

Processo MA(3) -----

$$MA(3) = y_t - \mu = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3}$$

$$\gamma_1 = E[(y_t - \mu)(y_{t-1} - \mu)]$$

$$\gamma_1 = E[(e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3})(e_{t-1} + \theta_1 e_{t-2} + \theta_2 e_{t-3} + \theta_3 e_{t-4})]$$

$$\gamma_1 = \theta_1 E(e_{t-1}^2) + \theta_1 \theta_2 E(e_{t-2}^2) + \theta_2 \theta_3 E(e_{t-3}^2) = (\theta_1 + \theta_1 \theta_2 + \theta_2 \theta_3) \sigma^2$$

$$\gamma_2 = E[(y_t - \mu)(y_{t-2} - \mu)]$$

$$\gamma_2 = E[(e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3})(e_{t-2} + \theta_1 e_{t-3} + \theta_2 e_{t-4} + \theta_3 e_{t-5})]$$

$$\gamma_2 = \theta_2 E(e_{t-2}^2) + \theta_1 \theta_3 E(e_{t-3}^2) = (\theta_2 + \theta_1 \theta_3) \sigma^2$$

$$\gamma_3 = E[(y_t - \mu)(y_{t-3} - \mu)]$$

$$\gamma_3 = E[(e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3})(e_{t-3} + \theta_1 e_{t-4} + \theta_2 e_{t-5} + \theta_3 e_{t-6})]$$

$$\gamma_3 = \theta_3 E(e_{t-3}^2) = \theta_3 \sigma^2$$

Processo MA(4) -----

$$MA(4) = y_t - \mu = e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3} + \theta_4 e_{t-4}$$

$$\gamma_1 = E[(e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3} + \theta_4 e_{t-4})(e_{t-1} + \theta_1 e_{t-2} + \theta_2 e_{t-3} + \theta_3 e_{t-4} + \theta_4 e_{t-5})]$$

$$\gamma_1 = \theta_1 E(e_{t-1}^2) + \theta_1 \theta_2 E(e_{t-2}^2) + \theta_2 \theta_3 E(e_{t-3}^2) + \theta_3 \theta_4 E(e_{t-4}^2) =$$

$$(\theta_1 + \theta_1 \theta_2 + \theta_2 \theta_3 + \theta_3 \theta_4) \sigma^2$$

$$\gamma_2 = E[(y_t - \mu)(y_{t-2} - \mu)]$$

$$\gamma_2 =$$

$$E[(e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3} + \theta_4 e_{t-4})(e_{t-2} + \theta_1 e_{t-3} + \theta_2 e_{t-4} + \theta_3 e_{t-5} + \theta_4 e_{t-6})]$$

$$\gamma_2 = \theta_2 E(e_{t-2}^2) + \theta_1 \theta_3 E(e_{t-3}^2) + \theta_2 \theta_4 E(e_{t-4}^2) = (\theta_2 + \theta_1 \theta_3 + \theta_2 \theta_4) \sigma^2$$

$$\gamma_3 =$$

$$E[(e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3} + \theta_4 e_{t-4})(e_{t-3} + \theta_1 e_{t-4} + \theta_2 e_{t-5} + \theta_3 e_{t-6} + \theta_4 e_{t-7})]$$

$$\gamma_3 = \theta_3 E(e_{t-3}^2) + \theta_1 \theta_4 E(e_{t-4}^2) =$$

$$(\theta_3 + \theta_1 \theta_4) \sigma^2$$

$$\begin{aligned}\gamma_4 &= \\ E[(e_t + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \theta_3 e_{t-3} + \theta_4 e_{t-4})(e_{t-4} + \theta_1 e_{t-5} + \theta_2 e_{t-6} + \theta_3 e_{t-7} + \theta_4 e_{t-8})] \\ \gamma_4 &= \theta_4 E(e_{t-4}^2) = \\ &\theta_4 \sigma^2\end{aligned}$$

Conclusão-----

MA(1):

$$\gamma_1 = \theta_1 \sigma^2$$
$$\gamma_j = 0, se j > 1$$

MA(2):

$$\gamma_1 = (\theta_1 + \theta_1 \theta_2) \sigma^2$$
$$\gamma_2 = \theta_2 \sigma^2$$
$$\gamma_j = 0, se j > 2$$

MA(3):

$$\gamma_1 = (\theta_1 + \theta_1 \theta_2 + \theta_2 \theta_3) \sigma^2$$
$$\gamma_2 = (\theta_2 + \theta_1 \theta_3) \sigma^2$$
$$\gamma_3 = \theta_3 \sigma^2$$
$$\gamma_j = 0, se j > 3$$

MA(4):

$$\gamma_1 = (\theta_1 + \theta_1 \theta_2 + \theta_2 \theta_3 + \theta_3 \theta_4) \sigma^2$$
$$\gamma_2 = (\theta_2 + \theta_1 \theta_3 + \theta_2 \theta_4) \sigma^2$$
$$\gamma_3 = (\theta_3 + \theta_1 \theta_4) \sigma^2$$
$$\gamma_4 = \theta_4 \sigma^2$$
$$\gamma_j = 0, se j > 4$$