CS 184/284A **Name:** ____________________

Spring 2016: Final **SID number:** ____________________

5/10/16 **WebAcct login:** ____________________

Time Limit: 180 Minutes

- This exam contains 15 pages (including this cover page) and 7 problems. Check for missing pages.
- Put your initials on the top of every page, in case the pages become separated.
- This exam is closed book, except for one 8.5×11 page of notes (double sided).
- This exam is 180 minutes long, and has a total of 128 points.
- Problem difficulty varies throughout the exam, so don't get stuck on a time-consuming problem till you have read through the entire exam.
- Answer each question in the space provided. Partial credit may be given on certain problems.
- To minimize distractions, do your best to avoid questions to staff. If you need to make assumptions to answer a question, write these assumptions into your answer.

Problem	Points	Score
1	20	
2	16	
3	18	
4	18	
5	20	
6	24	
7	12	
Total:	128	

1. (Total : 20points) True / False

   Mark each statement true or false. (1 point each)

   (a) (1 point) **F** Bump mapping is a texture mapping technique that alters the appearance of a surface by altering surface positions.

   (b) (1 point) **T** Trilinear interpolation in mipmapping can cause over-blurring of textures.

   (c) (1 point) **F** Supersampling by 8x8 eliminates all aliasing.

   (d) (1 point) **T** Barycentric interpolation of normal vectors across a triangle can result in vectors that are shortened.

   (e) (1 point) **T** The limit surface of Loop subdivision is C^2 continuous everywhere except extraordinary points.

   (f) (1 point) **F** $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ is a 2D rotation matrix.

   (g) (1 point) **F** Implicit surface representations are better than explicit for sampling the surface.

   (h) (1 point) **T** Catmull-Rom splines interpolate all their control points.

   (i) (1 point) **F** A bicubic Bezier surface patch has 9 control points.

   (j) (1 point) **F** A detailed teapot in a stadium is an example of a model that does NOT work well with a KD-Tree acceleration structure.

   (k) (1 point) **T** The units for radiance are watts per square meter per steradian.

   (l) (1 point) **T** Steradian is the dimensionless unit for solid angle.

   (m) (1 point) **T** Path tracing solves the rendering equation by Monte Carlo integration over all light paths.

   (n) (1 point) **T** CIELAB is a perceptually organized color space.

   (o) (1 point) **T** The index of refraction of glass varies as a function of wavelength.

   (p) (1 point) **T** In a dolly zoom, the focal length and the position of the camera must be changed at the same time.

   (q) (1 point) **T** At high shutter speeds, a focal plane shutter can result in image artifacts similar to electronic rolling shutter.

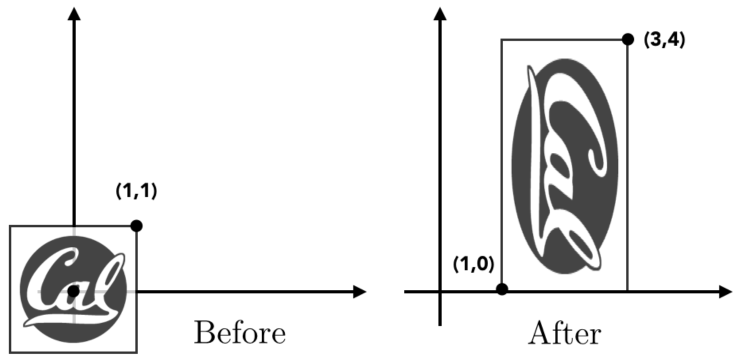
   (r) (1 point) **F** Photon shot noise has a variance equal to the square root of the signal.

   (s) (1 point) **T** A light field is a 4D function providing the radiance traveling along every ray in in some region of freespace.

   (t) (1 point) **T** Vergence is the phenomenon where the eyes rotate in their sockets so that an object of interest appears centered in each eye's field of view.

2. (Total : 16points) Rasterization Pipeline.

(a) (8 points) Which of the following sequences of basic transforms will produce the overall transform shown in this figure? Circle all that apply. (Rotations are counterclockwise.)



(i) Scale horizontally 2x; rotate 90 degrees; reflect across y-axis; translate by (2,2).
(ii) Translate (-1,2); rotate -90 degrees; scale 2x vertically.
(iii) Rotate -90 degrees; scale 2x vertically; translate by (2,2).
(iv) Scale horizontally 2x; translate (-2, 2); rotate 270 degrees.

Solution: The answer is (ii), (iii) and (iv).

(b) (4 points) If I record a spinning wagon wheel with a video camera at 60 frames per second (assume no motion blur), up to what rate can the wheel spin before I will begin to see aliasing? You can assume the wheel has 6 spokes.

(i) 1 rotations per second.
(ii) 5 rotations per second.
(iii) 10 rotations per second.
(iv) 60 rotations per second.

Solution: The answer is (ii).

(c) (4 points) Which of the following image processing operations can be implemented more quickly in the frequency domain than in the spatial domain (circle all that apply)?

(i) Blurring an image with a large Gaussian filter.
(ii) Applying a vignetting mask that darkens the corners of the image.
(iii) Reflecting the image about its x-axis.
(iv) Detecting edges by removing low frequencies.

Solution: The answer is (i) and (iv).

3. (Total : 18points) Geometry

(a) (2 points) A Bezier curve is always enclosed in the ______________ of its control points.

Solution: convex hull

(b) (2 points) A triangle mesh with N triangles will contain ______________ triangles after K applications of Loop subdivision.

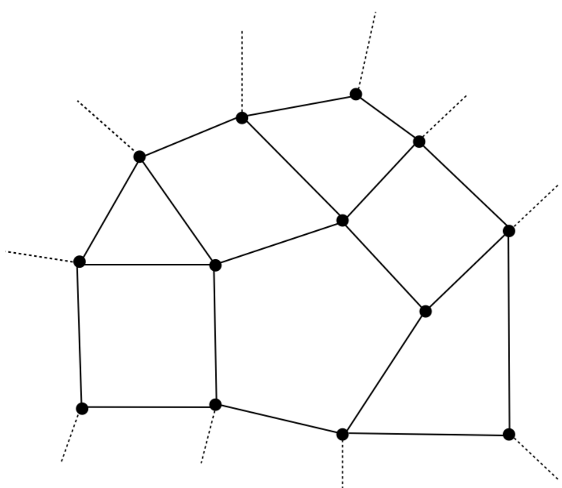
Solution: 4^K

(c) (8 points) Assume you have a cubic spline $g(t)$ defined on the interval $t \in [0, 1]$. Assume that the value at $t = 0$ and $t = 1$ are both 0, and that additionally the value and derivative at $t = 0.5$ are 0.5 and 0, respectively. Complete this formula for $g(t)$:

$$g(t) = \_\_\_\_\_ \ t^3 + \_\_\_\_\_ \ t^2 + \_\_\_\_\_ \ t + \_\_\_\_\_ . \quad (1)$$

Solution: The answer is $g(t) = 0t^3 - 2\ t^2 + 2\ t + 0$.

(d) (6 points) Here is a section of a mesh. Draw one level of Catmull-Clark subdivision, and indicate all extraordinary vertices on the resulting mesh. Only the topology matters. The final number of extraordinary points is ______**5**______.



Solution:

4. (Total : 18points) Rendering

(a) (6 points) Assume in a path tracing program that a ray is bouncing inside a diffuse sphere with Russian roulette termination probability p. Let X be the number of bounces before the ray terminates. Please fill in the following blanks.

i. The probability that X=0 is:

i. ______________

ii. The probability that X=2 is:

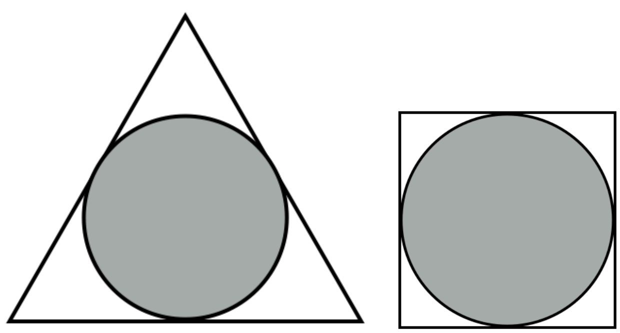
ii. ______________

iii. The expected value of X is:

iii. ______________

Solution:
i. p ii. $p(1-p)^2$ iii. $\frac{1-p}{p}$
Let $\bar{p} = 1 - p$
$E[X] = (\bar{p} + 2\bar{p}^2 + 3\bar{p}^3 + \dots)p$
$\bar{p}E[X] + (\bar{p}^2 + \bar{p}^3 + \dots)p = (2\bar{p}^2 + 3\bar{p}^3 + \dots)p$
$\bar{p}E[X] + \bar{p}^2 = E[x] - \bar{p}p$
$E[X] = \frac{1-p}{p}$

(b) (6 points) The figure below shows two methods for estimating π using rejection sampling. Method (a) generates random points uniformly in an equilateral triangle. Method (b) generates random points uniformly in a square. Both methods estimate π using the ratio of the number of points that lie within the shape's tangent circle to the total number of points sampled.



(a) Triangle (b) Square

Please answer the following questions.

i. In method (a), suppose there are k points out of all n points sampled that lie within the triangle's tangent circle. Then, a formula for the estimated value of π is:

i. ______________

ii. In method (b), suppose there are k points out of all n points sampled that lie within the square's tangent circle. Then, a formula for the estimated value of π is:

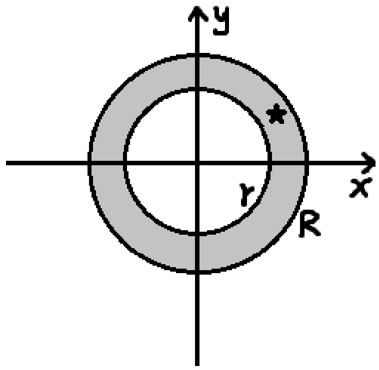
ii. ______________

iii. Which method ((a) or (b)) has lower variance with the same number of samples?

iii. ______________

Solution: $\frac{3\sqrt{3}k}{n}$; $\frac{4k}{n}$; (b)

(c) (6 points) In order to render a scene lit by a ring shaped light, we need to uniformly sample a random point on a 2D ring. In the figure below, suppose the inner and outer radii of the ring are r and R respectively.



We set up a coordinate system whose origin is aligned with the ring's center. You are given two real numbers u and v which are independently sampled from uniform distribution in range $[0, 1]$. Please fill in the coordinates of a random sample which has uniform distribution on the 2D ring below.

$x =$ ____________________________________

$y =$ ____________________________________

Solution: $\sqrt{r^2 + u(R^2 - r^2)}cos(2\pi v)$; $\sqrt{r^2 + u(R^2 - r^2)}sin(2\pi v)$ or other sin/cos combinations.

5. (Total : 20points) Color and Radiometry

(a) (2 points) Two spectral power distributions with the same perceived color are called ______________.

Solution: metamers

(b) (2 points) (True or false) In the color matching experiment, the color appearance of any input light can be matched by adjusting the brightness of three primary lights. ______________

Solution: False, it may be necessary to add some primary light to the input light if it is out of the gamut of the three primaries.

(c) (4 points) Which of the following photometric units are equivalent to 1 nit (circle all that apply)?

(i) 1 candela per square meter.
(ii) 1 lux per steradian.
(iii) 1 lumen per steradian per square meter.
(iv) 1 watt per steradian per square meter.

Solution: The answer is (i), (ii) and (iii).

(d) (12 points) In this multi-part question we will consider the color reproduction problem. Throughout this problem, assume that the spectral response curves of the three human cone cells, as a function of wavelength, are given by $S(\lambda)$, $M(\lambda)$, $L(\lambda)$. (3 points for each subpart of this question).

i. First, consider a target light with spectral power distribution (SPD) that we would like to reproduce, given by $C(\lambda)$. Write down expressions for the scalar response of each cone cell when exposed to $C(\lambda)$.

$s_C =$

$m_C =$

$l_C =$

Solution:

$$s_C = \int S(\lambda)C(\lambda)d\lambda$$
$$m_C = \int M(\lambda)C(\lambda)d\lambda$$
$$l_C = \int L(\lambda)C(\lambda)d\lambda$$

ii. Now consider a color reproduction system (e.g. a pixel on an RGB monitor) composed of three primary lights with SPDs given by functions $R(\lambda)$, $G(\lambda)$ and $B(\lambda)$. If we weight each of these primary lights by scalar values r, g, b, respectively, write down the scalar response from each cone cell type when exposed to these weighted primaries.

$s_{RGB} =$

$m_{RGB} =$

$l_{RGB} =$

Solution:

$$s_{RGB} = \int S(\lambda)\,(rR(\lambda) + gG(\lambda) + bB(\lambda))\,d\lambda$$
$$m_{RGB} = \int M(\lambda)\,(rR(\lambda) + gG(\lambda) + bB(\lambda))\,d\lambda$$
$$l_{RGB} = \int L(\lambda)\,(rR(\lambda) + gG(\lambda) + bB(\lambda))\,d\lambda$$

iii. Note that we can re-write the result from part (ii) in matrix form:

$$\begin{bmatrix} s_{RGB} \\ m_{RGB} \\ l_{RGB} \end{bmatrix} = \begin{bmatrix} & & \\ & M & \\ & & \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

As examples, write down expressions for m_{11} and m_{32}:

$m_{11} =$

$m_{32} =$

Solution:

$$m_{11} = \int S(\lambda)R(\lambda)d\lambda$$
$$m_{32} = \int L(\lambda)G(\lambda)d\lambda$$

iv. Finally, to complete the color matching procedure, let's determine how to choose values for r, g and b to match the perceived color of the input SPD $C(\lambda)$, assuming $C(\lambda)$ is in gamut. Write down a one-line matrix expression for such r, g and b values. You can use any variables defined in previous parts of this question, and you may also use matrix operations such as transpose and inverse in your solution if needed.

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} =$$

Solution: The goal is to have $(s_{RGB}, m_{RGB}, l_{RGB})^T = (s_C, m_C, l_C)^T$. This is accomplished by choosing:

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \begin{bmatrix} & & \\ & M & \\ & & \end{bmatrix}^{-1} \begin{bmatrix} s_C \\ m_C \\ l_C \end{bmatrix}$$

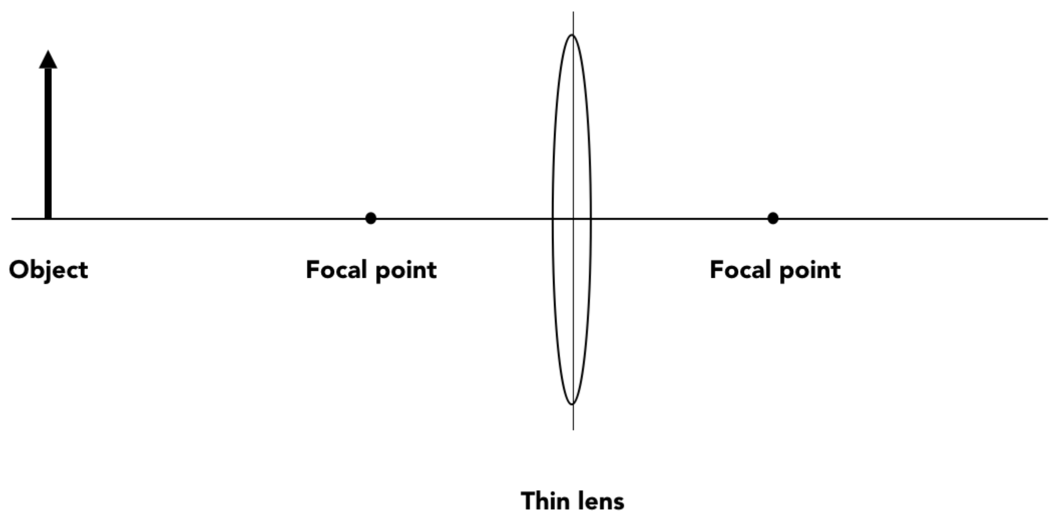
6. (Total : 24points) Cameras

   (a) (4 points) Which of the following cameras has the widest horizontal field of view on its focal plane? You can assume the cameras are all focused at infinity.

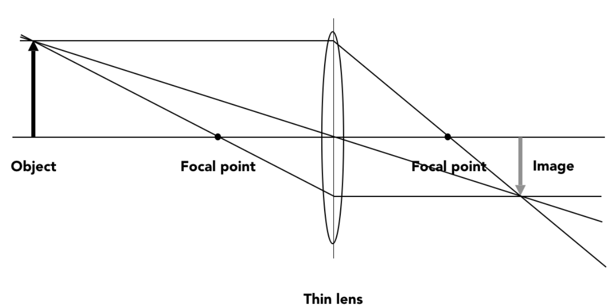
      (i) Cellphone camera with 6mm wide sensor and 5mm focal length lens.
      (ii) Full-frame camera with 36mm wide sensor and 50mm focal length lens.
      (iii) APS-C sized camera with 24mm wide sensor and 12mm focal length lens.
      (iv) They all have the same horizontal field of view.

   Solution: The answer is (iii), because it has the widest angular field of view and we are focused at infinity.

   (b) (4 points) Given the object and thin lens shown below, use Gauss' ray construction method to draw the image that would appear.



   Solution:



   (c) (4 points) Each line below describes a pair of different imaging conditions (assume a regular single-lens camera model, and in each line assume all unspecified camera model parameters are identical). Mark which of each pair will produce a photograph with a **larger** depth of field.

      (i) ______ Focus at 2 meters OR
      ______ Focus at 1 meter
      (ii) ______ Focus at hyperfocal with 35 mm focal-length lens OR
      ______ Focus at hyperfocal with 70 mm focal-length lens.

(iii) ______ F-stop of f/2.0 OR
______ F-stop of f/4.0

(iv) ______ 25x25 mm sensor and 50 mm focal-length lens OR
______ 5x5 mm sensor and 10 mm focal-length lens

Solution: The answers are, in order: A, A, B, B.

(d) (8 points) Assuming a regular single-lens camera with an ideal thin lens with 50 mm focal length, with an f/2.0 aperture, and which is focused 1 meter away. Now consider objects very far away (i.e. at optical infinity).

i. Draw a ray diagram representing this problem that includes the following items and labels them: a distant object, an object at 1 meter, lens, focal point, sensor, and rays from distant objects and label the circle of confusion. Your drawing should be neat and clear, but does not need to be drawn to scale.

Solution:

ii. What is the size of the circle of confusion (in mm) that these distant objects make on the sensor surface?

(i) $2.0 \times \left(\frac{1}{\frac{1}{50.0} - \frac{1}{1000.0}} - 50.0 \right)$ mm

(ii) $\frac{1}{2.0} \times \left(\frac{1}{\frac{1}{50.0} - \frac{1}{1000.0}} - 50.0 \right)$ mm

(iii) $2.0 \times \left(\frac{1}{50.0} - \frac{1}{1000.0} \right)$ mm

(iv) $\frac{1}{2.0} \times \left(\frac{1}{50.0} - \frac{1}{1000.0} \right)$ mm

Solution: The answer is (i)

(e) (4 points) In the following, assume that all the image sensors considered have 100% quantum efficiency and zero read noise (i.e. the only noise present in the signal is due to Poisson shot noise). Each of the following cameras is used to take a picture of a patch of a uniformly illuminated white wall, with the same shutter duration across all cameras. The pixels do not saturate in any camera. Which will produce a photograph with individual pixels having the highest SNR?

(i) 10x10mm sensor with 35mm f/2.0 lens and 8 million pixels.

(ii) 20x20mm sensor with 50mm f/4.0 lens and 16 million pixels.

(iii) 5x5mm sensor with 10mm f/2.0 lens and 1 million pixels.

(iv) All the other choices have the same expected SNR at a single pixel.

Solution:
The answer is (iii) because each of its pixels is expected to capture the largest number of photons, and SNR is equal to square root of the signal in a shot-noise-limited image. (iii) has SNR that is 2x as many as (i) and 4x as many as (ii).

7. (Total : 12points) Animation

   (a) (4 points) Jack wrote a mass-spring simulation program using explicit Euler's method. The program works well in scene A, but blows up after a few iterations in scene B. If Jack's implementation is correct, which of the following modification(s) could make his program stable in scene B?

   (i) Decrease simulation step size.
   (ii) Switch to implicit Euler implementation.
   (iii) Add some damping to the system.
   (iv) Increase stiffness of the springs.

   Solution: The answer is (i), (ii) and (iii).

   (b) (8 points) Suppose a particle travels with velocity $v(t, x(t))$ along x-axis, where $x(t)$ is the x-coordinate of the particle. Recall that to numerically simulate $x(T) = \int_0^T v(t, x(t))dt$, we divide T into n small time intervals $\Delta t (T = n\Delta t)$ and use the following numerical integration schemes.

   (i) Explicit Euler $x(t + \Delta t) = x(t) + v(t, x(t))\Delta t$
   (ii) Implicit Euler $x(t + \Delta t) = x(t) + v(t + \Delta t, x(t + \Delta t))\Delta t$

   Complete the following table with simulation and analytic formulas for $x(T)$ in the following scenes with initial condition $x(0) = 0$ in terms of n and Δt. The first row has been completed for you as an example.

	Explicit Euler	Implicit Euler	Analytic
$v(t, x(t)) = 1$	$n\Delta t$	$n\Delta t$	$n\Delta t$
$v(t, x(t)) = t$			

Solution:

	Explicit Euler	Implicit Euler	Analytic
$v(t, x(t)) = 1$	$n\Delta t$	$n\Delta t$	$n\Delta t$
$v(t, x(t)) = t$	$\frac{1}{2}n(n-1)\Delta t^2$	$\frac{1}{2}n(n+1)\Delta t^2$	$\frac{1}{2}n^2\Delta t^2$