CS 184/284A **Name:** ____________________

Spring 2018: Midterm 2 **SID number:** ____________________

April 24th, 2018 **cs184-??? login:** ____________________

Time Limit: 110 Minutes

- This exam contains 19 pages (including this cover page) and 6 problems. Check for missing pages.
- Put your initials on the top of every page, in case the pages become separated.
- This exam is closed book, except for one 8.5×11 page of notes (double sided), printed or handwritten.
- This exam is 110 minutes long, and has a total of 100 points.
- Problem difficulty varies throughout the exam, so don't get stuck on a time-consuming problem until you have read through the entire exam. Each problem's point value is roughly correlated with its expected difficulty.
- Answer each question in the space provided. Show all your work. Partial credit may be given on certain problems.
- To minimize distractions, do your best to avoid questions to staff. If you need to make assumptions to answer a question, write these assumptions into your answer.

Problem	Points	Score
1	20	
2	18	
3	12	
4	20	
5	16	
6	14	
Total:	100	

1. (Total : 20points) True / False

Mark the following statements true or false. (1 point each)

(a) (1 point) **T** Rasterization is generally believed more difficult to handle global illumination than path tracing.

(b) (1 point) **F** Metamerism would not be possible if our cone cells were each sensitive to a single wavelength of light.

(c) (1 point) **F** If my image sensor can count photons perfectly without noise, my photographs will be noise free.

(d) (1 point) **F** In image sensor design, pixel fill factor decreases as the pixel size increases.

(e) (1 point) **T** Low discrepancy sampling is used to relieve clumping of random samples.

(f) (1 point) **T** Mipmapping will result in overblurring when the pixel footprint in texture space is anisotropic.

(g) (1 point) **T** After N applications of loop subdivision, the number of triangles will increase by a factor of 4^N.

(h) (1 point) **T** Any rotation in 3D space can be decomposed into a sequence of rotations around each of the three coordinate axes.

(i) (1 point) **F** Antialiasing is the process of filtering out low frequencies before sampling.

(j) (1 point) **F** In Monte Carlo integration, importance sampling always reduces variance compared to uniform random sampling.

(k) (1 point) **T/F** When importance sampling an area light, we will get more noise in shadowed regions of the scene when the area light is larger.

(l) (1 point) **F** A microfacet BRDF in which the normal distribution function exhibits a directionality will result in an isotropic BRDF.

(m) (1 point) **T** Total internal reflection can only happen when light approaches a material with a smaller index of refraction.

(n) (1 point) **T** Depth of field increases as the distance from the world focal plane to the camera increases.

(o) (1 point) **T** In JPEG compression, the chroma channels are stored at lower resolution than the luma channel.

(p) (1 point) **F** For a sensor pixel that is in a defocused part of the image, the light arriving from different positions on the lens aperture is constant.

(q) (1 point) **T** CIELAB is a perceptually organized color space.

(r) (1 point) **T** An inverse kinematics system is harder to implement than forward kinematics.

(s) (1 point) **T** In a rectilinear spring mesh, adding diagonal cross links between masses will reduce shearing.

(t) (1 point) **T** In physical simulation, fully implicit methods can be unconditionally stable.

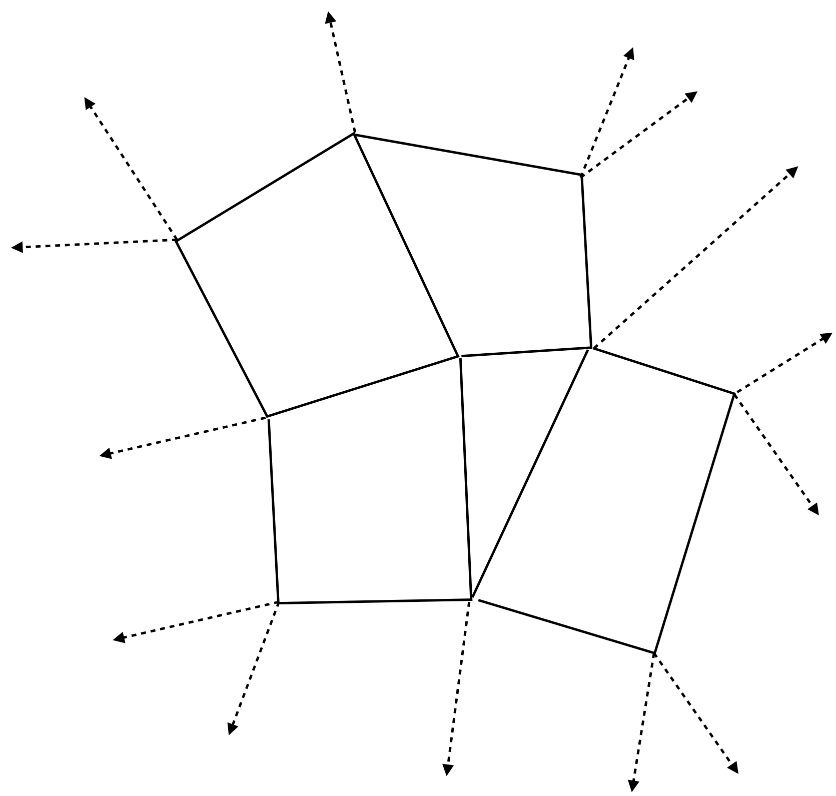
2. (Total : 18points) Rasterization and Geometry

   (a) (6 points) Below is a piece of mesh. Draw the mesh after one step of Catmull-Clark subdivision. Circle all the extraordinary vertices. Only the topology matters.

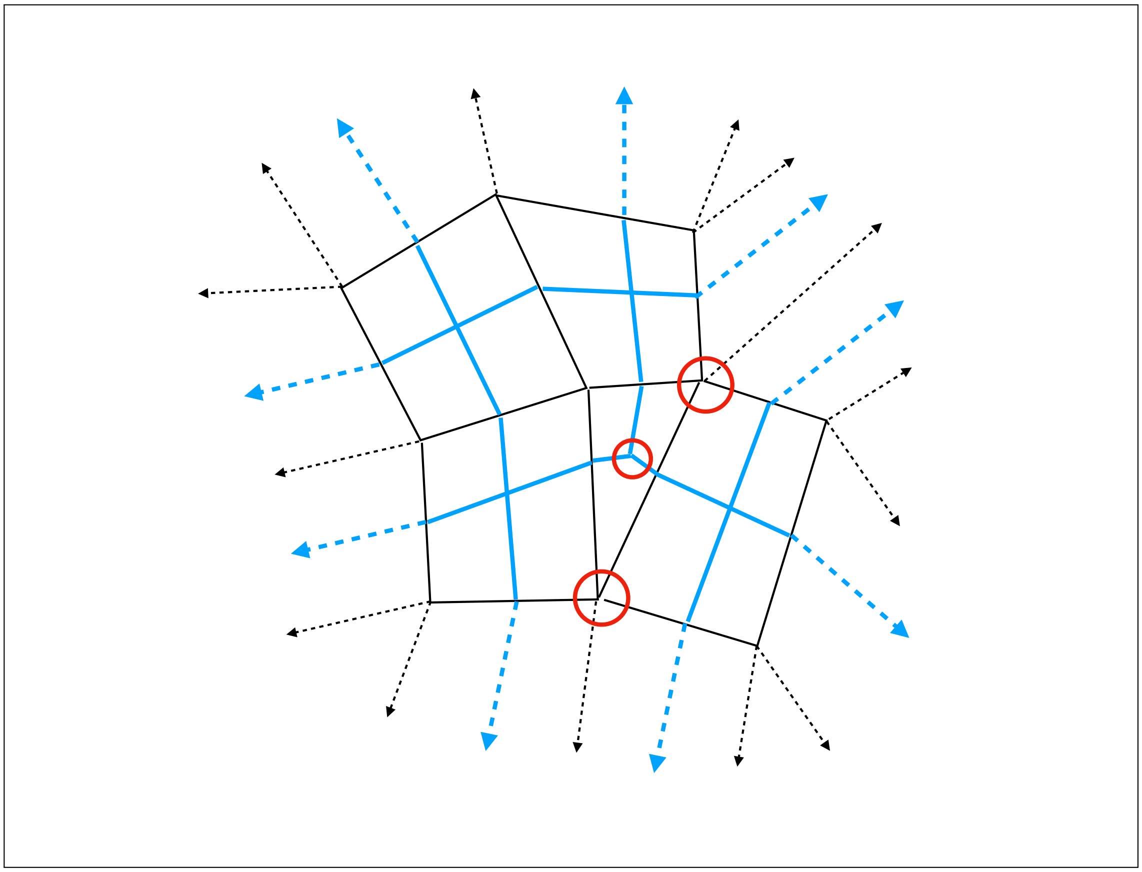
   How many extraordinary points were there originally? Answer: ______________

   How many extraordinary points (total) after one subdivision? Answer: ______________

   How many extraordinary points (total) after another subdivision? Answer: ______________

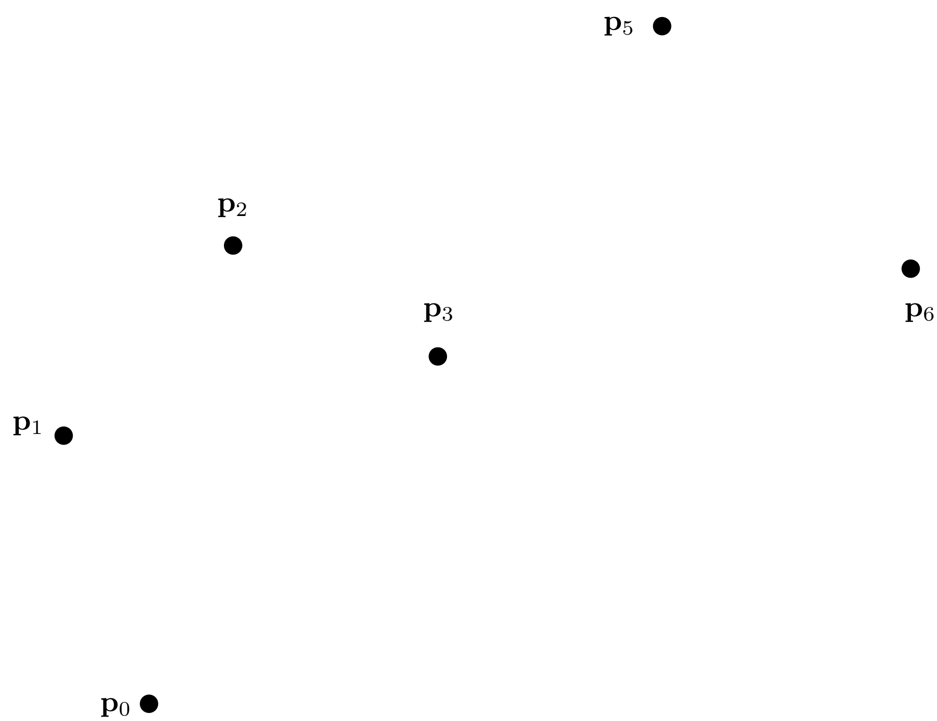


Solution:

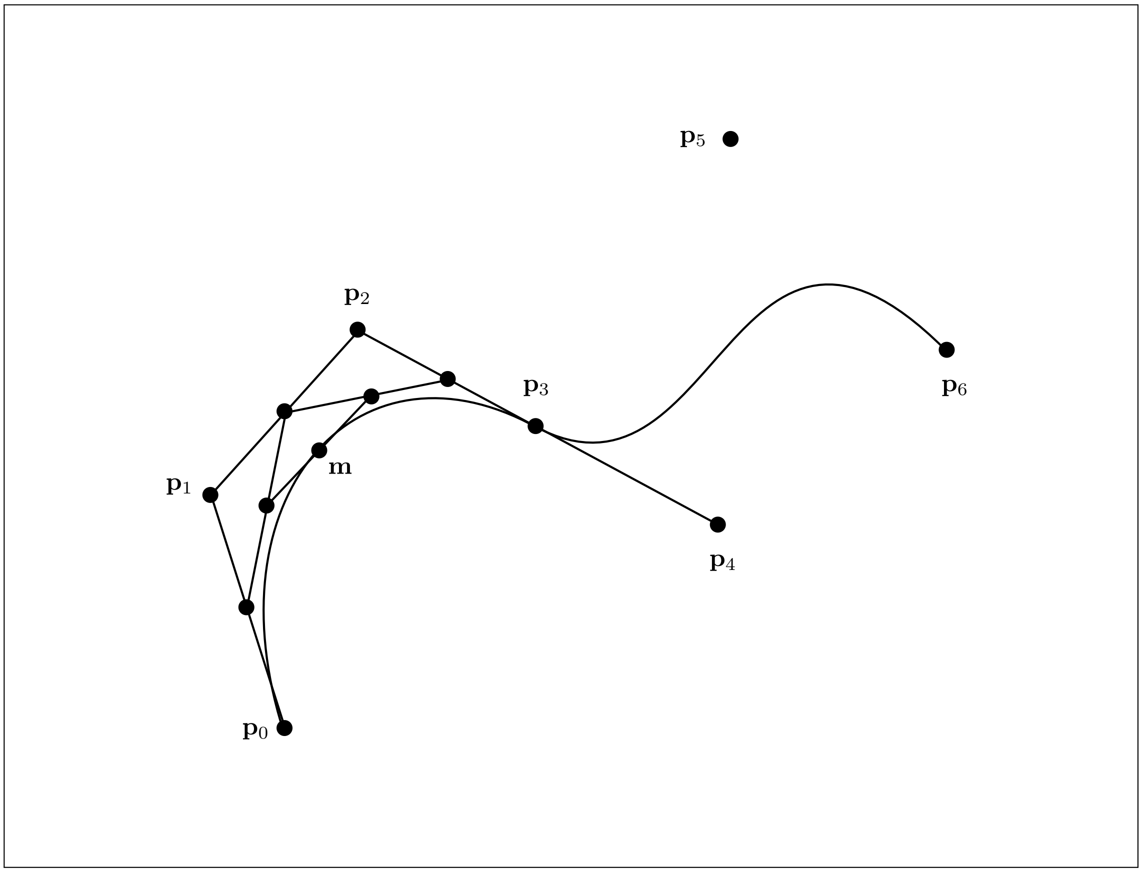


(b) (6 points) Consider two cubic Bézier curves with the control points shown below. The first curve has control points $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, and the second curve has control points $\mathbf{p}_3$, $\mathbf{p}_4$, $\mathbf{p}_5$, $\mathbf{p}_6$. Note that control point $\mathbf{p}_4$ is missing. Complete the following tasks:

1. Draw the application of de Casteljau's algorithm to determine the midpoint of the first Bézier curve. Label this midpoint $\mathbf{m}$.
2. Add point $\mathbf{p}_4$ on the diagram below, so that the overall spline curve is at least C_1 continuous at $\mathbf{p}_3$.
3. Draw the complete spline curve.



Solution:


p5
p2
p3
p6
m
p1
p4
p0

(c) (6 points) Find a quadratic polynomial $f(t)$ that satisfies $f(0) = \mathbf{p}_0, f'(0) = 2(\mathbf{p}_1 - \mathbf{p}_0), f(1) = \mathbf{p}_2$. Your answer should be in terms of t, $\mathbf{p}_0$ and $\mathbf{p}_1$ and $\mathbf{p}_2$.

Answer: $f(t) =$ ____________________________

Solution: $f(t) = at^2 + bt + c$. The three constraints give the following matrix:

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \mathbf{p}_0 \\ 2(\mathbf{p}_1 - \mathbf{p}_0) \\ \mathbf{p}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \end{pmatrix}$$

Solving for a, b, c gives $f(t) = (\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}_2)t^2 + (-2\mathbf{p}_0 + 2\mathbf{p}_1)t + \mathbf{p}_0$

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3. (Total : 12points) Ray Tracing, Rendering, Monte Carlo Integration

(a) (3 points) How and why do we use the Russian Roulette method in path tracing?

Solution: In recursively tracing rays to estimate radiance, we randomly terminate the ray with probability p. If we do not terminate, we weight the radiance returned by recursive call by $1/(1-p)$ to keep the integral estimate unbiased.

(b) (3 points) Assume ω_1 and ω_2 are the solid angles subtended by two spheres C_1 and C_2, respectively, from a given point P. Mark the following statements true or false.

F If the radius of C_1 is greater than C_2, then it must be true that $\omega_1 > \omega_2$.

F If $\omega_1 > \omega_2$ then C_2 must be further than C_1 from P.

T If C_1 and C_2 are the same size and their centers are the same distance from P, then ω_1 and ω_2 must be equal.

(c) (6 points) In Monte Carlo integral estimation of $\int_a^b f(x)dx$, we can use multiple importance sampling as discussed in class. In the general approach, we draw multiple random samples X_i from probability density functions $p_i(x)$, evaluate the function f at these samples to obtain values $f(X_i)$, and form a weighted average of these values to estimate the desired integral. This gives the following Monte Carlo integral estimator:

$$F = \sum_{i=0}^{N} w_i(X_i)\frac{f(X_i)}{p_i(X_i)}$$

Many choices of weighting factor $w_i(x)$ are possible (e.g. the "balance heuristic" weighting factor discussed in class is $w_i(x) = \frac{p_i(x)}{\sum_{k=1}^{N} p_k(x)}$ and gives an unbiased estimator F.) In general, what condition(s) on $w_i(x)$ are needed to produce an estimator F that is unbiased? Justify your answer mathematically.

Solution: F would be an unbiased estimator if the expected value of F is the integral, i.e. $E[F] = \int_a^b f(x)\,dx$. Based on the definition of F above,

$$\begin{aligned} E[F] &= \int_a^b \sum_{i=0}^{N} \frac{w_i(x)f(x)p_i(x)}{p_i(x)}dx \\ &= \int_a^b \sum_{i=0}^{N} w_i(x)f(x)dx \end{aligned}$$

From this analysis, we see that F will be unbiased if $\sum_{i=0}^{N} w_i(x) = 1$ for all x where $f(x)$ is not zero. (Technically we also need $w_i(x) = 0$ whenever $p_i(x) = 0$, but full credit will be given without this.)

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4. (Total : 20points) Radiometry, Cameras, Lenses

   (a) (2 points) Irradiance is measured in units of ________________

   Solution: W/m^2

   (b) (4 points) Compared to a phone camera, a DSLR camera has a focal length, sensor width and sensor height that are 4 times larger. The f-number is the same between the cameras. The DSLR camera has 4 times as many pixels. In this situation, each pixel in the DSLR camera will receive N times more light than a pixel at the corresponding location in the phone camera. What is N?

   Answer: $N =$ ____________

   Solution: $N = 64$. The DSLR lens aperture width is 4 times larger, so the area is 16 times larger, therefore the irradiance on the sensor is 16 times higher. Also, the DSLR pixels are 4 times larger in area (16 times larger sensor, and 4 times as many pixels). N is the product of these factors.

   (c) (2 points) In this problem assume that the phone and DSLR cameras in the previous question are photon-shot limited (that is, the device-related read noise is small compared to shot noise). What is the per-pixel SNR improvement of the DSLR camera over the phone camera? Very briefly justify your answer.

   Answer: ____________

   Solution: 8. In shot limited situation, the SNR is the square root of the signal level. 64 times higher signal in the DSLR camera is an 8 times improvement in SNR.

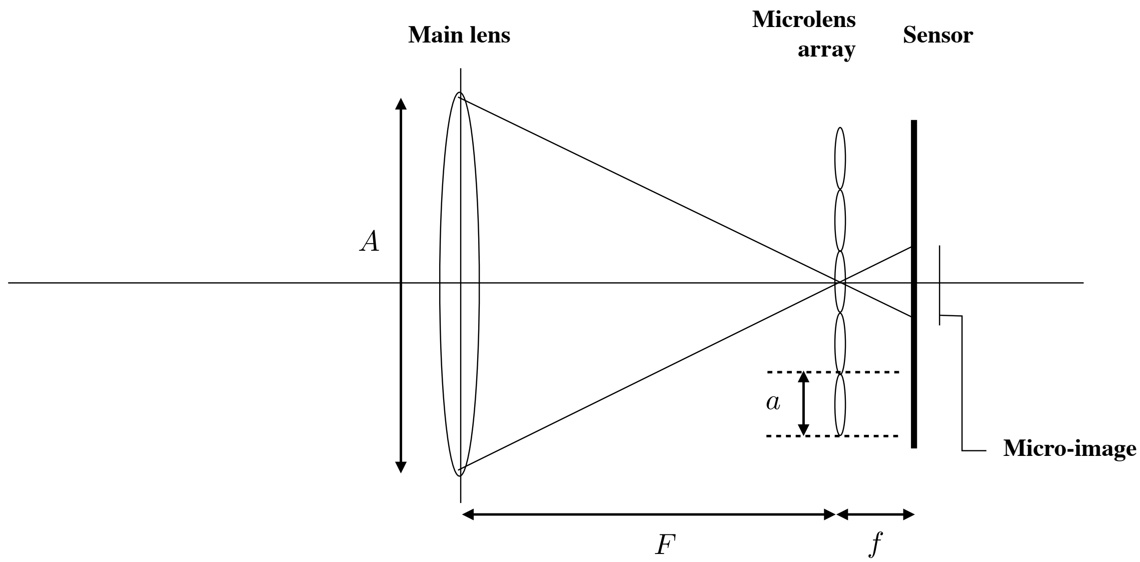
   (d) (2 points) Continuing the previous problem, if a photo of the same scene is taken with both cameras, and the photos are averaged down to the same display viewing resolution, what is the per-pixel improvement in SNR for the DSLR photo over the phone camera photo? Very briefly justify your answer.

   Answer: ____________

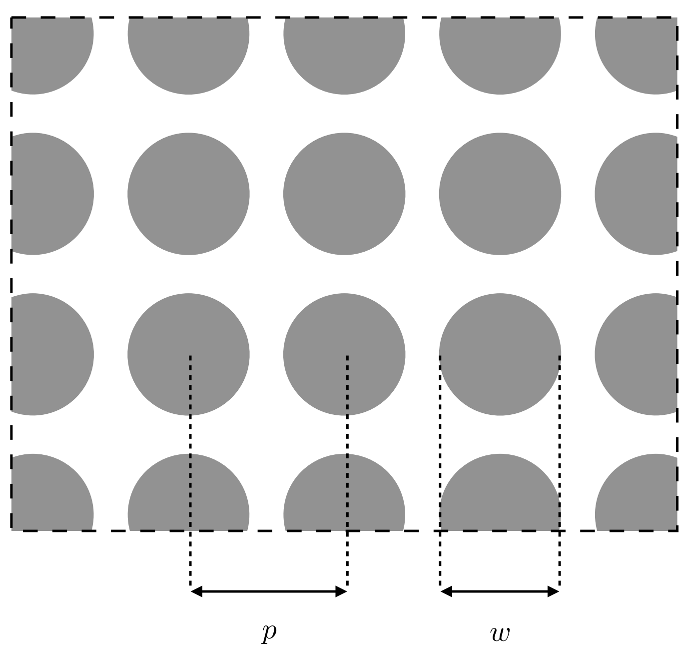
   Solution: SNR for DSLR is 4 times higher. The displayed image pixels for the DSLR represent 16 times more signal than for the phone camera (just the difference in the lens collecting power).

(e) Ms. B has built a plenoptic-sytle light field camera, with camera parameters A, F, a and f as defined below. She is wondering how to set the size (diameter A) of the aperture on the main lens.

As shown on the illustration below, the microlenses array is placed at the focal plane of the main lens (focal length F). The microlenses have an aperture of size a and focal length of f, with the sensor at the focal plane. Note that the illustration is not to scale, and in general the microlenses are tiny compared to the main lens, i.e. $F \gg f$ and $A \gg a$.



Recall that in this type of light field camera, each microlens acts like a tiny camera, and creates a round microimage of the main lens aperture onto the sensor. The illustration below shows a portion of the resulting sensor data, which shows the array of microimages.



i. (3 points) As shown on the diagram, the "pitch" of the microimage array, p, is the spacing between the centers of the microimages. Derive a formula for the pitch in terms of the camera parameters.

Answer: $p =$ ______________________________

Solution:

$$p = a\frac{F+f}{F} \approx a \text{ because } F \gg f$$

ii. (3 points) As shown on the diagram, each microimage has a diameter of w. Derive a formula for w in terms of the camera parameters:

Answer: $w =$ ______________________________

Solution:

$$w = A\frac{f}{F}$$

iii. (2 points) Ms. B decides to set the main lens aperture size to make the microimages as large as possible without overlapping. Calculate what diameter A for the main lens aperture she should choose in order to achieve this?

Answer: $A =$ ______________________________

Solution: She wants $p = w$, so

$$A = a\frac{F}{f}$$

iv. (2 points) Based on the previous answer, if the f-number of the microlenses is N, what f-number should Ms. B choose on the main lens?

Answer: ____

Solution: She should set the main lens number to N as well.

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5. (Total : 16points) Color

(a) (4 points) Consider two spectral power distributions, $s_1(\lambda)$ and $s_2(\lambda)$ that are metamers. For the following statements, fill in the blanks with true (T) or false (F).

T They appear the same color to a human observer.

F A linear combination of these two SPDs, $(a\, s_1(\lambda) + b\, s_2(\lambda))$, would also be a metamer, where a and b are scalar constants.

F They must carry the same total photon power, integrated over all visible wavelengths.

T They will be indistinguishable to a color-blind human observer who has only two types of cone cells instead of three.

(b) In this multi-part question we will consider the color reproduction problem. Throughout this problem, assume that the spectral response curves of the three human cone cells, as a function of wavelength, are given by $S(\lambda)$, $M(\lambda)$, $L(\lambda)$.

i. (3 points) First, consider a target light with spectral power distribution (SPD) that we would like to reproduce, given by $P(\lambda)$. Write down expressions for the scalar response of each cone cell when exposed to $P(\lambda)$.

$s_{target} =$

$m_{target} =$

$l_{target} =$

Solution:

$$s_{target} = \int S(\lambda)P(\lambda)d\lambda$$
$$m_{target} = \int M(\lambda)P(\lambda)d\lambda$$
$$l_{target} = \int L(\lambda)P(\lambda)d\lambda$$

ii. (3 points) Now consider a color reproduction system (e.g. a pixel on an RGB display) composed of three primary lights with SPDs given by functions $R(\lambda)$, $G(\lambda)$ and $B(\lambda)$. If we weight each of these primary lights by scalar values r, g, b, respectively, write down the scalar response from each cone cell type when exposed to these weighted primaries.

$s_{disp} =$

$m_{disp} =$

$l_{disp} =$

Solution:

$$s_{disp} = \int S(\lambda)\,(rR(\lambda) + gG(\lambda) + bB(\lambda))\,d\lambda$$
$$m_{disp} = \int M(\lambda)\,(rR(\lambda) + gG(\lambda) + bB(\lambda))\,d\lambda$$
$$l_{disp} = \int L(\lambda)\,(rR(\lambda) + gG(\lambda) + bB(\lambda))\,d\lambda$$

iii. (3 points) Note that we can re-write the result from part (ii) in matrix form:

$$\begin{bmatrix} s_{disp} \\ m_{disp} \\ l_{disp} \end{bmatrix} = \begin{bmatrix} & & \\ & M & \\ & & \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

As examples, write down expressions for m_{13} and m_{22}:

$m_{13} =$

$m_{22} =$

Solution:

$$m_{13} = \int S(\lambda)B(\lambda)d\lambda$$
$$m_{22} = \int M(\lambda)G(\lambda)d\lambda$$

iv. (3 points) Finally, to complete the color matching procedure, let's determine how to choose values for r, g and b to match the perceived color of the input SPD $P(\lambda)$, assuming $P(\lambda)$ is in gamut. Write down a one-line matrix expression for such r, g and b values. You can use any variables defined in previous parts of this question, and you may also use matrix operations such as transpose and inverse in your solution if needed.

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} =$$

Solution: The goal is to have $(s_{disp}, m_{disp}, l_{disp})^T = (s_{target}, m_{target}, l_{target})^T$. This is accomplished by choosing:

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} & & \\ & M & \\ & & \end{bmatrix}^{-1} \begin{bmatrix} s_{target} \\ m_{target} \\ l_{target} \end{bmatrix}$$

6. (Total : 14points) Animation and Physical Simulation

(a) (4 points) In inverse kinematics for a three-segment arm with three joints, we are given a desired position for the end position of the arm, and we must solve for the joint angle parameters. Mark the following statements true or false.

F There is always a unique solution.

F The set of solutions, if it exists, is continuous in parameter space.

T There might not be a solution.

T There might be a unique solution.

(b) (4 points) Mr. K is simulating a mass-spring system, and is using forward Euler steps. Unfortunately, the simulation is unstable. Mark the following true or false. The following steps would help to make the simulation more stable:

T Add damping to the springs.

T Decrease the stiffness of the springs.

F Increase the time steps.

T Use the derivatives at the end of each time step rather than the beginning.

(c) (6 points) Describe Position Based / Verlet integration, how it differs from the Forward Euler Method, and why we may choose to use Verlet in physical simulation for graphics.

Solution: In Forward Euler, we estimate the velocity and position of each particle for the next time step based on the acceleration and velocity of the particle at the current time step. This is simple, but is often unstable.

Verlet integration uses relatively low stiffness and modified Euler steps (i.e. using the derivative at the end of the step) to stabilize the forward step. Then, it applies constraints iteratively on the position of the particles to prevent the springs from being too stretched out. Finally, the velocities for each particle are calculated using the difference between the final, constrained positions and their positions at the previous time step.

We may choose to use Verlet integration because it gives a more stable simulation than Forward Euler. In Verlet, the constraints and method of estimating the velocity effectively throw away energy and stabilize the simulation.