CS170–Spring 2019 — Homework 11 Solutions

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1 Study Group

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2 Bipartite Vertex Cover

(a) **Main Idea**

Build the residual network based on maximum flow f. Then find a path from t to s that go through edge (v, u) in residual network. Push 1 unit flow into this path and fix the wrong capacity from c_{uv} to $c_{uv} - 1$. Then run the original max-flow algorithm starting from this residual network.

(b) **Proof of Correctness**

Pushing 1 unit back from t to s that go through edge (v, u) will make the flow go through edge (u, v) be valid. Then run the original max-flow algorithm again from this is sure will give an optimal solution.

(c) **Runtime Analysis**

Build residual network will cost $O(|E|)$ time. Find a path from t to s that go through edge (v, u) by DFS or BFS will cost $O(|V| + |E|)$ time. Then push 1 unit into this path will cost at most $|E|$ time. Since the max-flow solution after repairing capacity of edge (u, v) cannot be larger than the original one. At most 1 iteration is needed if run max-flow algorithm starting from this. So total runtime is still linear, which is $O(|V| + |E|)$.

3 Zero-Sum Battle

(a)

$$\max p$$

$$p \leq -10x_1 + 4x_2 + 6x_3 \text{ (payoff when trainer B chooses the ice Pokemon)}$$

$$p \leq 3x_1 - 1x_2 - 9x_3 \text{ (payoff when trainer B chooses the water Pokemon)}$$

$$p \leq 3x_1 - 3x_2 + 2x_3 \text{ (payoff when trainer B chooses the fire Pokemon)}$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$x_3 \geq 0$$

The optimal strategy is $(0.335, 0.563, 0.102)$ and the payoff is -0.48.

(b)

$$\min p$$

$$p \geq -10y_1 + 3y_2 + 3y_3 \text{ (payoff when trainer A chooses the dragon Pokemon)}$$

$$p \geq 4y_1 - 1y_2 - 3y_3 \text{ (payoff when trainer A chooses the steel Pokemon)}$$

$$p \geq 6y_1 - 9y_2 + 2y_3 \text{ (payoff when trainer A chooses the rock Pokemon)}$$

$$y_1 + y_2 + y_3 = 1$$

$$y_1 \geq 0$$

$$y_2 \geq 0$$

$$y_3 \geq 0$$

The optimal strategy is $(0.268, 0.323, 0.409)$ and the payoff is -0.48.

4 Domination

(a) It should be 0 since choosing E instead will always give a better payoff.

(b) It should also be 0 since choosing B instead will always give a better payoff(column player wants to minimize the payoff).

(c) Both of them should be $(0.5, 0.5)$, since they are completely symmetric