CS 170 DIS 11

Released on 2019-04-08

1   Weighted Rock-Paper-Scissors

You and your friend used to play rock-paper-scissors, and have the loser pay the winner 1 dollar. However, you then learned in CS170 that the best strategy is to pick each move uniformly at random, which took all the fun out of the game.

Your friend, trying to make the game interesting again, suggests playing the following variant: If you win by beating rock with paper, you get 5 dollars from your opponent. If you win by beating scissors with rock, you get 3 dollars. If you win by beating paper with scissors, you get 1 dollar.

(a) Draw the payoff matrix for this game.

(b) Write a linear program to find the optimal strategy.

2   Secret Santa

Imagine you are throwing a party and you want to play Secret Santa. Thus you would like to assign to every person at the party another partier to whom they must anonymously give a gift. However, there are some restrictions on who can give gifts to who. For instance, nobody should be assigned to give a gift to themselves or to their spouse. Since you are the host, you know all of these restrictions. Give an efficient algorithm that determines if you and your guests can play Secret Santa.

3   Existence of Perfect Matchings

Prove the following theorem: Let $G = (L \cup R, E)$ be a bipartite graph. Then G has a perfect matching if and only if, for every set $X \subseteq L$, X is connected to at least $|X|$ vertices in R. Note that you must prove both directions. (Hint: Use the max-flow-min-cut theorem.)