1. $\zeta(s) = \sum \frac{1}{n^s}$

$\left(\frac{1}{2}\right)^{2+i} = \underbrace{\left(\frac{1}{2}\right)^2}_{\text{determines the size}} \left(\frac{1}{2}\right)^i \rightarrow$ just dictate some rotation

Complex function that have a derivative every are called analytic

← equal to
angle-preserving

2. $\pi = \lim_{N \to \infty} \frac{4}{N} \sum_{n=1}^{N} \sum_{d|n} \chi(d)$

$n + mj$ → integers

this is Gaussian integers

5 can be factored into small Gussian integers → $(2+i)(2-i)$ ← this can't be factored smaller called "Gussian prime"

假设可以把 ζ $\frac{1}{2}$ 处映射到高维空间变成零点, use kernel

The circle of Gussian prime never hit any lattice points

→ some prime like $4k+3$ RGR can't factored.

$N = 3^4 \cdot 5^3 \cdot 13^2$

∵ $\overline{i}$ can be factored

$1\times(3+i)(2+i) \to$ lattice point number

if with number 2 $\quad (1+i)(1-i)=2$

and $(1+i)\cdot -i=(1-i)$

2^k doesn't change counts

← "multiplicative function $\quad \chi(3)\cdot\chi(5)=\chi(15)$

Define $\chi(n)$: $\begin{cases} \chi(1)=\chi(5)=\chi(9)=\chi(13)=\cdots=1 \\ \chi(3)=\chi(7)=\chi(11)=\chi(15)=\cdots=-1 \\ \chi(2)=\chi(4)=\chi(6)=\chi(8)=\cdots=0 \end{cases}$

$N=2^2\cdot3^4\cdot5^3$ ← use last formula

$1 \quad -1 \quad i \quad -i$

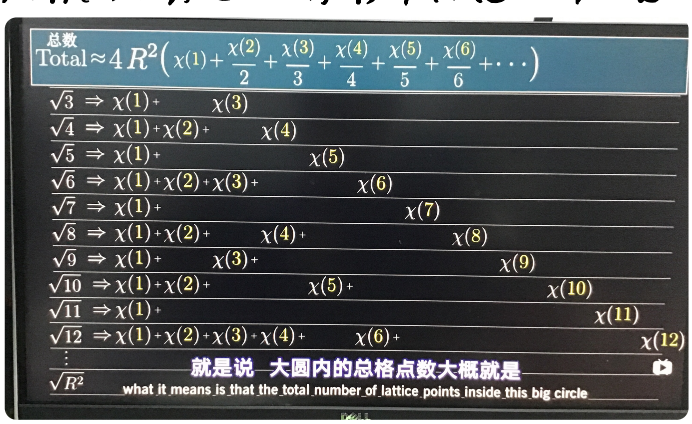
$=4\times1\times1\cdot(3+1)$

$\to \left(\overset{1}{\chi(3^0)}+\overset{-1}{\chi(3^1)}+\overset{1}{\chi(3^2)}+\overset{-1}{\chi(3^3)}+\overset{1}{\chi(3^5)}\right)$

$45=3^2\cdot5$

$=4(\chi(1)+\chi(3)+\chi(3^2))(\chi(1)+\chi(5))$ ← every divisors

$=4(\chi(1)+\chi(3)+\chi(5)+\chi(9)+\chi(15)+\chi(45))$



∵ $\chi(2)$ is 0

$\therefore$ Total $\doteqdot 4R^2\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\cdots\right) \doteqdot \pi R^2$