CS 184/284A

Spring 2019: Exam 2

April 25th, 2019

Time Limit: 110 Minutes

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Room: Pwinelle 155

- This exam contains 19 pages (including this cover page) and 5 parts. Check for missing pages.
- Put your initials on the top of every page, in case the pages become separated.
- This exam is closed book, except for one 8.5 × 11 page of notes (double sided), printed or handwritten.
- This exam is 110 minutes long, and has a total of 110 points.
- Problem difficulty varies throughout the exam, so don't get stuck on a time-consuming problem until you have read through the entire exam. Each problem's point value is roughly correlated with its expected difficulty.
- Answer each question in the space provided. Partial credit may be given on certain problems.
- To minimize distractions, do your best to avoid questions to staff. If you need to make assumptions to answer a question, write these assumptions into your answer.
- **For multiple choice questions, please completely fill the "bubble" containing the letter next to your answer. Do not just tick or circle.**

Problem	Points	Score
1	20	
2	22	
3	24	
4	22	
5	22	
Total:	110	

1. (Total : 20 points) True / False

Mark each statement true or false – indicate clearly. (1 point each)

(a) (1 point) F A shear transformation preserves distances.

(b) (1 point) T The Nyquist frequency is $\frac{1}{2}$ the sampling frequency.

(c) (1 point) T Considering only primary visibility (no shadow or global illumination effects) for a pinhole camera, rasterization and ray tracing produce equivalent results.

(d) (1 point) F Storing a mipmap takes 4/3 more memory than storing just the original resolution image.

(e) (1 point) T If I average the value of two unbiased Monte Carlo estimators, the resulting value is also an unbiased Monte Carlo estimator.

(f) (1 point) F A lens with an $f/2$ aperture always has a larger pupil than another lens with an $f/8$ aperture.

(g) (1 point) F If I increase the shutter duration of my camera from 1/500 to 1/250 of a second, I would have to decrease my aperture from $f/2$ to $f/4$ to maintain the same exposure.

(h) (1 point) F When maintaining focusing on a nearby object while moving closer to the object, the circle of confusion of the background in the resulting photo will increase.

(i) (1 point) T If I use a heart-shaped aperture in my lens, I can take a single photo that contains bokeh of a heart shape that is right side up or upside down, and of a continuous range of sizes.

(j) (1 point) T If I invent a perfectly noiseless image sensor, then the pixel values I capture with it will have a signal-to-noise-ratio (SNR) that increases proportional to the square root of the number of photo-electrons captured at that pixel.

(k) (1 point) F A 2D convolution filter can always be expressed as two 1D convolution filters.

(l) (1 point) T In JPEG compression, we compress the chroma channel more than the luma channel.

(m) (1 point) F Increasing the size of your time step is a useful way to increase the stability of your simulation.

(n) (1 point) T Verlet integration is a technique to increase the stability of simulations.

(o) (1 point) T With inverse kinematics, an animator specifies the desired position of an end effector (like a hand), and the computer solves for angles of the parent joints (like the shoulder and elbow) which would place the end effector there.

(p) (1 point) F Two beams of light that have exactly the same color must have exactly the same spectral power distribution.

(q) (1 point) F If I create light that is a linear combination of three metamers, the resulting light will also be a metamer.

(r) (1 point) F If I have pixels of three different colors (linearly independent power distributions) in my display, I can reproduce any color.

(s) (1 point) T The biological cause for normal human color vision being three-dimensional is the three types of cone cells in our retina.

(t) (1 point) F In the CIE 1931 xy chromaticity diagram, each point on the curved part of the "horseshoe" corresponds to monochromatic light of a different wavelength.

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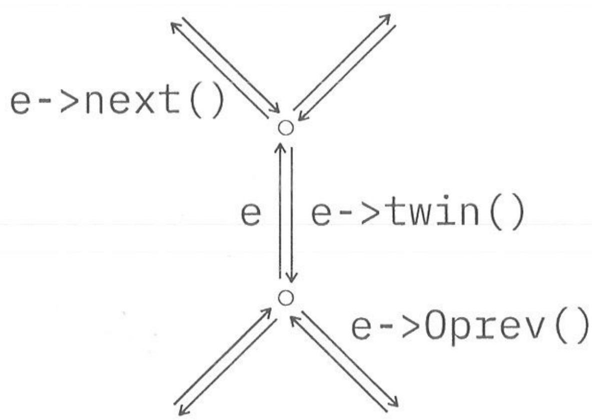
2. (Total : 22 points) Sampling, Geometry, Rendering

W

(a) (4 points) Consider a point light source that outputs 250 lumens of light uniformly in all directions. What is the irradiance (with correct units) at a point on the floor that is 4 meters below and 3 meters to the side of the light source?

$$\frac{250}{4\pi} \cdot \frac{1}{5^2} = \frac{5}{2\pi} \ \mathrm{W/m^2}$$

(b) (4 points) Some formulations of mesh data structures only provide three fields for each half-edge: `vertex()`, `Oprev()`, and `twin()`. The field e $\rightarrow$ `Oprev()` is defined as the next halfedge in the clockwise direction with the same `vertex()` as e, as shown in the diagram below.



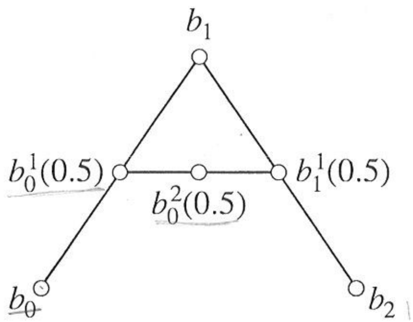
Give pseudocode to find e $\rightarrow$ `next()`, using only `vertex()`, `twin()`, and `Oprev()`.

e->next = e->twin()->Oprev() ;

(c) (6 points) Vector drawing programs often include the ability to subdivide Bezier curves by adding new control points. Let $f(t)$ be the quadratic Bezier curve defined by points b_0, b_1 and b_2. If we ran de Casteljau's algorithm to evaluate this curve, the following intermediate points would be calculated:

$$b_0^1(t) = (1-t)b_0 + tb_1 \tag{1}$$
$$b_1^1(t) = (1-t)b_1 + tb_2 \tag{2}$$
$$f(t) = b_0^2(t) = (1-t)^2 b_0 + 2t(1-t)b_1 + t^2 b_2 \tag{3}$$

Imagine we want to subdivide this curve in half at its midpoint. Let $g(u)$ be the quadratic Bezier curve defined by b_0, $b_0^1(0.5)$, and $b_0^2(0.5)$. Prove that $g(u)$ defines the piece of $f(t)$ between $t = 0$ and $t = 0.5$. The diagram below may help you visualize what's going on.



Denote: $X_0 = b_0$

$X_1 = b_0^1(0.5) = \frac{1}{2} b_0 + \frac{1}{2} b_1$

$X_2 = b_0^2(0.5) = \frac{1}{4} b_0 + \frac{1}{2} b_1 + \frac{1}{4} b_2$

By defination

$X_0^1 = (1-u)X_0 + uX_1$

$X_1^1 = (1-u)X_1 + uX_2$

$X_0^2 = (1-u)^2 X_0 + 2u(1-u)X_1 + u^2 X_2$

$= (1-u)^2 b_0 + 2u(1-u)(\frac{1}{2} b_0 + \frac{1}{2} b_1) + u^2(\frac{1}{4} b_0 + \frac{1}{2} b_1 + \frac{1}{4} b_2)$

$= \left[(1-u)^2 + u(1-u) + \frac{u^2}{4}\right] b_0 + \left[u(1-u) + \frac{1}{2} u^2\right] b_1 + \frac{u^2}{4} b_2$

$= \left[\frac{u^2}{4} - u + 1\right] b_0 + \left[u - \frac{1}{2} u^2\right] b_1 + \frac{u^2}{4} b_2$

let $u = 2t$, $t \in [0, 0.5]$: $\left[\frac{4t^2}{4} - 2t + 1\right] b_0 + \left[2t - \frac{1}{2} 4t^2\right] b_1 + \frac{4t^2}{4} b_2$

$= (1-t)^2 b_0 + 2t(1-t) b_1 + t^2 b_2$

$= f(t)$

(d) (2 points) Viewed from the center of the earth, the solid angle subtended by North America is closest (in steradians) to:

(iv)

~~(i) 4π.~~
~~(ii) 2π~~
(iii) π
(iv) $\frac{\pi}{2}$

$$\frac{c}{3}x^3 \Big|_0^1$$

$$\frac{c}{3} = 1 \quad c = 3$$

(e) (6 points) You are writing a program to do Monte Carlo integration of a function $f(x)$ for x between 0 and 1, and you want to use importance sampling. You have a hunch that $f(x)$ increases approximately quadratically, so you decide to do importance sampling with a probability density function proportional to the function $g(x) = x^2$.

Given v, a uniform random value between 0 and 1, derive an expression for a random value x (as a function of v) drawn from the desired probability density function.

① Scale $g(x)$, let $h(x) = c\,g(x) = cx^2$

$$\int_0^1 cx^2\,dx = 1 \Rightarrow c = 3$$

② Compute cdf.

$$H(x) = \int_0^x h(t)\,dt = \int_0^x 3t^2\,dt$$

$$= t^3 \Big|_0^x = x^3$$

③ let $H(x) = v$

$$x = \sqrt[3]{v}$$

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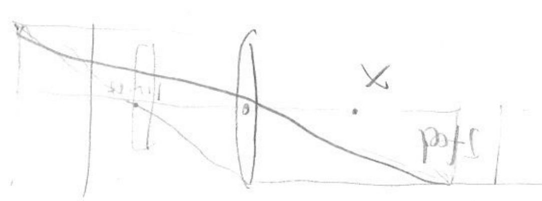
$$\frac{1}{f} = \frac{1}{z_i} + \frac{1}{z_o}$$

40mm 12.9mm ?

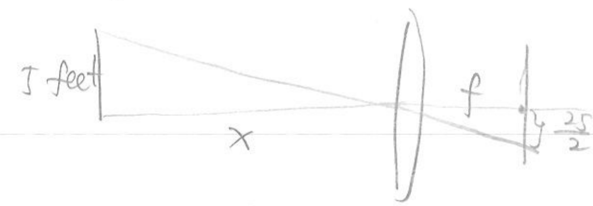


$\frac{1}{f} = \frac{1}{z_i} + \frac{1}{z_o}$

3. (Total : 24 points) Cameras and Imaging

(a) Focus and Field of View.

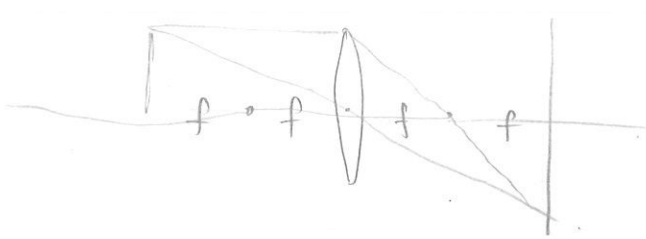
i. (2 points) You want to take a photo of your friend, who is 5 feet tall, and you want your friend to appear half the height of the full photo. You take the picture with a 50mm lens (recall this means that the focal length of the lens is 50mm) on a camera with a 25x25mm image sensor. Rounded to the nearest foot, how far away from your friend should you stand to take the photo? Show your work. Hint: for this part of the question you can safely assume that the camera is focused at infinity.



$$\frac{x}{5 \text{ feet}} = \frac{50 \text{ mm}}{\frac{25}{2} \text{ mm}}$$

$$x = 20 \text{ feet}$$

ii. (4 points) Unfortunately your friend is hit by a shrinking ray gun, and ends up exactly 12.5mm tall! You resolve to take a photo of your friend to document this tragedy, and again want your friend to appear half the height of the full photo. How far away from the lens should you place your friend to take the photo? Show your work. Hint: for this part of the question you cannot assume the camera will be focused at infinity.



$$x = 2f = 100 \text{ mm}$$

(b) (4 points) Exposure and Noise

In the following, assume that all the image sensors considered have 100% quantum efficiency and zero read noise (i.e. the only noise present in the signal is due to Poisson shot noise). Each of the following cameras is used to take a picture of a patch of the same, uniformly illuminated white wall, with the same shutter duration across all cameras. The pixels do not saturate in any camera. If the individual pixels in the first camera have an SNR as shown below, calculate the SNR for the individual pixels in the other two cameras. Show your work.

$2. = f = \frac{\text{focal length}}{\text{aperture}} = \frac{50}{x}$

$x = 25$

Camera 1: 10x10mm sensor with 50mm f/2.0 lens and 4 million pixels.

SNR = 100 $\frac{\mu}{\sigma} = \sqrt{X}$ $4.0 = \frac{50}{x'}$ $\boxed{x' = 12.5}$

(i) Camera 2: 20x20mm sensor with 50mm f/4.0 lens and 16 million pixels. $\frac{1}{4}$ Light

SNR = 50

(ii) Camera 3: 5x5mm sensor with 25mm f/1.0 lens and 2 million pixels.

SNR = 100

$1.0 = \frac{25}{x'}$

$\boxed{x' = 25}$

$(y_{in} f(y_{in}, y_{out}))_{out} = - f\, y_{out}, y_{in}$

$y_{in} y_{out} = f\, y_{in}, y_{out} - f\, y_{out}, y_{in}$

$f = \frac{y_{in}'}{y_{in}} \oplus \frac{y_{out}'}{y_{out}}$

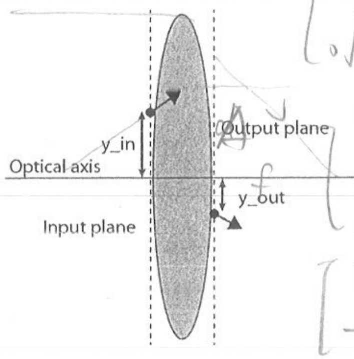
(c) Matrix Optics.

i. (6 points) Suppose we are raytracing a ray through a thin lens with focal length f, as in project 3-2. We can represent the incoming ray as a 2D column vector $r_{\text{in}} = [y_{\text{in}}, y'_{\text{in}}]^\top$, where y_{in} is the distance of the ray from the optical axis when it hits the lens, and y'_{in} is the slope of the ray (see figure below). Similarly, we represent the outgoing ray as $r_{\text{out}} = [y_{\text{out}}, y'_{\text{out}}]^\top$.

$\frac{1}{f} = \frac{1}{z_i} + \frac{1}{z_o}$

$z_o = \frac{y_{in}}{y_{in}'}$

$z_i = -\frac{y_{out}}{y_{out}'}$



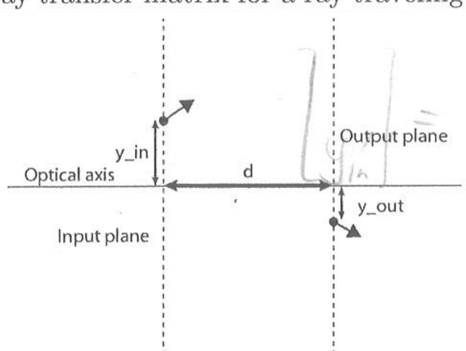
$\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

For a thin lens with focal length f, write down the matrix $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ such that $r_{\text{out}} = Mr_{\text{in}}$. The matrix M is called the *ray transfer matrix*, or ABCD matrix.

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

$\cancel{\begin{bmatrix} A \\ -\frac{A}{f} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} A \\ 0 \end{bmatrix}}$

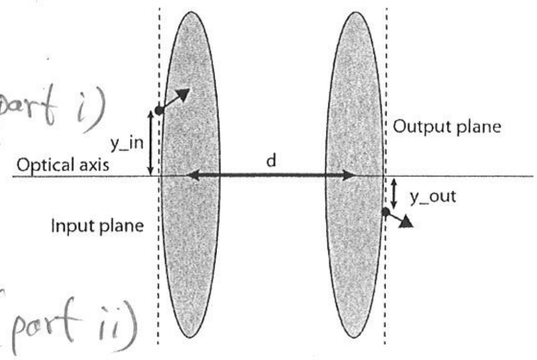
ii. (4 points) What is the ray transfer matrix for a ray traveling distance d in free space?



$= \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{in} \\ y_{in}' \end{bmatrix}$

$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

iii. (4 points) Ray transfer matrices are useful to analyze more complicated optical systems. Consider the figure below, with two thin lenses separated by distance d, having focal lengths f_1 and f_2, respectively. Write down the ray transfer matrix for this system.



$$M_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix} \quad \text{(part i)}$$

$$M_2 = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \quad \text{(part ii)}$$

$$M_3 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \quad \text{(part i)}$$

$$M = M_3 \cdot M_2 \cdot M_1$$

$$= \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d \\ -\frac{1}{f_2} & 1-\frac{d}{f_2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{d}{f_1} & d \\ -\frac{1}{f_2}-\frac{1}{f_1}+\frac{d}{f_1 f_2} & 1-\frac{d}{f_2} \end{bmatrix}$$

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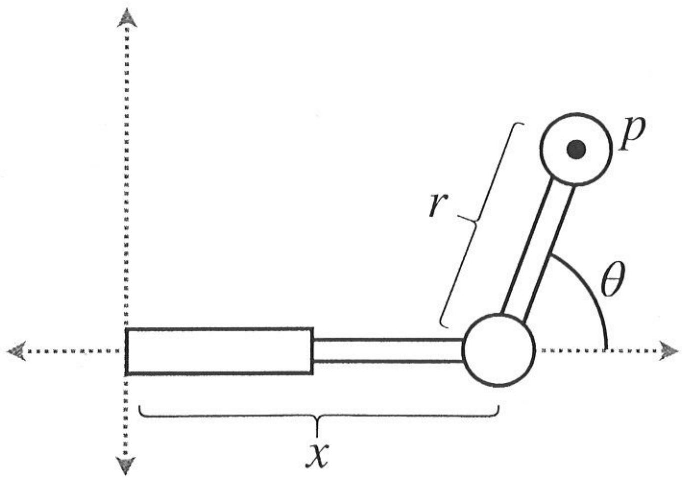
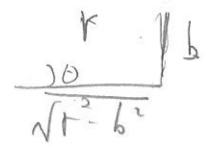
4. (Total : 22 points) Animation and Simulation

(a) (6 points) Consider the one segment arm of length r attached to a piston as shown in the diagram below. For each x (piston displacement) and θ (arm rotation angle), the end of the arm is at some point p in the plane.





For a point $p = (a, b)$ with $a \geq 0$, give an explicit formula for the x and θ in terms of a and b so that the end of the arm is at p. If there are multiple such pairs, find the one where x is minimal. You may assume that the point p can be reached using the arm, i.e. that at least one pair (x, θ) exists.

$$\theta = \arcsin\left(\frac{b}{r}\right)$$

$$x = a - \sqrt{r^2 - b^2}.$$

(b) (4 points) When interpolating between keyframes we often prefer not to use linear interpolation, because it may cause the animation to look clunky. Instead, it common to use an ease function: a function that smoothly interpolates between 0 and 1 while having the motion gradually start and stop.

Find the equation for the cubic ease function: the unique cubic polynomial $f(x)$ where $f(0) = 0, f(1) = 1$, and $f'(0) = f'(1) = 0$.

Let $f(x) = ax^3 + bx^2 + cx + d$

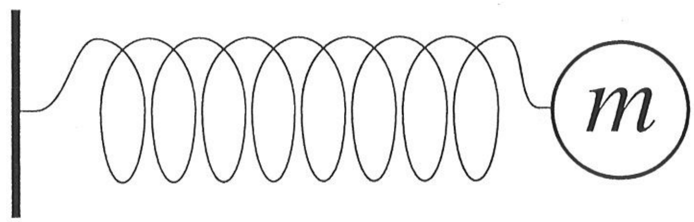
$f'(x) = 3ax^2 + 2bx + c$

$$\begin{cases} f(0) = d = 0 \\ f(1) = a + b + c + d = 1 \\ f'(0) = c = 0 \\ f'(1) = 3a + 2b + c = 0 \end{cases}$$

$$\Rightarrow \begin{cases} a = -2 \\ b = 3 \\ c = 0 \\ d = 0 \end{cases}$$

Therefore $f(x) = -2x^3 + 3x^2$

(c) Consider a spring with stiffness k, resting length L, and with a weight of mass m attached to the end:



i. (2 points) If the mass is displaced a distance $x_0 > 0$ (away from the wall) from its resting position, what force would the spring exert on the mass?

$F = -x_0 \cdot k$

ii. (6 points) Real world simulators often deal with systems much more complex than a single mass and spring. Thus, as we did in project 4, they use numerical (rather than analytical) integration schemes. Let the function $\hat{x}(t)$ be the mass's displacement from the resting position as a function of time, computed numerically.

Calculate $\hat{x}(\pi)$ using forward Euler integration with a time-step of $\Delta t = \frac{\pi}{2}$. Assume that $k = m = x_0 = 1$.

	Displacement	V	a	F
$t=0$	$x_0 = 1$	$V_0 = 0$	$a_0 = -1$	$F_0 = -1$
$t = \frac{\pi}{2}$	$x_1 = 1$	$V_1 = -\frac{\pi}{2}$	$a_1 = -1$	$F_1 = -1$
$t = \pi$	$x_2 = 1 - \frac{\pi^2}{4}$			

iii. (4 points) Using Hooke's law, we can also analytically compute the mass's displacement, given by the periodic function $x(t) = x_0 \cos\left(\sqrt{\frac{k}{m}}t\right)$. Compare $\hat{x}(\pi)$ computed with Euler's method above to the analytically computed "ground truth" $x(\pi)$. Which has the larger magnitude? Will the error ($|\hat{x}(t) - x(t)|$) grow or shrink over time?

The ground truth will has the larger magnitude

The error will grow over time

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5. (Total : 22 points) Color

(a) (2 points) Say whether the following statement is true or false, and give a one sentence explanation. In the color matching experiment, the color appearance of any input light can be matched by adjusting the brightness of three primary lights.

False.
Some color is out of gamut. it needs "negative" light.

(b) (2 points) Is it easier to find the color you want with a color picker based on HSV color space, or RGB color space? Briefly, why?

It is easier with HSV space.
because it's perceptual organized color space.

(c) (4 points) Reflected Color
Background: a good model to help think about the color of light reflected from a diffuse surface is the reflectance $r(\lambda)$ as a function of wavelength λ. Here, $r(\lambda)$ is the fraction of incident irradiance at wavelength λ that is reflected from the surface. If the incident irradiance at a point had spectral power distribution $I(\lambda)$, then the reflected spectral power distribution would be $I(\lambda)r(\lambda)$.
You decide to test two different online photo printing services. You send them a digital photo and obtain two matte (diffuse) prints in the mail. At first you look at the two photos near your window, under sunlight. The colors look identical to you. However, later that night, you happen to look at them under the fluorescent lighting of your kitchen and notice that some of the colors in the two photos now look quite different. What is happening?

Under the sunlight. $I(\lambda) r_1(\lambda)$ and $I(\lambda) r_2(\lambda)$ are accidentally equal. they are metamers.
However, under fluorescent lighting. they are different, they are not metamers.

(d) Color Blindness, Color Reproduction and Gamut

A terrible virus spreads rapidly through the world. Fortunately, it only affects the M cone cells in our retinas, turning them into L cone cells. As the last remaining humans on earth become red-green color-blind, you realize that the one silver lining of this sad situation is that you might be able to use just the red and blue pixels on your phone to show "full" color for humans.

Throughout this problem, assume that the spectral response curves of the remaining human cone cells, as a function of wavelength, are given by $S(\lambda)$ and $L(\lambda)$.

i. (3 points) First, consider a target light with spectral power distribution (SPD) that we would like to reproduce, given by $I(\lambda)$. Write down expressions for the scalar response of each cone cell when exposed to $I(\lambda)$.

$s_{target} =$ $I(\lambda) \cdot S(\lambda)$

$l_{target} =$ $I(\lambda) \cdot L(\lambda)$

ii. (3 points) Now consider the red and blue pixels on your phone, with SPDs given by functions $R(\lambda)$ and $B(\lambda)$. If we set the brightness of these pixels by scalar values r and b, respectively, write down the scalar response from each cone cell type when exposed to the resulting light.

$s_{disp} =$ $[rR(\lambda) + bB(\lambda)]\,S(\lambda)$

$l_{disp} =$ $[rR(\lambda) + bB(\lambda)]\,L(\lambda)$

iii. (3 points) Note that we can re-write the result from part (ii) in matrix form:

$$\begin{bmatrix} s_{disp} \\ l_{disp} \end{bmatrix} = \begin{bmatrix} & M & \end{bmatrix} \begin{bmatrix} r \\ b \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} r \\ b \end{bmatrix} = \begin{bmatrix} I(\lambda) \cdot S(\lambda) \\ I(\lambda) \cdot L(\lambda) \end{bmatrix}$$

Write down expressions for the elements of this matrix:

$m_{11} =$ $R(\lambda) \cdot S(\lambda)$ $\qquad m_{12} =$ $B(\lambda) \cdot S(\lambda)$

$m_{21} =$ $R(\lambda)\, L(\lambda)$ $\qquad m_{22} =$ $B(\lambda)\, L(\lambda)$

iv. (3 points) Finally, to complete the color matching procedure for us color-blind humans with just red and blue pixels, let's determine how to choose values for r and b to match the perceived color of the input SPD $I(\lambda)$, assuming $I(\lambda)$ is in gamut of the red and blue pixels. Write down a one-line matrix expression for such r and b values. You can use any variables defined in previous parts of this question, and you may also use matrix operations such as transpose and inverse in your solution if needed.

$$\begin{bmatrix} r \\ b \end{bmatrix} = M^{-1} \begin{bmatrix} S_{target} \\ L_{target} \end{bmatrix}$$

v. (2 points) Your friend says, "Hey! Why don't we make a display with just red and blue pixels!" You think this would actually work now. However, you realize there might still be a benefit to keeping the green pixels, even with everyone having only S and L cone cells. What is your reasoning?

It can enlarge the gamut.