**Probability**

**Random experiment**: Any experiment which is not sure to happen, also called undeterministic experiments. Ex: Tossing coin can leads to Heads or Tails, we are not sure about Heads and Tails.

Whenever a experiment(random experiment) is performed, a set which contains all possibilities is called Sample space(S). Ex: A die is rolled then S = {1, 2, 3, 4, 5, 6}

The subset of Sample space(S) is called event E. Ex: E = odd numbers when a die is rolled, E = {1, 3, 5}.

**Probability of Event E (P(E))** : It defines the chances of happening of that event, which can be mathematically expressed as ratio of number of favorable outcomes related that event to total possible outcomes (generally sample space).

**P(E) = favorable outcomes of event / sample space.**

It is more likely the counting of number of elements which is same as **Combinatorics**. So, we could use combinations and permutations when necessary.

Similarly, Probability in Geometry :

In 1D : P = favourable length / total length.

In 2D : P = favourable Area / total area.

In 3D : P = favourable volume / total volume.

Ex: A die is rolled and find the probability of getting even number.

Soln: S = {1, 2, 3, 4, 5, 6}, E = {2, 4, 6}. then P(E) = 3 / 6 => 1 / 2

**Types of Events:**

**Equally likely events**: Those events whose probability of occurrence is same.

Ex: A die is rolled, S = {1, 2, 3, 4, 5, 6}, Find the prob. of getting even and prob. getting odd.

Soln: E1 = {1, 3, 5}, E2 = {2, 4, 6},

P(E1) = 3/ 6 => 1 / 2

P(E2) = 3 / 6 => 1 / 2

As you can see P(E1) = P(E2), so they are Equally likely events.

**Mutually Exclusive events:** Those events which are disjoint and can’t occurr simultaneously. For ex: A die is rolled, E1 = coming out even number, E1 = coming out odd number. Now as you can see a die can show either an odd number or even number, not both simultaneoulsy. So if use set operation Intersection(Ⴖ),

|E1 Ⴖ E2 |= 0.

**Exhaustive events**: A set events E1, E2, E3… is called exhaustive events when the union of those events gives the sample sample(S), i.e., (E1 U E2 U E3…) = S.

Ex: A die is rolled, E1 = getting even number, E2 = getting odd number. E1 = {2, 4, 6}, E2 = {1, 3, 5}, E1 U E2 = {1, 2, 3, 4, 5, 6} same as S.

**Independent events:** Those events are called independent when they don’t effect each other or we can say, the occurrence of one event doesn’t effect the occurrence of other. Ex: 2 coins are tossed, the output on Coin-1 is independent of the ouput on coin-2.

**Theorems in Probability:**

1. **Addition of Probability**: As you learn in set theory:

| A U B | = |A| + |B| - |A Ⴖ B|, (**Using Venn’s diagram**) |X| = Cardinality of set X

Now, divide by |S| on both sides,

P(A U B) = P(A) + P(B) - P(A Ⴖ B),

**NOTE**: Venn’s diagram is very important in set as well as probability.

In **Mutually exclusive** events, P(A Ⴖ B) = 0, so,

P(A U B) = P(A) + P(B).

**Ex**: Given three events A, B and C of random experiment, then find the probability of occurrence of exactly two events from A, B, and C.

Soln: Using Venn’s diagram:

P(Required) = P(A Ⴖ B) + P(B Ⴖ C) + P(C Ⴖ A) - 3.P(A Ⴖ B Ⴖ C).

**NOTE**: You can use the points trick in in Venn’s diagram to find the above result.

**Ex**: P(A’ Ⴖ B) = P(B) - P(A Ⴖ B).

**Ex**: Find the probability of exactly one event occurr among A, B, and C.

**Soln**: Using Venn’s diagram,

P(Required) = P(A) + P(B) + P(C) - P(A Ⴖ B) - P(B Ⴖ C) - P(C Ⴖ A) + 3.P(A Ⴖ B Ⴖ C).

Ex: find the probability of exactly one event occurr among A & B.

Soln: Using Venn’s diagram,

P(Required) = P(A) + P(B) - 2.P(A Ⴖ B)

1. **Conditional Probability:** Suppose we performed a random experiment, and if we pick two events E1, E2 from experiment, and assume that E1 must occur then what is the probability that E2 will also occurr because of occurence of E1,

So, this is possible when both E1 and E2 are not mutually exclusive, or we can say, |**E1 Ⴖ E2| > 0.** denoted by **P(E2 | E1)** (probability of E2 due to occurence of E1).

**Formula: P(E2 | E1)** **= P(E1 Ⴖ E2) / P(E1)**, P(E1) != 0.

**Ex**: A die is tossed, E1 = getting an odd number, E2 = getting a prime number, now suppose E1 must occurr, then what is the probability of occurrence of E2 due to occurrence of E1.

Soln: S = {1, 2, 3, 4, 5, 6}, E1 = {1, 3, 5}, E2 = {2, 3, 5}.

Now, if E1 will must occurr and we want to find the probability of E2 when E1 will must occurr, ie., **P(E2 | E1).**

P(E1 Ⴖ E2) = {3, 5}.

P(E2 | E1) = 2 / 3.

**Some observations:**

1. P(A / B) + P(A’ / B) = 1
2. **Multiplication Theorem**: Suppose an experiment is performed and if we two events E1, E2, then find the probability of occurrence of E1 as well as E2 at at the same time (or we can say, ocurrence of E1 and E2 consectively, that is one after another).

To solve this problem we can use Venn’s Diagram or use common sense which says: P(E1 Ⴖ E2) = Prob. of occuring E1 and E2.

Ⴖ => Intersection(AND). P(E1 Ⴖ E2) = Occurrence of E1 and E2 simultaneouly (or we can say consectively).

Then, formula: **P(A Ⴖ B) = P(A). P(B | A).**

P(A Ⴖ B) => Probability of happening of A as well as B simultaneouly (or consecutively).

P(A) => Probability of A.

P(B | A) => probability of B when A is performed (Probability of B under effect of A).

**Ex**: Given a deck of card and pick 2 cards, find the probability of getting 1 king and 1 queen, which is **P(1K Ⴖ 1Q) = P(1K).P(1Q | 1K)**.

With replacement: P(1K Ⴖ 1Q) = 4/52 \* 4/52

Without replacetent = P(1K Ⴖ 1Q) = 4/52 \* 4 / 51

**Ex**: Given a box which has 5 red, 4 blue, 10 white balls, find the probability of getting 1 Red, blue and 1 white, 1 white ball, P(1R Ⴖ 1B Ⴖ 1W) = ?

P(1R Ⴖ 1B Ⴖ 1W) = P(1R) \* P(1B | 1R) \* P(1W | (1R Ⴖ 1B))

With replacement: P(1R Ⴖ 1B Ⴖ 1W) = 5/19 \* 4/19 \* 10 / 19.

Without replacement: P(1R Ⴖ 1B Ⴖ 1W) = 5/19 \* 4 / 18 \* 10 / 17

**Ex**: Given a box which has 5 red, 4 blue, 10 white balls, find the probability of getting 3 Red balls, P(1R’ Ⴖ 1R’’ Ⴖ 1R’’) = ? (Balls are picked one after after another consecutively).

P(1R’ Ⴖ 1R’’ Ⴖ 1R’’’) = P(1R’) \* P(1R’’ | 1R’) \* P(1R’’’ | (1R’ Ⴖ 1R’’’))

Similarly : Without replacement: P(1R’ Ⴖ 1R’’ Ⴖ 1R’’) = 5/19 \* 4/18 \* 3/17

In the above examples, the events may be dependent or independent according to the question, that means: for eg1: We picked 1 king card now place it back to the deck and now this time pick 1-queen. This type of replacement event is **Independent event**. So here occurrence of one event doesnot affects the occurence of another. But in case of **dependent events**, events affect on each other, for ex: pick 1-king card now don’t replace it back and pick the queen card.

**Note**: Here our meaning of simultaneoulsy is same as consectively that means 3 balls are picked one after another.

In **independent events:** P(A Ⴖ B) = P(A) . P(B), because they don’t effect on each other or (B | A) = P(B).

Similarly, concepts are try with complements:

P(A Ⴖ B’) = P(A) . P(B’)

P(A’ Ⴖ B’) = P(A’) . P(B’).

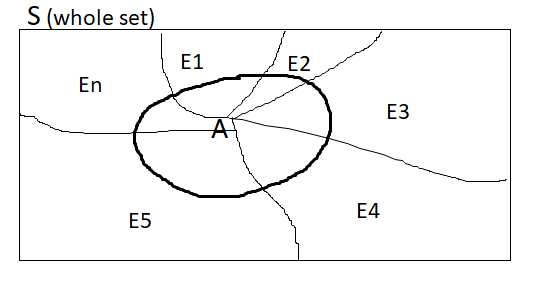
**Demorgan’s Law:** P(A’ Ⴖ B’ Ⴖ C’) = P((A U B U C)’) = 1 - P(A U B U C)

Similarly: P(A’ Ⴖ B’ ) = P((A U B)’)

**Total Probability:**

Suppose we have some **mutually exclusive** and **exhaustive** events E1, E2, E3..En and because they are exhaustive so, they form a set S (E1 U E2 U …En = S). There is a event ‘A’ which happened due to E1, E2…En events, that means there is some

|A Ⴖ E1|,|A Ⴖ E2|, |A Ⴖ E3|…|A Ⴖ En|. You can also say E1, E2…En are the **cause of event A** or **reason of event A.**



Then to find the probability of A:

P(A) = P(A Ⴖ E1) + P(A Ⴖ E2) + P(A Ⴖ E3)…P(A Ⴖ En). (By diagram)

P(A) = P(E1).P(A | E1) + P(E2).P(A | E2) + P(E3).P(A | E3) … P(En).P(A | En).

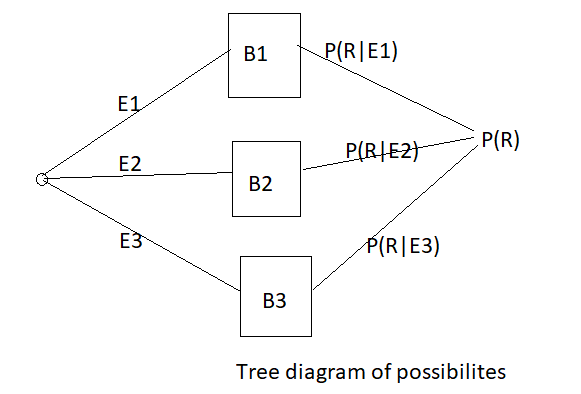
**P(A) = Ʃ P(Ei) . P(A | Ei)**

**Note**: So here the main goal is to recognize which one is event **A (goal)** and which events are the **reason / cause** of happening of event A. Those events (cause) must be mutually exclusive and exhaustive events.

**Ex**: Given 3 boxes B1, B2, B3 which contains (3Red, 4Blue balls), (5Red, 3Blue balls) and (10Red, 2Blue balls) respectively. Find the probability of getting 1-red ball.

**Soln**: Goal is to get the red ball, event A = getting 1 red ball, P(A) = P(R)

Reason/causes/sources to get the red balls are boxes B1, B2 & B3.



P(R) = P(RႶE1) + P(R Ⴖ E2) + P(R Ⴖ E3)

P(R) = P(E1).P(R|E1) + P(E2).P(R|E2) + P(E3).P(R|E3)

P(E1) => Probability to getting box-1

P(E2) => Probability to getting box-2

P(E3) => Probability to getting box-3

P(R|E1) => Probability of getting 1-Red when box-1 is chosen

P(R|E2) => Probability of getting 1-Red when box-2 is chosen

P(R|E3) => Probability of getting 1-Red when box-3 is chosen

P(R) = 1/3. (3/7) + 1/3. (5 / 8) + 1/3. (10/12)

**Note**:

1. Each path is mutually exclusive, picking the ball from one box doesn’t affect on another one and at a time we choose only one path (left to right).
2. Once a box is selected it doesn’t effects on another box.

**Bayes Theorem:** It is combination of conditional probability and total probability. In total probability we have given probabilites related to the event (cause) and we have to find the probability of the A (goal event). But in Bayes thorem, we have given the goal event A (must occur) and then we have to find what probability that some event E will be the cause/reason to happening the event A. So we can say that, Bayes theorem helps to determine the probability of some cause event or how much that event E is responsible of occurrence of A(goal).

**Ex:** In the previous example of total probablity, we have 3boxes and 1-red ball is drawn and we have to find the probability of that red ball. But in bayes theorem, suppose that, same those three boxes are given and a red ball is drawn, find the probablity that it was drawn from Box-1 or cause/reason/source was Box-1.

Remember here also, all cause events are mutually exclusive and exhaustive.

P(cause | A) => P(E | A) => Probability of event E such that it was the cause of occurrence of A.

**P(E | A) = P(E Ⴖ A) / P(A)** (By conditional probabilty)

P(A) = Probability of happening event A, this could be determined by Total probabilty theorem: **P(A) = Ʃ P(Ei) . P(A | Ei)**

P(E Ⴖ A) = P(E).P(A|E) (By multiplication theorem, which is same as one of the branch in Total probability)

**P(Ei | A) = P(Ei).P(A|Ei) / Ʃ P(Ei) . P(A | Ei)**

**Note**: Here again, is to recognize the **reasons/cause** events and goal **event A** which is performed. It is related to the total probabilty and conditioal probability, so the idea is same.

**Binomial Trials**: Suppose we have performed an experiment (any two values are possible: success or failure) **n-times**, we have picked the event E which is get the success r-times, then what will be probability to get success r-times:

**P(n, r) = C(n, r).pr.qn-r ,** p = probability of getting success when experiment is performed only once, q = probability of getting failure when experiment is performed only once. **p + q = 1**

**P(n, r)** = Probability of getting success r-times when experiment is performed n-times.

You can prove this formula easily:

Suppose we have **n** spaces, which contains **p** r-times and **q** n-r times, then how many ways we can arrange them is, C(n, r) and in each arrangement we have ‘p’ r-times and ‘q’ n-r times, so each arrangement value is in probability : pr.qn-r

**Ex**: A coin is tossed 6-times, find the probablity to get heads 4 times.

Soln: P(6, 3) = C(6, 4)(1/2)4.(1/2)2

**Ex**: A coin is tossed 8 times, find the probability to get atleast 6heads.

Soln: P(8, r > 5) = P(8, 6) + P(8, 7) + P(8, 8). solve them put the values and calculate the answer.