

CS170–Spring 2019 — Homework 2 Solutions

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1. Study Group

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2. Asymptotic Complexity Comparisons

(a) $3 \leq 7 \leq 2 \leq 5 \leq 4 \leq 9 \leq 8 \leq 6 \leq 1$

(b) (i) $\log_3 n = \Theta(\log_4 n)$

Proof:

$$\log_3 n = \log_{(4^{\log_4 3})} n = \left(\frac{1}{\log_4 3}\right) \log_4 n = \Theta(\log_4 n)$$

(ii) $n \log(n^4) = O(n^2 \log(n^3))$

Proof:

$$\lim_{n \rightarrow +\infty} \frac{n^2 \log(n^3)}{n \log(n^4)} = \lim_{n \rightarrow +\infty} \frac{3n^2 \log n}{4n \log n} = \lim_{n \rightarrow +\infty} \frac{3}{4} n = \infty$$

(iii) $\sqrt{n} = \Omega((\log n)^3)$

Proof: (Use L'Hôpital's rule)

$$\begin{aligned} \lim_{n \rightarrow +\infty} \frac{\sqrt{n}}{(\log n)^3} &= \lim_{n \rightarrow +\infty} \frac{\frac{1}{2\sqrt{n}}}{3(\log n)^2 \frac{1}{n}} = \lim_{n \rightarrow +\infty} \frac{\sqrt{n}}{(\log n)^2} \\ &= \lim_{n \rightarrow +\infty} \frac{\frac{1}{2\sqrt{n}}}{2(\log n) \frac{1}{n}} = \lim_{n \rightarrow +\infty} \frac{\sqrt{n}}{\log n} \\ &= \lim_{n \rightarrow +\infty} \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n}} = \lim_{n \rightarrow +\infty} \sqrt{n} = \infty \end{aligned}$$

(iv) $2^n = \Theta(2^{n+1})$

Proof:

$$2^{n+1} = 2 * 2^n = \Theta(2^n)$$

3. Understanding Academic Dishonesty

- (a) Not OK
- (b) Not OK
- (c) Not OK
- (d) OK