when all the conditions which might influence the measure are well controlled.
Monitoring measurements	are performed to assess the variation of a given quantity, possibly without control of the conditions by the operator.

Analog versus digital measurements

Analog measurements	give values which are based on the position of an indicator on a graduated scale, give values which vary with continuity in a given range.
Digital measurements	provide discrete, numerical values, i.e., they are limited to a limited (possibly high) number of values.

Most physical variables can be considered as continuous quantities
⇒ the quantities to be measured by an instrument should be considered as "analog" quantities
⇒ digital measurements are then obtained by analog-to-digital conversion with an ADC (Analog-to-Digital converter).

Analog versus digital measurements - ADC

ADCs are based on two steps

sampling and holding (S/H)	an analog signal is sampled with a regular sampling period (T_S) and the value attained during a given time window (t_w) is held.
quantizing and encoding (Q/E)	the held value is quantized, i.e., it is transformed into a numerical value in binary code.

⚠ The resolution of an ADC depends on the range A, i.e., the difference between the maximum and the minimum values that can be released by the ADC, and on the number of bits N. In fact, the number of possible output-values given by the ADC are 2^N, so that the resolution is $A \cdot 2^{-N}$.
⚠ Improving the performance of an ADC requires both decreasing resolution by increasing N and increasing the sampling rate by decreasing T_S.

Aliasing

When a continuous signal is sampled with a given sampling period T_S, it is possible to reconstruct the original signal in a perfect way from the discrete signal, only if the Fourier transform of the original signal is zero for $\nu > \nu_N$, where ν_N is the Nyquist frequency: $\nu_N = (2T_S)^{-1}$.

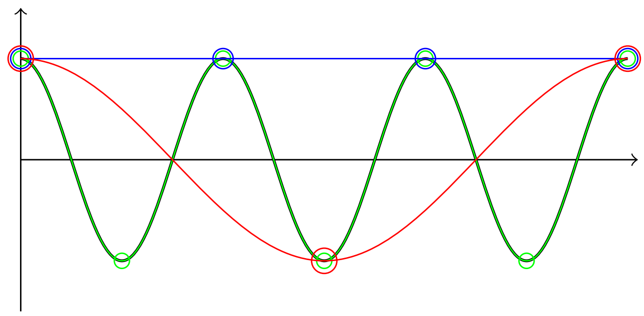


Figure: Example of the reconstruction of a continuous periodic signal with period T_0, after sampling at different frequencies: green line, $T_S = T_0/2$; blue line, $T_S = T_0$; red line, $T_S = 2T_0$.

Data storage and transmission

Data storage
In geophysics and environmental physics, it is important to consider the storage requirements and the power needed to feed the sensors and the data storage system. Two examples for a 16 bit ADC, with 10 channels:

- Sampling period: 10 minutes.
  24 hour/day $\times$ 60 minute/hour $\times$ 0.1 sample/minute $\times$ 10 datum/sample $\times$ 2 byte/datum $= 2,880$ byte/day $\simeq$ 3 kB/d.
- If the process is rapidly varying, so that it requires a small sampling period, e.g., 1 s, then the rate of memory storage becomes:
  24 hour/day $\times$ 60 minute/hour $\times$ 60 sample/minute $\times$ 10 datum/sample $\times$ 2 byte/datum $= 1,728,000$ byte/day $\simeq$ 1.65 MB/d.

Quantities which vary at high frequency, might require specifically designed storage devices or triggered systems.

Data storage and transmission

Data storage
Data transmission
A network of sensors or monitoring stations or individual remote monitoring stations send acquired data to a control center for data collection.
The designed system of data transmission strongly depends on
- the distances of sensors or monitoring stations from the control center,
- the required rate of data transfer and
- the availability of communication networks and electrical power lines close to the monitoring stations or sensors.

- Radio transmissions;
- GSM coverage can permit the use of specific SIM cards to transfer data with an IP protocol;
- Bluetooth systems can be used for data transfer in restricted areas.

Response time of a sensor

Example: a sensor to measure temperature.
T_0 is the initial temperature of the sensor in equilibrium with the body;
T_f is the temperature of the body due to an instantaneous change.
Assumption: the rate of change of the sensor temperature is proportional to the difference between the temperature of the sensor and the body temperature:

$$\frac{dT}{dt} = \frac{T_f - T}{\tau},$$

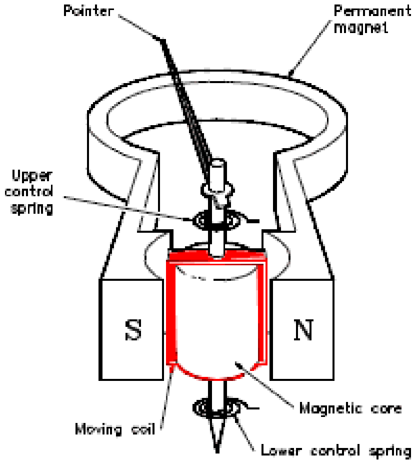
where τ is called the response time of the sensor.
Then the temperature of the sensor varies according to:

$$T(t) = T_0 + (T_f - T_0) \times \left[1 - \exp\left(-t/\tau\right)\right].$$

After a time $t = \tau$, $T(\tau) - T_f \simeq 0.368(T_f - T_0)$.

Amperometer

Sketch of the basic elements of an amperometer.



N turns of a conductive wire;

B radial magnetic induction field ($B = |\mathbf{B}|$);

I electrical current to be measured, flowing through the coil;

L length of the vertical side of the wire loop;

r "radius" of the wire loop;

C constant of elastic torsion;

θ torsion angle with respect to the equilibrium position of the coil when $I = 0$.

Amperometer

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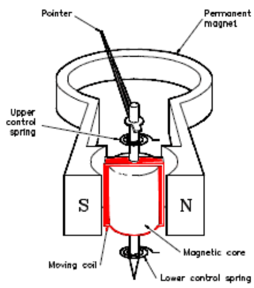


$$M_{\text{magnetic}} = 2NLr|\mathbf{B}|I$$

$$\updownarrow$$

$$M_{\text{torsion}} = C\theta$$

$$\Downarrow$$

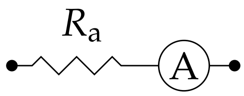
$$2NLrBI = C\theta \quad \Rightarrow \quad I = C \cdot (2NLrB)^{-1} \cdot \theta$$

Amperometer

$$I = \frac{C\theta}{2NLrB} \quad \Rightarrow \quad S = \frac{\Delta\theta}{\Delta I} = \frac{2NLrB}{C},$$

where $\Delta\theta$ is the variation of the equilibrium angle for a variation ΔI of the electrical current.

Electrical sketch of an amperometer

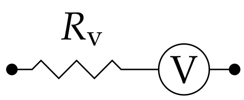


Voltmeter

If R_v is the coil resistance, the potential difference is computed as

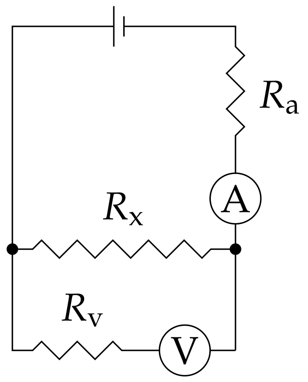
$$\Delta V = R_v I = R_v \frac{C\theta}{2NLrB}.$$

Scheme of a voltmeter, as an element of an electrical circuit



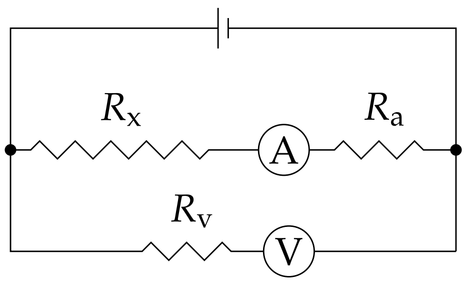
$$S = \frac{\Delta\theta}{\Delta V} = \frac{2NLrB}{R_v C}.$$

Measuring resistances - Volt-amperometric method



$$R_x = \frac{\Delta V}{I_x} = \frac{\Delta V}{I - \Delta V \cdot R_v^{-1}} = \left(\frac{I}{\Delta V} - \frac{1}{R_v}\right)^{-1}.$$

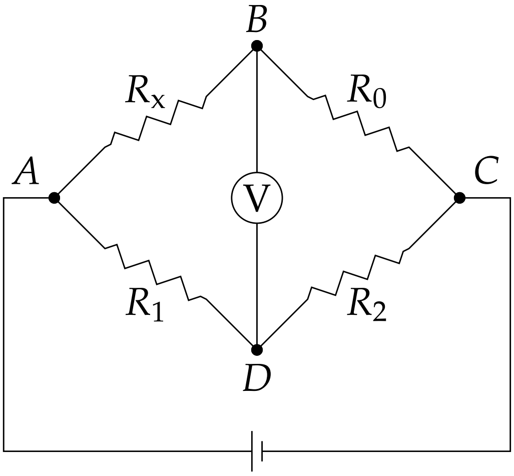
Measuring resistances - Volt-amperometric method for high resistances



$$\Delta V = (R_x + R_a)I \quad \Rightarrow \quad R_x = \frac{\Delta V}{I} - R_a.$$

Measuring resistances - Wheatstone and slide-wire bridges

Wheatstone bridge



R_0 is changed until $V_B = V_D$.
Under these conditions, $I_0 = I_x$ & $I_1 = I_2$.

$V_C - V_B = R_0 I_0 = V_C - V_D = R_2 I_2$

$\Rightarrow \quad R_0 I_0 = R_2 I_2.$

$V_B - V_A = R_x I_0 = V_D - V_A = R_1 I_2$

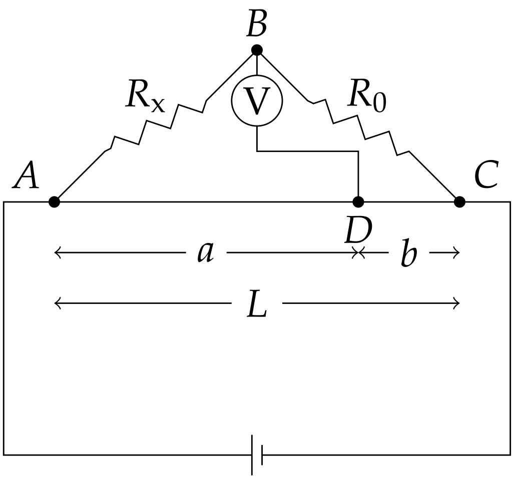
$\Rightarrow \quad R_x I_0 = R_1 I_2.$

$$\frac{I_0}{I_2} = \frac{R_2}{R_0} = \frac{R_1}{R_x} \quad \Rightarrow \quad R_x = R_0 \cdot \frac{R_1}{R_2}.$$

$$S = \frac{\Delta R_0}{\Delta R_x} = \frac{R_2}{R_1}.$$

Measuring resistances - Wheatstone and slide-wire bridges

Slide-wire bridge



$R_a = \rho a/\mathcal{S}$ ⊞ $R_b = \rho b/\mathcal{S}$

➔ $R_x = R_0 \cdot \dfrac{a}{L - a}$

$$S = \frac{\partial a}{\partial R_x} = \frac{L R_0}{(R_x + R_0)^2}$$

Basic properties of operational amplifiers

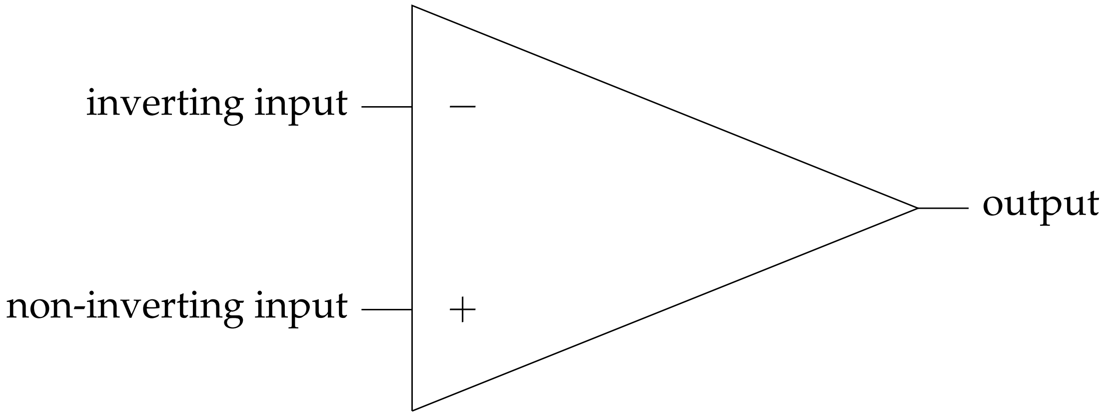


Figure: Symbol of the operational amplifier (OpAmp).

The ideal OpAmp

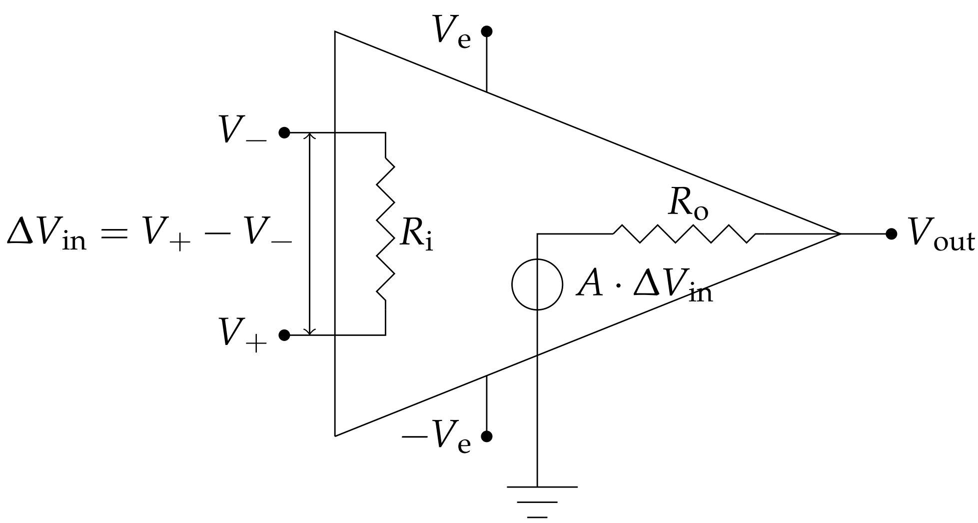
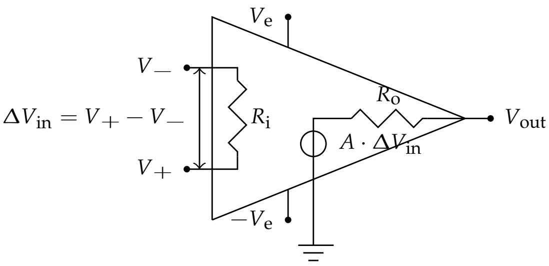


Figure: Scheme of the ideal operational amplifier.

The ideal OpAmp



- $I_i = 0$, $V_+ = V_-$, $\Delta V_{in} = 0$;
- $R_i = +\infty$, $R_o = 0$; $A = +\infty$.

Typical values of real operational amplifiers are the following:

- $10^5 < A < 10^8$;
- $10^6\ \Omega < R_i < 10^{13}\ \Omega$;
- $10\ \Omega < R_o < 100\ \Omega$;
- $5\ \mathrm{V} < V_e < 24\ \mathrm{V}$;
- $|A \cdot \Delta V_{in}| \leqslant V_e$.

Examples of simple circuits with OpAmp

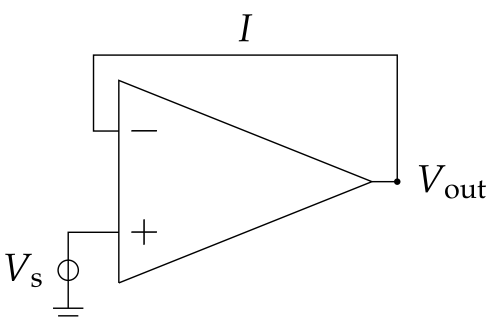


Figure: Non-inverting circuit.

$$V_s - A\Delta V_{in} = (R_i + R_o)\, I$$

$$\Delta V_{in} = R_i I$$

$$V_s = (AR_i + R_i + R_o)\, I$$

$$V_{out} = R_o I + A\Delta V_{in} = R_o I + AR_i I = (R_o + AR_i)\, I$$

$$V_{out} = \frac{R_o + AR_i}{AR_i + R_i + R_o} \cdot V_s \simeq V_s$$

Examples of simple circuits with OpAmp

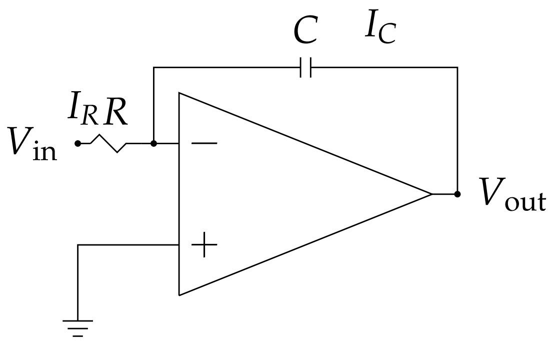


Figure: Integrating circuit.

$$-C\frac{\mathrm{d}}{\mathrm{d}t}(V_{out}-V_-) = I_C = I_R = \frac{V_{in}-V_-}{R}$$

In the ideal case $V_+ = V_- = 0$

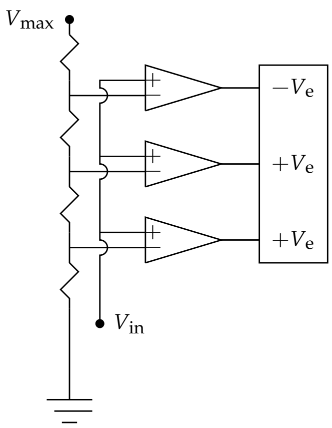
$$\frac{\mathrm{d}V_{out}}{\mathrm{d}t} = -\frac{1}{RC}V_{in}$$

$$V_{out}(t) - V_{out}(0) = -\frac{1}{RC}\int_0^t V_{in}(t')\,\mathrm{d}t' = -\frac{1}{\tau_{clock}}\int_0^t V_{in}(t')\,\mathrm{d}t',$$

where $\tau_{clock} = RC$ is the characteristic time.

Examples of ADCs - Flash ADC

Scheme of a *Flash ADC* with two bits.



Examples of ADCs - Dual-slope ADC

- Based on the use of OpAmps in integrating circuits;
- the integration phase proceeds for a time t_{int};
- a reference potential $-V_{\text{ref}}$ is applied as input;
- $V_{\text{out}} = 0$ after an additional time $t_{\text{de-int}}$ ("de–integration" phase).

$$V(t) = \begin{cases} V_{\text{in}} \dfrac{t}{\tau_{\text{clock}}} & t < t_{\text{int}}, \\ V_{\text{in}} \dfrac{t_{\text{int}}}{\tau_{\text{clock}}} - V_{\text{ref}} \dfrac{t - t_{\text{int}}}{\tau_{\text{clock}}} & t_{\text{int}} < t. \end{cases}$$

$$V\left(t_{\text{int}} + t_{\text{de-int}}\right) = 0 \quad \Rightarrow \quad V_{\text{in}} \frac{t_{\text{int}}}{\tau_{\text{clock}}} = V_{\text{ref}} \frac{t_{\text{de-int}}}{\tau_{\text{clock}}} \quad \Rightarrow \quad V_{\text{in}} = V_{\text{ref}} \frac{t_{\text{de-int}}}{t_{\text{int}}}$$