Recent Advances in 3D Computer Vision

SDF-Tracking

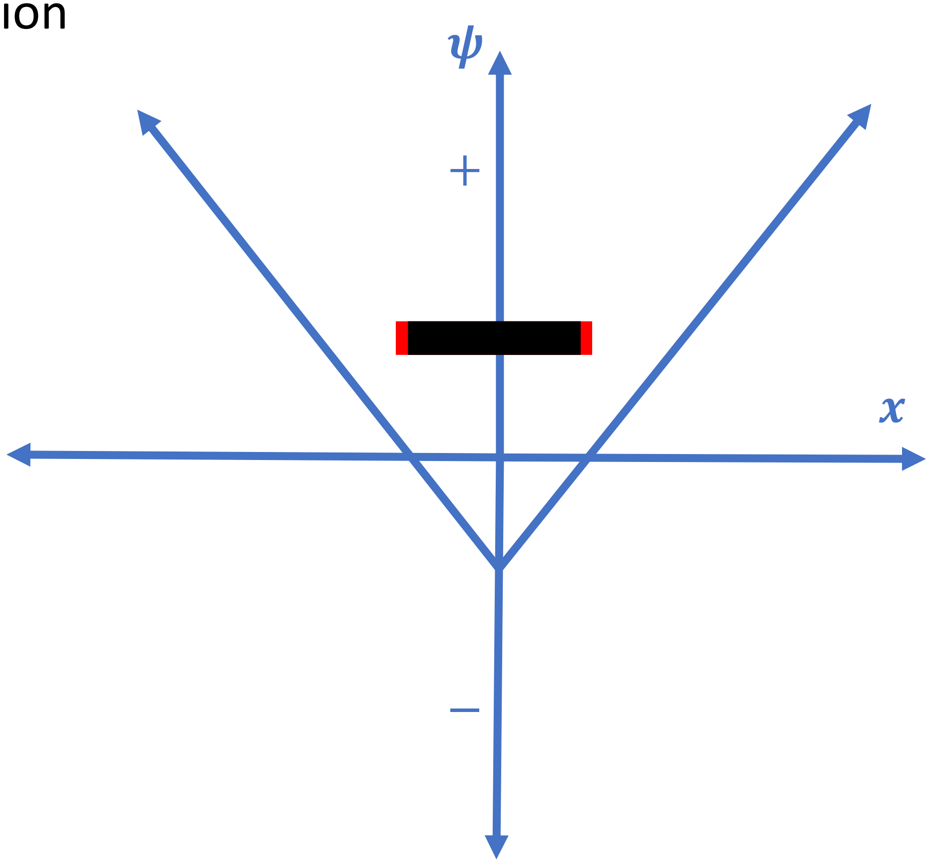
Leonardo Lerchenfeld

Real-Time Camera Tracking and 3D Reconstruction Using Signed Distance Functions

Erik Bylow, Jürgen Sturm, Christian Kerl, Fredrik Kahl and Daniel Cremers

https://www.youtube.com/watch?v=MzLdRFSrtuI

SDF-Tracking



Signed Distance Function (SDF)

Introduction and Problem Motivation

- **3D SLAM** is useful for
  - ➢ Robotics
  - ➢ Computer Vision
  - ➢ Architecture

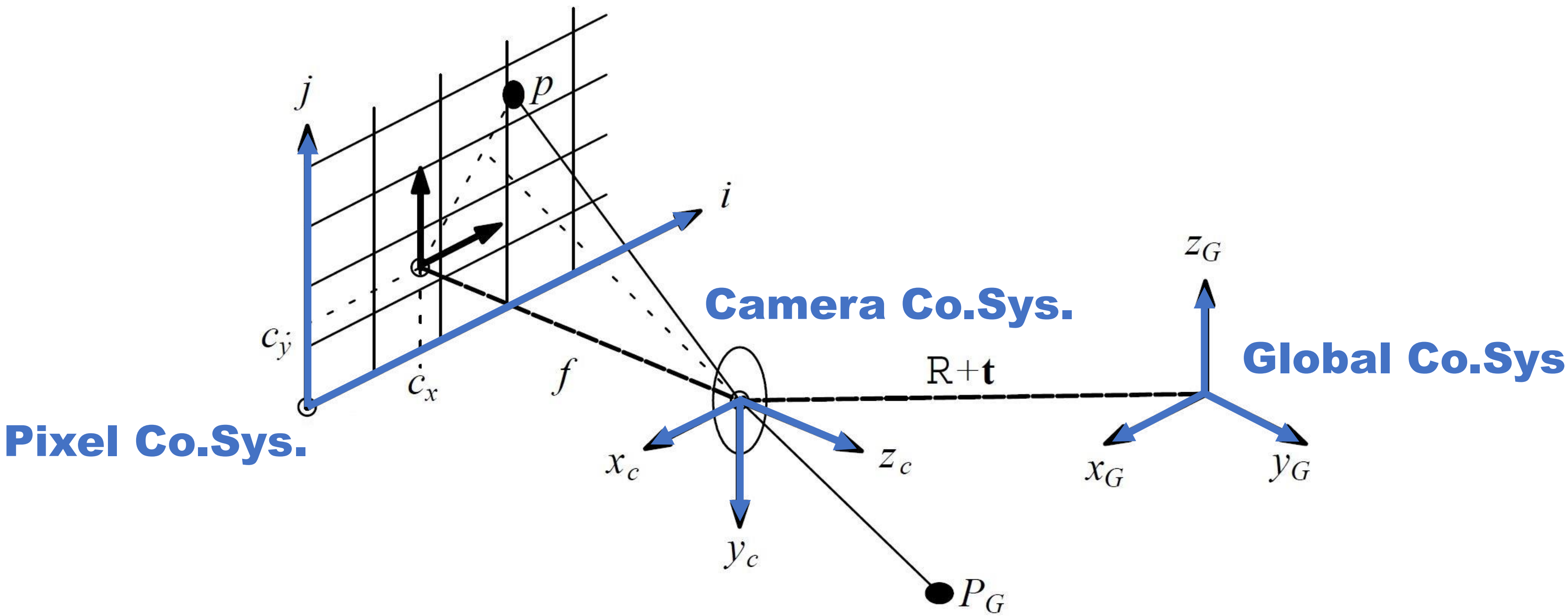
- **RGB-D sensors** output depth images
- **RGB-D SLAM** is accurate but no real-time
- **Kinect Fusion** in real-time but not as accurate

➔**SDF-Tracking**

Notation

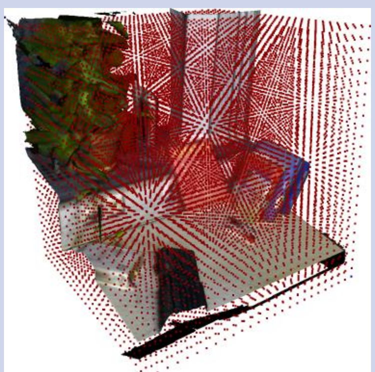
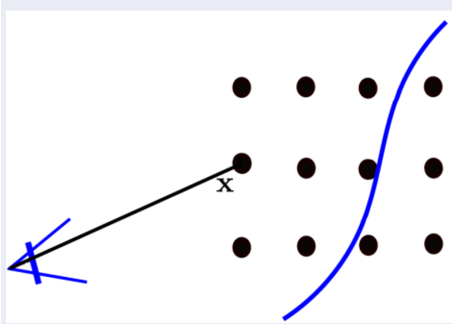
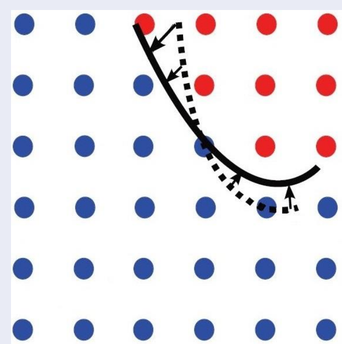
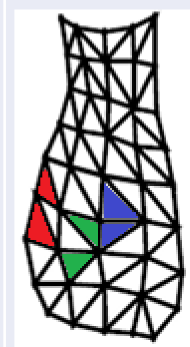
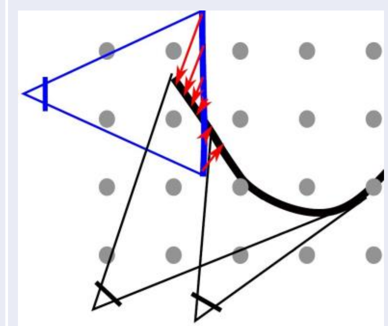
RGB-D Sensor	
Function for color images	$I_{RGB}\colon \mathbb{R}^2 \to \mathbb{R}^3$
Function for depth images	$I_d\colon \mathbb{R}^2 \to \mathbb{R}$
Camera and Global Coordinates	
3D point	$x \in \mathbb{R}^3$
Rotation of the camera	$R \in SO(3)$
And the translation	$\mathbf{t} \in \mathbb{R}^3$
from Camera to Global coordinates	$x_{ij}^{G} = Rx_{ij} + \mathbf{t}$
Camera Coordinates and the image plane coordinates	
focal length and the optical center	f_x, f_y, c_x, c_y

Coordinate Systems



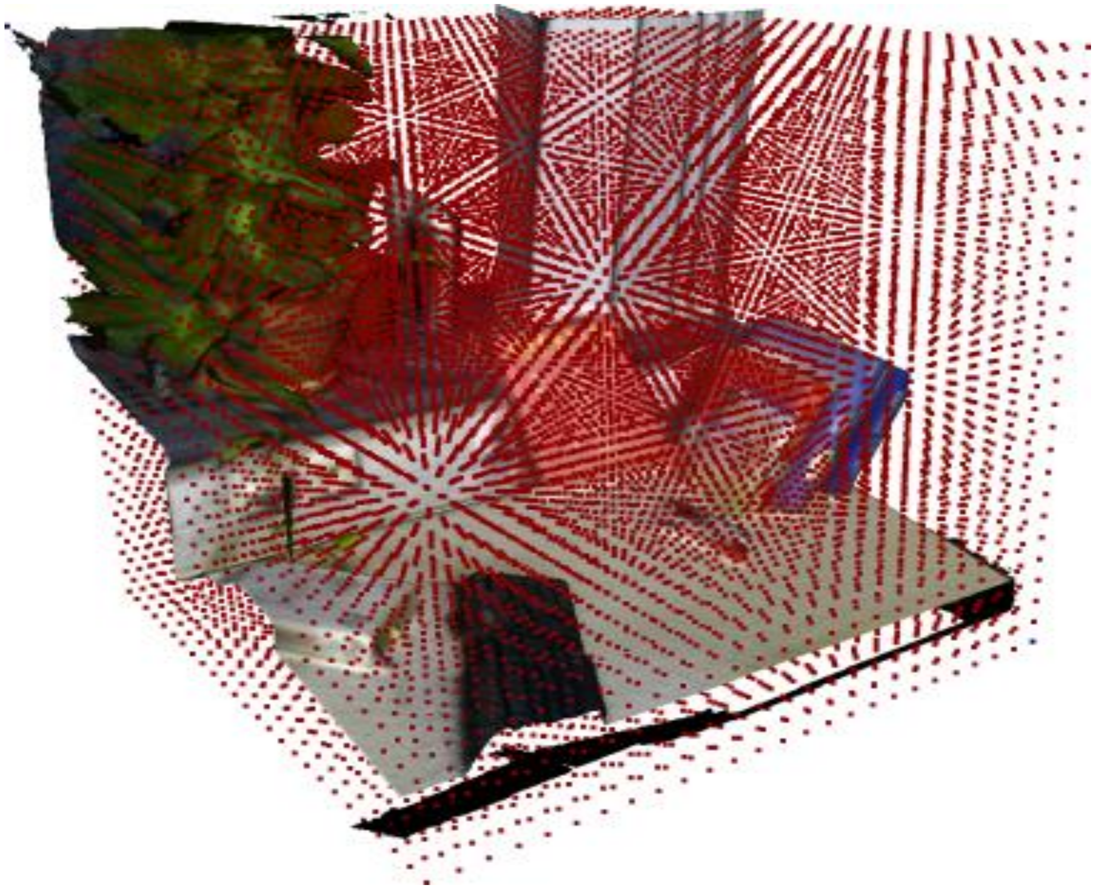
➔ Pixel coordinates	$\pi(\mathrm{x},\mathrm{y},\mathrm{z}) = \left(\frac{f_x x}{z} + c_x, \frac{f_y y}{z} + c_y\right)^T$
➔ Camera coordinates	$\rho(\mathrm{i},\mathrm{j},\mathrm{z}) = \left(\frac{(i - c_x)z}{f_x}, \frac{(j - c_y)z}{f_y}, z\right)^T$
➔ Global coordinates	$\mathrm{x}_{\mathrm{ij}}^{\mathrm{G}} = \mathrm{R}\mathrm{x}_{\mathrm{ij}} + \mathbf{t}$

Approach

Representation of the SDF	Distance and Weighting Functions	Data Fusion and 3D Reconstruction	Color and Mesh	Camera Tracking
Voxel grids	Projective distance Weighting functions	Optimal SDF ψ	Surface Color	Rotation R Translation **t**
				

Representation of the SDF

- Voxel grid of resolution *m*
- 6 channels
  - D: averaged **d**istance
  - W: sum of all **w**eights
  - R: **r**ed
  - G: **g**reen
  - B: **b**lue
  - W_c: **c**olor **w**eights
- $[0, \ldots, m-1]^3 \rightarrow \mathbb{R}$



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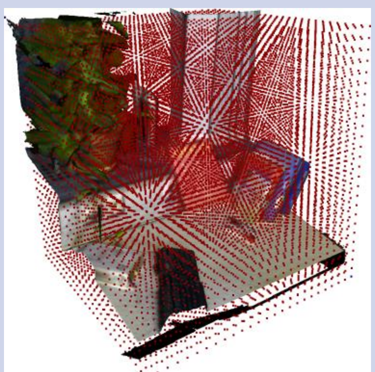
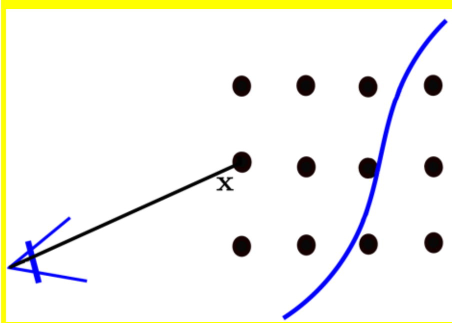
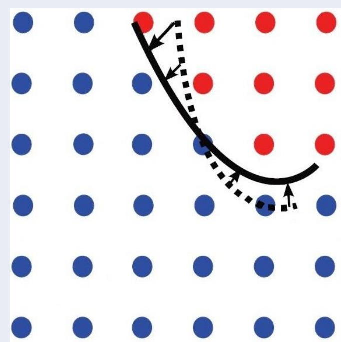
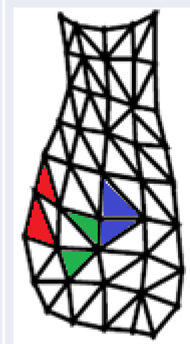
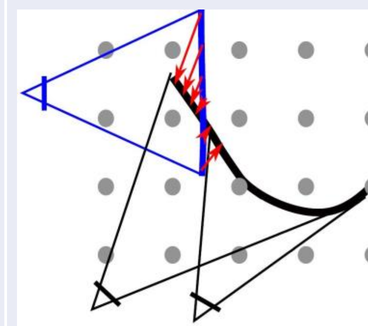
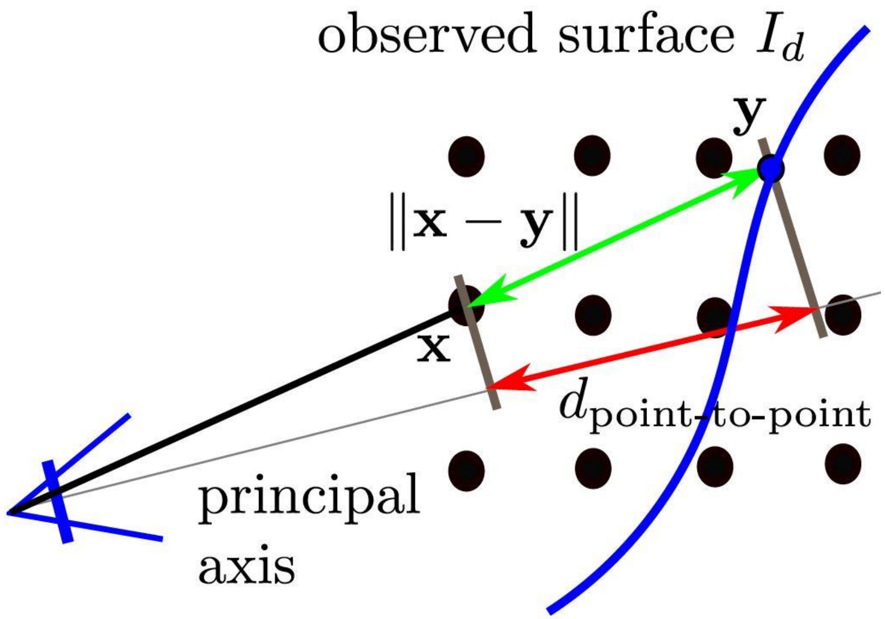
				

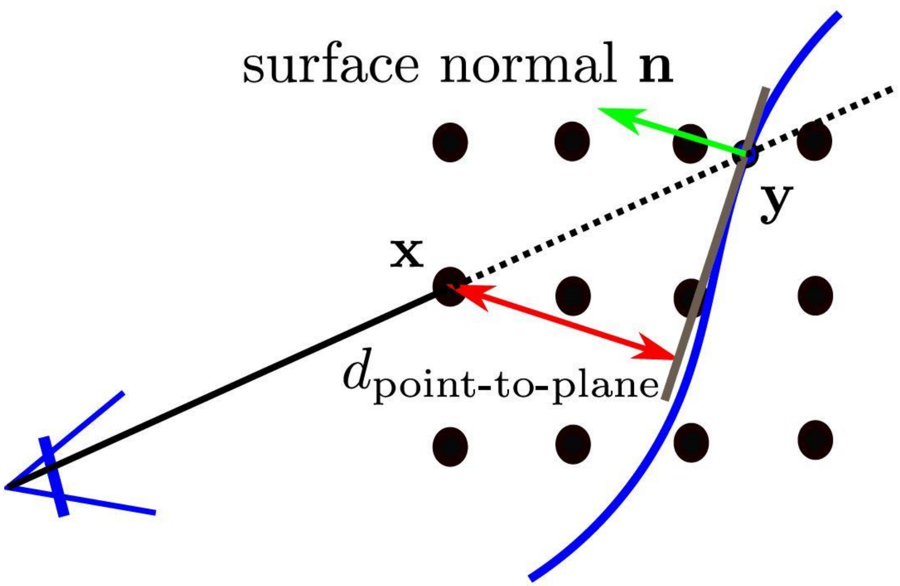
Projective Distances

Point-To-Point



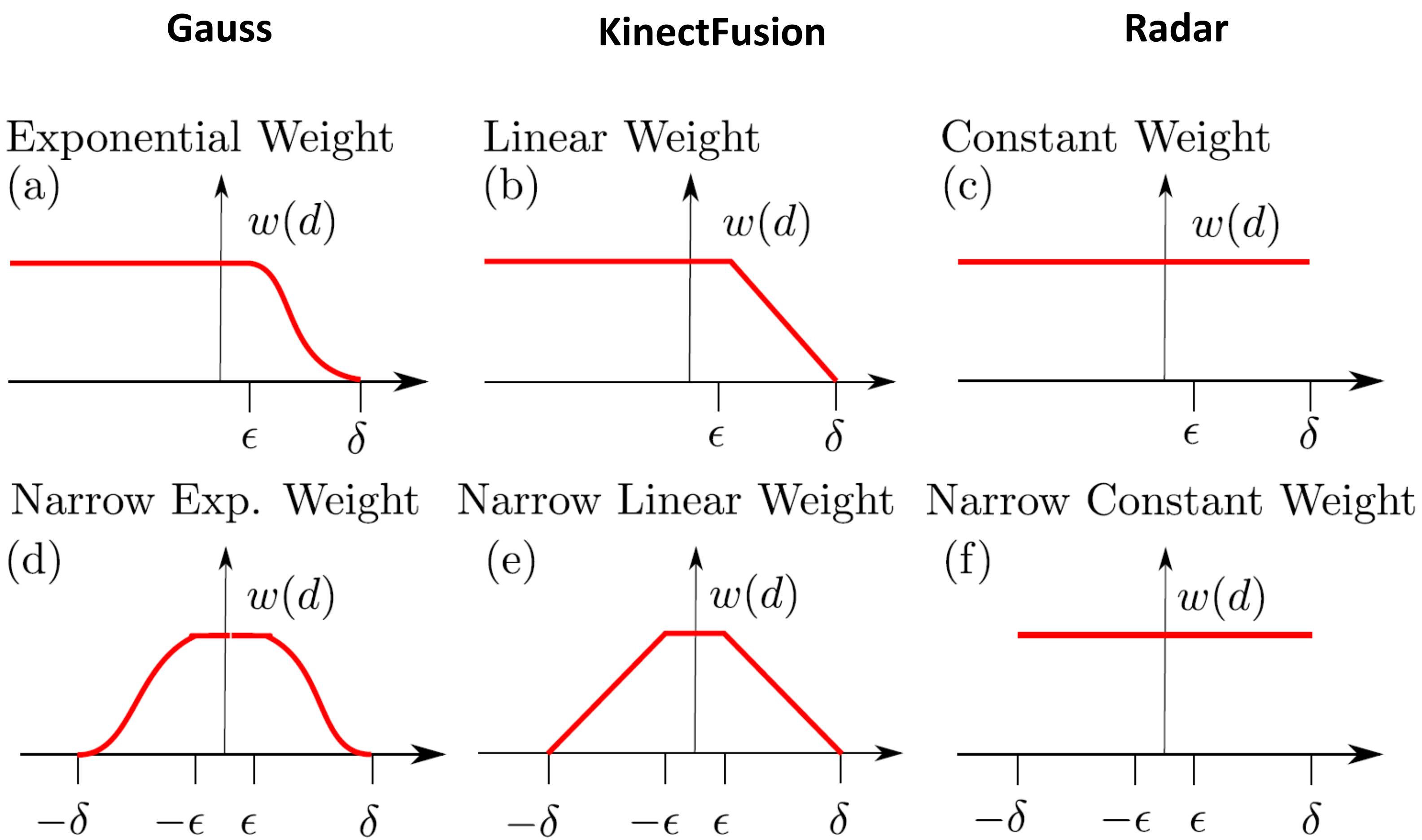
$$d_{point-to-point}(\boldsymbol{x}) = z - I_d(i,j)$$

Point-To-Plane



$$d_{point-to-plane}(\boldsymbol{x}) = (\boldsymbol{y} - \boldsymbol{x})^T \boldsymbol{n}(i,j)$$

Weighting Functions



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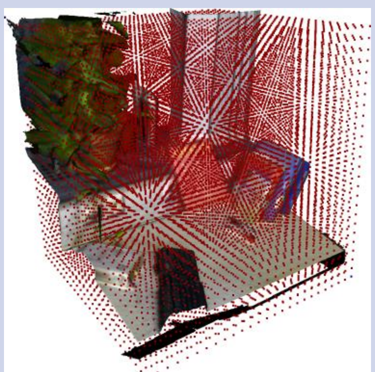
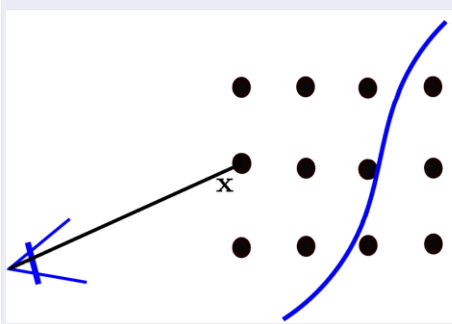
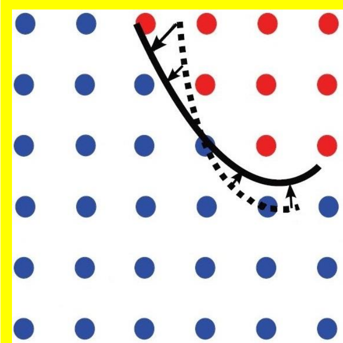
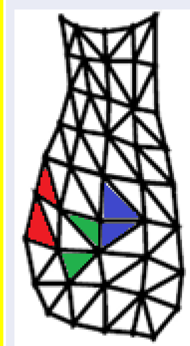
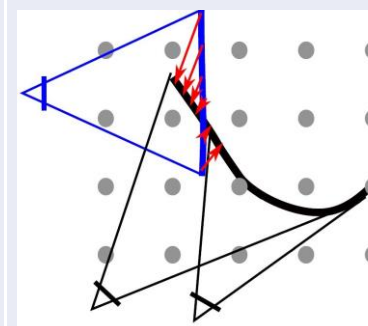
Voxel grids	Projective distance Weighting functions	Optimal SDF ψ	Surface Color	Rotation R Translation **t**
				

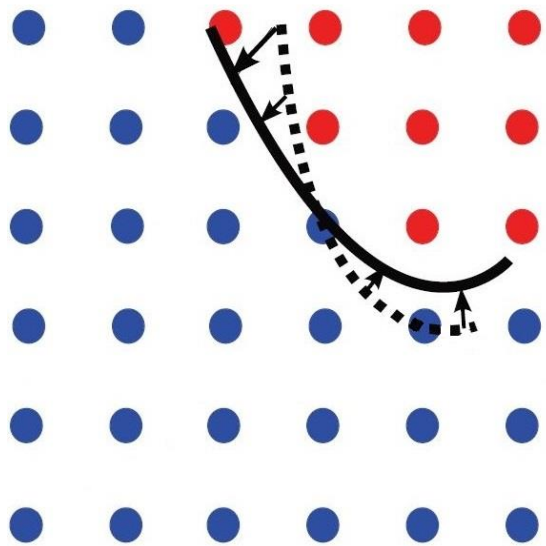
Data Fusion and 3D Reconstruction

SDF

$$\psi: \mathbb{R}^3 \rightarrow \mathbb{R}$$

Error Function

$$L(\psi) = \sum_{i=1}^{n} \frac{1}{2} w_i (\psi - d_i)^2$$



Optimal SDF

$$\psi = \frac{\sum_{i=1}^{n} w_i d_i}{\sum_{i=1}^{n} w_i}$$

Running Average

$$D \leftarrow \frac{WD + w_{n+1} d_{n+1}}{W + w_{n+1}}$$

$$W \leftarrow W + w_{n+1}$$

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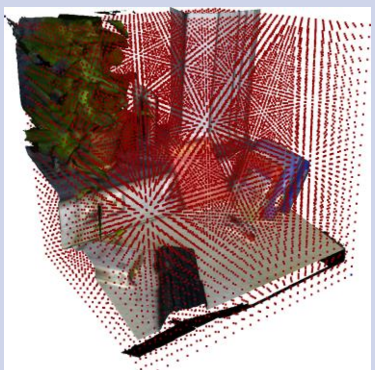
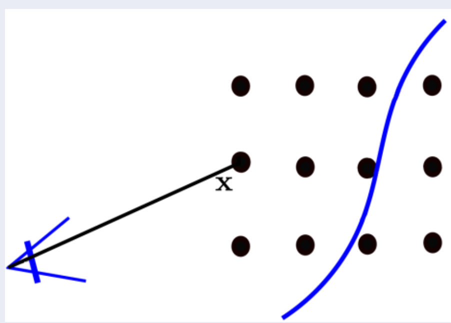
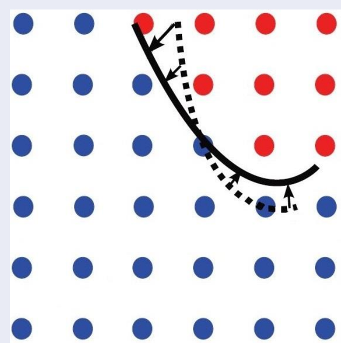
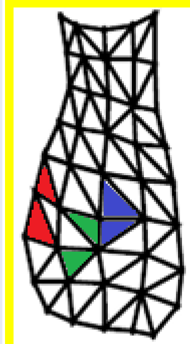
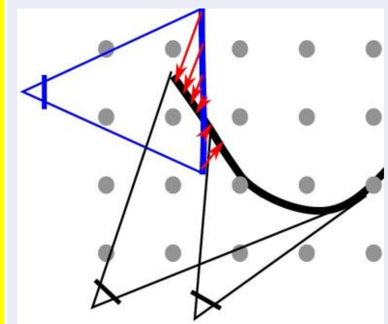
				

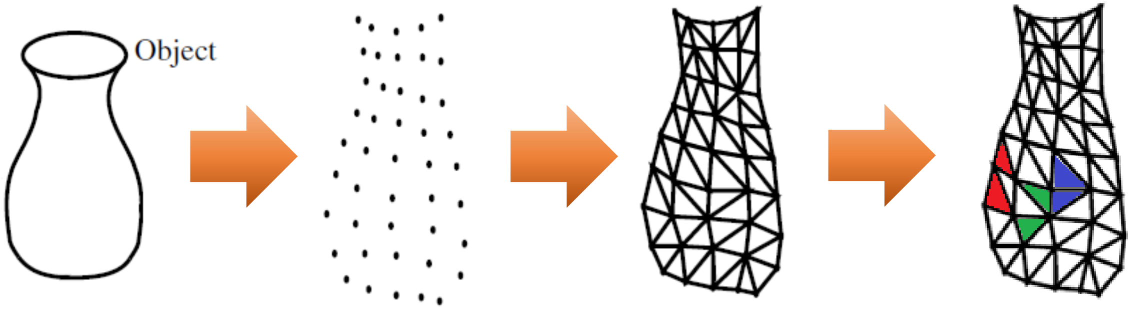
Color and Mesh

$$(r, g, b)^T = I_{RGB}(i, j)$$

$$G \leftarrow \frac{W_c G + w_c^{n+1} g}{W_c + w_c^{n-1}}$$

$$w_c^{n+1} = w_{n+1} cos\theta$$

Marching cubes algorithm



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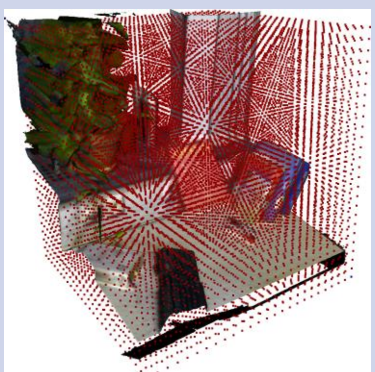
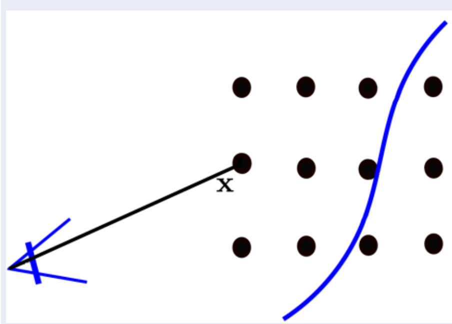
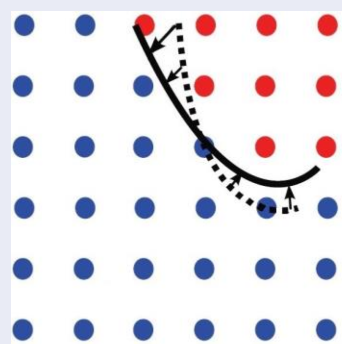
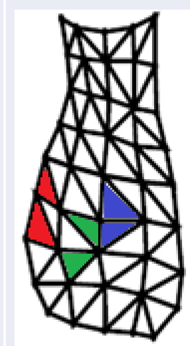
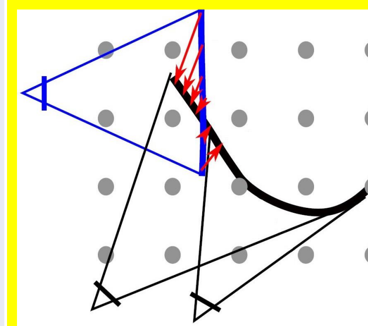
				

Camera Tracking

$$(x, y, z)^T = \rho(i, j, I_d(i, j))$$ Camera coordinates

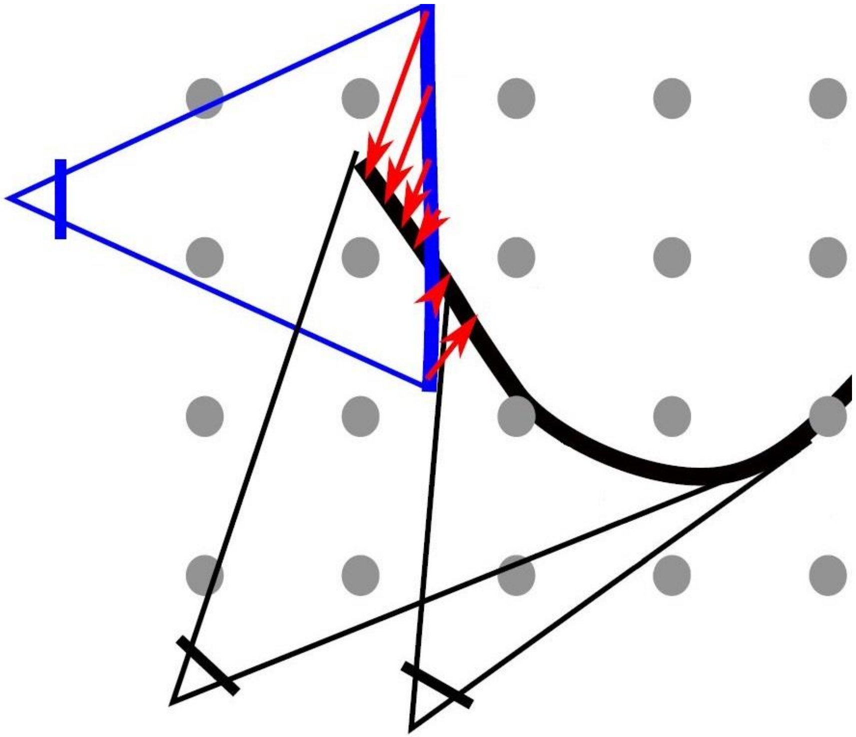
$$x_{ij}^G = Rx_{ij} + t$$ Global coordinates

$$\psi\colon \mathbb{R}^3 \to \mathbb{R}$$

$$\psi_{ij}(\xi) = \psi\left(\mathbf{R}x_{ij} + \mathbf{t}\right)$$

Twist Coordinates:

$$\xi = (\omega_1, \omega_2, \omega_3, v_1, v_2, v_3)$$



Gauss-Newton Method

$$E(\boldsymbol{\xi}) = E(\boldsymbol{R}, \boldsymbol{t}) = \sum_{i,j} \psi_{ij}(\boldsymbol{\xi})^2$$

$$\psi(\boldsymbol{\xi}) \approx \underbrace{\psi\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right)}_{1\text{x}1} + \underbrace{\nabla\psi\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right)^T}_{1\text{x}6} \underbrace{\left(\boldsymbol{\xi} - \boldsymbol{\xi}^{\boldsymbol{k}}\right)}_{6\text{x}1}$$

$$\sum_{i,j} \psi_{ij}(\boldsymbol{\xi})^2 \approx \sum_{i,j} \psi_{ij}\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right)^2 + 2\psi_{ij}\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right) \nabla\psi_{ij}\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right)^T \left(\boldsymbol{\xi} - \boldsymbol{\xi}^{\boldsymbol{k}}\right) + \left(\nabla\psi_{ij}\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right)^T \left(\boldsymbol{\xi} - \boldsymbol{\xi}^{\boldsymbol{k}}\right)\right)^2$$

$$\frac{d}{d\boldsymbol{\xi}} E_{approx}(\boldsymbol{\xi}) = \sum_{i,j} 2\psi_{ij}\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right) \nabla\psi_{ij}\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right) + 2\left(\nabla\psi_{ij}\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right) \nabla\psi_{ij}\left(\boldsymbol{\xi}^{\boldsymbol{k}}\right)^T \left(\boldsymbol{\xi} - \boldsymbol{\xi}^{\boldsymbol{k}}\right)\right) = 0$$

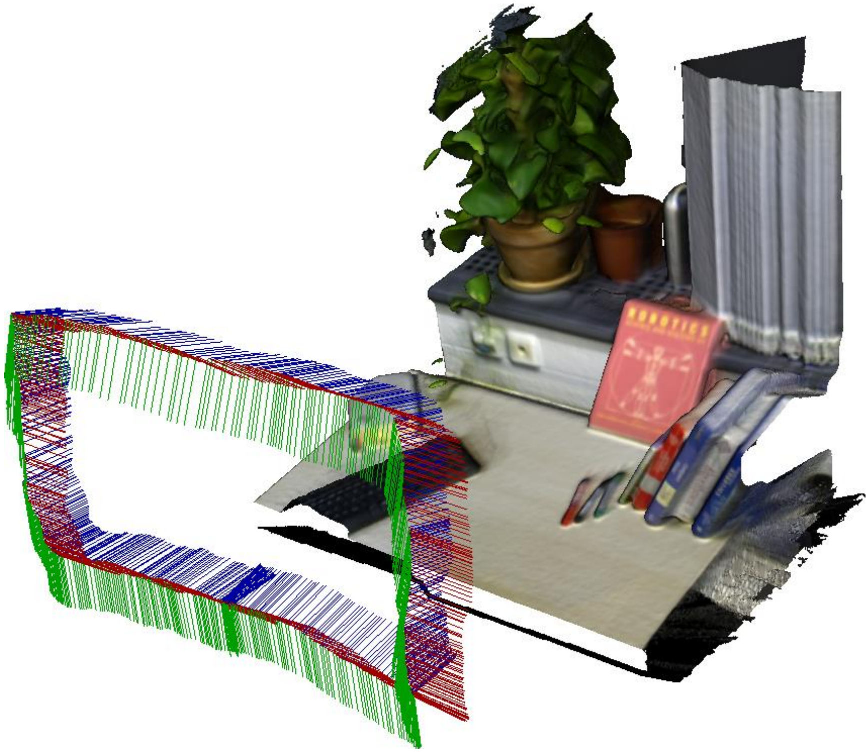
$$\boldsymbol{b} + \boldsymbol{A}\left(\boldsymbol{\xi} - \boldsymbol{\xi}^{\boldsymbol{k}}\right) = \boldsymbol{0}$$

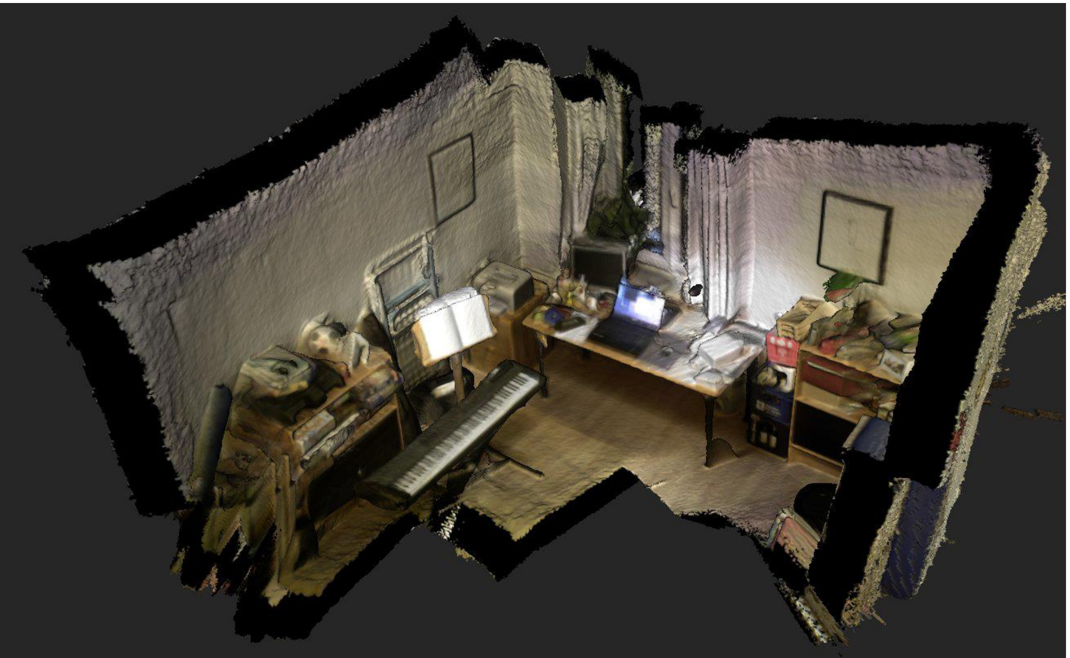
$$\boldsymbol{\xi}^{\boldsymbol{k+1}} = \boldsymbol{\xi}^{\boldsymbol{k}} - \boldsymbol{A}^{-1}\boldsymbol{b}$$

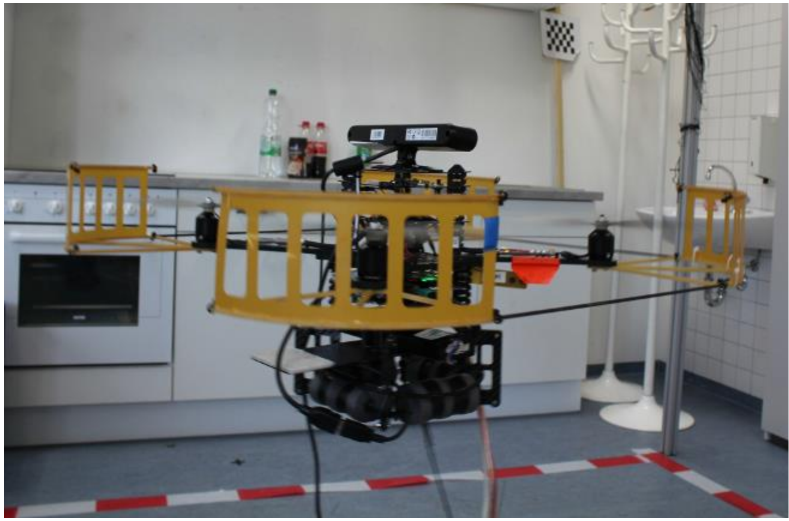
Experimental Results

Experimental Results

- Almost drift-free for small scenes
- Architects can use it for planning
- Navigation of robots







Parameter Study

- Truncation δ = 0.3m
- Weighting function
- Absolute trajectory error in m

Dataset	F1 Teddy		F1 Desk	
	RMSE	Max	RMSE	Max
Exp. Weight	0.088 m	**0.213 m**	**0.038 m**	0.088 m
Linear Weight	**0.083 m**	0.285 m	**0.038 m**	0.089 m
Constant Weight	0.093 m	0.242 m	0.040 m	0.089 m
Narrow Exp.	0.170 m	0.414 m	**0.038 m**	**0.083 m**
Narrow Linear	0.382 m	0.688 m	0.044 m	0.085 m
Narrow Constant	0.379 m	0.694 m	0.044 m	0.209 m

Method	Res.	Teddy	Desk	Desk2	Household	Floor	360	Room	Plant
KinFu	256	0.156	0.057	0.42	0.064	Failed	0.913	Failed	0.598
*Plane	256	**0.072**	0.087	0.078	0.053	0.811	0.533	0.163	0.047
*Point	256	0.086	0.038	**0.061**	**0.039**	0.641	0.420	0.121	0.047
KinFu	512	0.337	0.068	0.635	0.061	Failed	0.591	0.304	0.281
*Plane	512	0.101	0.059	0.623	0.053	0.640	0.206	0.105	**0.041**
*Point	512	0.08	**0.035**	0.062	0.04	**0.567**	**0.119**	**0.078**	0.043
RGB-D SLAM		0.111	0.026	0.043	0.059	0.035	0.071	0.101	0.061

- Point-To-Point metric better
- Similar performance compared to RGB-D SLAM
- Clearly outperforms KinectFusion

Runtime and Memory Consumption

Res.	RAM on the GPU		
	SDF	Color grid	
512	1 GB	2 GB	
256	128 MB	256 MB	
	Runtime		Comparison
	Pose optimization **O(i*j)**	Data fusion **O(m^3)**	
512	31.1 ms	21.6 ms	
256	19.4 ms	3.7 ms	= 23 ms
KinFu (256)			20 ms
RGB-D SLAM			100 – 250 ms

- For m=256, in real-time on 30 fps
- Speed as KinFu
- Faster than RGB-D SLAM

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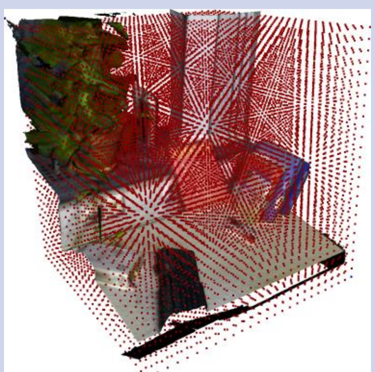	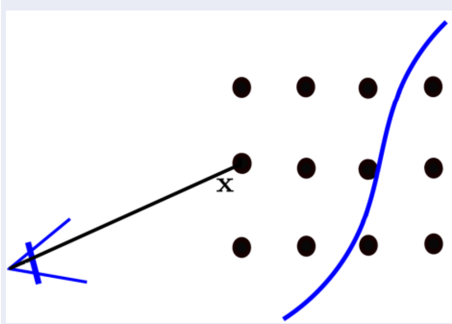	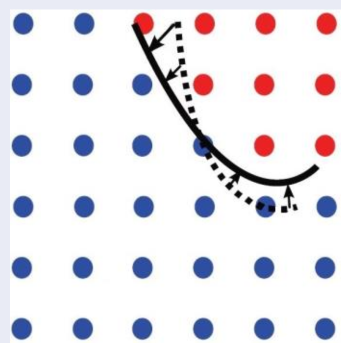	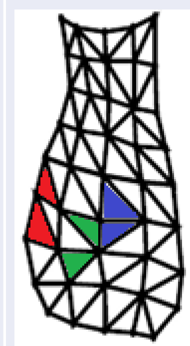	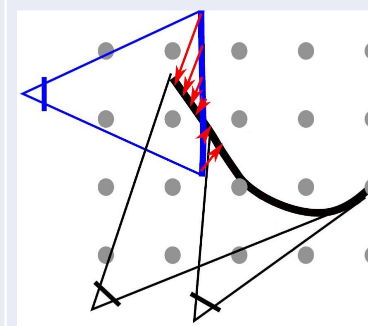

Discussion

Advantages +	Drawbacks -
RGB-D SLAM: feature extraction & bundle adjustment	
Very accurate	No real-time
Kinect Fusion: virtual camera & ICP	
Real time	Not as accurate
	Memory consumption
SDF-Tracking: direct estimation on SDF	
Real-time	Memory consumption
Very accurate	RGB values are not used for tracking

Questions?

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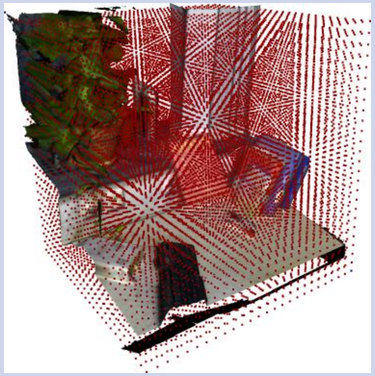	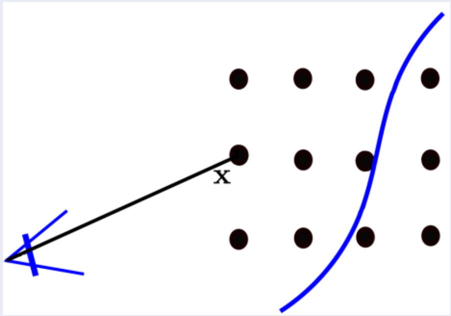	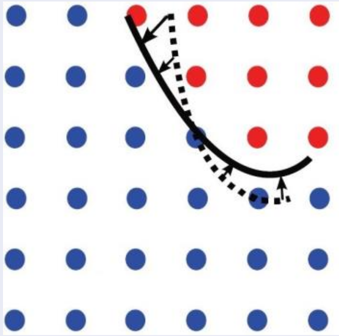	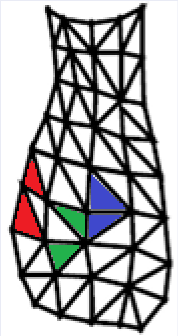	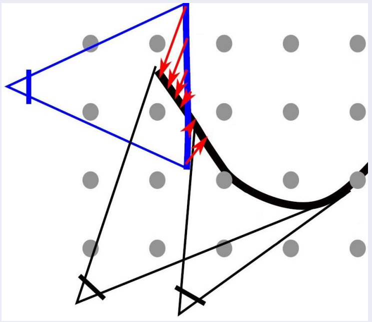

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