CS 184/284A **Name:** ____________________

Spring 2018: Midterm 1 **SID number:** ____________________

March 13th, 2018 **Room:** ____________________

Time Limit: 110 Minutes

- This exam contains 15 pages (including this cover page) and 6 problems. Check for missing pages.
- Put your initials on the top of every page, in case the pages become separated.
- This exam is closed book, except for one 8.5×11 page of notes (double sided), printed or handwritten.
- This exam is 110 minutes long, and has a total of 110 points.
- Problem difficulty varies throughout the exam, so don't get stuck on a time-consuming problem until you have read through the entire exam. Each problem's point value is roughly correlated with its expected difficulty.
- Answer each question in the space provided. Partial credit may be given on certain problems.
- To minimize distractions, do your best to avoid questions to staff. If you need to make assumptions to answer a question, write these assumptions into your answer.
- **For multiple choice questions, please fill the little bubble completely next to your answer. Do not just tick or circle.**

Problem	Points	Score
1	18	
2	18	
3	18	
4	20	
5	18	
6	18	
Total:	110	

1. (Total : 18points) True / False

   Mark each statement true or false. (1 point each)

   (a) (1 point) **F** If all three barycentric coordinates sum to 1, i.e. $\alpha + \beta + \gamma = 1$, then the point is guaranteed to be inside the triangle.

   (b) (1 point) **F** The Painter's Algorithm consistently resolves the visibility issue of which polygons are visible and which are obscured by others.

   (c) (1 point) **F** Jaggies, the wagon-wheel effect, and Moiré patterns are all artifacts of oversampling.

   (d) (1 point) **T** Storing a mipmap requires $\frac{1}{3}$ more memory than a regular texture.

   (e) (1 point) **F** A Bezier curve interpolates between all of its control points.

   (f) (1 point) **T** The Bernstein polynomials for a Bezier curve all have the same degree.

   (g) (1 point) **F** A subdivision surface is an implicit geometry representation.

   (h) (1 point) **F** The continuous surface defined by a subdivision surface in the limit where the number of subdivision steps approaches infinity is the same for all subdivision algorithms.

   (i) (1 point) **F** Rasterization algorithms are better suited for rendering implicit geometries than ray tracing algorithms.

   (j) (1 point) **F** Soft shadows and global illumination are prominent features of ray casting.

   (k) (1 point) **T** Path tracing is a good algorithm to use in cases where images can be created in advance of use.

   (l) (1 point) **T** The units of radiance are watts per meter squared per steradian.

   (m) (1 point) **F** In path tracing with Russian Roulette, if we forgot to normalize by dividing by the probability of terminating a ray, the resulting rendering will be too bright.

   (n) (1 point) **T** Brushed metal is an anisotropic BRDF.

   (o) (1 point) **T** In photographing my friend, if I increase the focal length of the lens on my camera he will look larger in my photo.

   (p) (1 point) **F** If I change the f-stop from 16 to 8, to have the same exposure, I need to change my shutter speed from 1/4 to 1/8 accordingly.

   (q) (1 point) **F** If we want to keep the subject the same size in the frame while zooming a zoom lens (decreasing the field of view), we would want to dolly (move) the zoom lens towards the subject.

   (r) (1 point) **T** Rays that pass through the outer portion of a lens (further from the optical axis) bend more.

2. (Total : 18points) Graphics Pipeline

(a) (2 points) If I record a spinning wagon wheel with a video camera at 60 frames per second (assume no motion blur), up to what rate can the wheel spin before I will begin to see aliasing? You can assume the wheel has 5 spokes.

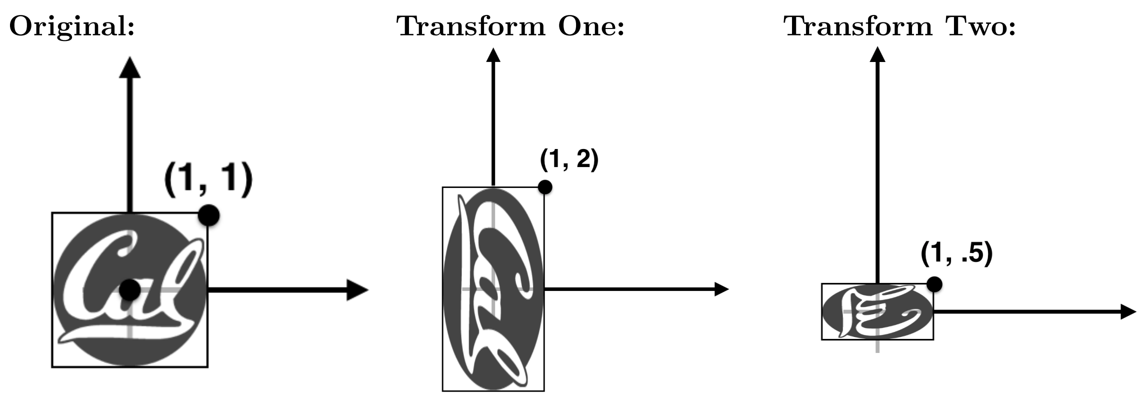
(i) 3 rotations per second.
(ii) 6 rotations per second.
(iii) 12 rotations per second.
(iv) 24 rotations per second.

Solution: The answer is (ii).

(b) (4 points) Suppose we have the following transformation matrices:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} .5 & 0 \\ 0 & 1 \end{bmatrix}$$

Given the Original image below, write a composition of the matrices above to implement each of Transform One and Transform Two. Note: both Transform One and Transform Two begin from the Original image, i.e. they are not consecutive.



Transform One:

(i) $B \cdot A$
(ii) $A \cdot B$
(iii) $C \cdot B$
(iv) $B \cdot C$

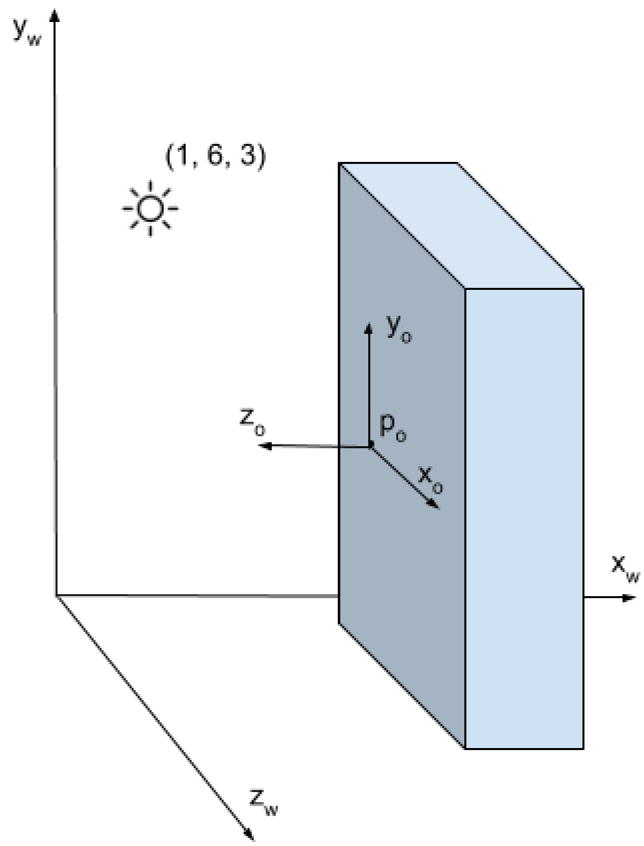
Solution: The answer is (ii).

Transform Two:

(i) $D \cdot B \cdot C$
(ii) $C \cdot B \cdot A$
(iii) $D \cdot E \cdot C$
(iv) $A \cdot E \cdot C$

Solution: The answer is (iii).

(c) Suppose we are trying to calculate the diffuse Lambertian term of the shaded color for a surface point p_o, as shown below.



In world coordinates, let $p_o = (3, 4, 3)$ be the origin of the object's coordinate system, and let the object's local axes be $x_o = (0, 0, 1), y_o = (0, 1, 0), z_o = (-1, 0, 0)$. The scene has a single point light, located at $p_l = (1, 6, 3)$, also in world coordinates.

i. (3 points) What is the homogeneous change of coordinates matrix from object to world space? That is, what is the matrix M_{o2w} such that $M_{o2w}\, x_{obj} = x_{world}$? Hint: it should be in the form

$$M_{o2w} = \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} & \mathbf{o} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution:

$$M_{o2w} = \begin{bmatrix} 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii. (3 points) The diffuse Lambertian term of a surface shading calculation has the form

$$L_d = k_d \left(\frac{I}{r^2}\right) \max(0, n \cdot l)$$

For this light, $k_d = 1$, $I = 16$. What is r^2 for the given point on the surface with respect to the light?

Solution: $r^2 = ((1, 6, 3) - (3, 4, 3))^2 = 8$

iii. (3 points) What is l in object coordinates, normalized?

Solution:

$$M_{w2o} = M_{o2w}^{-1} = \begin{bmatrix} 0 & 0 & 1 & -3 \\ 0 & 1 & 0 & -4 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d = p_l - p_o = (-2, 2, 0)$$

$$l = M_{w2o} \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

$$l_{normalized} = \begin{bmatrix} 0 \\ \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{bmatrix}$$

iv. (3 points) Putting it all together, what is the diffuse Lambertian term L_d?

Solution: $L_d = 1 \cdot \frac{16}{8} max(0, n \cdot l) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

3. (Total : 18points) Geometry

   (a) (12 points) Let's consider the process of subdividing a single Bezier curve into two separate Bezier curves that together make up the initial Bezier curve. The initial curve $B(t)$ has control points p_1, p_2, p_3, and p_4, and we subdivide $B(t)$ into two Bezier curves: $B_1(t)$ with control points q_1, q_2, q_3, and q_4 and $B_2(t)$ with control points r_1, r_2, r_3, and r_4. The initial curve $B(t)$ is subdivided at the point $t = z$, such that the curve $B_1(t)$ is geometrically equivalent to $B(t)$ from $t = 0$ to $t = z$, and the curve $B_2(t)$ is geometrically equivalent to $B(t)$ from $t = z$ to $t = 1$. We will skip the derivation here, but the expressions for the new control points are:

$$\begin{pmatrix} q_1 = p_1 \\ q_2 = (1-z)p_1 + zp_2 \\ q_3 = (1-z)^2 p_1 + 2z(1-z)p_2 + z^2 p_3 \\ q_4 = (1-z)^3 p_1 + 3z(1-z)^2 p_2 + 3z^2(1-z)p_3 + z^3 p_4 \end{pmatrix}$$

   and

$$\begin{pmatrix} r_1 = (1-z)^3 p_1 + 3z(1-z)^2 p_2 + 3z^2(1-z)p_3 + z^3 p_4 \\ r_2 = (1-z)^2 p_2 + 2z(1-z)p_3 + z^2 p_4 \\ r_3 = (1-z)p_3 + zp_4 \\ r_4 = p_4 \end{pmatrix}$$

   Circle the statements below that are true:

   (i) All the new control points are either control points of the original Bezier curve, or are evaluated when using de Casteljau's algorithm to compute $B(z)$.

   (ii) The first subdivided Bezier curve $B_1(t)$ interpolates between its first and last control points (q_1 and q_4).

   (iii) q_4, the last control point of $B_1(t)$, is the original Bezier curve $B(t)$ evaluated at $t = 1 - z$ ($q_4 = B(1 - z)$).

   (iv) The second and third control points (q_2 and q_3) of $B_1(t)$ are computed at the same recursive step of using de Casteljau's algorithm to evaluate the initial Bezier curve $B(t)$ at $t = z$.

   (v) The two resulting subdivided Bezier curves, $B_1(t)$ and $B_2(t)$, do not necessarily have the same derivatives with respect to t at $t = z$.

   (vi) q_3, q_4, and r_2 are always collinear.

   Solution: (i) Should be circled, and is evident from trying to compute de Casteljau's algorithm. (ii) Should be circled, as this is a property of all Bezier curves. (iii) Should not be circled (because this property actually holds at $t = z$ instead). (iv) Should not be circled, because they are calculated at different recursive steps (the second control point is computed in the recursive step before the third). (v) Should be circled, because the Bezier spline is only C_1 continuous if the distances are equal from q_3 to q_4 and from q_4 to r_2. (Think of the difference between geometric and parametric continuity). (vi) Should be circled, because the original Bezier curve was C_1 continuous (in fact, it was C_2 continuous).

   (b) I have a triangle mesh with n total triangles, and I would like to estimate the average position of the v vertices that neighbor a single input vertex.

i. (2 points) If my mesh is represented as a half-edge data structure, what is the asymptotic complexity of computing this average position?

Answer: ________________

Solution: O(v) because we just need to visit neighbor vertices using h.next().vertex() and update h as h.twin().next() until we arrive back at h.

ii. (2 points) If my mesh is represented as an unordered list of triangles, what is the asymptotic complexity of computing this average position?

Answer: ________________

Solution: O(n) because we need to check every triangle and see if it contains the input vertex as one of its vertices. We can continually add the other two vertices to a hashmap/dictionary to maintain the set of all neighbors.

iii. (2 points) If my mesh is represented as a triangle-neighbor data structure (each triangle contains pointers to its 3 vertices and pointers to the three neighboring triangles, and each vertex contains a pointer to its position and a pointer to a triangle that it belongs to), what is the asymptotic complexity of computing this average position?

Answer: ________________

Solution: O(v) because we just need to visit neighbor vertices by starting at the triangle that the input vertex points to, adding its other two vertices, then continually visiting the neighboring triangle that has not been visited and also still contains the input vertex as one of its vertices. We will still visit just v triangles, but it requires a bit more bookkeeping than using the half-edge data structure.

4. (Total : 20points) Ray Tracing

(a) Ray-Cylinder Intersection

i. (4 points) In lecture, we have seen how to determine the points of intersection of a ray with a box or a sphere. In this question we will perform a similar derivation of the intersection points of a ray and a cylinder. Recall the equation for an infinitely-long cylinder aligned along the z-axis with radius r:

$$x^2 + y^2 - r^2 = 0$$

Consider a ray with origin $\mathbf{o} = (-2, -2, -2)$ and direction $\mathbf{d} = (1, 1, 1)$, with a ray equation $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$. Calculate any and all t-value(s) for intersection with this ray and the infinitely-long cylinder aligned along the z-axis with radius 2. Write the smaller intersection time into t_0 and the larger into t_1.

$t_0 =$ ________________

$t_1 =$ ________________

Solution: The equation for the cylinder is $x^2 + y^2 - 4 = 0$
$\mathbf{r}(t) = (-2, -2, -2) + t(1, 1, 1)$
Substituting: $(-2 + t)^2 + (-2 + t)^2 = 4$
$2(-2 + t)^2 = 4$
$(-2 + t)^2 = 2$
$(-2 + t) = \pm\sqrt{2}$
$t = 2 \pm \sqrt{2}$

ii. (2 points) Now consider if we want the cylinder to have finite length, such that it has faces at $z = -1$ and $z = 1$. Find the t-value(s) of intersection of the same ray from part (ii) with the planes containing these two faces. Write the smaller intersection time into t_0 and the larger into t_1.

$t_0 =$ ________________

$t_1 =$ ________________

Solution: The equations for the two planes: $\mathbf{P_1} : z = -1$ and $\mathbf{P_2} : z = 1$.
The z-position of the ray: $\mathbf{r_z}(t) = -2 + t$
For $\mathbf{P_1}$: $-2 + t = -1 \implies t_1 = 1$
For $\mathbf{P_2}$: $-2 + t = 1 \implies t_2 = 3$

iii. (2 points) List the point(s) of intersection with the infinite cylinder and the point(s) of intersection with the face planes (That is, convert your t-value(s) to 3D coordinates).

For the cylinder:

$\mathbf{p}_0 =$ ________________

$\mathbf{p}_1 =$ ________________

For the caps:

$\mathbf{p}_2 =$ ________________

$\mathbf{p}_3 =$ ________________

Solution:
For the cylinder:
$\mathbf{p_0} = \mathbf{r}(2-\sqrt{2}) = (-\sqrt{2}, -\sqrt{2}, -\sqrt{2})$
$\mathbf{p_1} = \mathbf{r}(2+\sqrt{2}) = (\sqrt{2}, \sqrt{2}, \sqrt{2})$
For the caps:
$\mathbf{p_2} = \mathbf{r}(1) = (-1, -1, -1)$
$\mathbf{p_3} = \mathbf{r}(3) = (1, 1, 1)$

iv. (2 points) Which intersection points from part (iii) represent true intersection points with the finite cylinder? Which intersection point from part (iii) is the closest valid intersection of the ray with the cylinder?

Solution:
Both cylinder intersections are invalid as their z-coordinates both occur outside the caps of the cylinder.
Both cap intersections are valid as they are within a radius of 2 from the center of the cap.
The closest intersection is at time $t = 1$ at point $\mathbf{p_2} = (-1, -1, -1)$.

(b) We are given a triangle with the points $(-2,1,4)$, $(1,3,4)$, and $(1,5,2)$ that is transformed by the following homogeneous matrix:

$$M = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i. (6 points) Given the following ray

$$\mathbf{r}(t) = (1,-1,4) + t(-2,-2,2)$$

calculate any and all t-values of entry and exit of the ray through the axis-aligned bounding box of the transformed triangle.

Solution: The points of the transformed triangle are $(-4,-1,7), (2,-3,7), (2,-5,5)$.
The axis-aligned bounding box is then between $(-4,-5,5)$ and $(2,-1,7)$.
For the x-slab:
$t_{x0} = \frac{2-1}{-2} = -\frac{1}{2}$, $t_{x1} = \frac{-4-1}{-2} = \frac{5}{2}$
For the y-slab:
$t_{y0} = \frac{-1+1}{-2} = 0$, $t_{y1} = \frac{-5+1}{-2} = 2$
For the z-slab:
$t_{z0} = \frac{5-4}{2} = \frac{1}{2}$, $t_{z1} = \frac{7-4}{2} = \frac{3}{2}$
For the bounding box:
$t_0 = max\{t_{x0}, t_{y0}, t_{z0}\} = \frac{1}{2}$
$t_1 = min\{t_{x1}, t_{y1}, t_{z1}\} = \frac{3}{2}$

ii. (4 points) We add a sphere centered at the origin with radius 2 to this scene and transform it using M. We cast a ray into our scene from the same origin as the ray above, but with a directional component of $(-1,1,-1)$. Calculate any and all t-values of the ray intersection with this sphere.

Solution: The equation of the transformed sphere is $\frac{x^2}{4} + (-y)^2 + (z-3)^2 = 4$
The ray equation is $(1,-1,4) + t(-1,1,-1)$. Substituting:
$\frac{(1-t)^2}{4} + (1-t)^2 + (1-t)^2 = 4$
$(1-t)^2 + 4(1-t)^2 + 4(1-t)^2 = 16$
$9(1-t)^2 = 16$
$1 - t = \pm\frac{4}{3}$
$t = 1 \mp \frac{4}{3}$

5. (Total : 18points) Rendering

   (a) Monte Carlo Sampling and Integration

      i. (2 points) What's the name of the sampling method that generates random samples according to a given probability density function? (Hint: for this method, we can either do inversion or rejection.)

      Answer: ________________________________

      Solution: Importance sampling.

      ii. (3 points) In order to beat the curse of dimensionality, we turn to Monte Carlo integration with random samples. With Monte Carlo, we use the standard deviation (square root of variance) as a measure of expected noise levels. Suppose that we use n independent random samples and get a noisy result with standard deviation of σ. How many samples do we need (in total) so that the standard deviation becomes $\sigma/2$?

      (i) $\sqrt{2}n$
      (ii) $2n$
      (iii) $2\sqrt{2}n$
      (iv) $4n$

      Solution: The answer is (iv). The variance (σ^2) is inversely proportional to n.

   (b) Radiometry

      i. (3 points) Suppose I have a cube that measures 1 m on each side, and I cover all 6 faces with solar panels that are 100% efficient. At noon, the sun lies directly overhead, casting rays of light perpendicular to the ground such that any point has incident irradiance 1 kW/m^2.
      If I go outside at noon, what is the maximum energy in kilojoules (kilowatts are kilojoules per second) that my cube can collect in one second?

      (i) 1 kJ
      (ii) $\sqrt{2}\ kJ$
      (iii) $\sqrt{3}\ kJ$
      (iv) 2 kJ

      Solution: When the cube's longest diagonal is perpendicular to the ground, the cube will have projected area $\sqrt{3}\ m$ perpendicular to the sunlight, so it will collect $\sqrt{3}\ kW$ of power.

      ii. (3 points) Imagine a point light with a radiant flux of 10 watts, what is the irradiance at a point 2 meters away from the light? (Please specify the units too. Result as a fraction is OK.)

      Answer: ________________________________

      Solution: Since $E = \mathrm{d}\Phi/\mathrm{d}A$, the result is $10W/(4\pi(2m)^2) = 10/(16\pi)[W \cdot m^{-2}]$.

(c) Light transport and materials

i. (2 points) In order to avoid infinite number of bounces along a light path, what technique do we use to stop bouncing?

Answer: ______________________

Solution: Russian Roulette.

ii. (2 points) For direct illumination, we can either importance sample the light source, or importance sample the BRDF. Among all the following cases, what is the best suitable for importance sampling the BRDF?

(i) Large light source, diffuse surface.

(ii) Large light source, glossy surface.

(iii) Small light source, diffuse surface.

(iv) Small light source, glossy surface.

Solution: The answer is (ii). When the light is large, sampling the light is less efficient. When the surface is diffuse, BRDF sampling is less efficient.

iii. (3 points) For the microfacet BRDF model, which term is most capable of introducing anisotropy?

Answer: ____________

Solution: The D term, or NDF, or distribution of normals.

Which term is able to explain Snell's window/circle?

Answer: ____________

Solution: The F term, or Frensel term/reflection.

If we use the microfacet BRDF model to represent a rusty, brushed metal bunny with spatially-varying color over its surface, what is the dimension of this BRDF?

Answer: ____________

Solution: 6D. Anisotropic BRDF at any surface point is 4D with 2D incident and 2D outgoing. And the BRDF can be different at different places on the surface, that's another 2D.

6. (Total : 18points) Cameras and Lenses

   (a) (3 points) Recall that in a dolly zoom maneuver in cinematography, the camera is moved closer or further from the subject while the lens is zoomed to either a longer or shorter focal length. In Steven Spielberg's Jaws, a dolly zoom is used to enlarge the field of view of the background while keeping a man's face the same size. This was achieved by:

      (i) Moving in while zooming to shorter focal length.
      (ii) Moving in while zooming to longer focal length.
      (iii) Moving out while zooming to shorter focal length.
      (iv) Moving out while zooming to longer focal length.

   Solution: The answer is (i).

   (b) (3 points) Which of the following cameras has the widest field of view on its focal plane? You can assume the cameras are all focused at a distance equal to twice their focal length (such that we have unit magnification), and that the lenses are ideal thin lenses.

      (i) Cellphone camera with 6mm wide sensor and 5mm focal length lens.
      (ii) Full-frame camera with 36mm wide sensor and 50mm focal length lens.
      (iii) APS-C sized camera with 24mm wide sensor and 12mm focal length lens.
      (iv) They all have the same horizontal field of view.

   Solution: The answer is (ii), because it has largest sensor and this is 1:1 magnification.

   (c) (2 points) In order to increase the depth of field when taking a picture, you should

      (i) Use a higher f-number and a longer exposure time
      (ii) Use a lower f-number and a longer exposure time
      (iii) Use a higher f-number and a shorter exposure time
      (iv) Use a lower f-number and a shorter exposure time
      (v) Use a longer exposure time and a higher gain (ISO) on the image sensor

   Solution: The answer is (i) we want to use a smaller aperture and also allow for more light.

(d) A photographer takes a picture focused on the full moon rising on the distant horizon. A firefly, tiny but bright, flies into the frame just as the photographer takes the shot. The moon is sharp and the firefly is defocused. Does the moon appear larger, or the blurred firefly – we'll calculate the answer in this question.

You can assume the camera lens has a focal length of $f = 50$mm and an f-number of $f/2$; that the sensor is "full-frame" 36 x 24mm large with a pixel resolution of 3600 x 2400 pixels.

Note: no calculators on this question; leave fraction expressions if you need to.

i. (3 points) The moon is (very) approximately 2000 miles in diameter at a distance of 200,000 miles. Assuming these are the correct numbers, how many pixels wide is the moon in the photo?

Answer: ____________

Solution: 2000 mi / 200,000 mi * 50 mm * (3600 px / 36mm) = 50 pixels

ii. (2 points) What is the distance between the sensor and the lens? (Please round to the nearest millimeter.)

Answer: ____________

Solution: 50 mm because focused at infinity.
Or using the thin lens equation, 1/(1/50-1/200000) = 50.013 mm $\approx$ 50 mm.

iii. (3 points) At what distance behind the lens does the firefly come to focus? You can assume the firefly is at a distance of 1.3 meters from the camera.

Answer: ____________

Solution: Need to use thin lens equation, 1/(1/50-1/1300) = 1/(26/1300 - 1/1300) = 1/(25/1300) = 52 mm

iv. (2 points) Based on the previous two questions, what is the width of the blur of the firefly in the photo? Is the moon larger or the blurry firefly?

Answer: ____________

Solution: (52 mm - 50mm) * 25mm / 50mm *(3600px /36mm) = 100 pixels