CS170–Spring 2019 — Homework 2 Solutions

Ran Liao, SID 3034504227

Collaborators: YiLin Wu, YeFan Zhou

1. Study Group

Name	SID
Ran Liao	3034504227
YiLin Wu	3034503863
YeFan Zhou	3034503824

2. Asymptotic Complexity Comparisons

(a) $3 \leq 7 \leq 2 \leq 5 \leq 4 \leq 9 \leq 8 \leq 6 \leq 1$

(b) (i) $\log_3 n = \Theta(\log_4 n)$

Proof:

$$\log_3 n = \log_{(4^{\log_4 3})} n = (\frac{1}{\log_4 3}) \log_4 n = \Theta(\log_4 n)$$

(ii) $n \log(n^4) = O(n^2 \log(n^3))$

Proof:

$$\lim_{n \to +\infty} \frac{n^2 \log(n^3)}{n \log(n^4)} = \lim_{n \to +\infty} \frac{3n^2 \log n}{4n \log n} = \lim_{n \to +\infty} \frac{3}{4} n = \infty$$

(iii) $\sqrt{n} = \Omega((\log n)^3)$

Proof: (Use L'Hôpital's rule)

$$\begin{aligned}
\lim_{n \to +\infty} \frac{\sqrt{n}}{(\log n)^3} &= \lim_{n \to +\infty} \frac{\frac{1}{2\sqrt{n}}}{3(\log n)^2 \frac{1}{n}} = \lim_{n \to +\infty} \frac{\sqrt{n}}{(\log n)^2} \\
&= \lim_{n \to +\infty} \frac{\frac{1}{2\sqrt{n}}}{2(\log n) \frac{1}{n}} = \lim_{n \to +\infty} \frac{\sqrt{n}}{\log n} \\
&= \lim_{n \to +\infty} \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n}} = \lim_{n \to +\infty} \sqrt{n} = \infty
\end{aligned}$$

(iv) $2^n = \Theta(2^{n+1})$

Proof:

$$2^{n+1} = 2 * 2^n = \Theta(2^n)$$

(v) $n = \Omega((\log n)^{\log \log n})$

Proof:(Use L'Hôpital's rule)

Take the logarithm of both functions

$$\log(\log n)^{\log \log n} = (\log \log n)^2$$

Therefore, it is equivalent to compare $(\log \log n)^2$ and $\log n$

$$\begin{aligned}
\lim_{n \to +\infty} \frac{(\log \log n)^2}{\log n} &= \lim_{n \to +\infty} \frac{2(\log \log n) \frac{1}{\log n} \frac{1}{n}}{\frac{1}{n}} = \lim_{n \to +\infty} \frac{2(\log \log n)}{\log n} \\
&= \lim_{n \to +\infty} \frac{\frac{1}{\log n} \frac{1}{n}}{\frac{1}{n}} = \lim_{n \to +\infty} \frac{1}{\log n} = 0
\end{aligned}$$

(vi) $n + \log n = \Theta(n + (\log n)^2)$

Proof:(Use L'Hôpital's rule)

$$\begin{aligned}
\lim_{n \to +\infty} \frac{n + \log n}{n + (\log n)^2} &= \lim_{n \to +\infty} \frac{1 + \frac{1}{n}}{1 + 2(\log n) \frac{1}{n}} = \lim_{n \to +\infty} \frac{n + 1}{n + 2 \log n} \\
&= \lim_{n \to +\infty} \frac{1}{1 + \frac{2}{n}} = 1
\end{aligned}$$

(vii) $\log(n!) = O(n \log n)$

Proof: Since $n \log n = log(n^n)$, it's equivalent to compare $n!$ and n^n. Obviously, $n! = O(n^n)$.

3. Understanding Academic Dishonesty

(a) Not OK

(b) Not OK

(c) Not OK

(d) OK