CS 188
Spring 2019

Final Review Logic Solutions

Q1. Propositional Logic

(Taken from Russell and Norvig 7.15) This question considers representing satisfiability (SAT) problems as CSPs.

(a) Draw the constraint graph corresponding to the SAT problem

$$(\neg X_1 \vee X_2) \wedge (\neg X_2 \vee X_3) \wedge ... \wedge (\neg X_{n-1} \vee X_n)$$

for the particular case $n = 5$.

The graph is simply a connected chain of 5 nodes, one per variable.

(b) How many solutions are there for this general SAT problem as a function of n?

$n + 1$ solutions. Once any X_i is true, all subsequent X_js must be true. Hence, the solutions are i falses followed by $n - i$ trues, for $i = 0, ..., n$.

(c) Suppose we apply `Backtracking-Search` to find *all* solutions to a SAT CSP of the type given in (a). (To find *all* solutions to a CSP, we simply modify the basic algorithm so it continues searching after each solution si found.) Assume that variables are ordered $X_1, ..., X_n$ and *false* is ordered before *true*. How much time will the algorithm take to terminate? (Write an $O(\text{cot})$ expressino as a function of n.)

The complexity is $O(n^2)$. This is somewhat tricky. Consider what part of the complete binary tree is explored by the search. The algorithm must follow all solution sequences, which themselves cover a quadratic-sized portion of the tree. Failing branches are all those trying a *false* after the preceding variable is assigned *true*. Such conflicts are detected immediately, so they do not change the quadratic cost.

Q2. First Order Logic

(Taken from Russell and Norvig 8.10) Consider a vocabular with the following symbols:

- $Occuption(p, o)$: Predicate. Person p has occuption o.
- $Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
- $Boss(p1, p2)$: Predicate. Person $P1$ is a boss of person $p2$.
- $Doctor, Surgeon, Lawyer, Actor$: Constants denoting occupations.
- $Emily, Joe$: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

a Emily is either a surgeon or a lawyer. $O(E, S) \vee O(E, L)$

b Joe is an actor, but he also holds another job. $O(J, A) \wedge \exists p\ p \neq A \wedge O(J, p)$

c All surgeons are doctors. $\forall p\ O(p, S) \Rightarrow O(p, D)$

d Joe does not have a lawyer (i.e., is not a customer of any lawyer). $\neg\exists p\ C(J, p) \wedge O(p, L)$

e Emily has a boss who is a lawyer. $\exists p\ B(p, E) \wedge O(p, L)$

f There exists a lawyer all of whose customers are doctors. $\exists\ pO(p, L) \wedge \forall\ qC(q, p) \Rightarrow O(q, D)$

g Every surgeon has a lawyer. $\forall p\ O(p, S) \Rightarrow \exists\ qO(q, L) \wedge C(p, q)$