CS 184/284A **Name:** ______________________

Spring 2018: Midterm 2 **SID number:** ______________________

April 24th, 2018 **cs184-??? login:** ______________________

Time Limit: 110 Minutes

- This exam contains 16 pages (including this cover page) and 6 problems. Check for missing pages.
- Put your initials on the top of every page, in case the pages become separated.
- This exam is closed book, except for one 8.5×11 page of notes (double sided), printed or handwritten.
- This exam is 110 minutes long, and has a total of 100 points.
- Problem difficulty varies throughout the exam, so don't get stuck on a time-consuming problem until you have read through the entire exam. Each problem's point value is roughly correlated with its expected difficulty.
- Answer each question in the space provided. Show all your work. Partial credit may be given on certain problems.
- To minimize distractions, do your best to avoid questions to staff. If you need to make assumptions to answer a question, write these assumptions into your answer.

Problem	Points	Score
1	20	
2	18	
3	12	
4	20	
5	16	
6	14	
Total:	100	

1. (Total : 20points) True / False

   Mark the following statements true or false. (1 point each)

   (a) (1 point) ___ Rasterization is generally believed more difficult to handle global illumination than path tracing.

   (b) (1 point) ___ Metamerism would not be possible if our cone cells were each sensitive to a single wavelength of light.

   (c) (1 point) ___ If my image sensor can count photons perfectly without noise, my photographs will be noise free.

   (d) (1 point) ___ In image sensor design, pixel fill factor decreases as the pixel size increases.

   (e) (1 point) ___ Low discrepancy sampling is used to relieve clumping of random samples.

   (f) (1 point) ___ Mipmapping will result in overblurring when the pixel footprint in texture space is anisotropic.

   (g) (1 point) ___ After N applications of loop subdivision, the number of triangles will increase by a factor of 4^N.

   (h) (1 point) ___ Any rotation in 3D space can be decomposed into a sequence of rotations around each of the three coordinate axes.

   (i) (1 point) ___ Antialiasing is the process of filtering out low frequencies before sampling.

   (j) (1 point) ___ In Monte Carlo integration, importance sampling always reduces variance compared to uniform random sampling.

   (k) (1 point) ___ When importance sampling an area light, we will get more noise in shadowed regions of the scene when the area light is larger.

   (l) (1 point) ___ A microfacet BRDF in which the normal distribution function exhibits a directionality will result in an isotropic BRDF.

   (m) (1 point) ___ Total internal reflection can only happen when light approaches a material with a smaller index of refraction.

   (n) (1 point) ___ Depth of field increases as the distance from the world focal plane to the camera increases.

   (o) (1 point) ___ In JPEG compression, the chroma channels are stored at lower resolution than the luma channel.

   (p) (1 point) ___ For a sensor pixel that is in a defocused part of the image, the light arriving from different positions on the lens aperture is constant.

   (q) (1 point) ___ CIELAB is a perceptually organized color space.

   (r) (1 point) ___ An inverse kinematics system is harder to implement than forward kinematics.

   (s) (1 point) ___ In a rectilinear spring mesh, adding diagonal cross links between masses will reduce shearing.

   (t) (1 point) ___ In physical simulation, fully implicit methods can be unconditionally stable.

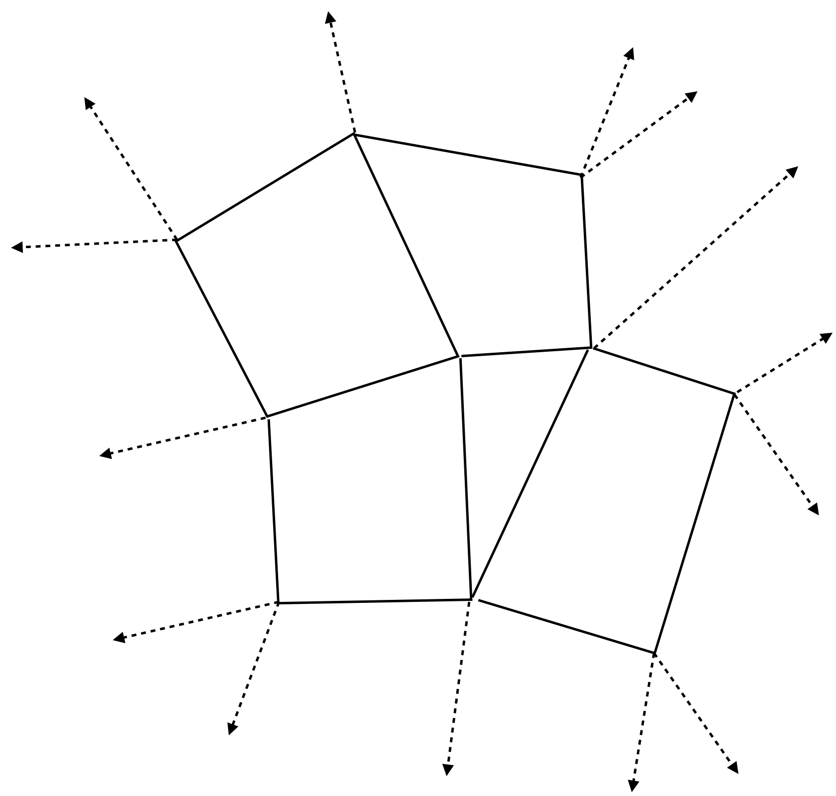
2. (Total : 18points) Rasterization and Geometry

   (a) (6 points) Below is a piece of mesh. Draw the mesh after one step of Catmull-Clark subdivision. Circle all the extraordinary vertices. Only the topology matters.

   How many extraordinary points were there originally? Answer: ________________

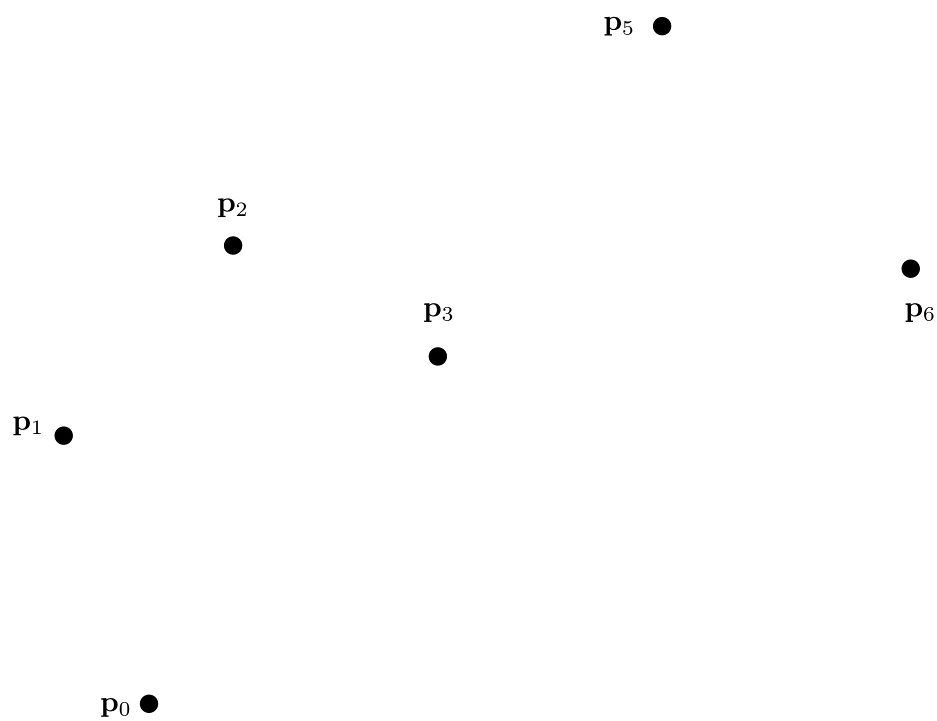
   How many extraordinary points (total) after one subdivision? Answer: ________________

   How many extraordinary points (total) after another subdivision? Answer: ________________



(b) (6 points) Consider two cubic Bézier curves with the control points shown below. The first curve has control points $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, and the second curve has control points $\mathbf{p}_3$, $\mathbf{p}_4$, $\mathbf{p}_5$, $\mathbf{p}_6$. Note that control point $\mathbf{p}_4$ is missing. Complete the following tasks:

1. Draw the application of de Casteljau's algorithm to determine the midpoint of the first Bézier curve. Label this midpoint $\mathbf{m}$.
2. Add point $\mathbf{p}_4$ on the diagram below, so that the overall spline curve is at least C_1 continuous at $\mathbf{p}_3$.
3. Draw the complete spline curve.



(c) (6 points) Find a quadratic polynomial $f(t)$ that satisfies $f(0) = \mathbf{p}_0, f'(0) = 2(\mathbf{p}_1 - \mathbf{p}_0), f(1) = \mathbf{p}_2$. Your answer should be in terms of t, $\mathbf{p}_0$ and $\mathbf{p}_1$ and $\mathbf{p}_2$.

Answer: $f(t) =$ ______________________________

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3. (Total : 12points) Ray Tracing, Rendering, Monte Carlo Integration

   (a) (3 points) How and why do we use the Russian Roulette method in path tracing?

   (b) (3 points) Assume ω_1 and ω_2 are the solid angles subtended by two spheres C_1 and C_2, respectively, from a given point P. Mark the following statements true or false.

   ____ If the radius of C_1 is greater than C_2, then it must be true that $\omega_1 > \omega_2$.

   ____ If $\omega_1 > \omega_2$ then C_2 must be further than C_1 from P.

   ____ If C_1 and C_2 are the same size and their centers are the same distance from P, then ω_1 and ω_2 must be equal.

(c) (6 points) In Monte Carlo integral estimation of $\int_a^b f(x)dx$, we can use multiple importance sampling as discussed in class. In the general approach, we draw multiple random samples X_i from probability density functions $p_i(x)$, evaluate the function f at these samples to obtain values $f(X_i)$, and form a weighted average of these values to estimate the desired integral. This gives the following Monte Carlo integral estimator:

$$F = \sum_{i=0}^{N} w_i(X_i) \frac{f(X_i)}{p_i(X_i)}$$

Many choices of weighting factor $w_i(x)$ are possible (e.g. the "balance heuristic" weighting factor discussed in class is $w_i(x) = \frac{p_i(x)}{\sum_{k=1}^{N} p_k(x)}$ and gives an unbiased estimator F.) In general, what condition(s) on $w_i(x)$ are needed to produce an estimator F that is unbiased? Justify your answer mathematically.

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4. (Total : 20points) Radiometry, Cameras, Lenses

   (a) (2 points) Irradiance is measured in units of ____________________

   (b) (4 points) Compared to a phone camera, a DSLR camera has a focal length, sensor width and sensor height that are 4 times larger. The f-number is the same between the cameras. The DSLR camera has 4 times as many pixels. In this situation, each pixel in the DSLR camera will receive N times more light than a pixel at the corresponding location in the phone camera. What is N?

   Answer: $N =$ _______________

   (c) (2 points) In this problem assume that the phone and DSLR cameras in the previous question are photon-shot limited (that is, the device-related read noise is small compared to shot noise). What is the per-pixel SNR improvement of the DSLR camera over the phone camera? Very briefly justify your answer.

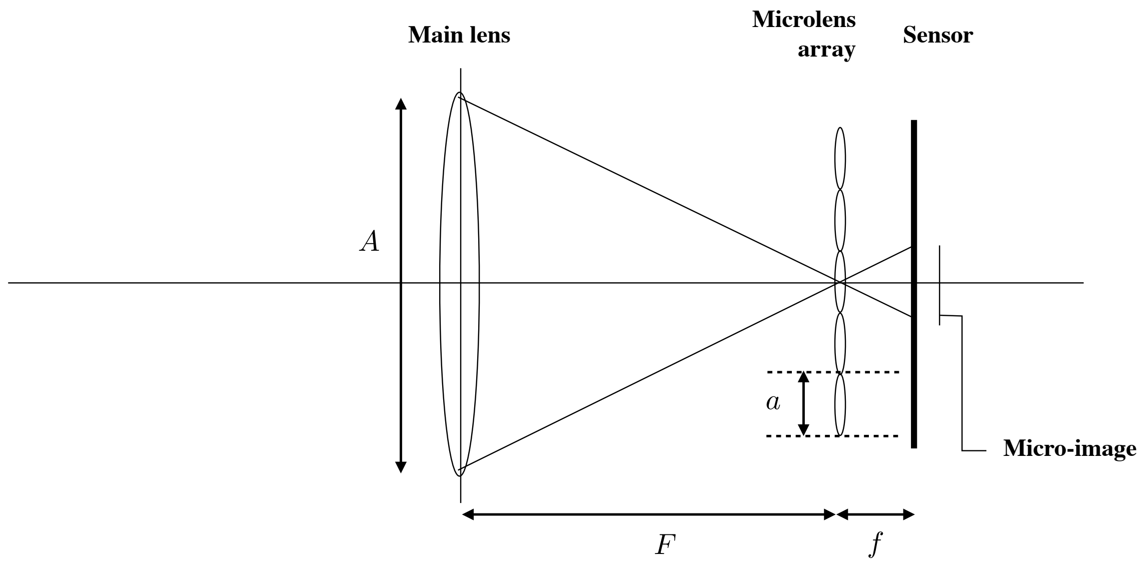
   Answer: _______________

   (d) (2 points) Continuing the previous problem, if a photo of the same scene is taken with both cameras, and the photos are averaged down to the same display viewing resolution, what is the per-pixel improvement in SNR for the DSLR photo over the phone camera photo? Very briefly justify your answer.

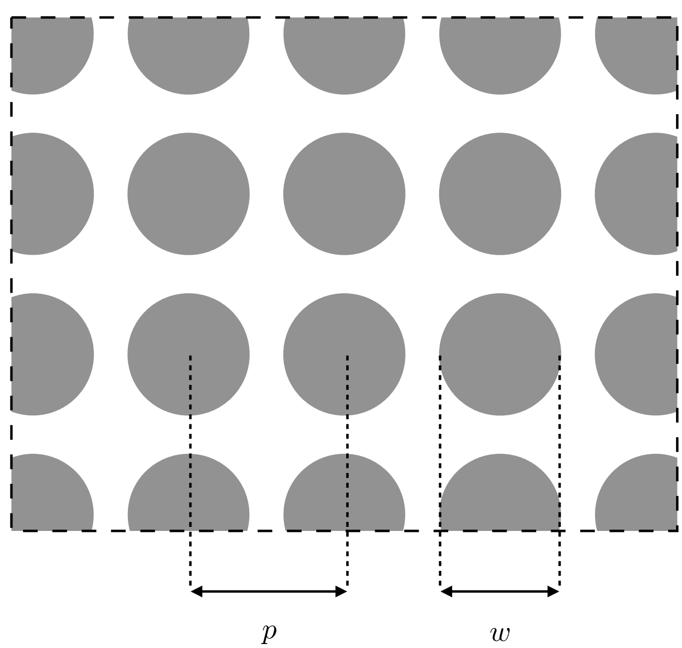
   Answer: _______________

(e) Ms. B has built a plenoptic-sytle light field camera, with camera parameters A, F, a and f as defined below. She is wondering how to set the size (diameter A) of the aperture on the main lens.

As shown on the illustration below, the microlenses array is placed at the focal plane of the main lens (focal length F). The microlenses have an aperture of size a and focal length of f, with the sensor at the focal plane. Note that the illustration is not to scale, and in general the microlenses are tiny compared to the main lens, i.e. $F \gg f$ and $A \gg a$.



Recall that in this type of light field camera, each microlens acts like a tiny camera, and creates a round microimage of the main lens aperture onto the sensor. The illustration below shows a portion of the resulting sensor data, which shows the array of microimages.



i. (3 points) As shown on the diagram, the "pitch" of the microimage array, p, is the spacing between the centers of the microimages. Derive a formula for the pitch in terms of the camera parameters.

Answer: $p =$ ______________________

ii. (3 points) As shown on the diagram, each microimage has a diameter of w. Derive a formula for w in terms of the camera parameters:

Answer: $w =$ ______________________

iii. (2 points) Ms. B decides to set the main lens aperture size to make the microimages as large as possible without overlapping. Calculate what diameter A for the main lens aperture she should choose in order to achieve this?

Answer: $A =$ ______________________

iv. (2 points) Based on the previous answer, if the f-number of the microlenses is N, what f-number should Ms. B choose on the main lens?

Answer: ____________________________________

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5. (Total : 16points) Color

(a) (4 points) Consider two spectral power distributions, $s_1(\lambda)$ and $s_2(\lambda)$ that are metamers. For the following statements, fill in the blanks with true (T) or false (F).

____ They appear the same color to a human observer.

____ A linear combination of these two SPDs, $(a\, s_1(\lambda) + b\, s_2(\lambda))$, would also be a metamer, where a and b are scalar constants.

____ They must carry the same total photon power, integrated over all visible wavelengths.

____ They will be indistinguishable to a color-blind human observer who has only two types of cone cells instead of three.

(b) In this multi-part question we will consider the color reproduction problem. Throughout this problem, assume that the spectral response curves of the three human cone cells, as a function of wavelength, are given by $S(\lambda)$, $M(\lambda)$, $L(\lambda)$.

i. (3 points) First, consider a target light with spectral power distribution (SPD) that we would like to reproduce, given by $P(\lambda)$. Write down expressions for the scalar response of each cone cell when exposed to $P(\lambda)$.

$s_{target} =$

$m_{target} =$

$l_{target} =$

ii. (3 points) Now consider a color reproduction system (e.g. a pixel on an RGB display) composed of three primary lights with SPDs given by functions $R(\lambda)$, $G(\lambda)$ and $B(\lambda)$. If we weight each of these primary lights by scalar values r, g, b, respectively, write down the scalar response from each cone cell type when exposed to these weighted primaries.

$s_{disp} =$

$m_{disp} =$

$l_{disp} =$

iii. (3 points) Note that we can re-write the result from part (ii) in matrix form:

$$\begin{bmatrix} s_{disp} \\ m_{disp} \\ l_{disp} \end{bmatrix} = \begin{bmatrix} & & \\ & M & \\ & & \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

As examples, write down expressions for m_{13} and m_{22}:

$m_{13} =$

$m_{22} =$

iv. (3 points) Finally, to complete the color matching procedure, let's determine how to choose values for r, g and b to match the perceived color of the input SPD $P(\lambda)$, assuming $P(\lambda)$ is in gamut. Write down a one-line matrix expression for such r, g and b values. You can use any variables defined in previous parts of this question, and you may also use matrix operations such as transpose and inverse in your solution if needed.

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} =$$

6. (Total : 14points) Animation and Physical Simulation

   (a) (4 points) In inverse kinematics for a three-segment arm with three joints, we are given a desired position for the end position of the arm, and we must solve for the joint angle parameters. Mark the following statements true or false.

   ___ There is always a unique solution.

   ___ The set of solutions, if it exists, is continuous in parameter space.

   ___ There might not be a solution.

   ___ There might be a unique solution.

   (b) (4 points) Mr. K is simulating a mass-spring system, and is using forward Euler steps. Unfortunately, the simulation is unstable. Mark the following true or false. The following steps would help to make the simulation more stable:

   ___ Add damping to the springs.

   ___ Decrease the stiffness of the springs.

   ___ Increase the time steps.

   ___ Use the derivatives at the end of each time step rather than the beginning.

   (c) (6 points) Describe Position Based / Verlet integration, how it differs from the Forward Euler Method, and why we may choose to use Verlet in physical simulation for graphics.