Complexity: neural networks.

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Plan

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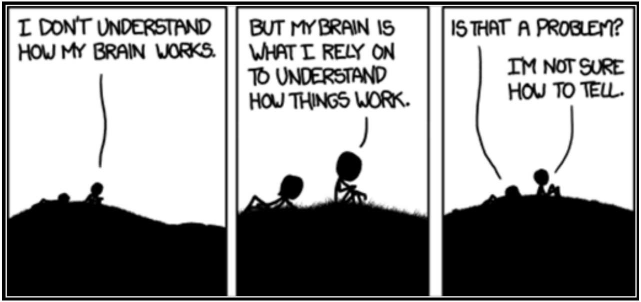
1. Motivation
2. Development of basic ideas
3. Hopfield network

Plan

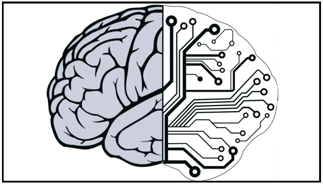
Ultimate complexity

Brain?



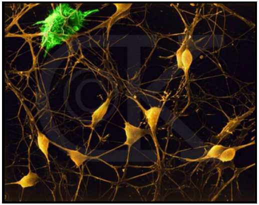
"If the human brain were so simple that we could understand it, we would be so simple that we couldn't" E.M. Pugh

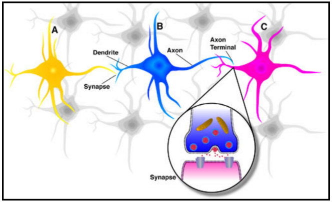
Brain vs. computer



200×10^9 neurons	10^9 bytes of RAM
32×10^{12} synapses	10^{12} bytes on disk
Element size: $10^{-6}\mathrm{m}$	Element size: $10^{-9}\mathrm{m}$
Energy use: $25\mathrm{W}$	Energy use: $30 - 90\mathrm{W}$
Processing speed: $100\mathrm{Hz}$	Processing speed: $10^9\mathrm{Hz}$
Parallel, distributed	Serial, centralized
Fault tolerant	not Fault tolerant
Learns: Yes	Learns: Some

Neurons in the brain





Although heterogeneous,
at a low level the brain is composed of neurons.

Neurons communicate with each other by passing electrochemical signals. Typical neuron: up to 10000 connections.

When the input exceeds certain voltage – the neron "fires" an electrical spike.

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Neural networks - history

- 1933 - Edward Thorndike: human learning consists in the strengthening of some (then unknown) property of neurons
- 1949 - Donald Hebb: it is specifically a strengthening of *connections* between neurons in the brain that accounts for learning

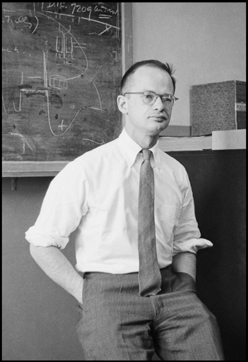
„When an axon of cell A is near enough to excite cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased."

D. Hebb, The Organization of Behavior, 1949

or "Neurons that fire together wire together."

(Hebb's Law)

Walter Pitts and Warren McCulloch



1923-1969

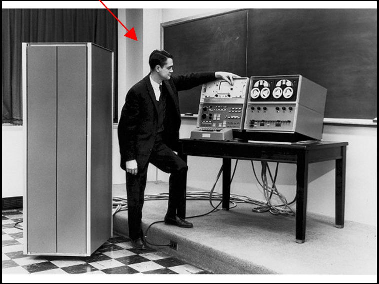
„A logical calculus of the ideas immanent in nervous activity"
Bulletin of Mathematical Biophysics, 5:115-133.

1898-1969

- each neuron in the brain is a simple digital processor and the brain as a whole is a form of computing machine

Belmont Farley and Wesley Clark

- the first computer simulations of small neural networks, training the networks containing 128 neurons to recognise simple patterns
- they discovered that the random destruction of up to 10% of the neurons in a trained network does not affect the network's performance



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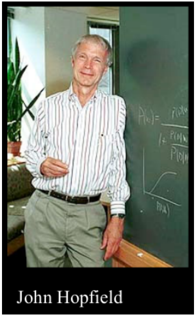
Recurrent Neural Network (1982)



Proc. Natl. Acad. Sci. USA
Vol. 79, pp. 2554–2558, April 1982
Biophysics

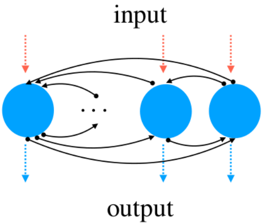
Neural networks and physical systems with emergent collective computational abilities

(associative memory/parallel processing/categorization/content-addressable memory/fail-soft devices)

J. J. HOPFIELD

Division of Chemistry and Biology, California Institute of Technology, Pasadena, California 91125; and Bell Laboratories, Murray Hill, New Jersey 07974

Contributed by John J. Hopfield, January 15, 1982



Model of associative memory – binary Hopfield network.

Inspired by physics of spin glasses, neuron variable $x_i = \pm 1$.

Hebb's rule

"Neurons that fire together, wire together. Neurons that fire out of sync, fail to link".

We learn N binary (spin) patterns:

$$W_{ij} = \frac{1}{N} \sum_n x_i^{(n)} x_j^{(n)} - \delta_{ij}.$$

If the bits corresponding to neurons i and j are equal in the pattern, the product will be positive. This, in turn, will have a positive effect on the weight W, and the values of i and j will tend to become equal. The opposite happens when the bits corresponding to neurons i and j are different.

Updating the network

The energy of the spins is $E = \sum_i E_i$, written for $i\mathrm{th}$ spin:

$$E_i = -\frac{1}{2} x_i \sum_j W_{ij} x_j = -\frac{1}{2} \sum_i x_i h(i).$$

We update the spins $x_i(t)$ to reach the energy minimum. If the product $x_i h_i$ is negative, then the field h_i would try to flip the spin:

$$x(t+1) = \mathrm{sgn}(Wx(t))$$

- trained patterns become attractors
- updates lead to convergance
- spurious patterns can arise as (local) energy minima
- the capacity of the network ≈ 0.138 pattern per node