CS 184/284A **Name:** ____________________

Spring 2017: Midterm 2 **SID number:** ____________________

April 20th, 2017 **cs184-??? login:** ____________________

Time Limit: 110 Minutes

- This exam contains 15 pages (including this cover page) and 6 problems. Check for missing pages.
- Put your initials on the top of every page, in case the pages become separated.
- This exam is closed book, except for one 8.5×11 page of notes (double sided), printed or handwritten.
- This exam is 110 minutes long, and has a total of 110 points.
- Problem difficulty varies throughout the exam, so don't get stuck on a time-consuming problem until you have read through the entire exam. Each problem's point value is roughly correlated with its expected difficulty.
- Answer each question in the space provided. Partial credit may be given on certain problems.
- To minimize distractions, do your best to avoid questions to staff. If you need to make assumptions to answer a question, write these assumptions into your answer.

Problem	Points	Score
1	20	
2	18	
3	18	
4	18	
5	18	
6	18	
Total:	110	

1. (Total : 20points) True / False

   Mark each statement true or false. (1 point each)

   (a) (1 point) **F** The Nyquist frequency is double the sampling frequency.

   (b) (1 point) **F** Storing a mipmap takes twice as much storage as storing only the full resolution texture.

   (c) (1 point) **T** Only 6 scalars are needed to specify a unique rigid transformation in three dimensions.

   (d) (1 point) **T** Implicit geometry works well for ray tracing and works poorly for rasterization.

   (e) (1 point) **F** If we switch the incident and outgoing directions, an anisotropic BRDF will give us different values.

   (f) (1 point) **T** When performing ray-scene intersection, a Bounding Volume Hierarchy (BVH) is not always more efficient than uniform grids.

   (g) (1 point) **F** Photon mapping is a biased approach, so given the same rendering time, it introduces more noise than path tracing.

   (h) (1 point) **T** Compared to uniform random sampling, pseudo-random sampling usually gives us low discrepancy sequences, but at the cost of introducing patterns.

   (i) (1 point) **T** In an f/4 light field camera with $N \times N$ pixels per microlens, the effective f-number of a sub-aperture image is f/$4N$.

   (j) (1 point) **F** A lens fabricated with perfect manufacturing precision will exhibit no optical aberrations.

   (k) (1 point) **T** Assuming sensor-related noise is very small, a pixel that receives 10,000 photons would have an SNR of 100.

   (l) (1 point) **T** For an ideal thin lens used at unit magnification, the distance from the subject to the lens, and the distance from the sensor to the lens, are both equal to twice the focal length of the lens.

   (m) (1 point) **T** A spectral power distribution describes how much power is contributed by each wavelength of light.

   (n) (1 point) **F** The rods in a human eye are primarily responsible for perceiving color.

   (o) (1 point) **F** In color sensors, the Bayer pattern uses more green pixels than red or blue pixels because the majority of light in the world have wavelengths close to green.

   (p) (1 point) **T** The majority of RGB values in an output image produced by demosaicking algorithms are interpolated values from individual CMOS sensors.

   (q) (1 point) **F** As time goes on, the path of a particle calculated using explicit Euler integration will approach the "true" path of that particle.

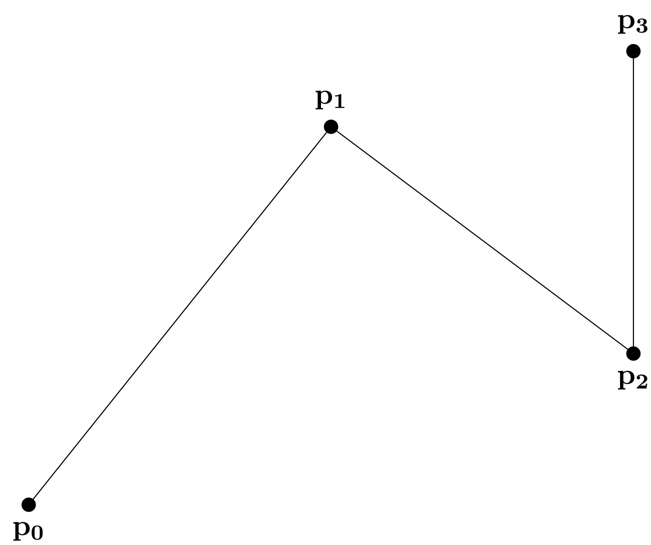
   (r) (1 point) **T** Rigging is the process of adding higher-level constructs to 3D models, making them easier to animate.

   (s) (1 point) **F** Fully implicit methods can be conditionally stable but not unconditionally stable.

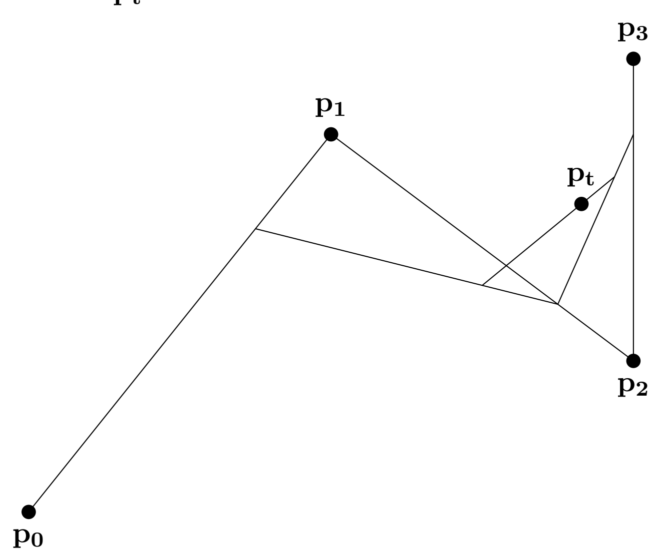
   (t) (1 point) **F** A cloth simulation using structural and shearing constraints will be able to resist out-of-plane bending.

2. (Total : 18points) Rasterization and Geometry

   (a) (2 points) Use de Casteljau's algorithm to find the point where $t = 3/4$ on the Bézier curve defined by these control points.



Solution: Solution is at $\mathbf{p_t}$.



(b) (6 points) Find a cubic polynomial $f(x)$ that satisfies $f(0) = 0, f(1) = 0, f'(0) = k, f'(1) = 0$. Your answer should be in terms of x and k.

Solution: $f(x) = ax^3 + bx^2 + cx + d$. Matrix:

$$\begin{pmatrix} 0 \\ 0 \\ k \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

Solve one at a time. Immediately, $d = 0$ and $c = k$. Then $a + b + k = 0$ and $3a + 2b + k = 0$ together yield $a = k$ and $b = -2k$. Thus

$$f(x) = kx^3 - 2kx^2 + kx = kx(x-1)^2$$

(c) (2 points) Consider a triangle in three dimensional space with vertices p_0, p_1, and p_2 and some point p on the triangle that barycentric coordinates α, β, and γ. Write an expression that will give the position of p (don't overthink this).

$p =$ ______________________________

Solution: $\alpha p_0 + \beta p_1 + \gamma p_2$

(d) (3 points) Now, we place a pinhole camera at the origin looking along the negative z-axis, with the positive y axis as "up," and with its image plane at $z = -d$. Recall that we can project a world point p with z coordinate p_z onto the image plane of the camera by mapping it to $\frac{d}{-p_z}p$, after which the z coordinate will be equal to $-d$ and the two other coordinates will correspond to the location of the point on the two dimensional image plane.

Thus the three points of the triangle p_0, p_1, p_2 from problem (e) would project to

$$q_0 = \frac{d}{-p_{0z}}p_0, \quad q_1 = \frac{d}{-p_{1z}}p_1, \quad q_2 = \frac{d}{-p_{2z}}p_2$$

on the image plane. However, p does **not** project to $\alpha q_0 + \beta q_1 + \gamma q_2$! Write an expression for the correct projection $q = \frac{d}{-p_z}p$ in terms of only the following quantities:

$$p_0, p_1, p_2, p_{0z}, p_{1z}, p_{2z}, \alpha, \beta, \gamma, \text{ and } d.$$

$q =$ ______________________________

Solution:

$$q = \frac{d}{-(\alpha p_{0z} + \beta p_{1z} + \gamma p_{2z})}(\alpha p_0 + \beta p_1 + \gamma p_2)$$

(e) (5 points) Now, in terms of only $p_{0z}, p_{1z}, p_{2z}, \alpha, \beta$, and γ, find three values α', β', and γ' such that $q = \alpha' q_0 + \beta' q_1 + \gamma' q_2$ and $\alpha' + \beta' + \gamma' = 1$. (The difference between these and the original barycentric coordinates underlies the principle of *perspective-correct texture mapping.*)

$\alpha' =$ ______________________________

$\beta' =$ ______________________________

$\gamma' =$ ______________________________

Solution:

$$\alpha' = \frac{\alpha p_{0z}}{\alpha p_{0z} + \beta p_{1z} + \gamma p_{2z}}, \beta' = \frac{\beta p_{1z}}{\alpha p_{0z} + \beta p_{1z} + \gamma p_{2z}}, \gamma' = \frac{\gamma p_{2z}}{\alpha p_{0z} + \beta p_{1z} + \gamma p_{2z}}$$

3. (Total : 18points) Ray Tracing and Rendering

(a) (2 points) What is the name of the equation that can be used to compute the fraction of light reflected from a smooth dielectric surface (hint: the equation predicts 100% reflection at grazing angles).

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Solution: Fresnel equation.

(b) (2 points) What is the name of the law that can be used to compute the direction of a refracted ray entering a dielectric material?

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Solution: Snell's law.

(c) (2 points) What is the name of the effect that path tracing is hard to resolve but but photon mapping can easily deal with?

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Solution: Caustics.

(d) (2 points) Briefly explain the difference between bump mapping and displacement mapping.

Solution: Displacement mapping actually changes the surface, while bump mapping only changes the normal. So they'll give different silhouettes to the same object. (Partial credit if students just say that displacement mapping is difficult to implement.)

(e) (10 points) A point light at $(8, 9, 0)$ with radiant flux (power) 50 illuminates a surface location at $(5, 5, 0)$ with normal $(0, 1, 0)$. Assume that the surface is diffuse with color/albedo/reflectance $(0.6, 0.8, 1.0)$. How much light is reflected from the shading point? Express your answer as a unitless 3-tuple representing the RGB spectrum of the light reflected.

Solution: The un-normalized $\omega_i' = (8, 9, 0) - (5, 5, 0) = (3, 4, 0)$.
The distance between the shading point and the light is then $\sqrt{3^2 + 4^2} = 5$.
Then the incident radiance is $I/r^2 = 50/(5^2) = 2$.
The BRDF is $(0.6, 0.8, 1.0)$.
The cosine term is $n \cdot \omega_i = (0, 1, 0) \cdot \frac{(3,4,0)}{5} = 0.8$.
Then the light reflected is $2 * (0.6, 0.8, 1.0) * 0.8 = (0.96, 1.28, 1.6)$

4. (Total : 18points) Cameras and Lenses

   (a) (4 points) In a light field camera, the main lens and microlenses are usually designed so that the disk micro-images that appear under each microlens on the sensor are as large as possible without overlapping. Assume the main lens has aperture diameter A, focal length F and is focused at infinity. If the micro-images on the sensor have a pitch (center-to-center spacing) of Δx, then write down a formula for f, the ideal spacing between the microlenses and the sensor. Hint: recall that each micro-image is an image of the circular aperture of the main lens.

   $f =$ ______________________________

   Solution: The answer is $f = F/A * \Delta x$, which can be derived from a ray-diagram that shows by similar triangles that $F/A = f/\Delta x$.

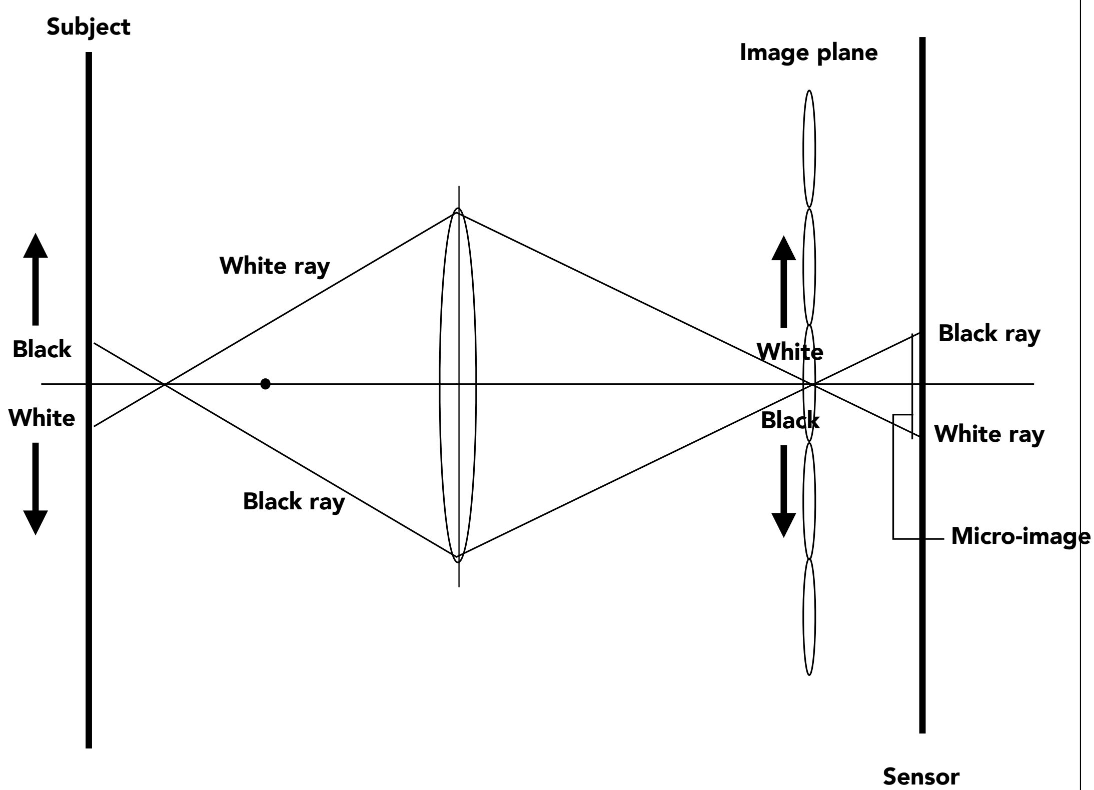
   (b) (4 points) Assume we start with the light field camera in the previous question, including the correct value for f. Sarah changes the main lens focus depth from infinity to a depth that achieves a subject magnification of 1.0. After she does this she notices that the micro-images no longer meet the criterion of being as large as possible without overlapping. Derive the appropriate new main lens aperture diameter, A' that Sarah should choose so that the micro-images are again as large as possible without overlapping.

   $A' =$ ______________________________

   Solution: The answer is that the aperture should be doubled in size, $A' = 2A$. This is because unit magnification is achieved when the gap between lens and sensor is twice the focal length.

(c) (10 points) Consider a photograph that shows an edge with white on the bottom and black on the top. Now consider shooting this scene with a light field camera (plenoptic type), and also consider changing the focus so that the lens is focused slightly *closer* than this white / black edge. In the light field sensor's pixel data, the disk micro-images located near the edge will also show a black-white edge inside the disk micro-image. Within these micro-images, will it be white above or below the edge? Justify your answer with a ray diagram and brief explanation.

Solution:



If a photo would be white on bottom, the sensor image must have white on the top due to optical inversion, as shown. Since the lens is focused in front of the subject, the ray-diagram shows that there will be another optical inversion for the micro-image so that in the micro-image the white portion will appear opposite from the photograph orientation. Therefore, in the micro-images the white portion will be on the top.

5. (Total : 18points) Color and Imaging

   (a) (2 points) In color reproduction, what is the name given to two different spectral power distributions that have the same perceived appearance?

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   Solution: metamers

   (b) (2 points) What is the name of the color mixing model that uses the C, M, and Y primaries?

   ____

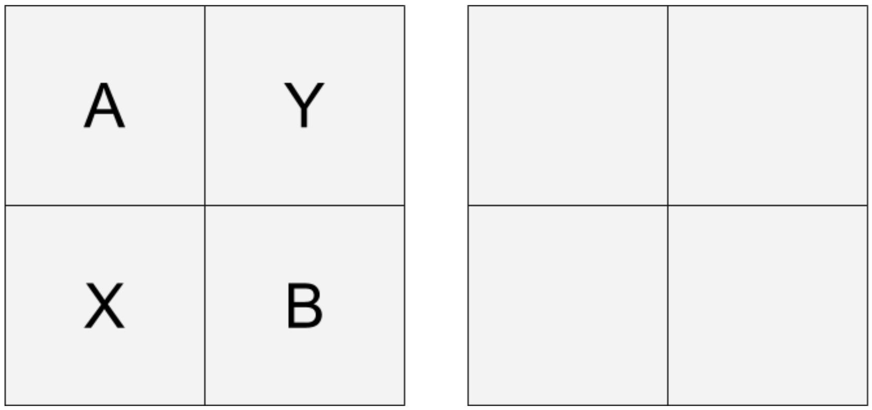
   Solution: subtractive

   (c) (2 points) What do we call the set of chromaticities generated by a set of primaries?

   ____

   Solution: gamut

(d) (2 points) Suppose that there are 4 colors that represent the basis of a color space: A, B, X, and Y. We know that A and B are color-opponent pairs and that X and Y are also color-opponent pairs. If I stare at the image on the left for a few seconds and then look away to an empty screen, what is the afterimage that I will see? Label the empty image on the right with the afterimage seen.



Solution:
B, X
Y, A

(e) (2 points) Which of the following are perceptually organized color spaces? Circle all that apply.

(a) HSV
(b) sRGB
(c) YCbCr
(d) CIELAB

Solution: (a), (c), and (d) are perceptually organized color spaces.

(f) (2 points) In an image sensor, what is the name of the phenomenon in which incoming light rays meant for one pixel are accidentally detected by a different, nearby pixel?

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Solution: cross-talk

(g) (4 points) Which of the following are true about JPEG compression? Circle all that apply.

(a) It more aggressively downsamples chromaticity components than it does the luminance component

(b) It takes advantage of the fact that the human visual system is less sensitive to high frequency sources of error

(c) Using a quantization matrix with higher values in it will result in a higher quality JPEG compressed image

(d) Loss of information only occurs when quantizing the results of the discrete cosine transform

Solution: (a) and (b) are true

(h) (2 points) What does convolution with the following filter do to an image?

$$\begin{pmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{pmatrix}$$

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Solution: extracts the edges (edge detection), high pass filter, sharpen

6. (Total : 18points) Animation and Physical Simulation

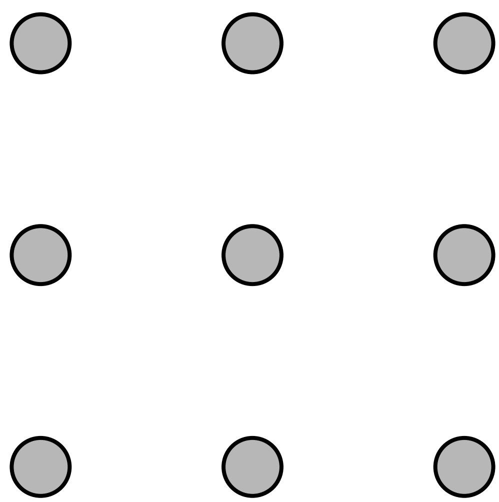
   (a) (2 points) Tweening is a process used to interpolate between ____________.

   Solution: Keyframes

   (b) (2 points) Suppose that we are trying to animate a three-jointed robotic arm that has to pick up an object in one location and move it realistically to another location. What method are we likely to use to compute the path that the arm takes?

   Solution: Inverse kinematics

   (c) (4 points) Recall that there are three types of constraints used in our mass-spring cloth. For the **top-left node** in the diagram below, draw one of each kind of constraint attached to that node and clearly label each with one word that describes what kind of constraint it is.

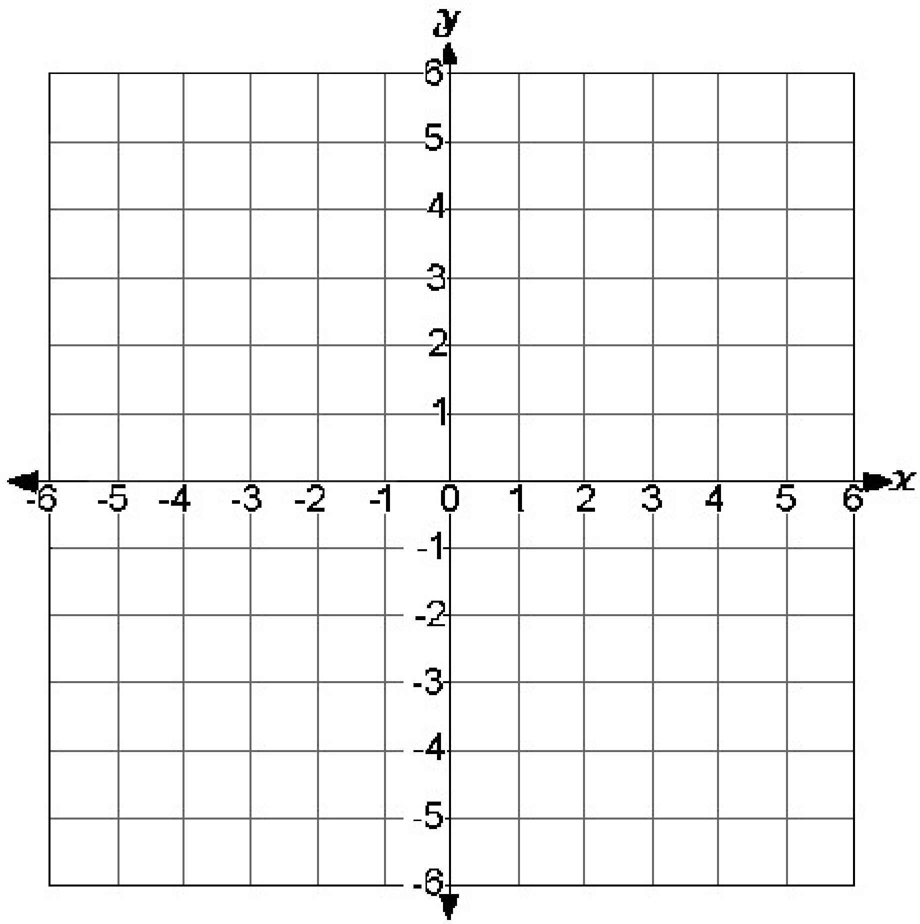


(d) (10 points) Suppose we have a particle of mass $m = 0.5$ with the following initial conditions:

$$(x^0, y^0) = (2, 0) \qquad (\dot{x}^0, \dot{y}^0) = (0, 2)$$

This particle is subject to a force due to a spring attached to the origin at one end and the particle at the other. The spring's position at the origin is fixed, whereas the particle is allowed to move around. The spring constant of this spring is $k = 1$, and its rest length is 2.

We wish to find the trajectory of this particle using explicit (non-modified) Euler integration with a timestep of $\Delta t = 1$. **Draw the first three steps of this particle (not including its position at $t = 0$) and write out its final position.**



Solution: Forward Euler integration works according to the equations:

$$x^{t+\Delta t} = x^t + \Delta t \dot{x}^t$$
$$\dot{x}^{t+\Delta t} = \dot{x}^t + \Delta t \ddot{x}^t$$

Using these, we can derive the following values based on the initial conditions:

$$
\begin{aligned}
x^0 &= (2, 0) & \dot{x}^0 &= (0, 2) & \ddot{x}^0 &= (0, 0) \\
x^1 &= (2, 2) & \dot{x}^1 &= (0, 2) & \ddot{x}^1 &= (2\sqrt{2} - 4, 2\sqrt{2} - 4) \\
x^2 &= (2, 4) & \dot{x}^2 &= (2\sqrt{2} - 4, 2\sqrt{2} - 2) & & \\
x^3 &= (2\sqrt{2} - 2, 2\sqrt{2} + 2) & & & &
\end{aligned}
$$

The trick to finishing this question in a reasonable time is to realize that we only need the first two velocities and the first acceleration.