CS 188
Spring 2019

Final Review HMMs

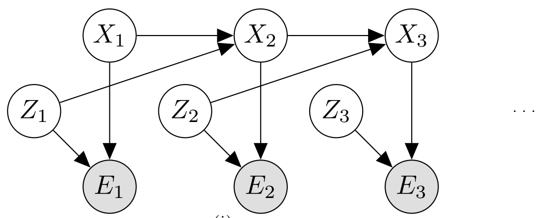
Q1. Modified HMM Updates

(a) Recall that for a standard HMM the Elapse Time update and the Observation update are of the respective forms:

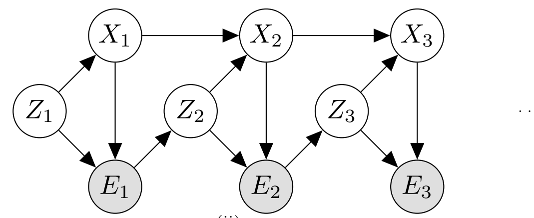
$P(X_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(X_t \mid x_{t-1}) P(x_{t-1} \mid e_{1:t-1})$

$P(X_t \mid e_{1:t}) \propto P(X_t \mid e_{1:t-1}) P(e_t \mid x_t)$

We now consider the following two HMM-like models:



(i)



(ii)

Mark the modified Elapse Time update and the modified Observation update that correctly compute the beliefs from the quantities that are available in the Bayes' Net. (Mark one of the first set of six options, and mark one of the second set of six options for (i), and same for (ii).)

(i)

- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}, z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t \mid x_{t-1}, z_{t-1}) P(Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}, z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t \mid x_{t-1}, z_{t-1})$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}, z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t, Z_t \mid x_{t-1}, z_{t-1})$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t \mid x_{t-1}, z_{t-1}) P(Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t \mid x_{t-1}, z_{t-1})$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t, Z_t \mid x_{t-1}, z_{t-1})$

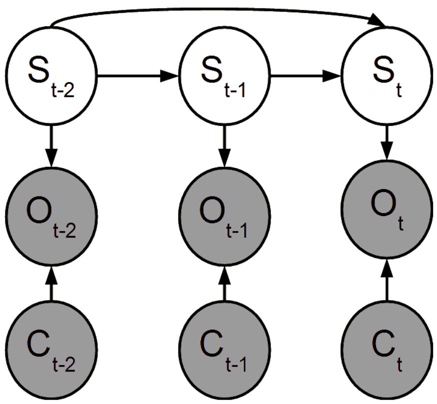
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t, Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto \sum_{X_t} P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t, Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto \sum_{Z_t} P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t, Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t) P(e_t \mid Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) \sum_{X_t} P(e_t \mid X_t)$

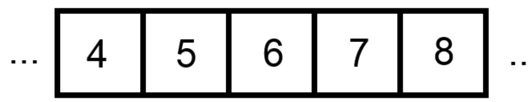
(ii)

- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}, z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t \mid x_{t-1}, z_{t-1}) P(Z_t \mid e_{t-1})$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}, z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(Z_t \mid e_{t-1}) P(X_t \mid x_{t-1}, Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}, z_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t, Z_t \mid x_{t-1}, e_{t-1})$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t \mid x_{t-1}, z_{t-1}) P(Z_t \mid e_{t-1})$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(Z_t \mid e_{t-1}) P(X_t \mid x_{t-1}, Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}, z_{t-1} \mid e_{1:t-1}) P(X_t, Z_t \mid x_{t-1}, e_{t-1})$

- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t, Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto \sum_{X_t} P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t, Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto \sum_{Z_t} P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t, Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t) P(e_t \mid Z_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) P(e_t \mid X_t)$
- ◯ $P(X_t, Z_t \mid e_{1:t}) \propto P(X_t, Z_t \mid e_{1:t-1}) \sum_{X_t} P(e_t \mid X_t)$

Q2. Particle Filtering





Pacman is trying to hunt a ghost in an infinite hallway with positions labeled as in the picture above. He's become more technologically savvy, and decided to locate find the ghosts actual position, $\mathbf{S_t}$, using some sensors he set up. From the sensors, Pacman can find, at each time step, a noisy reading of the ghost's location, $\mathbf{O_t}$. However, just as Pacman has gained technology, so has the ghost. It is able to cloak itself at each time step, given by $\mathbf{C_t}$, adding extra noise to Pacman's sensor readings.

Pacman has generated an error model, given in the table below, for the sensor depending on whether the ghost is cloaked or not.

Pacman has also generated a dynamics model, given in the table below, that takes into account the position of the ghost at the two previous timesteps.

Dynamics model:

$P(S_t|S_{t-1}, S_{t-2}) = F(D_1, D_2)$
$D_1 = |S_t - S_{t-1}|$
$D_2 = |S_t - S_{t-2}|$

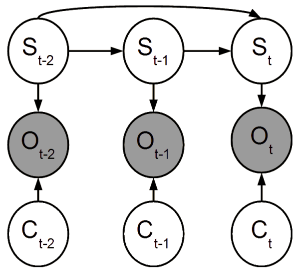
D_1	D_2	F(D_1, D_2)
0	0	0.7
0	1	0.2
0	2	0
1	0	0.3
1	1	0.3
1	2	0.5

Observation model:

$P(O_t|S_t, C_t) = E(C_t, D)$
$D = |O_t - S_t|$

C	D	E(C, D)
+	0	0.4
+	1	0.2
+	2	0.1
-	0	0.6
-	1	0.2
-	2	0

(a) Assume that you currently have the following two particles: $(S_6 = 7, S_7 = 8)$ and $(S_6 = 6, S_7 = 6)$. Compute the weights for each particle given the observations $C_6 = +, C_7 = -, O_6 = 5$, $O_7 = 8$:



C	P(C)
+	0.5
-	0.5

$(S_6 = 7, S_7 = 8)$	
$(S_6 = 6, S_7 = 6)$	

(b) Assume that Pacman can no longer see whether the ghost is cloaked or not, but assumes that it will be cloaked at each timestep with probability 0.5. Compute the weights for each particle given the observations $O_6 = 5$, $O_7 = 8$:

$(S_6 = 7, S_7 = 8)$	
$(S_6 = 6, S_7 = 6)$	

(c) To prevent error propagation, assume that after weighting the particles and resampling, one of the particles you end up with is $(S_6 = 6, S_7 = 7)$.

(i) What is the probability that after passing this particle through the dynamics model it becomes $(S_7 = 6, S_8 = 6)$?

(ii) What is the probability the particle becomes $(S_7 = 7, S_8 = 8)$?

(d) To again decouple this part from previous parts, assume that you have the following three particles with the specified weights.

Particle	weight
$(S_7 = 5, S_8 = 6)$	.1
$(S_7 = 7, S_8 = 6)$	.25
$(S_7 = 7, S_8 = 7)$	.3

What is Pacman's belief for the ghost's position at time $t = 8$?

Position	P(S_8)
$S_8 = 5$	
$S_8 = 6$	
$S_8 = 7$	
$S_8 = 8$	