Modeling an analog quantum computer.

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Plan

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1. Motivation
2. Computation: spins and qubits
3. Simulation: adiabatic quantum computer

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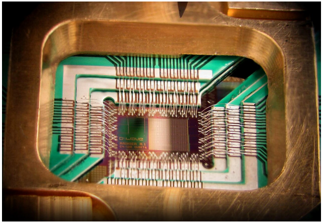
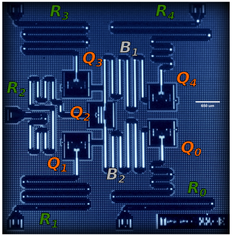
Why quantum computation?

- crossroad of physics and computer science
- cirquit model of QC (IBM, IonQ, Rigetti)
- analog model of QC: D-Wave company
- Google, IBM, Microsoft
  $\rightarrow$ extreme effort to find a practical application

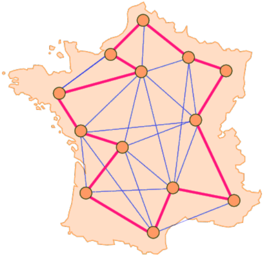




What to solve?

typically combinatorial optimization problems

- traveling salesman problem
- knapsack problem
- graph colouring, SAT logical problems



complexity classes:
examples of **NP**-complete problems – particularly hard to solve

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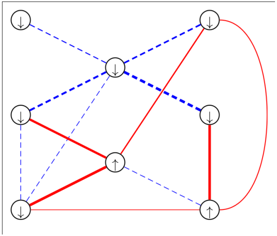
Optimization for random Ising problem

- optimization problem related to physics of magnetism
- N spins, which can take only values ± 1 (or $\uparrow, \downarrow$)
- energy of the system is our cost function
  (increases for a pair of misplaced objects)

$$H_0 = E(s_1, \ldots, s_N) = -\sum_{ij} J_{ij} s_i s_j - \sum_i h_i s_i$$



- competing links connecting spins: F or AF

Can nature solve e.g. spin-glass problem?

- G. Parisi (Nobel 2021)
  random Ising model → physical phase of spin-glass with very slow relaxation
- annealing algorithm: MC with thermal fluctuations, lowering the temperature → system freezes in metastable states
- first indications: an improvement with quantum annealing

Article | Published: 19 April 2023

Quantum critical dynamics in a 5,000-qubit programmable spin glass

Andrew D. King, Jack Raymond, Trevor Lanting, Richard Harris, Alex Zucca, Fabio Altomare, Andrew J. Berkley, Kelly Boothby, Sara Ejtemaee, Colin Enderud, Emile Hoskinson, Shuiyuan Huang, Eric Ladizinsky, Allison J. R. MacDonald, Gaelen Marsden, Reza Molavi, Travis Oh, Gabriel Poulin-Lamarre, Mauricio Reis, Chris Rich, Yuki Sato, Nicholas Tsai, Mark Volkmann, Jed D. Whittaker, ... Mohammad H. Amin

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Abstract

Experiments on disordered alloys[1,2,3] suggest that spin glasses can be brought into low-energy states faster by annealing quantum fluctuations than by conventional thermal annealing. Owing to the importance of spin glasses as a paradigmatic computational testbed, reproducing this phenomenon in a programmable system has remained a central challenge in

Spin as qubit – elementary building block

- states of the form: $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ (computational basis)
- important state: $|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$
  random results along z, oriented along x

Spin operators:

along z-axis: $s_z|\uparrow\rangle = |\uparrow\rangle$, $s_z|\downarrow\rangle = -|\downarrow\rangle$
along x-axis: $s_x|\uparrow\rangle = |\downarrow\rangle$, $s_x|\downarrow\rangle = |\uparrow\rangle$
simple check: what is $s_x|\rightarrow\rangle =$?

Exponential curse or opportunity?

- spin represented as a qubit; N-qubit system discribed by 2^N-component vector
- probablitiy amplitudes do really exist (entanglement)
- make quantum system "explore" all combinatorial possibilities at once $\longrightarrow$ quantum speed up
- general idea:
  system with $H = -f(t)H_0 - g(t)\sum_i s_i^x$
  from $f(t) = 0$ untill $g(T_{\max}) = 0$ (D-wave hardware)

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Abstract engineering

TARGET: quantum form of the spin-glass problem

$$H_0 = -\sum_{ij} J_{ij} s_i^z s_j^z - \sum_i h_i s_i^z$$

the corresponding ground state is $|\mathrm{GS}\rangle_0 = |s_1, s_2, \ldots\rangle$ where $\{s_i\}$ minimizes the target function $E(s_1, s_2, \ldots, s_N)$.

FLUCTUATIONS: most democratic state as the groundstate of

$$H_1 = -\sum_i s_i^x$$

with $|\mathrm{GS}\rangle_1 = |\rightarrow_1, \rightarrow_2, \rightarrow_3\rangle$
$= \frac{1}{\sqrt{2}^3} \left(|\uparrow,\uparrow,\uparrow\rangle + |\uparrow,\uparrow,\downarrow\rangle + |\uparrow,\downarrow,\uparrow\rangle + |\uparrow,\downarrow,\downarrow\rangle + \ldots\right)$
for $N = 3$ as an example.

Adiabatic computation

Transversal field Ising model

$$H(\lambda) = (1-\lambda)H_1 + \lambda H_0$$

where we control the "experimental" knob, which is $\lambda \in [0, 1]$.

Evolve slowly: ensure the system stays in the ground state. That is Adiabatic Quantum Computer.

Worst case scenario:
when ΔE close to zero, but we have to avoid exciting the system (energy gap $\Delta E(\lambda)$ for lowest eigenenergies of $H(\lambda)$).

How quickly can we run adiabatic computation?

Landau-Zener formula

$$\frac{d\lambda}{dt} \propto (\Delta E(\lambda))^2$$

guaranties that speed is low enough.

(check: https://arxiv.org/abs/2306.11633 “Landau Zener problem in the classroom”)

- study the scaling of T_{adiab} with N
- few analytical results eg. database (Grover) search $T_{\text{adiab}} \propto \sqrt{2^N}$
- amazing link with quantum phase transitions: $\Delta E \to 0$ for $N \to \infty$

Final remarks

- trotterization gives the mapping:
  adiabatic evolution $\rightarrow$ circuit model
- adiabatic QC idea is useful as a sub-routine e.g. in QC studies of molecules (new VQE, QAOA approaches)

Thesis subjects available

- QMC for (1+1)D chiral fermions.
- Transfer matrix: from simple to artisitc geometry.
- Optimal controll for a transmon qubit.
- Limits of quantum computing in a chaotic system.
- Strain bubbles in graphene.