CS 184 Discussion 5

Radiometry & Photometry

A Bag of Words

- Radiant flux/power
- Radiant intensity
- Irradiance
- Radiance
- Solid angles
- Steradians
- Elevation angle
- Azimuth angle

Q3.1 Walkthrough

Lecture Definitions

Which of these change as we get further from the light source?

Hint: We know power is constant. For the others, look at the units. Which change based on distance?

Radiant Energy and Flux (Power)

Definition: Radiant (luminous*) energy is the energy of electromagnetic radiation. It is measured in units of joules, and denoted by the symbol:

$$Q \ [\mathrm{J} = \mathrm{Joule}]$$

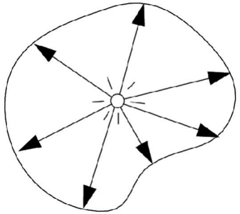
Definition: Radiant (luminous*) flux is the energy emitted, reflected, transmitted or received, per unit time.

$$\Phi \equiv \frac{\mathrm{d}Q}{\mathrm{d}t} \ [\mathrm{W} = \mathrm{Watt}] \ [\mathrm{lm} = \mathrm{lumen}]^{*}$$

*** Definition slides will provide photometric terms in parentheses and give photometric units**

Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle emitted by a point light source.



$$I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

$$\left[\frac{\mathrm{W}}{\mathrm{sr}}\right] \left[\frac{\mathrm{lm}}{\mathrm{sr}} = \mathrm{cd} = \mathrm{candela}\right]$$

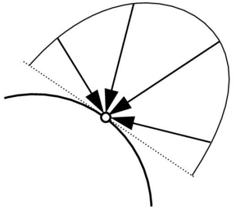
The candela is one of the seven SI base units.

Irradiance

Definition: The irradiance (illuminance) is the power per unit area incident on a surface point.

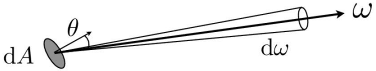
$$E(\mathbf{x}) \equiv \frac{\mathrm{d}\Phi(\mathbf{x})}{\mathrm{d}A}$$

$$\left[\frac{\mathrm{W}}{\mathrm{m}^2}\right] \left[\frac{\mathrm{lm}}{\mathrm{m}^2} = \mathrm{lux}\right]$$



Surface Radiance

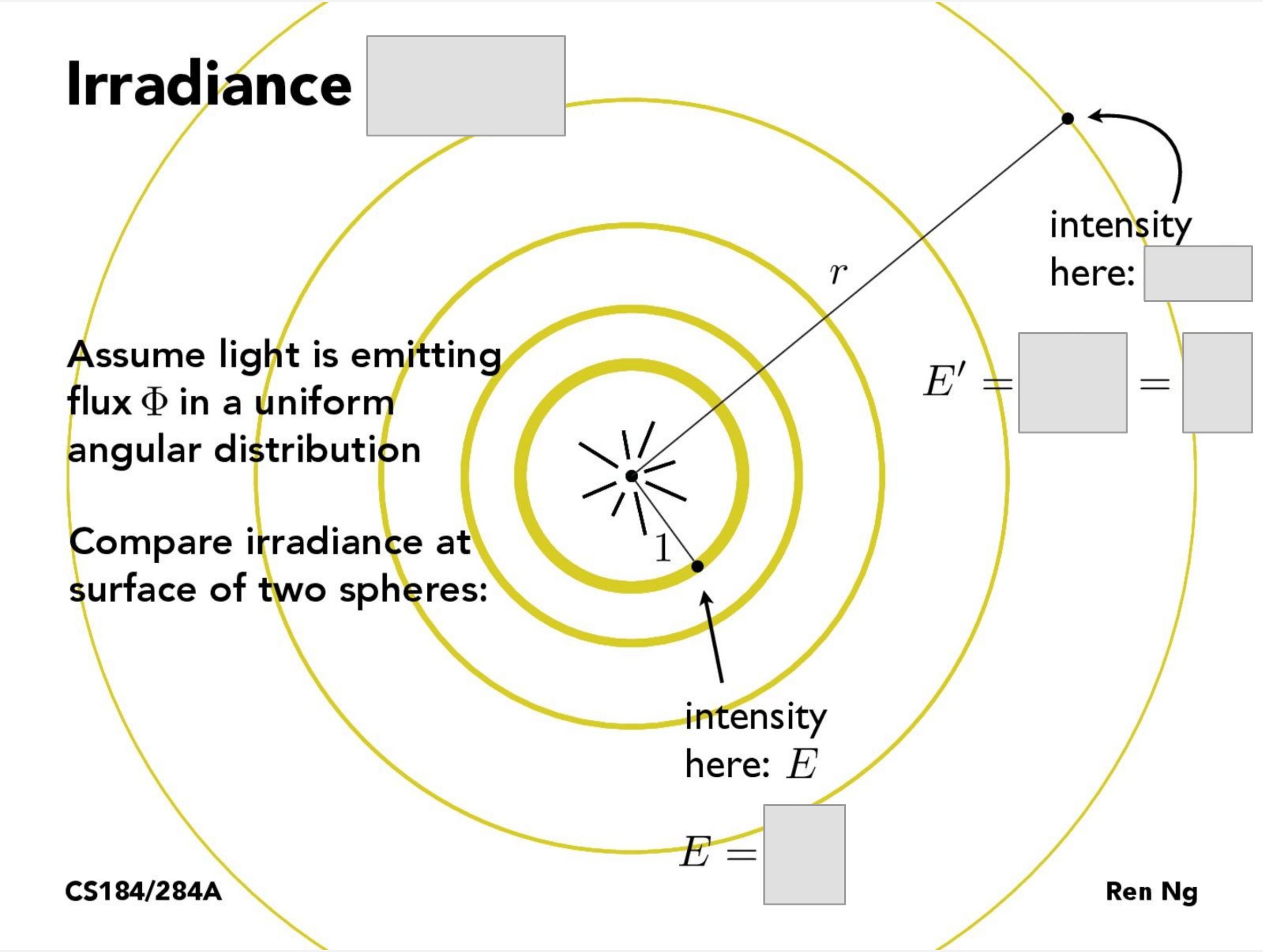
Definition: The radiance (luminance) is the power emitted, reflected, transmitted or received by a surface, per unit solid angle, per unit projected area.



$$L(\mathrm{p}, \omega) \equiv \frac{\mathrm{d}^2\Phi(\mathrm{p}, \omega)}{\mathrm{d}\omega \, \mathrm{d}A \cos\theta}$$

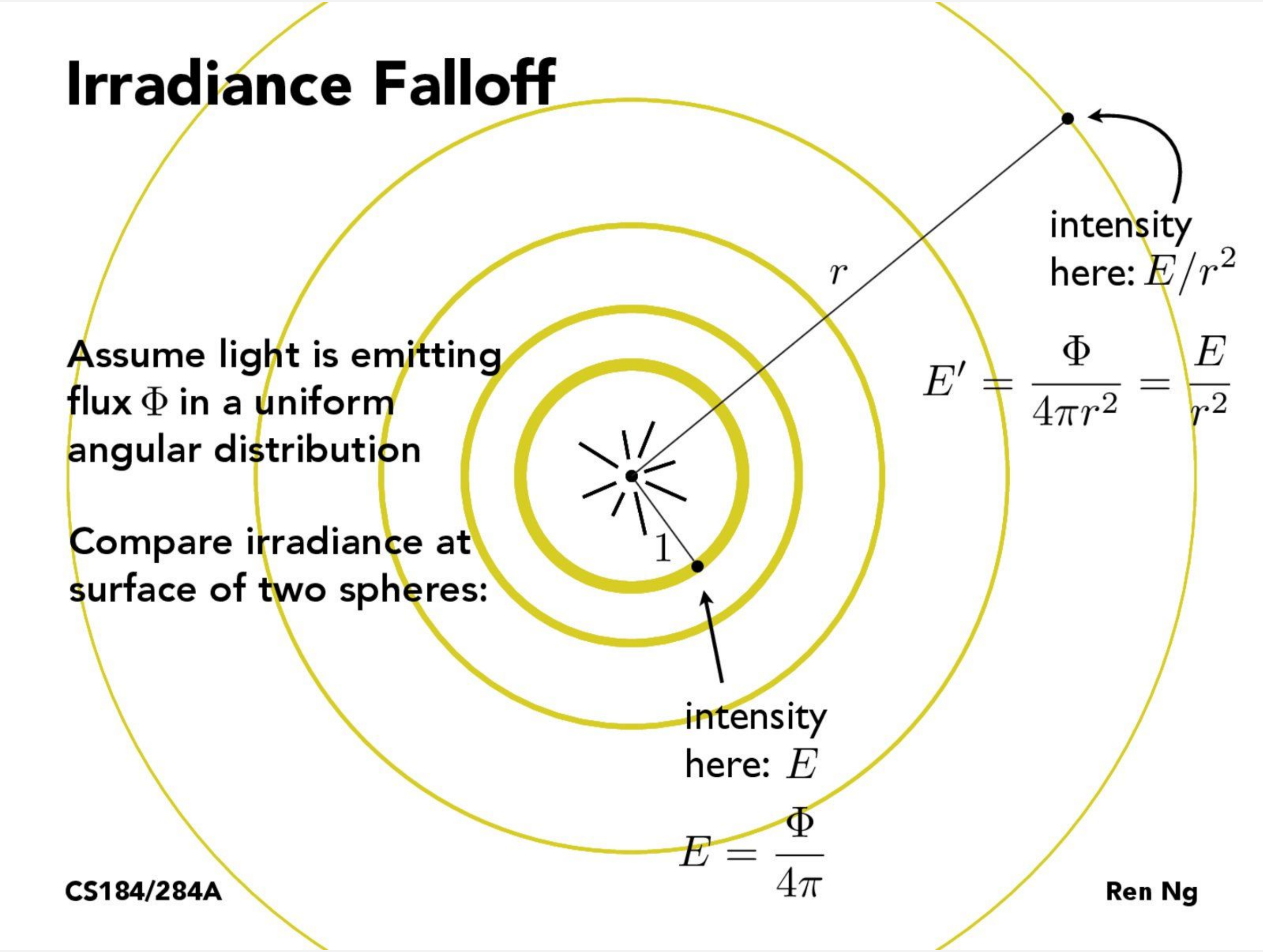
$\cos\theta$ accounts for projected surface area

$$\left[\frac{\mathrm{W}}{\mathrm{sr}\,\mathrm{m}^2}\right] \left[\frac{\mathrm{cd}}{\mathrm{m}^2} = \frac{\mathrm{lm}}{\mathrm{sr}\,\mathrm{m}^2} = \mathrm{nit}\right]$$



Let's start with Irradiance: How does Irradiance change with distance?

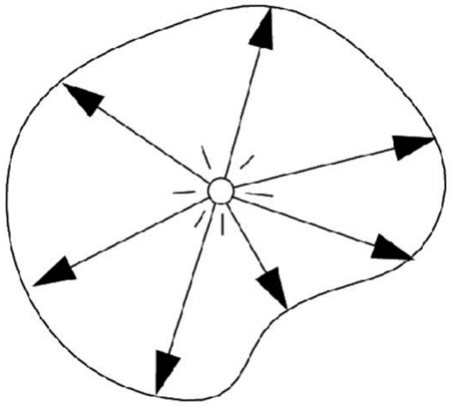
Hint: Radiant flux is fixed, and equally spread across the area of the sphere



Since we spread the same flux across a larger area, irradiance at any single point decreases as we get further away.

Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle emitted by a point light source.



$$I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

$$\left[\frac{\mathrm{W}}{\mathrm{sr}}\right] \left[\frac{\mathrm{lm}}{\mathrm{sr}} = \mathrm{cd} = \mathrm{candela}\right]$$

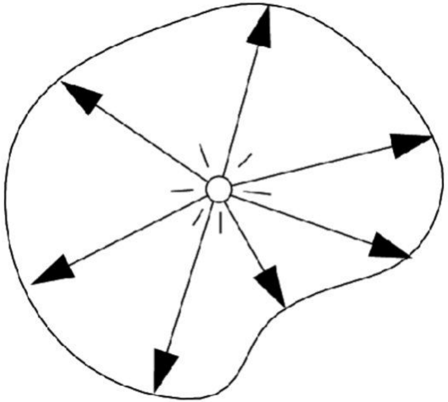
The candela is one of the seven SI base units.

What about the radiant intensity at the surface of the two spheres?

Hint: We're now spreading the flux across steradians rather than area. Does the total number of steradians in a sphere change with radius?

Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle emitted by a point light source.



$$I(\omega) \equiv \frac{\mathrm{d}\Phi}{\mathrm{d}\omega}$$

$$\left[\frac{\mathrm{W}}{\mathrm{sr}}\right] \left[\frac{\mathrm{lm}}{\mathrm{sr}} = \mathrm{cd} = \mathrm{candela}\right]$$

The candela is one of the seven SI base units.

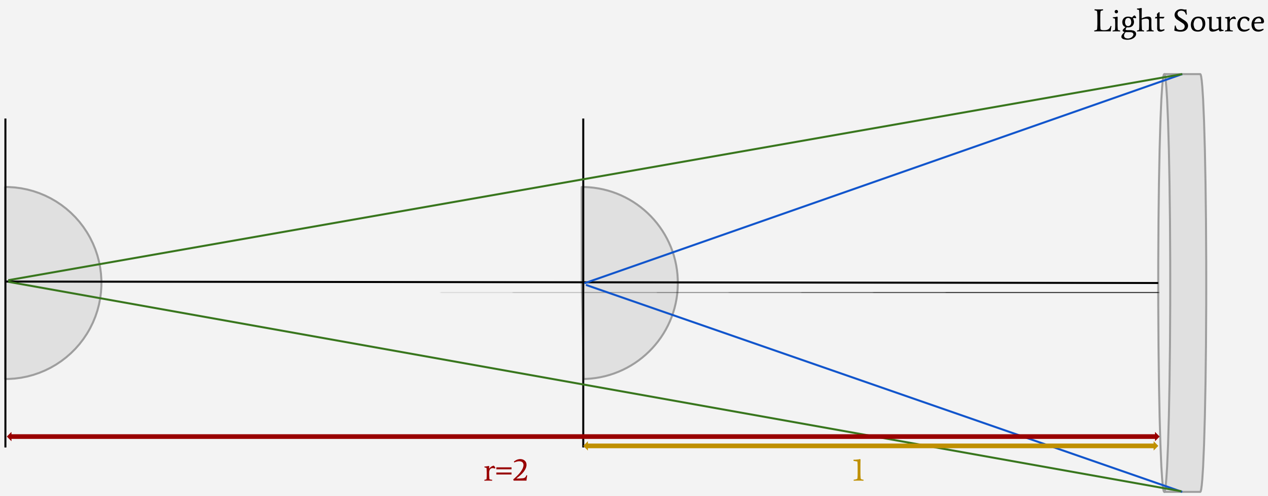
The number of steradians doesn't change, so the radiant intensity will not change.

What about Radiance?

We know from before that the irradiance decreases by a factor of r^2 when we increase the distance from 1 to r.

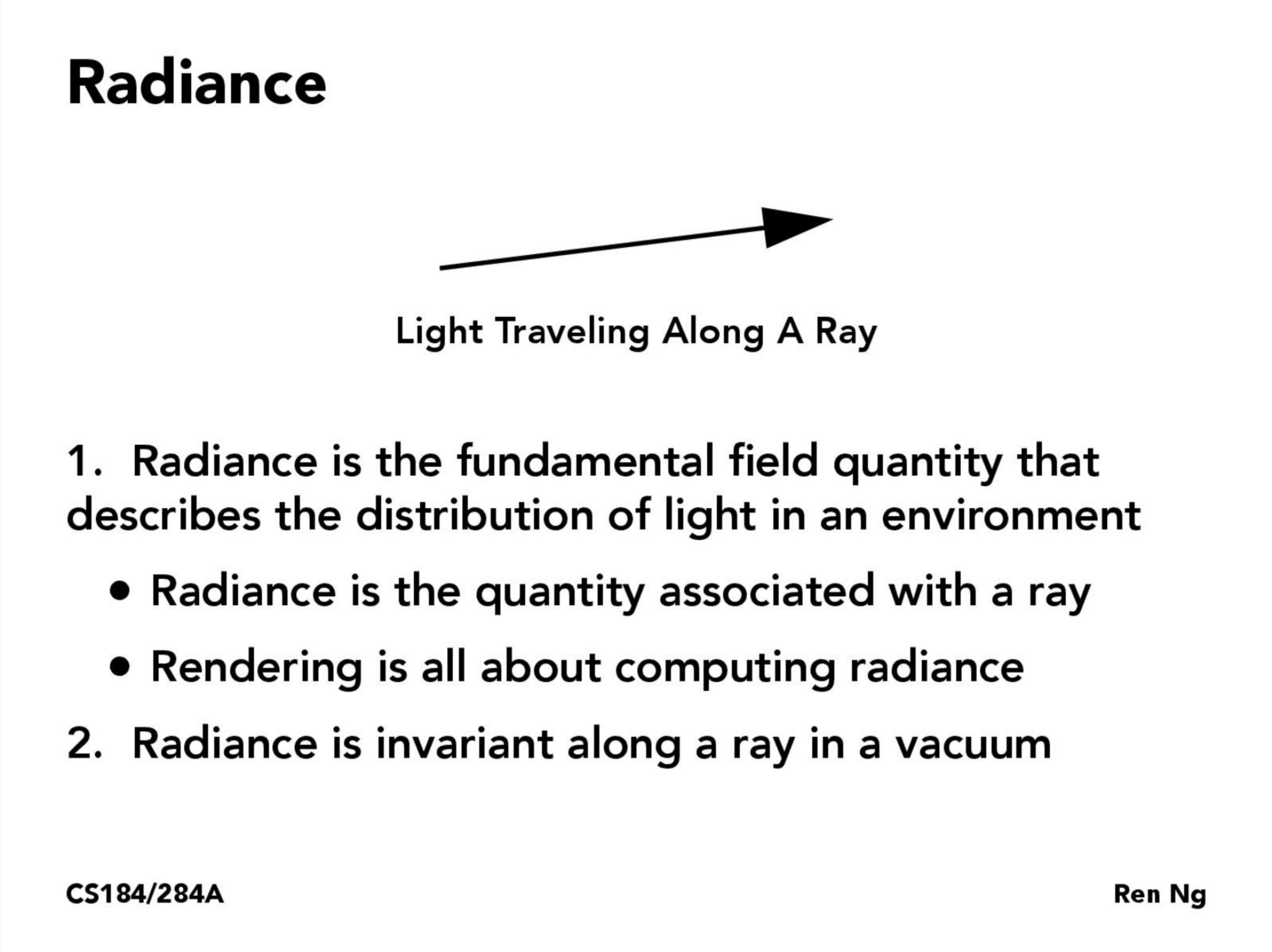
From this diagram, note that the solid angle that receives radiance from the light source also decreases by a factor of r^2.

Assuming a the light source is a disk that emits uniform radiance, what does this imply about the radiance received at the two different distances?



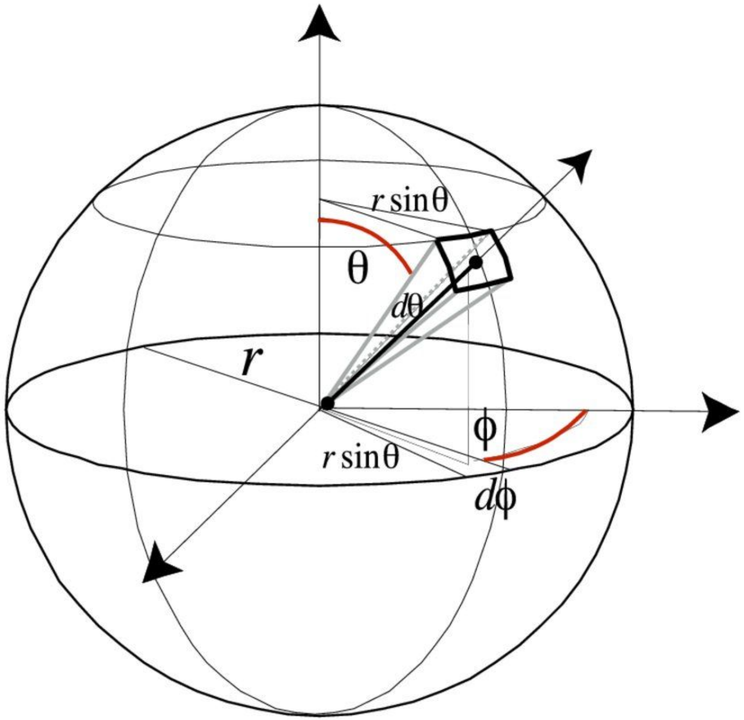
What about Radiance?

The decrease in irradiance is 100% the result of receiving less rays, as we integrate over fewer steradians. The actual radiance along each ray is constant, regardless of distance traveled.



Q3.2 Walkthrough

Differential Solid Angles



$$\mathrm{d}A =$$

$$=$$

$$\mathrm{d}\omega =$$

Differential Solid Angles

What's the radius of this circle?

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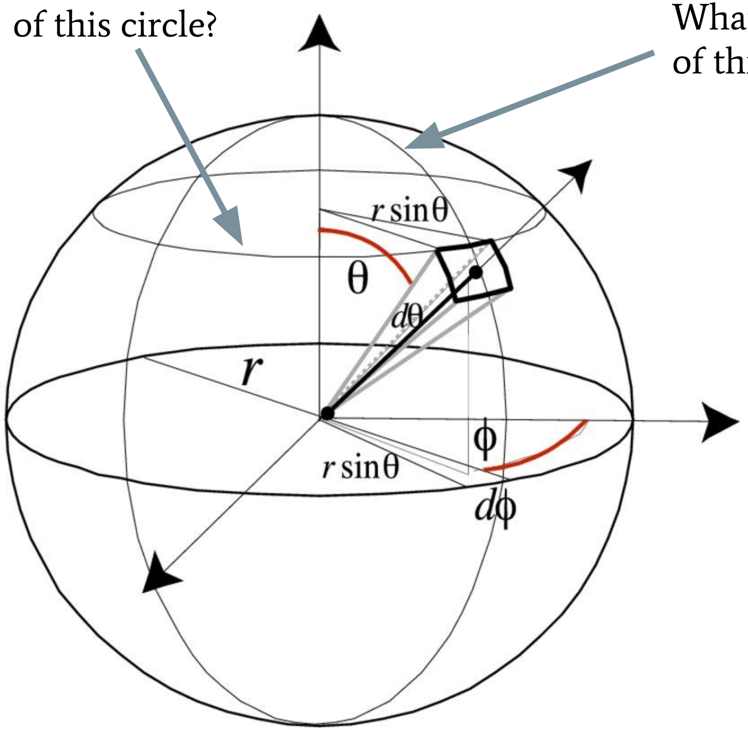


$\mathrm{d}A =$

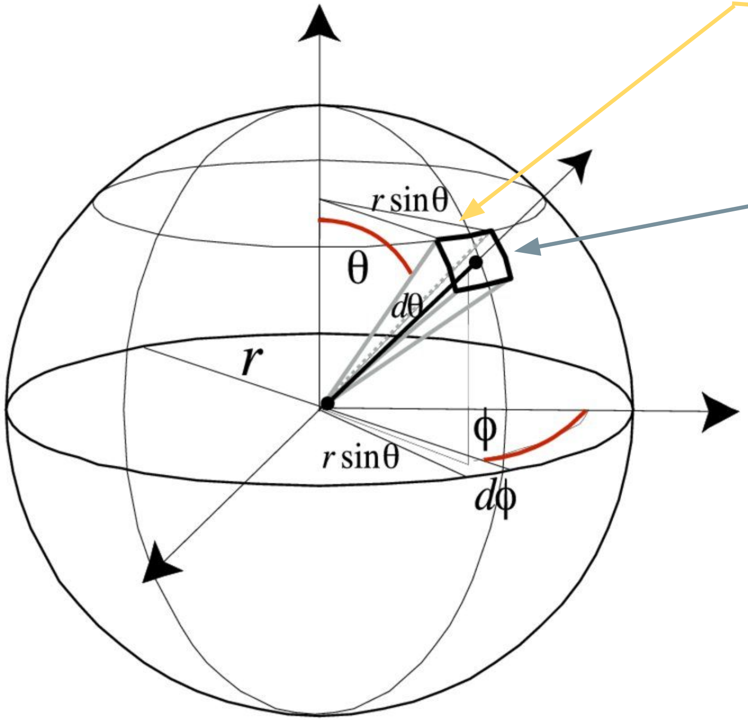
$=$

$\mathrm{d}\omega =$

Approximate this value as a ...?

Recall the arclength of a circle:
$L = r \cdot \theta$

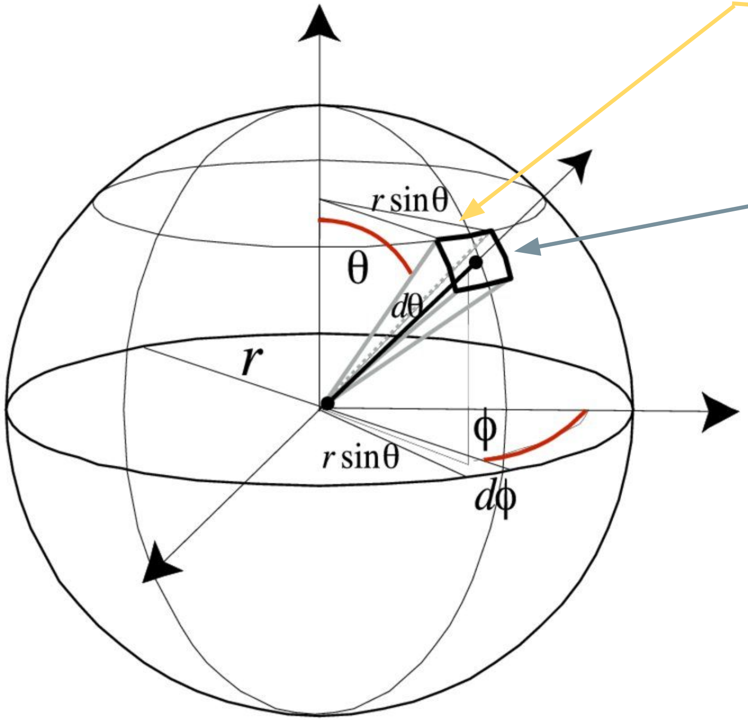
Differential Solid Angles



$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$\mathrm{d}\omega = \boxed{}$$

Differential Solid Angles

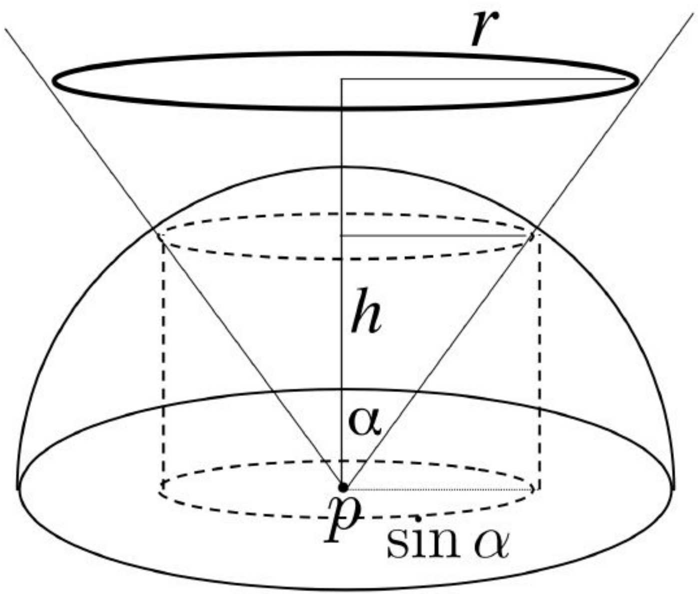


$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

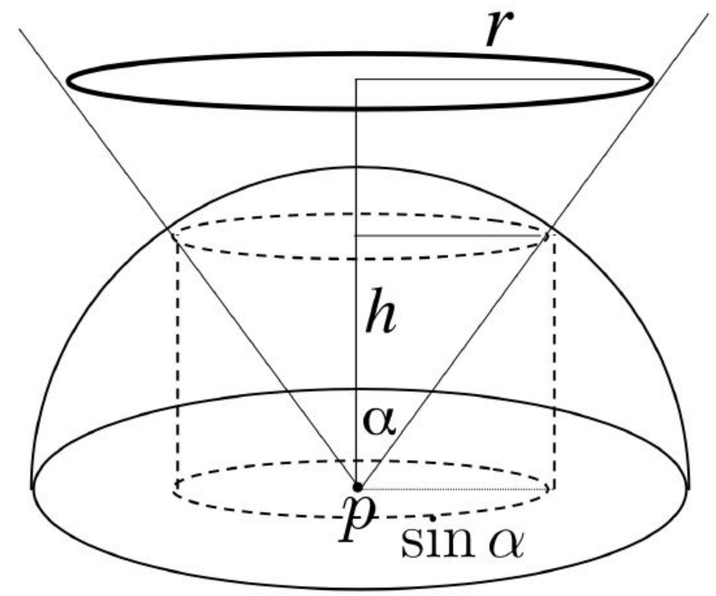
Q3.3 Walkthrough

Calculate the irradiance at point p from a disk area light overhead with uniform radiance L. (Hint: irradiance is an integral of incoming radiance over the hemisphere: $E(p) = \int_{H^2} L_i(p, \omega) \cos\theta \, \mathrm{d}\omega$.)



Note: We integrate over a hemisphere because we assume our point is on a surface, and can only receive light from one side of that surface.

Calculate the irradiance at point p from a disk area light overhead with uniform radiance L. (Hint: irradiance is an integral of incoming radiance over the hemisphere: $E(p) = \int_{H^2} L_i(p, \omega) \cos\theta \, d\omega$.)



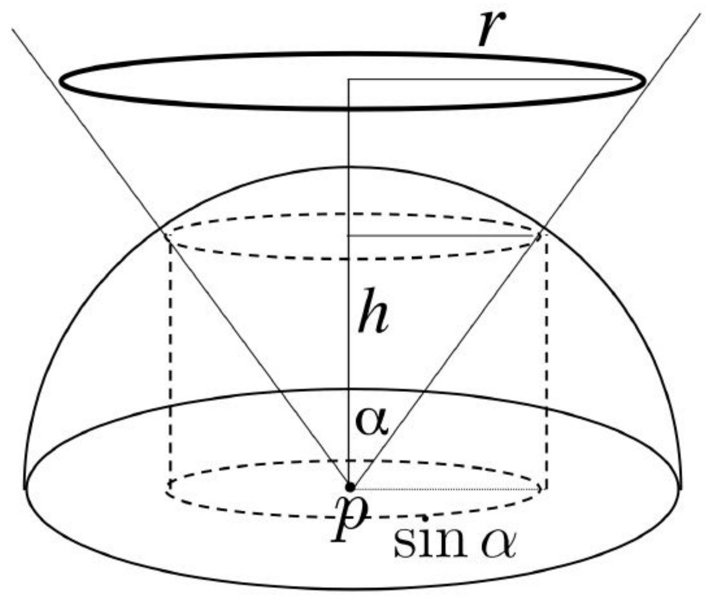
$$E(p) = \int_{H^2} L_i(p, \omega) \cos\theta \, \underbrace{d\omega}$$

$$= \boxed{}$$

1. What can we break $d\omega$ into? How does this change our integration "conditions"?

2. Looking at the diagram, which parts of the hemisphere actually receive any radiance at all from the disk light? Do we really need to integrate over the whole hemisphere?

Calculate the irradiance at point p from a disk area light overhead with uniform radiance L. (Hint: irradiance is an integral of incoming radiance over the hemisphere: $E(p) = \int_{H^2} L_i(p, \omega) \cos\theta \, d\omega$.)

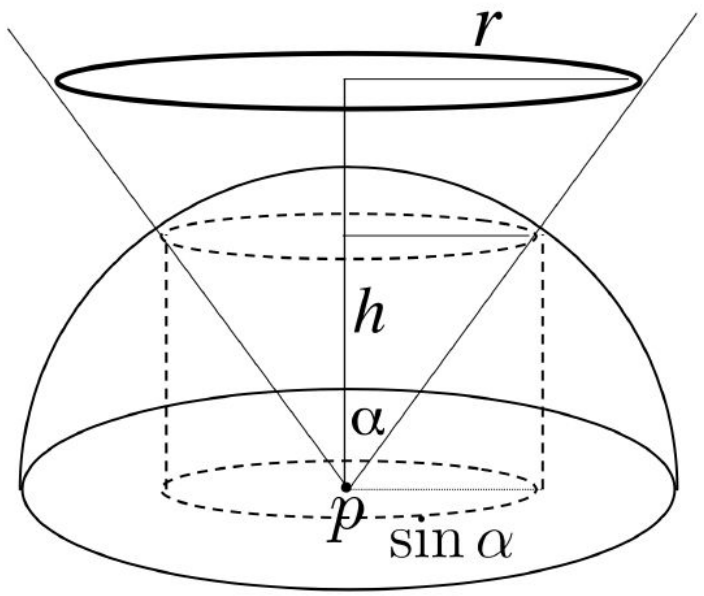


$$E(p) = \int_{H^2} L_i(p, \omega) \cos\theta \, \mathrm{d}\omega$$
$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\alpha} L \cos\theta \sin\theta \, \mathrm{d}\theta \mathrm{d}\phi$$

We make a transform to a double integral, and only integrate over the portion of the hemisphere with non-zero incoming radiance. Now all that's left is to evaluate.

Note: The additional *sinθ* term results from our change of variables. The specifics aren't too important here, but in summary we multiply by the determinant of the Jacobian matrix.

Calculate the irradiance at point p from a disk area light overhead with uniform radiance L. (Hint: irradiance is an integral of incoming radiance over the hemisphere: $E(p) = \int_{H^2} L_i(p,\omega)\cos\theta\,\mathrm{d}\omega$.)



$$
\begin{aligned}
E(p) &= \int_{H^2} L_i(p,\omega)\cos\theta\,\mathrm{d}\omega \\
&= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\alpha} L\cos\theta\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi \\
&= 2\pi L\left.\frac{\sin^2\theta}{2}\right|_0^{\alpha} \\
&= \pi L\sin^2\alpha \\
&= \frac{\pi L r^2}{r^2+h^2}
\end{aligned}
$$