CS 184/284A **Name:** ____________________

Spring 2017: Midterm 1 **SID number:** ____________________

March 16th, 2017 **cs184-??? login:** ____________________

Time Limit: 110 Minutes

- This exam contains 13 pages (including this cover page) and 6 problems. Check for missing pages.
- Put your initials on the top of every page, in case the pages become separated.
- This exam is closed book, except for one 8.5×11 page of notes (double sided), printed or handwritten.
- This exam is 110 minutes long, and has a total of 110 points.
- Problem difficulty varies throughout the exam, so don't get stuck on a time-consuming problem until you have read through the entire exam. Each problem's point value is roughly correlated with its expected difficulty.
- Answer each question in the space provided. Partial credit may be given on certain problems.
- To minimize distractions, do your best to avoid questions to staff. If you need to make assumptions to answer a question, write these assumptions into your answer.

Problem	Points	Score
1	20	
2	18	
3	18	
4	18	
5	18	
6	18	
Total:	110	

1. (Total : 20points) True / False

   Mark each statement true or false. (1 point each)

   (a) (1 point) **T** Using a z-buffer allows us to correctly identify the triangle nearest to the screen for each pixel in linear time (in the number of triangles).

   (b) (1 point) **T** If you have not implemented mipmaps, an alternative way to antialias your textures is to increase the supersampling rate.

   (c) (1 point) **T** Texture magnification is when there are multiple pixel samples per texel sample.

   (d) (1 point) **F** For a two dimensional triangle: if the first two barycentric coordinates α and β are between 0 and 1, then the third coordinate γ will also be between 0 and 1.

   (e) (1 point) **T** An orthographic projection simulates a camera at an infinite distance away from the scene.

   (f) (1 point) **F** Rotation, shearing, and translation are all examples of linear transforms.

   (g) (1 point) **T** Implicit surface representations make it easy to perform inside/outside tests.

   (h) (1 point) **F** Piecing together many low-order Bézier curves results in a Bézier spline that is difficult to control.

   (i) (1 point) **F** A Catmull-Rom spline interpolates all of its control points with C2 continuity.

   (j) (1 point) **T** Two objects with the same geometry, but different topology can still have the same shape.

   (k) (1 point) **T** The units of irradiance are watts per meter squared.

   (l) (1 point) **F** Increasing the size of an area light while keeping its outgoing radiance constant can decrease the amount of noise in your rendered image.

   (m) (1 point) **F** Using rejection sampling to generate random samples in the unit circle is guaranteed to require no more than 4 calls to your random number generator.

   (n) (1 point) **T** The asymptotic complexity of tracing a ray is $O(n)$ with no acceleration structure and $O(\log n)$ when using a properly constructed bounding volume hierarchy (BVH).

   (o) (1 point) **F** You create a Cornell box scene containing two spheres. The first sphere has a glossy BRDF with a small and bright highlight. The second has a glossy BRDF with a larger and dimmer highlight. Claim: the first sphere will benefit more than the second (in terms of decreased noise) if the samples per light are increased.

   (p) (1 point) **T** A sensor is placed at the focal point to focus a camera at infinity.

   (q) (1 point) **F** In a camera, magnification increases as the distance between the sensor and lens decreases.

   (r) (1 point) **F** All else being equal, field of view increases as the focal length of the lens increases.

   (s) (1 point) **T** The diameter of the circle of confusion is directly proportional to the diameter of the lens aperture.

   (t) (1 point) **T** If I change the f-stop from F/2.8 to F/1.4, to keep the exposure constant I must change the shutter speed from 1/100 second to 1/400 second.

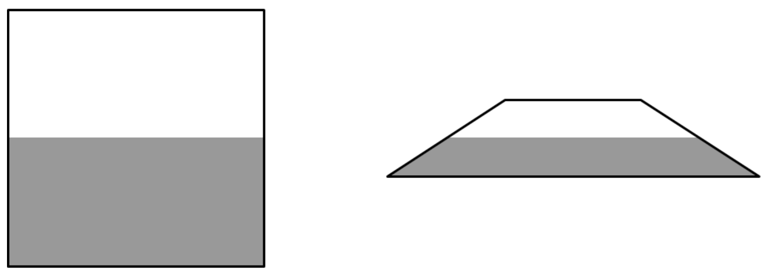
2. (Total : 18points) Rasterization

(a) (2 points) If I record a spinning wagon wheel with a video camera at 60 frames per second (assume no motion blur), up to what rate can the wheel spin before I will begin to see aliasing? You can assume the wheel has 6 spokes.

(i) 1 rotations per second.
(ii) 5 rotations per second.
(iii) 10 rotations per second.
(iv) 60 rotations per second.

Solution: The answer is (ii).

(b) (2 points) Suppose I have a textured rectangle that is tilted at a steep angle downwards in 3D space as shown below. If I use a mipmapped texture, what would happen to the rasterized pixels along the top edge of the tilted rectangle? (1-2 words)

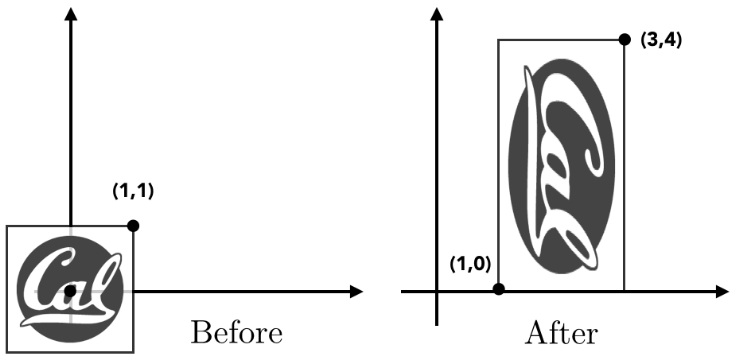


Solution: Overblurring (anisotropic mipmaps would be required for the texture to be rendered properly).

(c) (2 points) The centroid of a triangle is defined as a point that represents its center of mass. That is, if lines were drawn from each vertex through the centroid to the opposing face, the resulting six triangles would all have equal areas. What are its barycentric coordinates?

Solution: $(\alpha, \beta, \gamma) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

(d) (4 points) Which of the following sequences of basic transforms will produce the overall transform shown in this figure? Circle all that apply. (Rotations are counterclockwise.)



(i) Scale horizontally 2x; rotate 90 degrees; reflect across y-axis; translate by (2,2).

(ii) Translate (-1,2); rotate -90 degrees; scale 2x vertically.

(iii) Rotate -90 degrees; scale 2x vertically; translate by (2, 4).

(iv) Scale horizontally 2x; translate (-2, 2); rotate 270 degrees.

Solution: The answer is (ii) and (iv).

(e) (8 points) Recall from class that 2D rotations can be performed using a single transformation matrix. However, it is also possible to rotate a 2D object using only shears: an X shear, then a Y shear, and finally another X shear. This is fast and convenient when image data is represented using rows and columns.

The relevant transformation matrices are given below for your convenience:

$$Shear(X) = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \qquad Shear(Y) = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \qquad Rot(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

To rotate an object by a counterclockwise angle θ, what are the shearing factors α, β, and γ for the first, second, and third shears, respectively?

Solution:

$$\alpha = \frac{\cos\theta - 1}{\sin\theta} = \frac{-\tan\theta}{2}, \beta = \sin\theta, \gamma = \frac{\cos\theta - 1}{\sin\theta} = \frac{-\tan\theta}{2}$$

3. (Total : 18points) Geometry

(a) (2 points) A triangle mesh with N triangles will contain ________ triangles after $K > 0$ applications of Loop subdivision.

Solution: $N * 4^K$

(b) (2 points) A general mesh starts with V extraordinary vertices and P non-quad polygons. After K applications of Catmull-Clark subdivision, it will have ________ extraordinary vertices.

Solution: $V + P$

(c) (6 points) Use de Casteljau's algorithm to derive the following matrix formulation of a quadratic Bezier curve with control points P_0, P_1, P_2:

$$\begin{pmatrix} 1 & t & t^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ P_2 \end{pmatrix}$$

Solution: Repeatedly apply de Casteljau algorithm to the control points to obtain an explicit expression for $B(t)$ through a series of nested linear interpolations.

$$\begin{aligned}
B(t) &= (1-t)\{(1-t)P_0 + tP_1\} + t\{(1-t)P_1 + tP_2\} \\
&= (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2 \\
&= \begin{pmatrix} (1-t)^2 & 2t(1-t) & t^2 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ P_2 \end{pmatrix} \\
&= \begin{pmatrix} (1-2t+t^2) & (2t-2t^2) & t^2 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ P_2 \end{pmatrix} \\
&= \begin{pmatrix} 1 & t & t^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ P_2 \end{pmatrix}.
\end{aligned}$$

(d) (8 points) Instead of using a halfedge data structure, I implement my triangle mesh with a simpler data structure in which I have an unordered triangle list `vector<vector<Vector3D>>`. Each triangle in the list is represented by a vector of length 3 containing the positions of the triangle's 3 vertices. Which of the following operations will be significantly more inefficient (i.e. at least $O(n)$ times worse) with this mesh representation? Circle all that apply.

(a) Collapsing an edge in mesh simplification

(b) Topology change in Loop subdivision (splitting each triangle into 4 smaller triangles without updating vertex positions)

(c) Computing new vertex positions in Loop subdivision

(d) Centering vertices to improve overall triangle shape during mesh regularization

Solution: (a) should be circled because collapsing an edge requires knowledge of the neighboring triangles that share the vertices on the two ends of the edge being collapsed. As such, we would need to loop through our unordered list of triangles to look for the required triangles.

(b) should **not** be circled since it does require any knowledge of neighboring triangles or vertices. Given the triangle we wish to split into 4 smaller triangles, we simply have to replace that triangle in our list with four new triangles with their own respective vertices by adding the new ones to the end of the triangle list.

(c) should be circled because computation of the new vertex positions requires knowledge of neighboring vertices and their positions, which is topology information that this mesh representation lacks. To compute these, we would need to loop through our triangle list in search of the neighboring vertices.

(d) should be circled since our mesh representation does not share vertices between triangles. If we center a vertex, we need to loop through our list to find all other triangles that share the same vertex and update their vertex's position as well. This is one of the advantages we get from using a point-and-triangle representation over this "triangle soup".

4. (Total : 18points) Ray Tracing

   (a) (4 points) Given a triangle with vertices A, B and C, derive the equation for a plane that goes through A, B and C (using cross and dot products). The facing side of the plane doesn't matter.

   Solution: Equation for a plane: $N \cdot (P - X) = 0$, where X is a point on the plane.
   Normal of the plane/triangle: $N = \frac{(B-A)\times(C-A)}{\|(B-A)\times(C-A)\|}$.
   One point on the plane is A, so the equation for this plane is $N \cdot (P - A) = 0$

   (b) (7 points) There is a sphere with radius 1 at the origin. The sphere is transformed by the following homogeneous matrix

$$M = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

   Compute the closest intersection of this transformed sphere and a ray with origin $(1, 1, 1)$ and direction $(-1, -1, -1)$. (Hint: You don't have to multiply the matrix.)

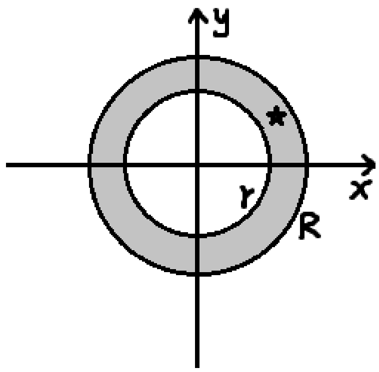
   Solution: Equation for the unit sphere: $x^2 + y^2 + z^2 = 1$.
   Equation for the transformed sphere: $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 + z^2 = 1$.
   Equation for the ray: $r = (1, 1, 1) + t(-1, -1, -1)$.
   For the intersection: $\left(\frac{1-t}{4}\right)^2 + \left(\frac{1-t}{3}\right)^2 + (1 - t)^2 = 1$.
   Solving it, we have $t = 1 \pm 12/13$.
   For the closest intersection, $t = 1 - 12/13 = 1/13$.
   So the closest intersection is $(1, 1, 1) + (-1, -1, -1)/13 = (12/13, 12/13, 12/13)$.

(c) (7 points) Given a box with corners $(-1,-1,-1)$ and $(2,-2,-2)$. Compute the entry and exit point of this box for a ray that has origin $(-3,5,-5)$ and direction $(1,-2,1)$.

Solution:
$t_{x1} = \frac{-1-(-3)}{1} = 2$
$t_{x2} = \frac{2-(-3)}{1} = 5$
$t_{y1} = \frac{-1-5}{-2} = 3$
$t_{y2} = \frac{-2-5}{-2} = 3.5$
$t_{z1} = \frac{-1-(-5)}{1} = 4$
$t_{z2} = \frac{-2-(-5)}{1} = 3$
Swap t_{z1} and t_{z2} so that the smaller value comes first, i.e.,
$t_{z1} = 3$
$t_{z2} = 4$
So, the entry and exit times are:
$t_{\text{min}} = \max(2,3,3) = 3$
$t_{\text{max}} = \min(5,3.5,4) = 3.5$
Finally, the entry and exit points are:
$p_{\text{entry}} = (-3,5,-5) + 3 \cdot (1,-2,1) = (0,-1,-2)$
$p_{\text{exit}} = (-3,5,-5) + 3.5 \cdot (1,-2,1) = (0.5,-2,-1.5)$

5. (Total : 18points) Rendering

(a) (8 points) In order to render a scene lit by a ring shaped light, we need to uniformly sample a random point on a 2D ring. In the figure below, suppose the inner and outer radii of the ring are r and R respectively.



We set up a coordinate system whose origin is aligned with the ring's center. You are given two real numbers u and v which are independently sampled from uniform distribution in range $[0, 1]$. Please fill in the coordinates of a random sample which has uniform distribution on the 2D ring below.

$x =$ ________________________________

$y =$ ________________________________

> **Solution:** $\sqrt{r^2 + u(R^2 - r^2)}cos(2\pi v)$; $\sqrt{r^2 + u(R^2 - r^2)}sin(2\pi v)$ or other sin/cos combinations.

(b) (4 points) Suppose that you used rejection sampling above: you have access to a function that generates points uniformly at random in a circle with radius R, and you reject those points until one is given that lies in the shaded ring. In this case, what is the expected value of the number of calls you will have to make to your random point generating function in order to get a sample inside the ring?

> **Solution:** $\frac{R^2}{R^2 - r^2}$.

(c) (6 points) If I give you a uniform random number U on the interval $[0, 1]$, what function g can I apply to U so that $X = g(U)$ is a random sample on the interval $[-\pi/2, \pi/2]$ with probability density function $p(x) = \frac{1}{2}\cos x$?

Solution: CDF: $F(x) = \int_{-\pi/2}^{x} \frac{1}{2}\cos x' dx' = \frac{1}{2}\sin x + \frac{1}{2}$.
Inverse CDF: $F^{-1}(x) = \sin^{-1}(2x - 1)$.
Answer: $X = g(U) = \sin^{-1}(2U - 1)$.

6. (Total : 18points) Cameras and Lenses

   (a) (4 points) In which of the images taken with the following cameras would a person's head appear largest?

      (i) Cellphone camera with 6mm wide sensor and 5mm focal length lens.
      (ii) Full-frame camera with 36mm wide sensor and 50mm focal length lens.
      (iii) APS-C sized camera with 24mm wide sensor and 12mm focal length lens.
      (iv) The head appears the same size in all images.

      Solution: The answer is (ii), because it has the smallest angular field of view.

   (b) (4 points) In which of the images taken with the following cameras would a grain of rice appear largest?

      (i) Cellphone camera with 6mm wide sensor and 5mm focal length macro lens attachment, focused at unit magnification.
      (ii) Full-frame camera with 36mm wide sensor and 50mm focal length macro lens focused at a magnification of 6.0.
      (iii) APS-C sized camera with 24mm wide sensor and 12mm focal length macro lens focused at a magnification of 4.0?
      (iv) The grain of rice appears the same size in all images.

      Solution: The answer is (iv), because all cameras will have a field of view of 6mm across the image.

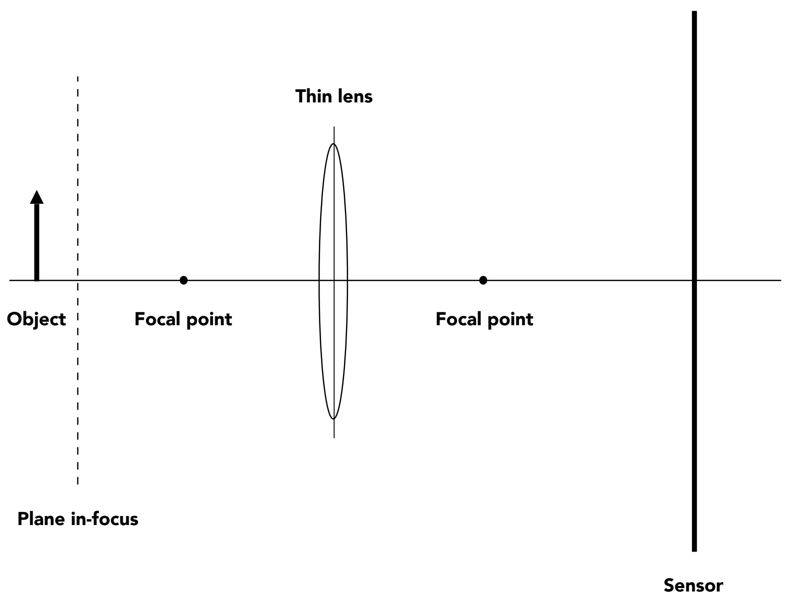
(c) (10 points) Consider an object placed an equal distance on either side of the plane at which a camera is focused in the world.

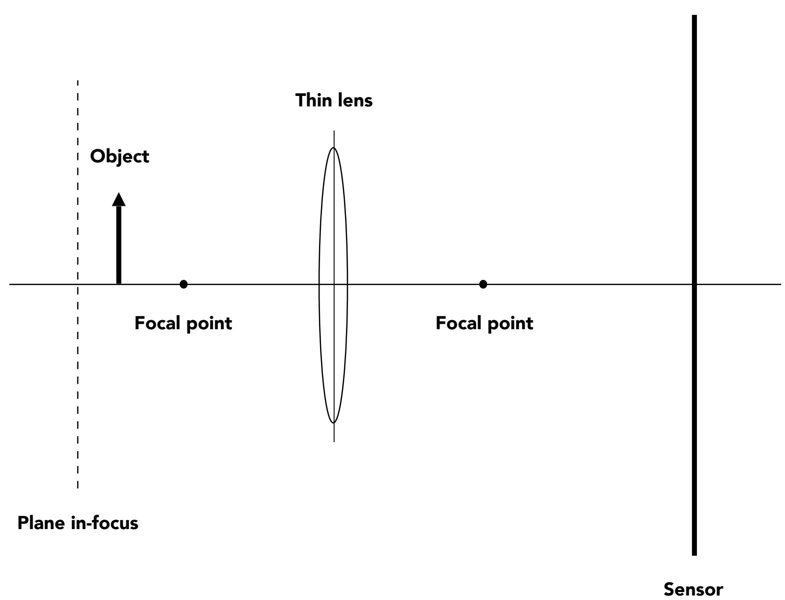
In the following ray diagrams:

i. Use Gauss' ray construction to draw the image that would appear.

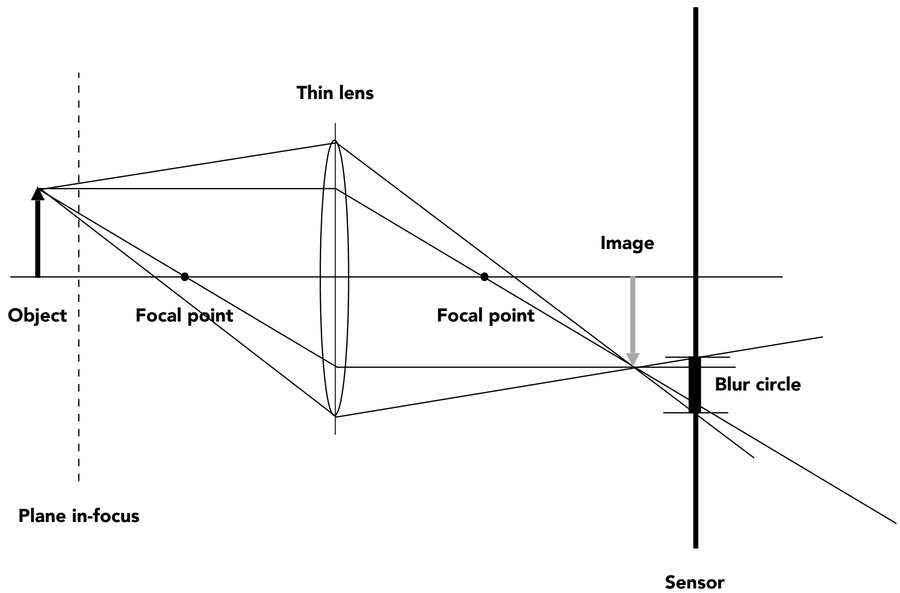
ii. Draw additional rays to graphically calculate the size of the blur circle formed by the tip of the object on the sensor plane, and clearly indicate on your drawing the size of this blur on the sensor plane (note: you do not need to solve for a formula or numerical size).

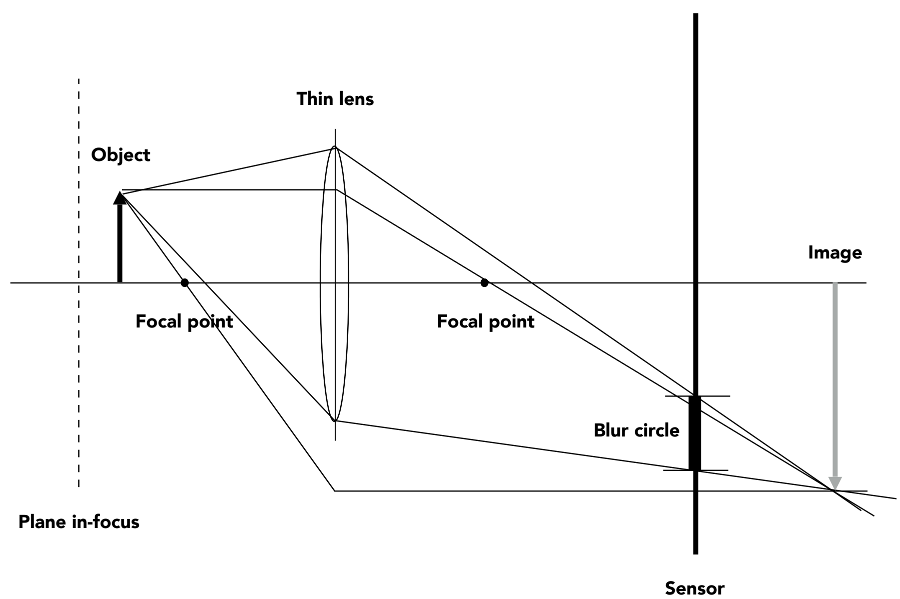
iii. Is the object more blurry when positioned closer or further from the camera?





Solution:





The object is blurrier when closer to the camera.