

**Due:** Monday 2/4/2019 at 11:59pm (submit via Gradescope).

Leave self assessment boxes blank for this due date.

**Self assessment due:** Monday 2/11/2018 at 11:59pm (submit via Gradescope)

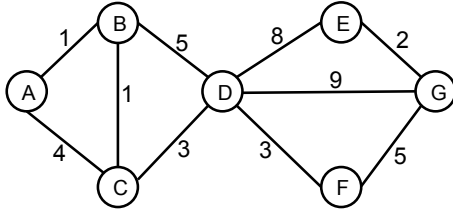
For the self assessment, **fill in the self assessment boxes in your original submission** (you can download a PDF copy of your submission from Gradescope). For each subpart where your original answer was correct, write “correct.” Otherwise, write and explain the correct answer.

**Policy:** Can be solved in groups (acknowledge collaborators) but must be written up individually

**Submission:** Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

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Collaborators	None

# Q1. Search



Node	$h_1$	$h_2$
A	9.5	10
B	9	12
C	8	10
D	7	8
E	1.5	1
F	4	4.5
G	0	0

Consider the state space graph shown above. A is the start state and G is the goal state. The costs for each edge are shown on the graph. Each edge can be traversed in both directions. Note that the heuristic  $h_1$  is consistent but the heuristic  $h_2$  is not consistent.

## (a) Possible paths returned

For each of the following graph search strategies (*do not answer for tree search*), mark which, if any, of the listed paths it could return. Note that for some search strategies the specific path returned might depend on tie-breaking behavior. In any such cases, make sure to mark *all* paths that could be returned under some tie-breaking scheme.

Search Algorithm	A-B-D-G	A-C-D-G	A-B-C-D-F-G
Depth first search	✓	✓	✓
Breadth first search	✓	✓	
Uniform cost search			✓
A* search with heuristic $h_1$			✓
A* search with heuristic $h_2$			✓

## (b) Heuristic function properties

Suppose you are completing the new heuristic function  $h_3$  shown below. All the values are fixed except  $h_3(B)$ .

Node	A	B	C	D	E	F	G
$h_3$	10	?	9	7	1.5	4.5	0

For each of the following conditions, write the set of values that are possible for  $h_3(B)$ . For example, to denote all non-negative numbers, write  $[0, \infty]$ , to denote the empty set, write  $\emptyset$ , and so on.

(i) What values of  $h_3(B)$  make  $h_3$  admissible?

$[0, 12]$

(ii) What values of  $h_3(B)$  make  $h_3$  consistent?

$[9, 10]$

(iii) What values of  $h_3(B)$  will cause A\* graph search to expand node A, then node C, then node B, then node D in order?

$\emptyset$

## Q2. $n$ -pacmen search

Consider the problem of controlling  $n$  pacmen simultaneously. Several pacmen can be in the same square at the same time, and at each time step, each pacman moves by at most one unit vertically or horizontally (in other words, a pacman can stop, and also several pacmen can move simultaneously). The goal of the game is to have all the pacmen be at the same square in the minimum number of time steps. In this question, use the following notation: let  $M$  denote the number of squares in the maze that are not walls (i.e. the number of squares where pacmen can go);  $n$  the number of pacmen; and  $p_i = (x_i, y_i) : i = 1 \dots n$ , the position of pacman  $i$ . Assume that the maze is connected.

(a) What is the state space of this problem?

The position of all pacmen. i.e.  $p_i = (x_i, y_i)$  ( $i = 1 \dots n$ )

(b) What is the size of the state space (not a bound, the exact size)?

$M^n$

(c) Give the tightest upper bound on the branching factor of this problem.

$5^n$

Each pacman has 5 choices.

i.e. stay, move upward, move downward, move leftward, move rightward.

(d) Bound the number of nodes expanded by uniform cost *tree* search on this problem, as a function of  $n$  and  $M$ . Justify your answer.

Number of nodes expanded  $\leq (5^n M^n - 1) / (5^n - 1)$

All arcs in this problem is 1. So the behavior of UCS in this problem is as same as BFS. The search ends when the shortest path is found. The length of this path is the depth of search tree. There're  $M$  possible position in maze. Therefore, the shortest path should be no longer than  $M$ . Since the branching factor is bound by  $5^n$ .

Number of nodes expanded  $\leq 1 + 5^n + (5^n)^2 + \dots + (5^n)^{M-1} = (5^n M^n - 1) / (5^n - 1)$

(e) Which of the following heuristics are admissible? Which one(s), if any, are consistent? Circle the corresponding Roman numerals and briefly justify all your answers.

1. The number of (ordered) pairs  $(i, j)$  of pacmen with different coordinates:

$$h_1(p_1, \dots, p_n) = \sum_{i=1}^n \sum_{j=i+1}^n \mathbf{1}[p_i \neq p_j] \quad \text{where} \quad \mathbf{1}[p_i \neq p_j] = \begin{cases} 1 & \text{if } p_i \neq p_j \\ 0 & \text{otherwise} \end{cases}$$

(i) Consistent? (ii) Admissible?

Neither consistent nor admissible.



If 4 pacmen are in 4 adjacent squares,  $h_1 = 3 + 2 + 1 = 6$ , but the actual cost is only 2.

And  $h_1$  could decrease more than 1 in each step, which means it's not consistent

2.  $h_2((x_1, y_1), \dots, (x_n, y_n)) = \frac{1}{2} \max \{ \max_{i,j} |x_i - x_j|, \max_{i,j} |y_i - y_j| \}$

☒ (i) Consistent? ☒ (ii) Admissible?

Both consistent and admissible.

Even without walls, it will take at least  $h_2$  steps for the farthest two pacman meet.

So it's admissible.

And  $h_2$  could at most decrease 1 in each step, so it's consistent.