CÁLCULO III - DIOMARA

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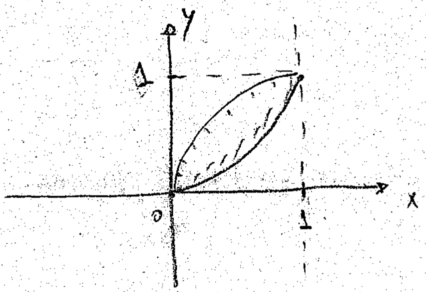
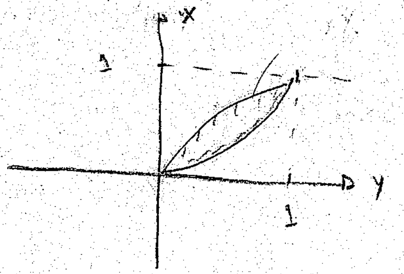
5.4 1) a) $\displaystyle\int_0^1 dx \int_{x^3}^{\sqrt{x}} f(x,y)\,dy = \int_0^1 dy \int_{y^2}^{\sqrt[3]{y}} f(x,y)\,dx$

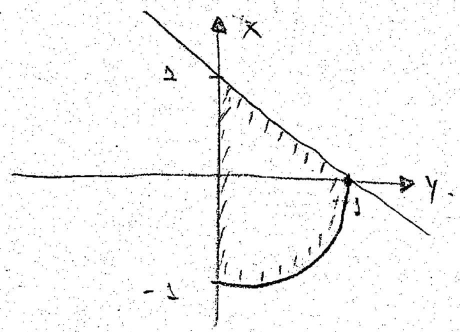
$y = \sqrt{x} \to x = y^2$

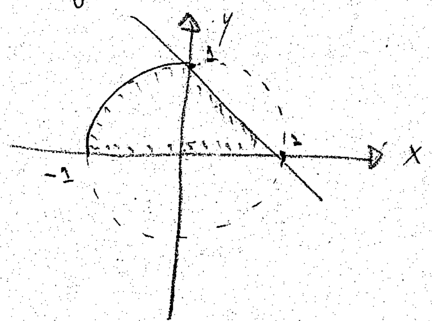
$y = x^3 \to x = \sqrt[3]{y}$



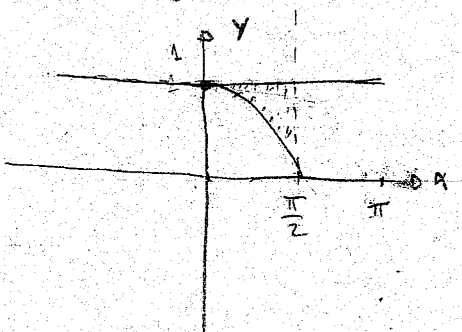


b) $\displaystyle\int_0^1 dy \int_{-\sqrt{1-y^2}}^{1-y} f(x,y)\,dx = \int_{-1}^{0} dx \int_0^{\sqrt{1-x^2}} f(x,y)\,dy + \int_0^1 dx \int_0^{1-x} f(x,y)\,dy$





c) $\displaystyle\int_0^{\frac{\pi}{2}} dx \int_{\cos x}^{1} f(x,y)\,dy = \int_0^1 dy \int_{\arccos y}^{\pi/2} f(x,y)\,dx$



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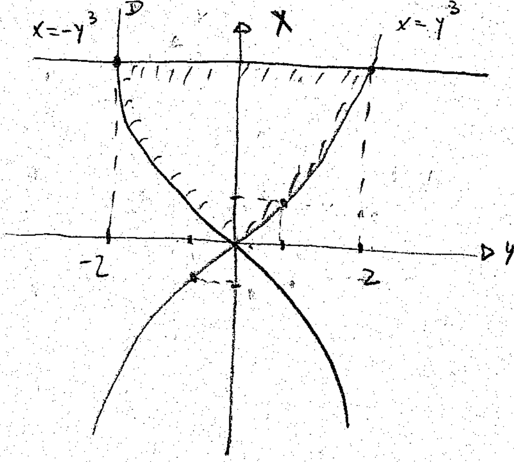
3) a) $\int dy \int y^2 \operatorname{sen}(x^2)\, dx = \int_{-2}^{0} dy \int_{-y^3}^{8} y^2 \operatorname{sen}(x^2)\, dx \Rightarrow \int_0^8 dx \int_{-\sqrt[3]{x}}^{\sqrt[3]{x}} y^2 \operatorname{sen} x^2\, dy \Rightarrow$



$$\Rightarrow \int_0^8 \operatorname{sen} x^2 dx \cdot \left[\frac{y^3}{3} \Big|_{-\sqrt[3]{x}}^{\sqrt[3]{x}} \right]$$

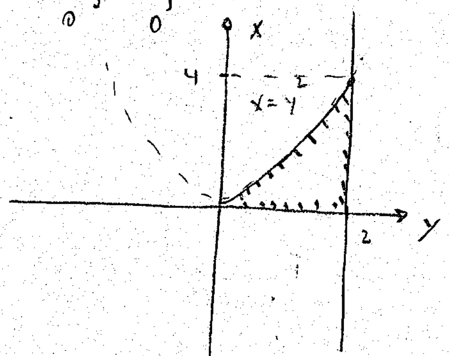
$$\Rightarrow \int_0^8 \operatorname{sen} x^2 dx \cdot 2\frac{x}{3} = \frac{1}{3}\int_0^8 \operatorname{sen} x^2 \cdot 2x\, dx = \frac{1}{3}\int_{u=0}^{u=2\sqrt{2}} \operatorname{sen} u\, du$$

$x^2 = u \to 2x\,dx = du$

$$\Rightarrow -\frac{\cos u}{3}\Big|_0^{2\sqrt{2}} = -\frac{\cos x^2}{3}\Big|_0^8 = \frac{1-\cos 64}{3}$$

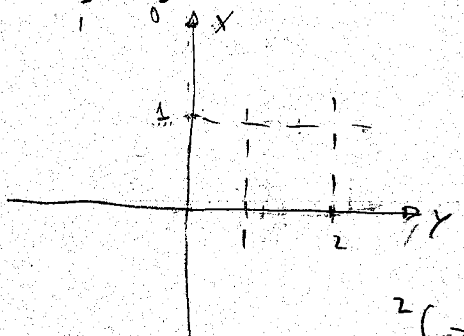
b) $\int_0^2 dy \int_0^{y^2} \cos y^3\, dx = \int_0^2 \cos y^3 dy \cdot \left(x \Big|_0^{y^2} \right) = \int_0^2 \cos y^3 \cdot y^2 dy = \frac{1}{3}\int_0^8 \cos u\, du =$

$y^3 = u$
$3y^2 dy = du$



$$= \frac{\operatorname{sen} u}{3}\Big|_0^8 = \frac{\operatorname{sen} 8}{3}$$

d) $\int_1^2 dy \int_0^1 y^{-2} \cdot e^{\frac{x}{\sqrt{y}}}\, dx = \int_1^2 dy \int_0^1 y^{-2} \cdot \sqrt{y}\, du : \int_1^2 y^{-2}\sqrt{y}\, dy \cdot \left[e^{\frac{x}{\sqrt{y}}} \Big|_0^1 \right] = \int_1^2 y^{-3/2} \cdot e^{\frac{1}{\sqrt{y}}} dy - \int_1^2 y^{-\frac{3}{2}} dy$



$$\left\{ \begin{array}{l} e^{\frac{1}{\sqrt{y}} \cdot x} = u \\ e^{\frac{x}{\sqrt{y}}} \cdot \frac{1}{\sqrt{y}} dx = du \end{array} \right. \qquad \left\{ \begin{array}{l} e^{\frac{1}{\sqrt{y}}} = v \\ -e^{\frac{1}{\sqrt{y}}} \cdot \frac{y^{-\frac{3}{2}}}{2} dy = dv \end{array} \right.$$

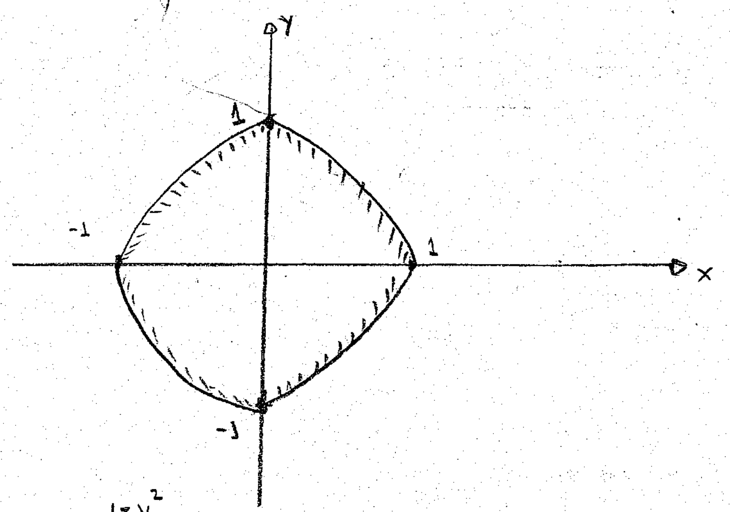
$$= -\int_1^2 2\, dv + 2 \cdot y^{-\frac{1}{2}} \Big|_1^2 = 2v \Big|_2^1 + 2 \cdot \left(\frac{1}{\sqrt{2}} - 1 \right)$$

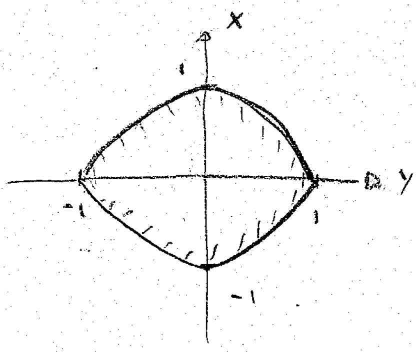
$$= 2 \cdot \left(e - e^{\frac{1}{\sqrt{2}}} \right) + \frac{2}{\sqrt{2}} - 2 = 2\left(e - e^{\frac{1}{\sqrt{2}}} + \frac{1}{\sqrt{2}} - 1 \right)$$

4) a) $\int_{-1}^{0} dx \int_{-\sqrt{x+1}}^{\sqrt{x+1}} f\,dy + \int_{0}^{1} dx \int_{-\sqrt{1-x}}^{\sqrt{1-x}} f\,dy = I$

$y=\sqrt{x+1} \to x = y^2-1\ (y \geq 0)$
$y=-\sqrt{x+1} \to x = y^2-1\ (y \leq 0)$
$y=\sqrt{1-x} \to x = 1-y^2$

2



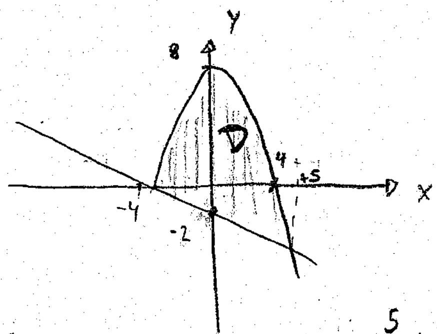


$$I = \int_{-1}^{1} dy \int_{y^2-1}^{1-y^2} f(x,y)\,dx$$

b) $$\int_{-1}^{1} dy \int_{y^2-1}^{1-y^2} xy\,dx = \int_{-1}^{1} y\,dy \cdot \left[\frac{x^2}{2}\right]_{y^2-1}^{-(y^2-1)} = \int_{-1}^{1} \frac{y\,dy}{2} \cdot \left(\cancel{(1-y^2)^2 - (y^2-1)^2}\right) = 0$$

0

5) a)



$$\int_{-4}^{5} dx \int_{\frac{-x-4}{2}}^{\frac{16-x^2}{2}} f(x,y)\,dy$$

$f(x,y) = 1 \iff \iint dx\,dy = D$

$\frac{16-x^2}{2} = \frac{-x-4}{2}$

$16-x^2 = -x-4$

$20-x^2 = -x$

$x^2-x-20 = 0$

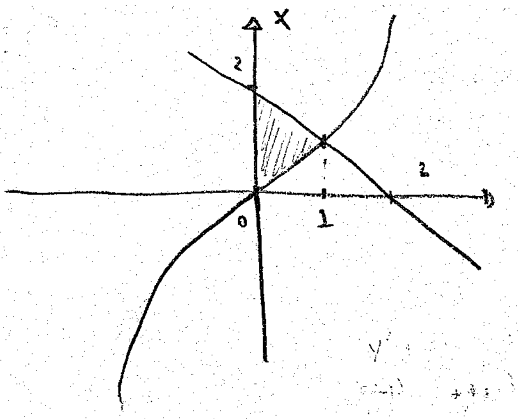
$(x-5)(x+4) = 0$

$$\begin{array}{r} 1107 \\ -\ 122 \\ \hline 985 \end{array}$$

$$\int_{-4}^{5} dx \left[\frac{16-x^2}{2} + \frac{(x+4)}{2}\right] = \frac{1}{2}\int_{-4}^{5} 20 + x - x^2 = \frac{20x\Big|_{-4}^{5} + \frac{x^2}{2}\Big|_{-4}^{5} - \frac{x^3}{3}\Big|_{-4}^{5}}{2}$$

$$= \frac{100 + 80 + \frac{25}{2} - \frac{16}{2} - \left(\frac{125}{3} + \frac{64}{3}\right)}{2} = \frac{180 + \frac{9}{2} - \frac{189}{3}}{2} = \frac{1080 + 27 - 378}{12} = \frac{729}{12} = \frac{243}{4}$$

5)

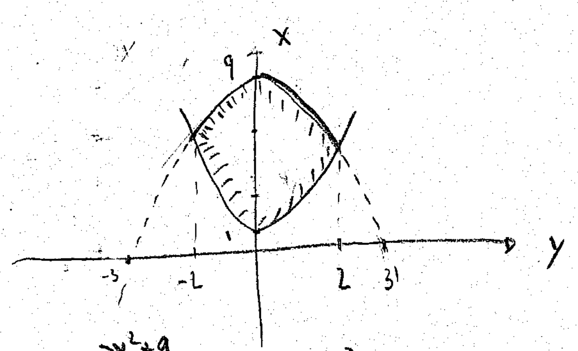


$$\int_0^1 dy \int_{y^3}^{2-y} 1\,dx = \int_0^1 (2-y)-(y^3)\,dy$$

$$\int_0^1 (2-y-y^3)\,dy = 2y\Big|_0^1 - \frac{y^2}{2}\Big|_0^1 - \frac{y^4}{4}\Big|_0^1$$

$$= \underset{1/4}{2} - \underset{2/2}{\frac{1}{2}} - \frac{1}{4} = \frac{8-2-1}{4} = \frac{5}{4}$$

6)



$y^2+1 = -y^2+9$

$2y^2 = 8$

$y^2 = 4$

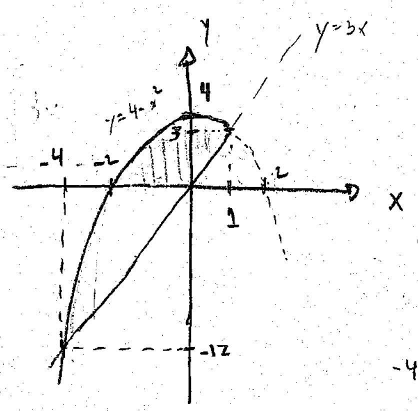
$y = \pm 2$

$$\int_{-2}^{2} dy \int_{y^2+1}^{-y^2+9} (4-y^2)\,dx = \int_{-2}^{2} (4-y^2)\,dy \cdot (-y^2+9-y^2-1) = \int_{-2}^{2} (4-y^2)(-2y^2+8)\,dy$$

$$= \int_{-2}^{2} (-10y^2+8+2y^4)\,dy = -\frac{10y^3}{3}\Big|_{-2}^{2} + 8y\Big|_{-2}^{2} + \frac{2y^5}{5}\Big|_{-2}^{2} = -\underset{3/5}{\frac{160}{3}} + \underset{1/15}{32} + \underset{5/3}{\frac{128}{5}} = \frac{-800+480+384}{15}$$

$$= \frac{64}{15}$$

7)



interseção: $4-x^2 = 3x$

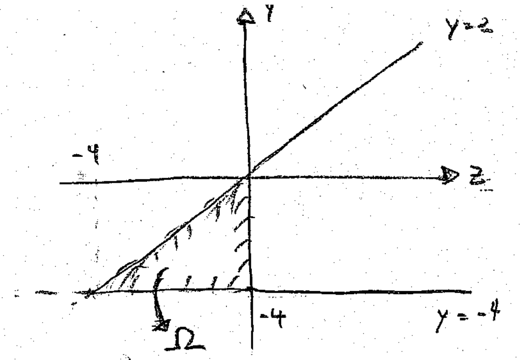
$x^2+3x-4=0 \rightarrow x=1 \quad x=-4$

$$\int_{-4}^{1} dx \int_{3x}^{4-x^2} (x+4)\,dy = \int_{-4}^{1} dx \cdot (xy+4y)\Big|_{3x}^{4-x^2} = \int_{-4}^{1} dx \left[\begin{matrix} 4x-x^3+16-4x^2 \\ -3x^2-12x \end{matrix}\right]$$

$$\int_{-4}^{1} (-x^3-7x^2-8x+16)\,dx = -\frac{x^4}{4} - \frac{7x^3}{3} - 4x^2 + 16x\Big|_{-4}^{1} = -\frac{1}{4} - \frac{7}{3} - 4 + 16 - \left(-64 + \frac{448}{3} - 64 - 64\right)$$

$$= -\underset{4/3}{\frac{1}{4}} - \underset{3/4}{\frac{7}{3}} + \underset{1/12}{12+64} - \underset{3/4}{\frac{448}{3}} + \underset{1/12}{128} \; \frac{-3-28+912-1792+1536}{12} = \frac{625}{12}$$

9)



$z = 4 - x^2 \to x^2 = 4 - z$

$x = \pm\sqrt{4-z}$

para $z \geq 0 \to x \geq 0$

$$\int_{-4}^{0} dy \int_{-y}^{0} \sqrt{4-z}\, dz = \int_{-4}^{0} dy \cdot -\int_{y}^{0} u^{1/2}\, du = \int_{-4}^{0} dy \cdot -\left(\frac{2(4-z)^{3/2}}{3}\right)\Bigg|_{y}^{0} = -\int_{-4}^{0} dy \left[\frac{2}{3}\cdot 4^{3/2} - \frac{2}{3}\cdot (4-y)^{3/2}\right]$$

$4 - z = u$

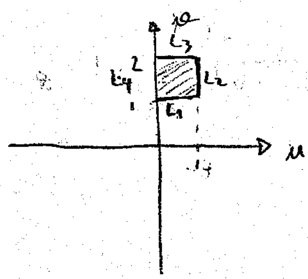
$-dz = du$

$$= -\frac{2}{3}\int_{-4}^{0}\left[4^{3/2} - (4-y)^{3/2}\right] dy = \left[-\frac{2}{3}\cdot 4^{3/2}\, y - \frac{2}{3}(4-y)^{5/2}\cdot\frac{2}{5}\right]_{-4}^{0} = \frac{4^{7/2}}{15} - \left(\frac{2}{3}\cdot 4^{5/2} - \frac{2}{3}\cdot 8^{5/2}\cdot\frac{2}{5}\right)$$

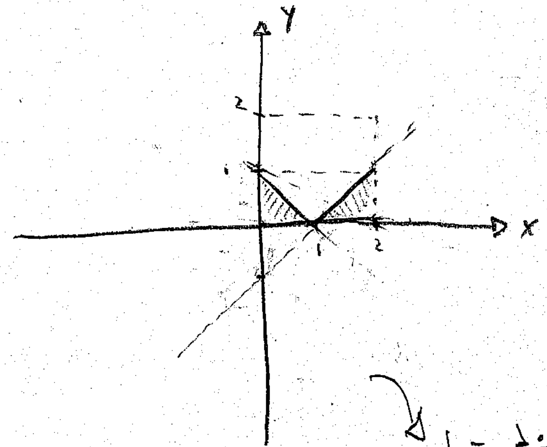
$$= \frac{2^7}{15} - \frac{2}{3}\cdot\left(2^5 - 2^{19/2}\right)$$

$$\frac{128}{15} - \frac{2^6}{3} + \frac{2^{19/2}}{3} = \frac{128}{15} - \frac{64}{3} + \frac{2^{19/2}}{15}$$

5.6 | 1)



$\Rightarrow$



$b = \frac{1\cdot 1}{2} + \frac{1\cdot 1}{2} = 1$

$\begin{cases} x = uv \\ y = v - u \end{cases}$

L_1: para $v = 1 \to (0 \leq u \leq 1)$

$\Rightarrow x = u \to (0 \leq x \leq 1)$

$y = 1 - u \to (0 \leq y \leq 1)$

$x + y = 1$

L_2: $u = 1$ $(1 \leq v \leq 2)$

$x = v \to x - y = 1 \to y = x - 1$

$y = v - 1$ $(1 \leq x \leq 2)$; $(0 \leq y \leq 1)$

L_3: para $v = 2 \to (0 \leq u \leq 1)$

$x = 2u \to (0 \leq x \leq 2)$

$y = 2 - u \to (1 \leq y \leq 2)$

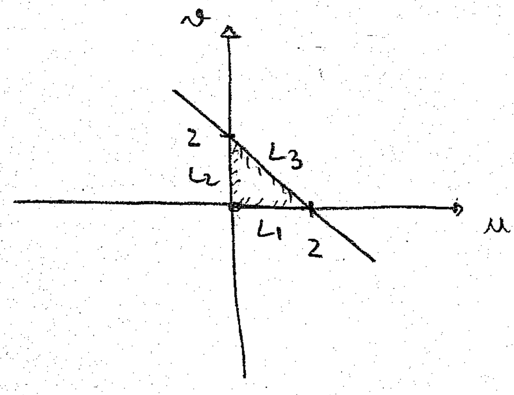
$x + 2y = 2 \to \boxed{y = 1 - \frac{x}{2}}$?

L_4: $u = 0$ $(1 \leq v \leq 2)$

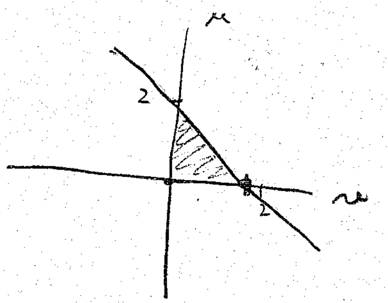
$x = 0 \to$

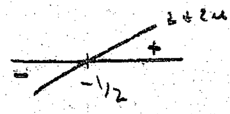
$y = v \to 1 \leq y \leq 2$

2)



$u = 2 - v$





$$\begin{cases} x = u + v \\ y = v - u^2 \end{cases}$$

↓

$$J(u,v) = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ -2u & 1 \end{vmatrix} = 1 + 2u \Rightarrow |J(u,v)| = 1 + 2u \quad (u>0)$$

$$\int dy \int \left(x - y + \frac{1}{4}\right)^{-1/2} dx = \int_0^2 dv \int_0^{2-v} \left(u^2 + u + \frac{1}{4}\right)^{-1/2} (1+2u)\, du \Rightarrow \int_0^2 dv \int_0^{2-v} (w)^{-1/2} \, dw = 2\int_0^2 dv \cdot w^{1/2}\Big|_0^{2-v}$$

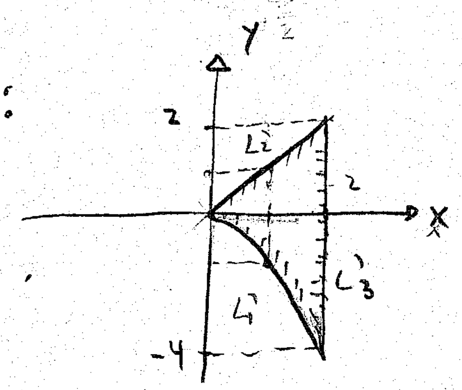
$u^2 + u + \frac{1}{4} = w$

$(2u+1)du = dw$

$$= 2\int_0^2 dv \cdot \left(u^2 + u + \frac{1}{4}\right)^{1/2}\Big|_0^{2-v} = 2\int_0^2 dv \cdot \left[\sqrt{(2-v)^2 + 2 - v + \frac{1}{4}} - \frac{1}{2}\right] = 2\int_0 dv \left[\sqrt{\frac{25}{4} - 5v + v^2} - \frac{1}{2}\right]$$

$$2\int_0^2 \left(\left(v - \frac{5}{2}\right) - \frac{1}{2}\right) dv = 2\int_0^2 (v-3)\, dv = \cancel{2} \cdot \frac{(v-3)^2}{\cancel{2}}\Big|_0^2 = 1 - 9 = -8$$

b) D:



L_1: $v = 0$ e $0 \le u \le 2$

$x = u$

$y = -u^2 \Rightarrow y = -x^2 \rightarrow \boxed{0 \le x \le 2}$

L_2: $u = 0$ e $0 \le v \le 2$

$x = v$, $y = v \rightarrow x - y = 0 \rightarrow x = y$

L_3: $u + v = 2$

$\boxed{x = 2}$

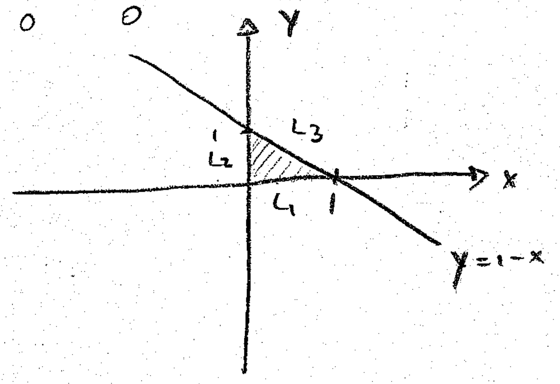
$y = 2 - u - u^2 \Rightarrow u = 0 \rightarrow y = 2$

$u = 2 \rightarrow y = -4$

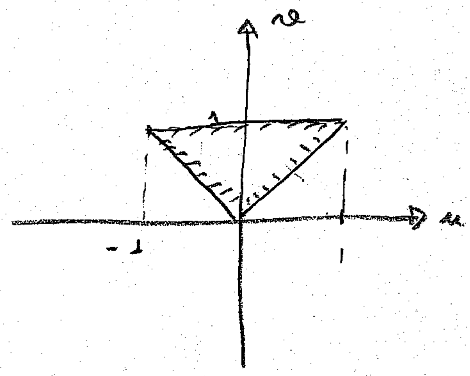
$\boxed{-4 \le y \le 2}$

$$\int_0^2 dx \int_{-x^2}^{x} \left(x - y + \frac{1}{4}\right)^{-1/2}$$

3) $\int_0^1 dy \int_0^1 \cos\left(\frac{x-y}{x+y}\right) dx$



$\Rightarrow$



$\begin{cases} u = x - y \\ v = x + y \end{cases} \rightarrow \begin{cases} x = \frac{u+v}{2} \\ y = \frac{v-u}{2} \end{cases} \Rightarrow J(u,v) = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2}$

Para desenhar $[u \times v]$:

L_1: $y = 0$ e $0 \leq x \leq 1$

$\begin{matrix} u = x \\ v = x \end{matrix} \rightarrow u = v \begin{cases} 0 \leq u \leq 1 \\ 0 \leq v \leq 1 \end{cases}$

; L_2: $x = 0$ e $0 \leq y \leq 1$

$\begin{cases} u = -y \\ v = y \end{cases} \rightarrow u = -v \begin{cases} 0 \leq v \leq 1 \\ -1 \leq u \leq 0 \end{cases}$

L_3: $y + x = 1$

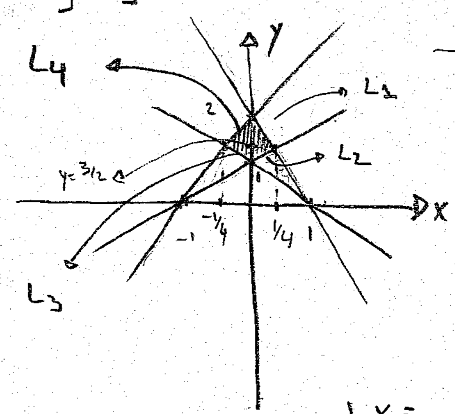
$u = x - y$

$v = 1$

$\frac{1}{2} \int_0^1 dv \int_{-v}^{v} \cos\left(\frac{u}{v}\right) \cdot du = \frac{1}{2} \int_0^1 dv \cdot \operatorname{sen}\left(\frac{u}{v}\right) \cdot v \Big|_{-v}^{v} = \frac{1}{2} \int_0^1 dv \cdot v\left(\operatorname{sen} 1 - \operatorname{sen}(-1)\right)$

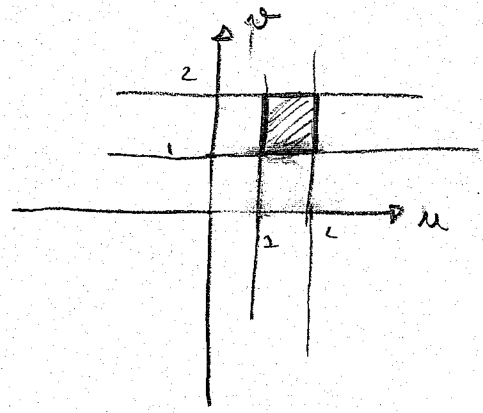
$\frac{1}{\cancel{2}} \cdot \cancel{2}\operatorname{sen} 1 \cdot \frac{v^2}{2} \Big|_0^1 = \frac{\operatorname{sen} 1}{2}$ //

4) $\int dy \int \frac{y+2x}{\sqrt{y-2x-1}}\, dx\, dy$



$$\begin{aligned} y+2x&=2\\ y-2x&=1\\ \hline 2y&=3 \end{aligned}$$

$\frac{3}{2}-2=-2x$



$$\begin{cases} u = y+2x \\ v = y-2x \end{cases} \to \begin{cases} x = \frac{u-v}{4} \\ y = \frac{u+v}{2} \end{cases} \Rightarrow J(u,v) = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4}$$

L_1: $y+2x=2$ $\begin{cases} 0 \le x \le 1/4 \\ \frac{3}{2} \le y \le 2 \end{cases}$

$\boxed{u=2}$

L_2: $y-2x=1$

$v=1$

$y+2x=u$

$2x+1+2x=u$

$1+4x=u$

$0 \le x \le 1/4 \to \begin{matrix} u=1 \\ u=2 \end{matrix} \to 1 \le u \le 2$

L_3: $y+2x=1$

$\boxed{u=1}$

$y-2x=v$

$1-2x-2x=v \to v=1-4x$

$-1/4 \le x \le 0 \Rightarrow v = 1+1=2$

$v=1-0=1$

$0 \le v \le 1$

L_4: $y-2x=2$

$\boxed{v=2}$ $\to$ $y+2x=u$

$2+2x+2x=u$

$u=2+4x$

$-1/4 \le x \le 0 \Rightarrow u=2-1=1$

$u=2$

$1 \le u \le 2$

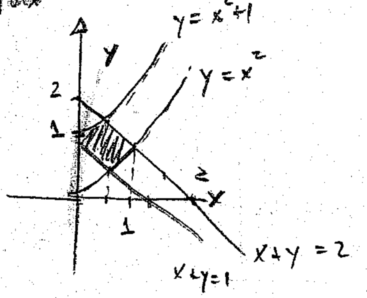
$y-2x=v$

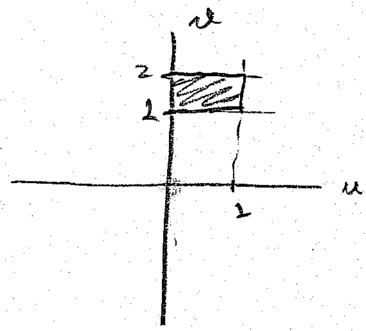
$2-2x-2x=v \to v=2-4x$ $\begin{cases} v=2 \\ v=1 \end{cases}$ $1 \le v \le 2$

$0 \le x \le 1/4$

$$\frac{1}{4}\int_1^2 dv \int_1^2 \frac{u}{\sqrt{v-1}}\,du = \frac{1}{4}\int_1^2 \frac{1}{\sqrt{v-1}}\,dv \cdot \frac{u^2}{2}\Big|_1^2 = \frac{1}{4}\int_1^2 \frac{dv}{\sqrt{v-1}} \cdot \frac{3}{2} = \frac{3}{\cancel{8}_4} \cdot \cancel{2} \cdot (v-1)^{1/2}\Big|_1^2 = \frac{3}{4}$$

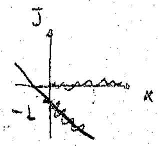
5) $\int dy \int (2x+1)\,dx$





$$\begin{cases} u = y - x^2 \longrightarrow 0 \le u \le 1 \\ v = x + y \longrightarrow 1 \le v \le 2 \end{cases}$$

$$J(x,y) = \begin{vmatrix} -2x & 1 \\ 1 & 1 \end{vmatrix} = -2x - 1$$

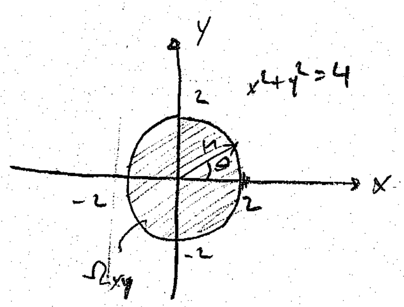
$x > 0$ (1º Q) $\rightarrow$  $\rightarrow |J(x,y)| = -(-2x-1)$

$|J(x,y)| = 2x+1$

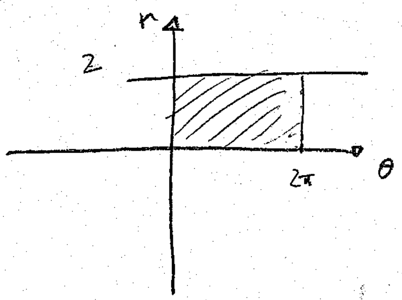
$$\int_0^1 du \int_1^2 (2x+1)\cdot |J(u,v)|\,dv = \int_0^1 du \int_1^2 \cancel{(2x+1)} \cdot \frac{1}{\cancel{2x+1}}\,dv$$

$= 1 \cdot 1 = 1$ //

7)



$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases} \qquad J(r,\theta) = r$$



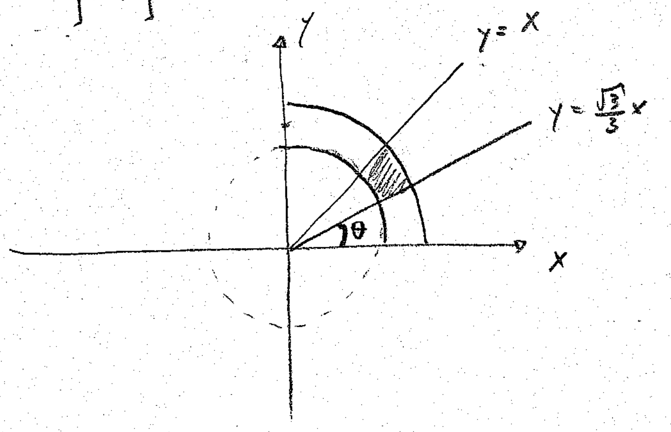
$$\int_0^{2\pi} d\theta \int_0^2 \frac{dr}{(1+r^2)^2}$$

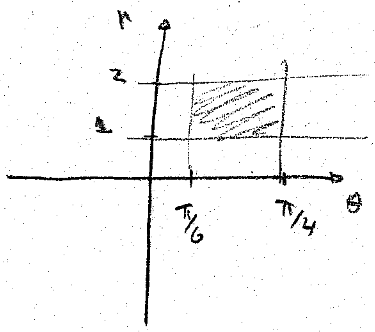
$$= \int_0^{2\pi} d\theta \int_0^2 \frac{r}{(1+r^2)^2} \cdot \frac{d[r^2]}{2} = \int_0^{2\pi} d\theta \int_0^2 \frac{r}{(1+r^2)^2}\,dr = \frac{1}{2}\int_0^{2\pi} d\theta \int_0^2 \frac{du}{(1+u)^2} = -\frac{1}{2}\int_0^{2\pi} (1+r^2)^{-1}\Big|_0^2$$

$u = r^2 \rightarrow du = 2r\,dr \rightarrow r\,dr = \frac{du}{2}$

$$= -2\pi \cdot \frac{1}{2}\left(\frac{1}{5} - 1\right) = \frac{4\pi}{5} \;//$$

b) $\int dy \int (x^2+y^2)\,dx$





$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases} \rightarrow |J(r,\theta)| = r$$

$$\begin{cases} r^2 = 1 \\ r^2 = 4 \end{cases} \rightarrow \begin{matrix} 1 \le r \le 2 \\ \pi/6 \le \theta \le \pi/4 \end{matrix}$$

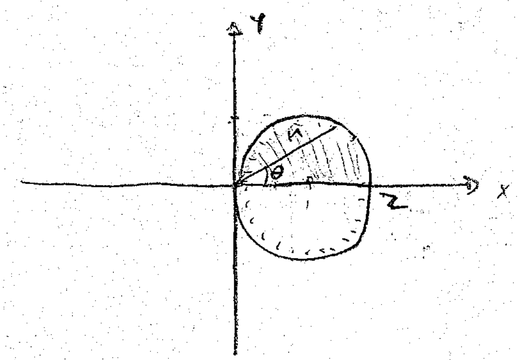
$$\int_{\pi/6}^{\pi/4} d\theta \int_1^2 r^2 \cdot r \cdot dr = \left(\frac{16}{4} - \frac{1}{4}\right)\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{15}{4} \cdot \frac{\pi}{12}$$

$$= 5\pi/16$$

$a + b = 2\theta$
$a - b = \theta$

c) $\int dx \int x^2 y\,dy$

$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases} \longrightarrow J(r,\theta) = r$$

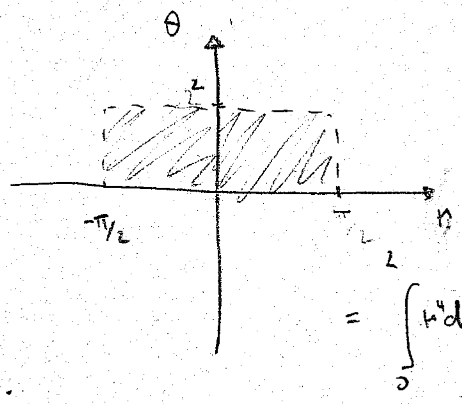


$(x-1)^2 + y^2 \le 1 \Rightarrow (r\cos\theta - 1)^2 + r^2\text{sen}^2\theta \le 1$

$r^2 - 2r\cos\theta \le 0 \rightarrow r \le 2\cos\theta$

$\cos\theta \geq \frac{r}{2}$ $(\cos\theta \geq 0)$

- $0 \le r \le 2$
- $0 \le \cos\theta \le 1 \rightarrow -\pi/2 \le \theta \le \pi/2$

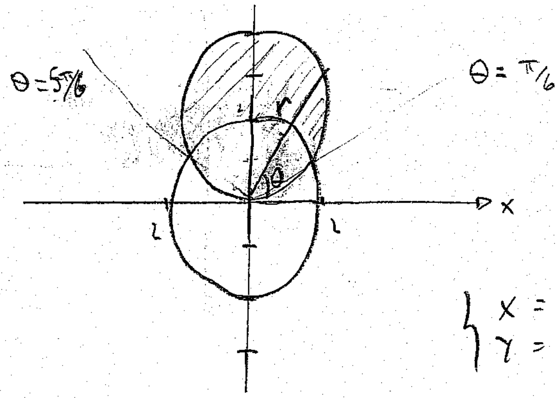


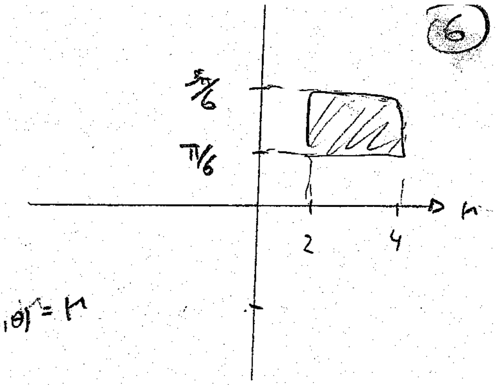
$$\int_0^2 dr \int_{-\pi/2}^{\pi/2} r^2\cos^2\theta \cdot r\,\text{sen}\,\theta \cdot r\,d\theta = \int_0^2 r^4 dr \int_{-\pi/2}^{\pi/2} \text{sen}\,\theta\cos^2\theta\,d\theta$$

$$= \int_0^2 r^4 dr \int_{-\pi/2}^{\pi/2} \cos^2\theta \cdot \underbrace{\text{sen}\,\theta\,d\theta}_{=-d[\cos\theta]} = -\int_0^2 r^4 dr \int_{-\pi/2}^{\pi/2} \cos^2\theta \cdot d[\cos\theta] = -\int_0^2 r^4 dr \cdot \frac{\cos^3\theta}{3}\Bigg|_{-\pi/2}^{\pi/2}$$

$$= -\frac{1}{3}\int_0^2 r^4 dr \cdot \underbrace{\left(\cos^3\frac{\pi}{2} - \cos^3\left(-\frac{\pi}{2}\right)\right)}_{0} = 0$$

8)





$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases} \rightarrow J(r,\theta) = r$$

$$\begin{cases} x^2 + (y-2)^2 \leq 4 \rightarrow r^2 - 4r\,\text{sen}\,\theta \leq 0 \rightarrow r \leq 4\,\text{sen}\,\theta \\ x^2 + y^2 \geq 4 \rightarrow r^2 \geq 0 \rightarrow r \geq 2 \\ \qquad\qquad (r \geq 0) \end{cases}$$

• $2 \leq r \leq 4\,\text{sen}\,\theta$

$\leq \text{sen}\,\theta \leq$
4

interseção: $x^2 + (y-2)^2 = \cancel{x^2} + \cancel{y^2} \rightarrow -4y + 4 = 0 \rightarrow \boxed{y = \pm 1} \rightarrow \boxed{x = \pm\sqrt{3}}$

$$\frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}$$

$$\int_{\pi/6}^{5\pi/6} d\theta \int_2^{4\,\text{sen}\,\theta} r\,dr = \int d\theta \cdot \frac{r^2}{2}\Big|_2^{4\,\text{sen}\,\theta} = \frac{1}{2}\int d\theta \cdot (16\,\text{sen}^2\theta - 4)$$

$$\frac{1}{2}\int 16\,\text{sen}^2\theta - 2\int d\theta = 8 \cdot \int \frac{1-\cos 2\theta}{2}\,d\theta - 2\theta\Big|_{\pi/6}^{5\pi/6}$$

$$= 4\int d\theta - 4\int \cos 2\theta\,d\theta - \frac{4\pi}{3} = \frac{8\pi}{3} - \frac{4\pi}{3} - \frac{4}{2}\cdot \text{sen}\,2\theta\Big|_{\pi/6}^{5\pi/6}$$

$$= \frac{4\pi}{3} - 2\,\text{sen}\,\frac{5\pi}{3} + 2\,\text{sen}\,\frac{\pi}{3} = \frac{4\pi}{3} + 2\,\text{sen}\,\frac{2\pi}{3} + 2\,\text{sen}\,\frac{2\pi}{3} = \frac{4\pi}{3} + 4\underbrace{\text{sen}\,\frac{2\pi}{3}}_{\sqrt{3}/2}$$

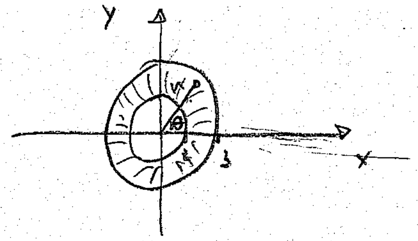
$$= \frac{4\pi}{3} + 2\sqrt{3}$$

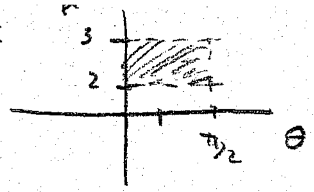
9) $\begin{cases} u = \frac{x}{2} \longrightarrow x = 2u \\ v = \frac{y}{3} \longrightarrow y = 3v \end{cases}$

$(u^2+v^2)^2 = u^2 - v^2 \rightarrow u^4 + 2u^2v^2 + v^4 = u^2 - v^2$

$-b$

10) a)



1ºQ: 

$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases} \to J(r,\theta) = r$

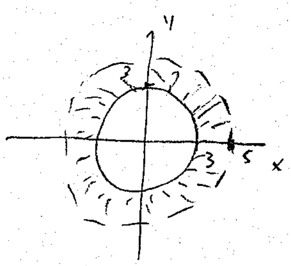
$x^2+y^2 = z \to r^2 = z$

$z = 10 \to r^2 = 10$

$$\int_0^{\pi/2} d\theta \int_2^3 r\,dr \int_{r^2}^{10} dz = \frac{\pi}{2}\int_2^3 r\,dr\,.\,(10-r^2) = -\frac{\pi}{2}\cdot\frac{1}{2}\int_2^3 (10-r^2)\,d(10-r^2)$$

$$-\frac{\pi}{4}\,\frac{(10-r^2)^2}{2}\Big|_2^3 = -\frac{\pi}{8}\left[(10-9)^2-(10-4)^2\right] = -\frac{\pi}{8}(1-36) = 35\pi/8 \;//$$

b) $\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$



$r^2 \geq 9 \to r \geq 3$

$z \leq \sqrt{25-r^2} \to$ para $z = 0$

$r^2 \leq 25 \to r \leq 5$

$$\int_0^{2\pi} d\theta \int_3^5 r\sqrt{25-r^2}\,dr$$

$$-\frac{2\pi}{2}\int_3^5 \sqrt{25-r^2}\,d[25-r^2] = -\frac{2\pi}{\cancel{2}}\cdot\frac{\cancel{2}}{3}(25-r^2)^{3/2}\Big|_3^5 = 2\pi\cdot\frac{(25-9)^{3/2}}{3} = \frac{128\pi}{3}$$

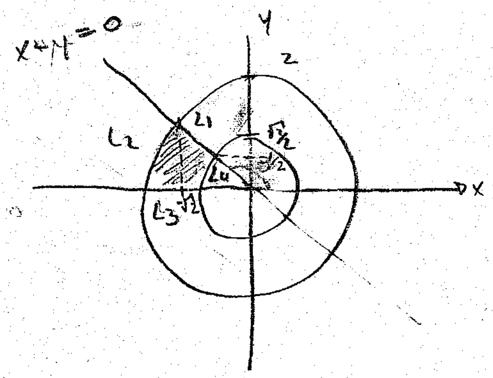
c) $\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases} \to$ $r^2 = 2r\,\text{sen}\,\theta \to r = 2\,\text{sen}\,\theta$

$$\int_0^{\pi} d\theta \int_0^{2\,\text{sen}\,\theta} r\,.\,r\,dr \to \int_0^{\pi} d\theta\,\frac{r^3}{3}\Big|_0^{2\,\text{sen}\,\theta} = \frac{1}{3}\int_0^{\pi} 8\,\text{sen}^3\theta\,d\theta = \frac{8}{3}\int_0^{\pi}\text{sen}^3\theta\,d\theta$$

$$\frac{8}{3}\int_0^{\pi}\text{sen}^2\theta\,.\,\text{sen}\,\theta\,d\theta = -\frac{8}{3}\int_0^{\pi}(1-\cos^2\theta)\,.\,d[\cos\theta] = -\frac{8}{3}\left[\cos\theta\Big|_0^{\pi} - \frac{\cos^3\theta}{3}\Big|_0^{\pi}\right] = -\frac{8}{3}\cdot\left[-2-\left(-\frac{1}{3}-\frac{1}{3}\right)\right]$$

$$-\frac{8}{3}\left[-2+\frac{2}{3}\right] = \frac{8}{3}\left(2-\frac{2}{3}\right) = \frac{8}{3}\cdot\frac{4}{3} = \frac{32}{9}$$

11) $\int dy \int \frac{e^{\frac{x+y}{x-y}}}{(x-y)^2}\, dx$ $\qquad \frac{1}{2} \le x^2+y^2 \le 4$



$\begin{cases} x^2+y^2 = u \\ x+y = v \end{cases} \rightarrow J_{(x,y)} = \begin{vmatrix} 2x & 2y \\ 1 & +1 \end{vmatrix} = -2x-2y = -2(x+y)$

$|J_{(u,v)}| = \left|\frac{1}{-2(x+y)}\right| = \frac{1}{2(x+y)}$

$(x+y \geq 0)$

$\underline{L1}$: $\begin{cases} x^2+y^2 \geq \frac{1}{2} \rightarrow u \geq 1/2 \\ x^2+y^2 \le 4 \rightarrow u \le 4 \\ x+y \geq 0 \rightarrow v \geq 0 \end{cases}$ $\qquad \frac{1}{2} \le u \le 4$, $v \geq 0$

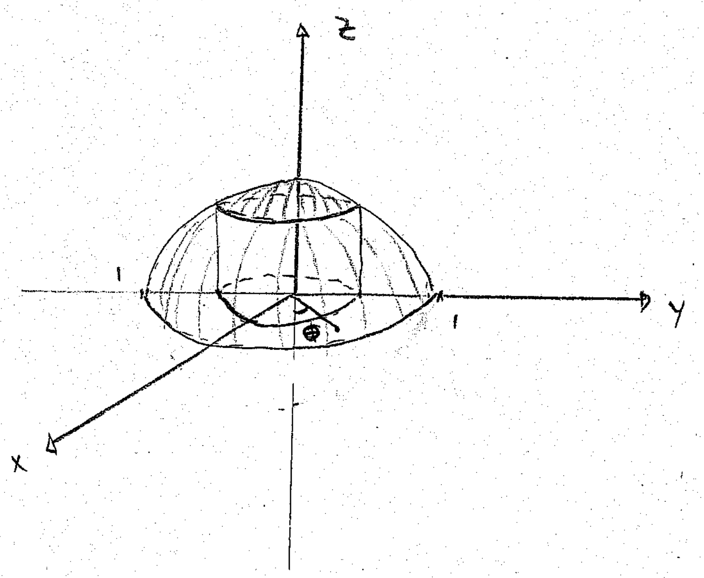
$\underline{L2}$: $x^2+y^2 = 4$, $x+y \le 0$, $y \geq 0$ $\rightarrow$ $u=4$, $\cancel{x} \le 0$

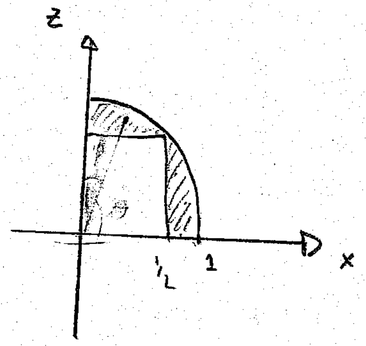
$\underline{L3}$: $\begin{cases} y=0 \\ x^2+y^2 \le 4 \rightarrow u \le 4 \\ x^2+y^2 \geq 1/2 \quad u \geq 1/2 \end{cases}$ $\quad x \geq 0$

5.11

7.1

4) a) W: $\underset{(1)}{x^2+y^2+z^2 \leq 1}$; $\underset{(2)}{z \geq 0}$; $\underset{(3)}{x^2+y^2 \geq \frac{1}{4}}$





Usando a mudança cilíndrica:

$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \\ z = z \end{cases} \rightarrow |J(r,\theta)| = r$$

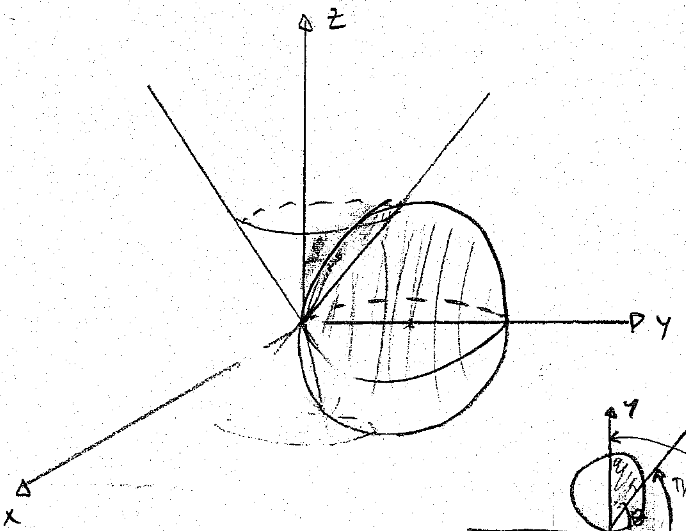
$$\left.\begin{array}{l} (1)\; 0 \leq z^2 \leq 1 - r^2 \rightarrow 1-r^2 \geq 0 \rightarrow r^2 \leq 1 \rightarrow 0 \leq r \leq 1 \\ (3)\; r^2 \geq \frac{1}{4} \rightarrow r \geq 1/2 \end{array}\right\} 1/2 \leq r \leq 1$$

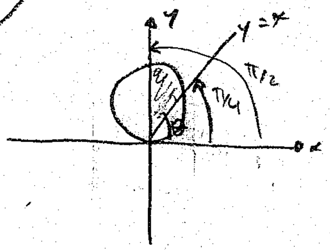
$$\int_0^{2\pi} d\theta \int_{1/2}^{1} r\,dr \int_0^{\sqrt{1-r^2}} z\,dz = 2\pi \cdot \int_{1/2}^{1} r(1-r^2)\,dr = -\frac{2\pi}{2}\int_{1/2}^{1}(1-r^2)\,d(1-r^2)$$

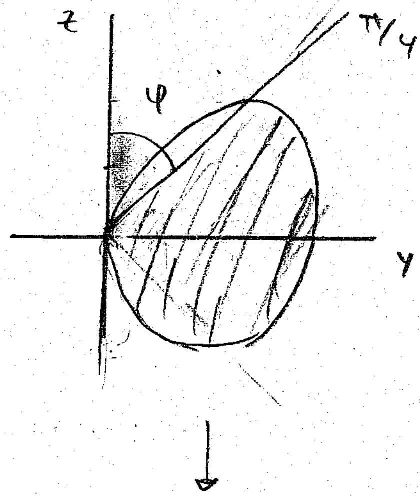
$$-\pi \cdot \left.\frac{(1-r^2)^2}{2}\right|_{1/2}^{1} = \frac{9\pi}{64}$$

) $\iiint \frac{dx\,dy\,dz}{x^2+y^2+z^2}$

$W: \underbrace{x^2+(y-1)^2+z^2 \le 1}_{(1)} \;;\; \underbrace{z \le \sqrt{x^2+y^2}}_{(2)} \;;\; \underbrace{y \geq x \text{ e } x \geq 0}_{(3)}$







$\frac{\pi}{4} \le \varphi \le \pi$

$x = r\,\text{sen}\varphi\cos\theta$
$y = r\,\text{sen}\varphi\,\text{sen}\theta \rightarrow |J_{(r,\varphi,\theta)}| = r^2\,\text{sen}\varphi$
$z = r\cos\varphi$

$r^{\cancel{2}} \le 2\cancel{r}\,\text{sen}\varphi\,\text{sen}\theta \rightarrow r \le 2\,\text{sen}\varphi\,\text{sen}\theta$

$\cancel{r}\cos\varphi \le \cancel{r}\,\text{sen}\varphi \rightarrow \text{tg}\varphi \geq 1 \rightarrow$

$\cancel{r\,\text{sen}\varphi}\,\text{sen}\theta \geq \cancel{r\,\text{sen}\varphi}\cos\theta \rightarrow \text{tg}\theta \geq 1 \rightarrow \frac{\pi}{4} \le \theta \le \frac{\pi}{2}$ ✓

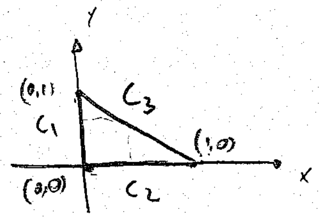
$\int_{\pi/4}^{\pi/2} d\theta \int_{\pi/4}^{\pi} d\varphi \int_0^{2\,\text{sen}\varphi\,\text{sen}\theta} \frac{\cancel{r^2}\,\text{sen}\varphi}{\cancel{r^2}} = \int_{\pi/4}^{\pi/2} d\theta \int_{\pi/4}^{\pi} 2\,\text{sen}^2\varphi\,\text{sen}\theta\,d\varphi = 2\int_{\pi/4}^{\pi/2} \text{sen}\theta\,d\theta \int_{\pi/4}^{\pi} \text{sen}^2\varphi\,d\varphi$

$2\int \text{sen}\theta\,d\theta \cdot \left[\frac{\varphi}{2} - \frac{1}{4}\cdot\text{sen}\,2\varphi\right]_{\pi/4}^{\pi} = \sqrt{2}\cdot\left(\frac{\pi}{2/4} - \frac{\pi}{8} + \frac{1}{4/2}\right) = \frac{\sqrt{2}}{8}(3\pi+2)$

CAPÍTULO VI:

6.3

1) a) $f(x,y) = x+y$



Parametrizando:

C_1: $x=0$; $0\le y\le 1 \rightarrow \sigma_1(t) = (0,t)$ $\longleftrightarrow F(\sigma_1(t)) = t \longleftrightarrow |\sigma_1'(t)| = |(0,1)| = 1$

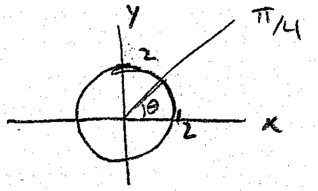
C_2: $y=0$; $0\le x\le 1 \rightarrow \sigma_2(t) = (t,0)$ $\longleftrightarrow F(\sigma_2(t)) = t \longleftrightarrow |\sigma_2'(t)| = |(1,0)| = 1$

C_3: $y+x=1 \longrightarrow \sigma_3(t) = (t;(1-t))$ $\longleftrightarrow F(\sigma_3(t)) = 1 \longleftrightarrow |\sigma_3'(t)| = |(1,1)| = \sqrt{2}$

$$\int_C f\,ds = \int_{C_1} f\,ds + \int_{C_2} f\,ds + \int_{C_3} f\,ds = \int_{0\,C_1}^{1} t\cdot 1\,dt + \int_{0\,C_2}^{1} t\cdot 1\,dt + \int_{0\,C_3}^{1} 1\cdot\sqrt{2}\,dt$$

$$= \frac{t^2}{2} + \frac{t^2}{2} + \sqrt{2}\,t\Big|_0^1 = 1+\sqrt{2}$$

b) $f(x,y) = x^2 - y^2$



Parametrizando: $\begin{cases} x = 2\cos\theta \\ y = 2\,\text{sen}\,\theta \end{cases}$ $\rightarrow \sigma(\theta) = (2\cos\theta, 2\,\text{sen}\,\theta)$

$\sigma'(\theta) = (-2\,\text{sen}\,\theta, 2\cos\theta) \rightarrow |\sigma'(\theta)| = 2$

$0\le\theta\le\pi/2$

$F(\sigma(\theta)) = 4(\cos^2\theta - \text{sen}^2\theta) = 4\cos 2\theta$

$$\int_C f\,ds = \int F(\sigma(\theta))\cdot|\sigma'(\theta)| = \int_{0\,C}^{2\pi} 4\cos 2\theta\cdot 2\,d\theta = 8\int_0^{2\pi}\cos 2\theta\,d\theta = 8\cdot\frac{\text{sen}\,2\theta}{2}\Big|_0^{2\pi} = 0$$

d) $f(x,y,z) = e^{\sqrt{z}}$; $C: \vec{r}(t) = (1,2,t^2) \rightarrow \vec{r}'(t) = (0,0,2t)$

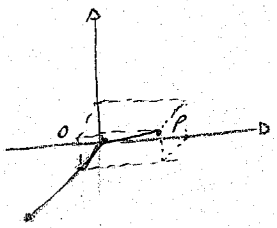
$0 \leq t \leq 1$ $\qquad |\vec{r}'(t)| = \sqrt{4t^2} = 2\sqrt{t^2} = 2|t|$

$F(\vec{r}(t)) = e^{\sqrt{t^2}} = e^{|t|}$

$$\int_C f\,ds = \int_0^1 e^{|t|}\, 2t\,dt = 2\int_0^1 \underbrace{e^{|t|}}_{dv} \cdot \underbrace{t}_{u} = -2\int_0^1 e^{|t|}dt + 2t \cdot e^{|t|}\Big|_0^1 = A$$

$A = -2 \cdot (e-1) + 2e = 2$ //

e) $f(x,y,z) = yz$; C :



$\vec{OP} = (1,3,2)$

$0 \leq t \leq 1$

$\vec{r}(t) = (t, 3t, 2t)$

$\vec{r}'(t) = (1,3,2)$ $\qquad F(\vec{r}(t)) = 6t^2$

$|\vec{r}'(t)| = \sqrt{14}$

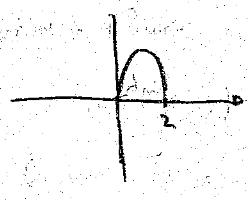
$$\int_0^1 6t^2 \cdot \sqrt{14}\,dt = 6\sqrt{14} \cdot \frac{t^3}{3}\Big|_0^1 = 2\sqrt{14}$$ //

2) $C: x^2 + y^2 + (2-x)^2 = 4$; $y \geq 0$

$2x^2 - 4x + \cancel{4} + y^2 = \cancel{4}$

$\boxed{2x^2 - 4x + y^2 = 0} \rightarrow x^2 - 2x + 1 + \frac{y^2}{2} = 1 \rightarrow \boxed{(x-1)^2 + \frac{y^2}{2} = 1}$ (C)

forma do arame: C



$x = \cos\theta + 1$

$0 \leq x \leq 2 \rightarrow 0 \leq \cos\theta + 1 \leq 2$

$-1 \leq \cos\theta \leq 1$

$\hookrightarrow 0 \leq \theta \leq \pi$

$$\begin{cases} x = \cos\theta + 1 \\ y = \sqrt{2}\,\text{sen}\theta \\ z = 2 - x = 1 - \cos\theta \end{cases}$$

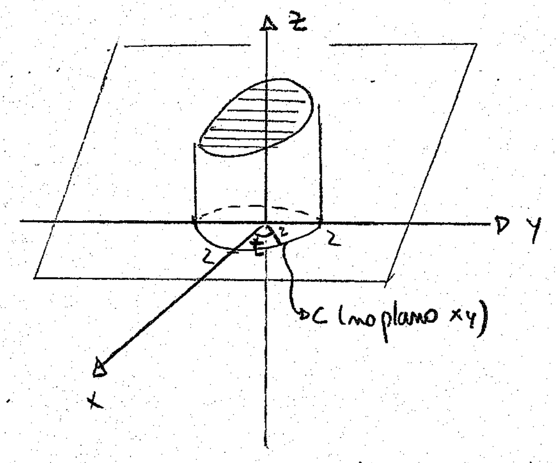
densidade do arame

$f(x,y,z) = xy = (\cos\theta + 1)\sqrt{2}\,\text{sen}\theta = \frac{\sqrt{2}}{2}\text{sen}2\theta + \sqrt{2}\,\text{sen}\theta = \frac{\sqrt{2}}{2}(\text{sen}2\theta + 2\,\text{sen}\theta)$

$\vec{r}(\theta) = (\cos\theta + 1, \sqrt{2}\,\text{sen}\theta, 1 - \cos\theta) \rightarrow \vec{r}'(\theta) = (-\text{sen}\theta, \sqrt{2}\cos\theta, \text{sen}\theta) \rightarrow |\vec{r}'(\theta)| = \sqrt{2\text{sen}^2\theta + 2\cos^2\theta} = \sqrt{2}$

$$\int f(\vec{r}(\theta)) \cdot |\vec{r}'(\theta)| = \frac{\cancel{\sqrt{2}}}{\cancel{2}}\int_0^\pi (\text{sen}2\theta + 2\,\text{sen}\theta)\,\cancel{\sqrt{2}} = -\frac{\cos 2\theta}{2}\Big|_0^\pi - 2\cos\theta\Big|_0^\pi = -\frac{1}{2} + \frac{1}{2} + 2 + 2 = 4$$ //

3) $x^2+y^2=4$; $z=0$; $x+y+z=2$; $z \geq 0$



$$A = \int_C (2-x-y)\,ds = \int_0^{2\pi} (2-2\cos t-2\operatorname{sen} t).\,2\,dt$$

$\frac{1}{2}\left(1-\frac{1}{2}-\frac{1}{2}\right)$

$\frac{1}{2}-\frac{1}{4}-\frac{1}{4}$

$\begin{cases} x = 2\cos t \\ y = 2\operatorname{sen} t \end{cases} \rightarrow \vec{r}(t) = (2\cos t, 2\operatorname{sen} t) \rightarrow \vec{r}\,'(t) = (-2\operatorname{sen} t, 2\cos t) \rightarrow |\vec{r}\,'(t)| = \sqrt{4} = 2$

$$A = 4\int_0^{2\pi} (1-\cos t-\operatorname{sen} t)\,dt = 4.\left(t-\operatorname{sen} t+\cos t\right)\Big|_0^{2\pi} = 4\left[2\pi - 0 + \cancel{1} - 0 + 0 - \cancel{1}\right] = 8\pi$$

preço = 8π M reais

4) a) $F(x,y) = (x^2-2xy,\ y^2-2xy)$

$C:\ y = x^2$ de $(-2,4)$ a $(1,1)$ $\rightsquigarrow \begin{cases} A = (-2,4) \\ B = (1,1) \end{cases}$

'arametrizando:

$$C: \begin{cases} x = t \\ y = t^2 \end{cases} \sim -2 \le t \le 1$$

$$\gamma(t) = (t, t^2) \longrightarrow \gamma'(t) = (1, 2t)$$

$$F(\gamma(t)) = (t^2-2t^3,\ t^4-2t^3)$$

$$\int_{-2}^{1} (t^2-2t^3, t^4-2t^3)\cdot(1,2t)\,dt = \int_{-2}^{1} (t^2-2t^3+2t^5-4t^4)\,dt =$$

$$= \left[\frac{t^3}{3} - \frac{2t^4}{4} + \frac{2t^6}{6} - \frac{4t^5}{5}\right]_{-2}^{1} = \frac{1}{3} - \frac{2}{4} + \frac{2}{6} - \frac{4}{5} - \left(-\frac{8}{3} - \frac{\cancel{32}^{\,8}}{\cancel{4}} + \frac{\cancel{128}^{\,64}}{\cancel{6}_{\,3}} + \frac{128}{5}\right)$$

$$= \underset{/10}{\frac{2}{3}} - \underset{/15}{\frac{1}{2}} - \underset{/6}{\frac{4}{5}} + \underset{/10}{\frac{8}{3}} + \underset{/30}{8} - \underset{/10}{\frac{64}{3}} - \underset{/6}{\frac{128}{5}} = \frac{+20-15-24+80+240-640-768}{30}$$

$$= \frac{-1107}{30} = \frac{-369}{10}\ //$$

b) $F(x,y) = \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\right)$; $C: x^2+y^2=a^2$

$\begin{cases} x = a\cos t \\ y = a\,\text{sen}\, t \end{cases} \rightarrow \sigma(t) = (a\cos t, a\,\text{sen}\, t) \rightarrow \sigma'(t) = (-a\,\text{sen}\, t, a\cos t)$

$0 \leq t \leq 2\pi$

$F(\sigma(t)) = (\cos t, \text{sen}\, t)$

$$\int_0^{2\pi} (\cos t, \text{sen}\, t) \cdot (-a\,\text{sen}\, t, a\cos t) = \int_0^{2\pi} (-a\,\cancel{\text{sen}\, t\cos t} + a\,\cancel{\text{sen}\, t\cos t})\, dt = 0$$

c) $F(x,y) = (y+3x, 2y-x)$; $C: 4x^2+y^2 = 4 \rightsquigarrow x^2 + \frac{y^2}{4} = 1$

$\begin{cases} x = \cos t \\ y = 2\,\text{sen}\, t \end{cases} \rightarrow \sigma(t) = (\cos t, 2\,\text{sen}\, t) \rightarrow \sigma'(t) = (-\text{sen}\, t, 2\cos t)$

$0 \leq t \leq 2\pi$

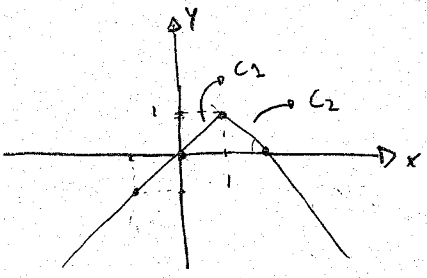
$F(x,y) = (2\,\text{sen}\, t + 3\cos t, 4\,\text{sen}\, t - \cos t)$

$$\int_C F\, dr = \int_0^{2\pi} (2\,\text{sen}\, t + 3\cos t, 4\,\text{sen}\, t - \cos t)(-\text{sen}\, t, 2\cos t)\, dt =$$

$$= \int_0^{2\pi} (-2\,\text{sen}^2 t - 3\,\text{sen}\, t\cos t + 8\,\text{sen}\, t\cos t - 2\cos^2 t) = \int_0^{2\pi} \left(\frac{5}{2}\,\text{sen}\, 2t - 2\right) dt$$

$$= \left[-\frac{5}{4}\cos 2t - 2t\right]_0^{2\pi} = -\cancel{\frac{5}{4}} - 4\pi - \left(-\cancel{\frac{5}{4}} - 0\right) = -4\pi$$

1) $F(x,y) = (x^2+y^2, x^2-y^2)$; $C: y = 1 - |1-x| \rightsquigarrow (0,0)$ a $(2,0)$



Parametrizando:

C_1 : $\boxed{y = x} \to \begin{cases} x = t \\ y = t \end{cases} \rightsquigarrow 0 \le t \le 1$

$\sigma_1(t) = (t,t) \to \sigma_1'(t) = (1,1)$

$F(\sigma_1(t)) = (2t^2, 0)$

C_2 : $\boxed{y = -x+2} \begin{cases} x = t \\ y = 2-t \end{cases} \to 1 \le t \le 2 \quad \sigma(t) = (t, 2-t) \to \sigma'(t) = (1,-1)$

$F(\sigma_2(t)) = (t^2+(2-t)^2, t^2-(2-t)^2)$

$$\int_C F \cdot dr = \int_0^1 (2t^2,0)(1,1)dt + \int_1^2 (2t^2-2t+4, +2t-4)(1,-1)\,dt$$

$$= \int_0^1 2t^2 + \int_1^2 (2t^2-4t+4-4t+4)\,dt = \frac{2t^3}{3}\Big|_0^1 + \left(\frac{2t^3}{3} - 4t^2 + 8t\right)\Big|_1^2$$

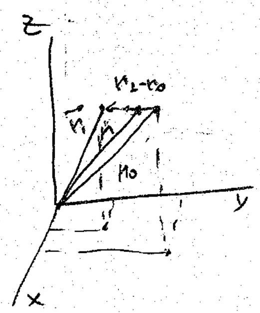
$$= \cancel{\frac{2}{3}} + \left(\frac{16}{3} - \cancel{\frac{2}{3}} - \cancel{16} + 4 + \cancel{16} - 8\right) = \frac{16}{3} - 4 = \frac{4}{3} \;//$$

2) $F(x,y,z) = (x, y, xz-y)$; $C \to$ segmento de reta de $(0,0,0)$ a $(1,2,4)$

Segmento de reta : $r(t) - r_0 = t(r_1 - r_0)$ onde $0 \le t \le 1$

$\vec{r_0} = (0,0,0)$

$\vec{r_1} = (1,2,4)$



$(x,y,z) - (0,0,0) = t(1,2,4) \to \begin{cases} x = t \\ y = 2t \\ z = 4t \end{cases} \to 0 \le t \le 1$

$\sigma(t) = (t, 2t, 4t) \to \sigma'(t) = (1,2,4)$

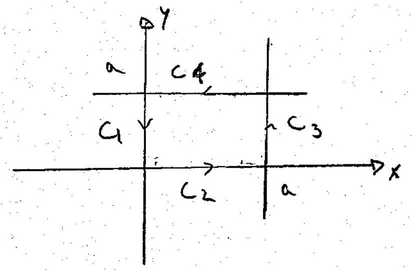
$F(\sigma(t)) = (t, 2t, 4t^2-2t)$

$$\int_0^1 (t, 2t, 4t^2-2t)\cdot(1,2,4)\,dt = \int_0^1 (t+4t+16t^2-8t)\,dt = \int_0^1 (16t^2-3t)\,dt = \left(\frac{16t^3}{3} - \frac{3t^2}{2}\right)\Big|_0^1 =$$

$$= \frac{16}{3} - \frac{3}{2} = \frac{23}{6} \;//$$

(11)

5) $F(x,y) = (x^2 - y^2, 2xy)$



Parametrizando:

$C_1: \quad x = 0 \ e \ 0 \leq y \leq a$

$\sigma_1(t) = (0, t) \longrightarrow \sigma'(t) = (0,1)$

$F(\sigma(t)) = (-t^2, 0)$

$C_2: \ 0 \leq x \leq a \ e \ y = 0$

$\sigma_2(t) = (t, 0) \rightarrow \sigma'(t) = (1,0)$

$F(\sigma(t)) = (t^2, 0)$

$C_3: x = a \ e \ 0 \leq y \leq a$

$\sigma_3(t) = (a, t) \rightarrow \sigma'(t) = (0,1)$

$F(\sigma(t)) = (a^2 - t^2, 2at)$

$C_4: 0 \leq x \leq a \ e \ y = a$

$\sigma_4(t) = (t, a) \rightarrow \sigma_4'(t) = (1,0)$

$F(\sigma_4(t)) = (t^2 - a^2, 2at)$

$$W = \int_C F ds = \int_a^0 (-t^2, 0)(0,1)\,dt + \int_0^a (t^2, 0).(1,0)\,dt + \int_0^a (a^2 - t^2, 2at).(0,1)\,dt + \int_a^0 (t^2 - a^2, 2at).(1,0) =$$

$$= 0 + \frac{t^3}{3}\Big|_0^a + at^2\Big|_0^a + \left[\frac{t^3}{3} - a^2 t\right]_a^0$$

$$= \cancel{\frac{a^3}{3}} + a^3 - \cancel{\frac{a^3}{3}} + a^3 = 2a^3 \ J$$

6) $F(x,y,z) = (y^2, z^2, x^2)$

$$C: \begin{cases} x^2 + y^2 + z^2 = a^2 \ ; \ z \geq 0 \ e \ a > 0 \\ x^2 + y^2 = ax \ (I) \end{cases} \Rightarrow ax + z^2 = a^2 \rightarrow (C)$$

$x^2 - ax + y^2 = 0$

$\left(x^2 - ax + \frac{a^2}{4}\right) + y^2 = \frac{a^2}{4}$

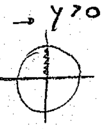
$\left(x - \frac{a}{2}\right)^2 + y^2 = \frac{a^2}{4}$

$$\begin{cases} x = a\,sen^2 t \\ z = a\cos t \end{cases} \rightarrow \sigma(t) = (a\,sen^2 t, a|sen 2t|, a\cos t)$$

$\sigma'(t) = (a\,sen 2t, 2a|\cos 2t|, -a\,sen t)$

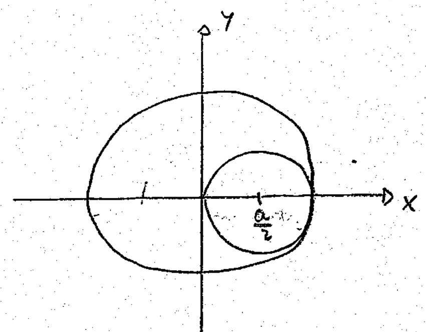
(I) $y^2 = ax - x^2 = a^2 sen^2 t - a^2 sen^4 t \rightarrow y = a|sen t \cos t| \rightarrow y > 0$

$\rightarrow y = a|sen 2t|$



$0 \leq 2t \leq \pi$

$0 \leq t \leq \pi/2$



$$W = \int_0^{\frac{\pi}{2}} (a^2 sen^2 2t, a^2 \cos^2 t, a^2 sen^4 t).(a\,sen 2t, 2a \cos 2t, -a\,sen t)\,dt$$

?

$$= \int_0^{\frac{\pi}{2}} a^3 sen^3 2t + 2a^3 \cos 2t . \cos^2 t - a^3 sen^5 t$$

OU: $\begin{cases} x = t \\ y = \sqrt{at-t^2} \\ z = \sqrt{a^2-at} \end{cases}$

C: $ax+z^2 = a^2$

$z \geq 0 \to a^2 - \cancel{a}t \geq 0 \to t \leq a$

$x \geq 0 \to t \geq 0 \to 0 \leq t \leq a$

$\sigma(t) = (t, \sqrt{at-t^2}, \sqrt{a^2-at})$

$\sigma'(t) = \left(1, \frac{(a-2t)}{2\sqrt{at-t^2}}, \frac{-a}{2\sqrt{a^2-at}}\right)$

$F(\sigma(t)) = (at-t^2, a^2-at, t^2)$

$$W = \int_0^a (at-t^2, a^2-at, t^2) \cdot \left(1, \frac{(a-2t)}{2\sqrt{at-t^2}}, \frac{-a}{2\sqrt{a^2-at}}\right) dt$$

$$W = \int_0^a \left[at-t^2 + (a^2-at) \cdot \frac{(a-2t)}{2\sqrt{at-t^2}} - \frac{at^2}{2\sqrt{a^2-at}}\right] dt$$

$$W = \int_0^a \left[at-t^2 + \frac{a^3-3a^2t+2at^2}{2\sqrt{at-t^2}} \; \frac{-at^2}{2\sqrt{a^2-at}}\right] dt$$

?

$\left.\begin{array}{l} x^2+y^2+z^2 = a^2 \\ x^2+y^2 = ax \end{array}\right\} \quad ax+z^2 = a^2 \; (\mathrm{II})$

$\left(x-\frac{a}{2}\right)^2 + y^2 = \frac{a^2}{4} \; (\mathrm{I})$

$\begin{cases} x = \frac{a}{2}(\cos t+1) \\ y = \frac{a}{2}\operatorname{sen} t \\ z = a^2 - ax \end{cases}$

$\sigma(t) = \left(\frac{a}{2}\cos t + \frac{a}{2}, \frac{a}{2}\operatorname{sen} t, a^2 - \frac{a^2}{2}(1-\cos t)\right)$

$\sigma'(t) = \left(-\frac{a}{2}\operatorname{sen} t, \frac{a}{2}\cos t, -\frac{a^2}{2}\operatorname{sen} t\right)$

$F(\sigma(t)) = \left(\frac{a^2}{4}\operatorname{sen}^2 t, a^4 - a^3 \cdot (\cos t+1) + \frac{a^3}{2} \cdot (\cos t+1)^2, \frac{a^2}{4}(\cos t+1)^2\right)$

$F(\sigma(t)) \cdot \sigma'(t) = \frac{-a^3}{4}\operatorname{sen}^3 t + \frac{a^5}{2}\cos t + \frac{a^4}{2}$

7) a) $F(x,y) = (e^x \operatorname{sen} y, e^x \cos y)$

$$\frac{\partial f}{\partial x} = e^x \operatorname{sen} y \xrightarrow{\int} f(x,y) = e^x \operatorname{sen} y + A(y) \rightarrow A(y) = B(x) = 0$$

$$\frac{\partial f}{\partial y} = e^x \cos y \xrightarrow{\int} f(x,y) = e^x \operatorname{sen} y + B(x)$$

$$\boxed{f(x,y) = e^x \operatorname{sen} y}$$

b) $F(x,y) = (2xy^2 - y^3, 2x^2y - 3xy^2 + 2)$

$$\frac{\partial f}{\partial x} = 2xy^2 - y^3 \xrightarrow{\int} f(x,y) = x^2y^2 - xy^3 + A(y)$$

$$\frac{\partial f}{\partial y} = 2x^2y - 3xy^2 + 2 \xrightarrow{\int} f(x,y) = x^2y^2 - xy^3 + 2y + B(x)$$

$$\begin{cases} A(y) = 2y \\ B(x) = 0 \end{cases}$$

$$\boxed{f(x,y) = x^2y^2 - xy^3 + 2y}$$

c) $F(x,y) = (3x^2 + 2y - y^2e^x, 2x - 2ye^x)$

$$\frac{\partial f}{\partial x} = 3x^2 + 2y - y^2e^x \xrightarrow{\int} f(x,y) = x^3 + 2yx - y^2e^x + A(y)$$

$$\frac{\partial f}{\partial y} = 2x - 2ye^x \xrightarrow{\int} f(x,y) = 2xy - y^2e^x + B(x)$$

$$\begin{cases} A(y) = 0 \\ B(x) = x^3 \end{cases}$$

$$\boxed{f(x,y) = x^3 + 2xy - e^xy^2}$$

d) $F(x,y,z) = (y+z, x+z, x+y)$

$$\frac{\partial f}{\partial x} = y+z \xrightarrow{\int} f(x,y,z) = xy + xz + A(y,z)$$

$$\frac{\partial f}{\partial y} = x+z \xrightarrow{\int} f(x,y,z) = xy + yz + B(x,z)$$

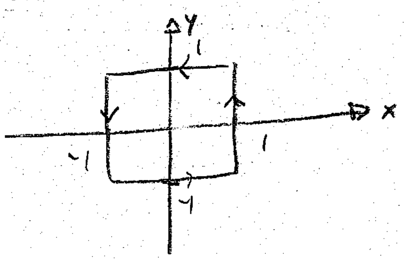
$$\frac{\partial f}{\partial z} = x+y \xrightarrow{\int} f(x,y,z) = xz + yz + C(x,y)$$

$$\begin{cases} A(y,z) = yz \\ B(x,z) = xz \\ C(x,y) = xy \end{cases}$$

$$\boxed{f(x,y,z) = xy + xz + yz}$$

$\boxed{6.5}$

1) a) $\oint \underbrace{y^2}_{Q} dx + \underbrace{x^2}_{P} dy$; $C: D = [-1,1] \times [-1,1]$



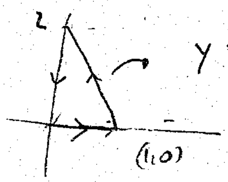
$Q = y^2 \longrightarrow \dfrac{\partial Q}{\partial y} = 2y$

$P = x^2 \longrightarrow \dfrac{\partial P}{\partial x} = 2x$

$$\oint y^2 dx + x^2 dy = \iint_D (P_x - Q_y)\, dxdy = \int_{-1}^{1} dy \int_{-1}^{1} (2x-2y)\, dx = \int_{-1}^{1} (x^2 - 2xy)\Big|_{-1}^{1} dy = \int_{-1}^{1} -4y\, dy$$

$$= -2y^2\Big|_{-1}^{1} = -2+2 = 0$$

b) $\oint \underbrace{(3x^2+y)}_{} dx + \underbrace{4y^2}_{} dy$ $\quad D: \Delta = \begin{cases} (0,0) \\ (1,0) \\ (0,2) \end{cases}$



$P = 3x^2+y \longrightarrow \dfrac{\partial P}{\partial y} = 1$

$Q = 4y^2 \longrightarrow \dfrac{\partial Q}{\partial x} = 0$

$$\oint P dx + Q dy = \iint_D -1\, dxdy = -\int_0^1 dx \int_0^{-2x+2} dy = -\int_0^1 (-2x+2)\, dx = -2\int_0^1 (-x+1)\, dx = -2\left(-\frac{x^2}{2}+x\right)\Big|_0^1$$

$$-2\left(-\frac{1}{2}+1\right) = 1-2 = -1$$

$\boxed{\text{OU}}$

$P = 3x^2+y \longrightarrow \dfrac{\partial P}{\partial x} = 6x$

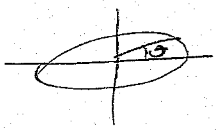
$Q = 4y^2 \longrightarrow \dfrac{\partial P}{\partial y} = 8y$

$$\iint_D (6x-8y)\, dxdy = \int_0^1 dx \int_0^{-2x+2} (6x-8y)\, dy = \int_0^1 (6xy - 4y^2)\Big|_0^{-2x+2} = \int_0^1 \left[6x(-2x+2) - 4(-2x+2)^2\right] dx =$$

$$= \int_0^1 (-28x^2 + 44x - 16) = \left[\frac{-28x^3}{3} + 22x^2 - 16x\right]_0^1 = \frac{-28}{3} + 22 - 16 = \frac{-28+66-48}{3} = \frac{-10}{3}$$

(13)

c) $\oint \underbrace{(e^x-3y)}_{Q}dx + \underbrace{(e^y-6x)}_{P}dy$; $C: x^2 + \frac{y^2}{1/4} = 1$



$$\begin{cases} x = r\cos\theta \\ y = \frac{r\,\text{sen}\,\theta}{2} \end{cases} \rightarrow 0 \le \theta \le 2\pi$$

$$\begin{cases} Q = e^x - 3y \longrightarrow \frac{\partial Q}{\partial y} = -3 \\ P = e^y - 6x \longrightarrow \frac{\partial P}{\partial x} = -6 \end{cases}$$

$$J_{(r,\theta)} = \begin{vmatrix} \cos\theta & \frac{\text{sen}\,\theta}{2} \\ -r\,\text{sen}\,\theta & \frac{r\cos\theta}{2} \end{vmatrix} = -\frac{r}{2}\text{sen}^2\theta - \frac{r}{2}\cos^2\theta = -\frac{r}{2}$$

$$|J_{(r,\theta)}| = \frac{r}{2}$$

$$\iint_C (-6+3)\,dxdy = \iint_C -3\,dxdy$$

$$\Rightarrow C: \; r^2\cos^2\theta + \frac{4r^2\text{sen}^2\theta}{4} \le 1$$

$$r^2 \le 1 \rightarrow \underset{r \ge 0}{r \le 1} \rightarrow 0 \le r \le 1$$

$$= \iint -3 \cdot \frac{r}{2}\,drd\theta$$

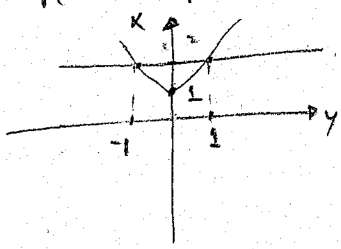
$$= -\frac{3}{2}\int_0^{2\pi} d\theta \int_0^1 r\,dr = -\frac{3}{2}\int_0^{2\pi} d\theta \left.\frac{r^2}{2}\right|_0^1 = \frac{-3}{4}\int_0^{2\pi} d\theta = -\frac{3\pi}{2}$$

d) $\oint \underbrace{x^{-1}e^y}_{Q}dx + \underbrace{(e^y \ln x + 2x)}_{P}dy$; C: $x = y^4 + 1$ e $x = 2$

$x - 1 = y^4 \rightarrow x - 1 \ge 0 \rightarrow \boxed{x \ge 1}$

$\sqrt{(x-1)} = y^2$

$y^4 + 1 \le x \le 2$



$$P = e^y \ln x + 2x \longrightarrow \frac{\partial P}{\partial x} = \frac{e^y}{x} + 2$$

$$Q = x^{-1}e^y \longrightarrow \frac{\partial Q}{\partial y} = x^{-1}e^y$$

$y^4 + 1 = 2$

$\boxed{y = \pm 1}$

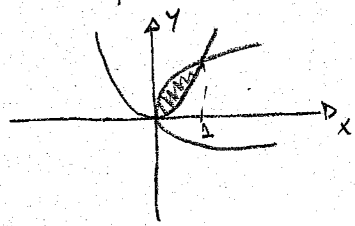
$$\iint_C \cancel{\frac{e^y}{x}} + 2 - \cancel{\frac{e^y}{x}}\,dxdy = \iint 2\,dxdy = 2\int_{-1}^{1} dy \int_{y^4+1}^{2} dx$$

$$2\int_{-1}^{1}(2 - y^4 - 1)\,dy = 4\int_0^1 (1-y^4)\,dy = 4\cdot\left.\left(y - \frac{y^5}{5}\right)\right|_0^1 = \frac{16}{5}$$

e) $\oint \underbrace{(2xy - x^2)}_{Q}dx + \underbrace{(x - y^2)}_{P}dy$; $C: \; y = x^2$ e $y^2 = x$ $\longrightarrow$ $x^4 = x \Rightarrow x(x^3 - 1) = 0$

$\begin{cases} x = 0 \\ x = +1 \end{cases}$ $(\pm i)$

$\hookrightarrow$



$P = x + y^2 \longrightarrow \dfrac{\partial P}{\partial x} = 1$

$Q = 2xy - x^2 \longrightarrow \dfrac{\partial Q}{\partial y} = 2x$

$$\iint\limits_C (1-2x)\,dx\,dy = \int_0^1 dx \int_{x^2}^{+\sqrt{x}} (1-2x)\,dy = \int_0^1 dx \,.\, (1-2x)y)\Big|_{x^2}^{+\sqrt{x}} = \int_0^1 (1-2x)(\sqrt{x} - x^2)\,dx$$

$$\int_0^1 \left(\sqrt{x} - x^2 - 2x^{3/2} + 2x^3\right) dx = \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3} - \frac{4x^{5/2}}{5} + \frac{x^4}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{3} - \frac{4}{5} + \frac{1}{2} =$$

$$= \frac{1}{3}_{/10} + \frac{1}{2}_{/15} - \frac{4}{5}_{/6} = \frac{10 + 15 - 24}{30} = \frac{1}{30} \;//$$

f) $\oint \underbrace{(x+y)}_{Q}dx + \underbrace{(y-x)}_{P}dy$; $C: \; (x-a)^2 + y^2 = a^2$

$$\begin{cases} x = r\cos\theta + a \\ y = r\,\mathrm{sen}\,\theta \end{cases} \rightarrow J_{(r,\theta)} = \begin{vmatrix} \cos\theta & \mathrm{sen}\,\theta \\ -r\,\mathrm{sen}\,\theta & r\cos\theta \end{vmatrix} = -r \Rightarrow |J_{(r,\theta)}| = r$$

$C: \; r^2 \leq a^2 \rightarrow \boxed{0 \leq r \leq a}$

$0 \leq \theta \leq 2\pi$

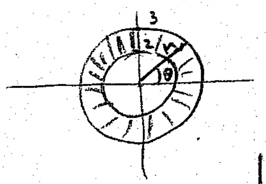
$Q = x + y \longrightarrow \dfrac{\partial Q}{\partial y} = 1$

$P = y - x \longrightarrow \dfrac{\partial P}{\partial x} = -1$

$$\iint\limits_C (-1-1)\,dx\,dy = -2\int_0^{2\pi} d\theta \int_0^a r\,dr = -\cancel{2}\int_0^{2\pi} \frac{a^2}{\cancel{2}}\,d\theta = -2\pi a^2 \;//$$

g) $\oint (\underbrace{2x-y^3}_{Q})\,dx - \underbrace{xy}_{P}\,dy$; C: $x^2+y^2=4$ e $x^2+y^2=9$

$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$$



$2 \leq r \leq 3$

$0 \leq \theta \leq 2\pi$

$|J(r,\theta)| = r$

$P = xy \longrightarrow \dfrac{\partial P}{\partial x} = y$

$Q = 2x - y^3 \longrightarrow \dfrac{\partial Q}{\partial y} = -3y^2$

$$\iint_C (y + 3y^2)\,dx\,dy = \int_0^{2\pi} d\theta \int_2^3 (r\,\text{sen}\,\theta + 3r^2\,\text{sen}^2\theta)\,r\,dr =$$

$$= \int_0^{2\pi} d\theta \cdot \left[\frac{r^3}{3}\,\text{sen}\,\theta + \frac{3r^4}{4}\,\text{sen}^2\theta\right]_2^3 = \int_0^{2\pi} \frac{19}{3}\,\text{sen}\,\theta\,d\theta + \int_0^{2\pi} \frac{195}{4}\,\text{sen}^2\theta\,d\theta$$

$$\cancel{-\frac{19}{3}\cdot\cos\theta\Big|_0^{2\pi}} \to 0 \; + \; \frac{195}{8}\int_0^{2\pi} 1 - \cos 2\theta = \frac{195}{8}\cdot\left[\theta\Big|_0^{2\pi} - \cancel{\frac{\text{sen}\,2\theta}{2}\Big|_0^{2\pi}}\right] \to 0$$

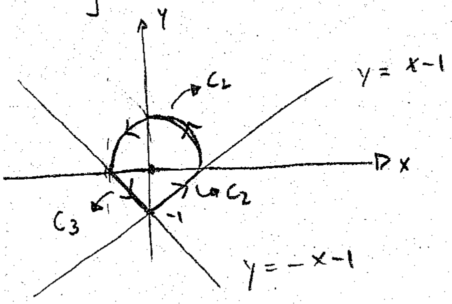
$$= \frac{195}{8}\cdot 2\pi = \frac{195\pi}{4}$$

2) $\oint (a_1 x + a_2 y + a_3) dx + (b_1 x + b_2 y + b_3) dy$

$$\begin{cases} P = b_1 x + b_2 y + b_3 \rightarrow \dfrac{\partial P}{\partial x} = b_1 \\ Q = a_1 x + a_2 y + a_3 \rightarrow \dfrac{\partial Q}{\partial y} = a_2 \end{cases}$$

$$= \iint_C \underbrace{(b_1 - a_2)}_{cte} dx dy = (b_1 - a_2) \iint dx dy = (b_1 - a_2) A$$

3) $\oint (x^2 + y + 2xy^3) dx + (5x + 3x^2y^2 + y) dy \qquad C: C_1 \cup C_2 \cup C_3$



$$\begin{cases} P = 5x + 3x^2y^2 + y \rightarrow \dfrac{\partial P}{\partial x} = 5 + 6xy^2 \\ Q = x^2 + y + 2xy^3 \rightarrow \dfrac{\partial Q}{\partial y} = 1 + 6xy^2 \end{cases}$$

C_1: $\begin{cases} x = r\cos\theta \\ y = r \operatorname{sen}\theta \end{cases} \rightarrow |J(r,\theta)| = r$

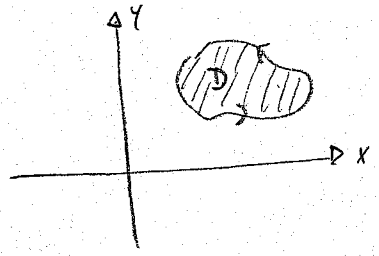
$$\int_0^\pi d\theta \int_0^1 (5 + \cancel{6xy^2} - 1 - \cancel{6xy^2}) r\, dr = 2\pi$$

C_2: $\int_0^1 dx \int_{x-1}^0 4 dy = 4 \int_0^1 dy \int_{x-1}^0 dx = 4 \int_0^1 (-x+1) dx = 4 \left(\frac{-x^2}{2} + x \right) \Big|_0^1 = 2$

C_3: $\int_{-1}^0 dx \int_{-x-1}^0 4 dy = 4 \int_{-1}^0 (x+1) dx = 4 \left(\frac{x^2}{2} + x \right) \Big|_{-1}^0 = 4\left(-\frac{1}{2} + 1\right) = 2$

$$\oint_C F \cdot dr = \int_{C_1} F dr + \int_{C_2} F dr + \int_{C_3} F dr = 2\pi + 2 + 2 = 2\pi + 4$$

4)

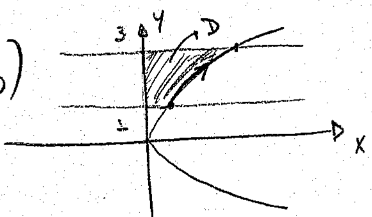


a) $\oint 0.dx + x\,dy$

$P = x \rightarrow \frac{\partial P}{\partial x} = 1 \quad \Rightarrow \quad \iint_D (1-0)\,dxdy = \iint_D dxdy = \text{área}(D)$ //

$Q = 0 \rightarrow \frac{\partial Q}{\partial y} = 0$

b)



$\iint_D 1\,dxdy = \oint 0\,dx + x\,dy$

$\int_1^3 dy \int_0^{y^2} dx = \int_1^3 y^2\,dy = \frac{y^3}{3}\Big|_1^3 = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$ //

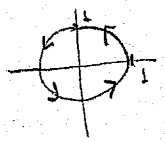
$\begin{cases} 1 \leq y \leq 3 \\ 0 \leq x \leq y^2 \end{cases}$

5) $F = (F_1, F_2) \rightarrow$ campo vetorial, menos em $(0,0)$ $\qquad C: \frac{x^2}{4} + \frac{y^2}{25} = 1$

↳ área $= 10\pi$

$\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} + 4 \rightarrow \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 4 \rightarrow \forall (x,y) \neq (0,0)$

Dado: $\oint_\gamma \bar{F}\,d\bar{s} = 6\pi \rightsquigarrow \gamma:$



$\oint_C \bar{F}\,d\bar{s} = \int_\gamma \bar{F}\,d\bar{s} + \oint_{C \cup \gamma^-} F\,ds \quad (\text{I})$

Em $\oint_{C \cup \gamma^-} F\,ds$, posso aplicar o teorema de Green: $\oint_{C \cup \gamma^-} F\,ds = \iint_D 4\,dxdy = 4\,\text{área}(D) =$

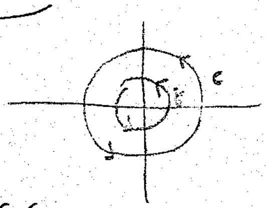
$= 4.(10\pi - \pi) = 36\pi$

Em (I): $\oint_C \bar{F}\,d\bar{s} = 6\pi + 36\pi = 42\pi$ //

6) $F(x,y) = \left(\underbrace{\dfrac{-y}{x^2+y^2}}_{P} , \underbrace{\dfrac{x}{x^2+y^2} + 3x}_{Q} \right)$

$$\left| \begin{array}{l} P_y = \dfrac{2xy}{(x^2+y^2)^2} \\ Q_x = \dfrac{2xy}{(x^2+y^2)^2} + 3 \end{array} \right. \rightarrow Q_x - P_y = 3$$

a) $c: \; x^2+y^2 = 4$



$$\oint_{C \cup \gamma^-} \bar{F} d\bar{s} = \iint_D (P_x - Q_y)\, dx\, dy = 3 \iint_D dx\, dy = 3 \text{ area } (D) = 3(4\pi - \pi) = 9\pi$$

$$\oint_{\gamma} \bar{F} d\bar{s} \; ; \left| \begin{array}{l} x = \cos t \\ y = \operatorname{sen} t \end{array} \right. \rightarrow \sigma(t) = (\cos t, \operatorname{sen} t) \rightarrow \sigma'(t) = (-\operatorname{sen} t, \cos t)$$

$$F(\sigma(t)) = (-\operatorname{sen} t, 4\cos t)$$

$$\oint_{\gamma} F ds = \int_0^{2\pi} (\operatorname{sen} t, 4\cos t) \cdot (-\operatorname{sen} t, \cos t) dt = \int_0^{2\pi} (\operatorname{sen}^2 t + 4\cos^2 t) dt = \int_0^{2\pi} (1 + 3\cos^2 t) dt$$

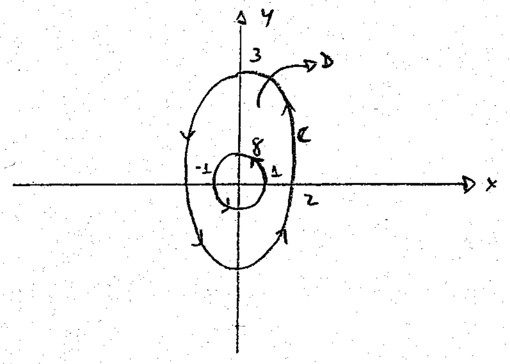
$$\cos^2 t = \frac{1 + \cos 2t}{2} \rightarrow 1 + 3\cos^2 t = 1 + \frac{3}{2}(1 + \cos 2t)$$

$$\int_0^{2\pi} \left(1 + \frac{3}{2} + \frac{3\cos 2t}{2}\right) dt = 2\pi + 3\pi + 0 = 5\pi$$

$$\Rightarrow \oint_C \bar{F} d\bar{s} = 9\pi + 5\pi = 14\pi$$

Usando Teorema de Green com restrições no domínio: (RESUMO) (16)

Ex: Seja um campo vetorial $F(x,y) = \left(\frac{-y}{x^2+y^2} , \frac{x}{x^2+y^2} + 2x \right)$ ao longo de $C: \frac{x^2}{4} + \frac{y^2}{9} = 1$ no sentido anti-horário, calcule $\oint_C \bar{F} d\bar{s}$:



$$\oint \underbrace{\frac{-y}{x^2+y^2}}_{P} dx + \underbrace{\left(\frac{x}{x^2+y^2} + 2x \right)}_{Q} dy$$

$$P_y = \frac{2xy}{(x^2+y^2)^2} \Rightarrow \boxed{Q_x - P_y = 2}$$

$$Q_x = \frac{2xy}{(x^2+y^2)^2} + 2$$

$$\int_{C \cup \gamma^-} \bar{F} d\bar{s} = \iint_D 2\, dx dy = 2 \cdot \text{area}(D) = 2 \left(\underbrace{\text{area}(c)}_{\text{elipse}} - \underbrace{\text{area}(\gamma)}_{\text{circunferência}} \right) = 2(6\pi - \pi) = 10\pi$$

Então: $\oint_C \frac{-y}{x^2+y^2} dx + \left(\frac{x}{(x^2+y^2)} + 2x \right) dy \Rightarrow \oint_C F ds$

$$\oint_{C \circlearrowleft} F ds - \oint_{\gamma \circlearrowleft} \bar{F} d\bar{s} = \underbrace{\oint_{C \cup \gamma^-} \bar{F} ds}_{10\pi} \Rightarrow \boxed{\oint_C \bar{F} d\bar{s} = 10\pi + \int_{\gamma \circlearrowleft} \bar{F} d\bar{s}} \; (\text{I})$$

Calculando $\int_{\gamma \circlearrowleft} \bar{F} d\bar{s}$ para achar $\oint_C \bar{F} d\bar{s} \Rightarrow$ (Lembrando que não pode aplicar green em γ pois $(x,y) \neq (0,0)$)

$$\int_\gamma \frac{-y}{x^2+y^2} dx + \left(\frac{x}{x^2+y^2} + 2x \right) dx \Rightarrow F = \left(\frac{-y}{x^2+y^2} , \frac{x}{x^2+y^2} + 2x \right)$$

$\begin{cases} x = \cos t \\ y = \text{sen}\, t \end{cases} \rightarrow \sigma(t) = (\cos t, \text{sen}\, t) \rightarrow \sigma'(t) = (-\text{sen}\, t, \cos t)$; $0 \leq t \leq 2\pi$

$$F(\sigma(t)) = \left(\frac{-\text{sen}\, t}{1} , \frac{\cos t}{1} + 2\cos t \right) = (-\text{sen}\, t, 3\cos t)$$

$$\int_0^{2\pi} (-\text{sen}\, t, 3\cos t) \cdot (-\text{sen}\, t, \cos t)\, dt = \int_0^{2\pi} (\text{sen}^2 t + 3\cos^2 t)\, dt = \int_0^{2\pi} (1 + 2\cos^2 t)\, dt = \int_0^{2\pi} (\cos 2t + 2)\, dt$$

$$= 4\pi + \cancel{\frac{\text{sen}\, 2t}{2}} \Big|_0^{2\pi} = 4\pi$$

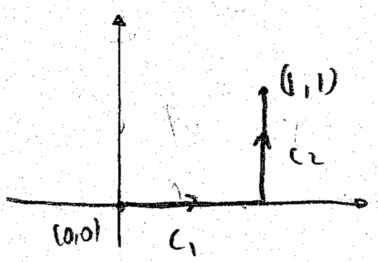
Daí: (Em I) $\oint_C \bar{F} d\bar{s} = 10\pi + 4\pi = 14\pi$

6.7

1) a) Independente do caminho → Campo potencial ou conservativo:

$$P_y = Q_x \to 2y - x = k(2x - 4y) \to 2y - x = 2k(x - 2y) \to \boxed{k = -1/2}$$

b) A = (0,0) até B = (1,1)



C_1: $x = t \to 0 \le t \le 1$; $y = 0 \to \gamma_1(t) = (t, 0) \to \gamma_1'(t) = (1,0)$

C_2: $y = t \to 0 \le t \le 1$; $x = 1 \to \gamma_2(t) = (1, t) \to \gamma_2'(t) = (0,1)$

$$F = \left(y^2 - xy, -\frac{x^2}{2} - 2xy\right)$$

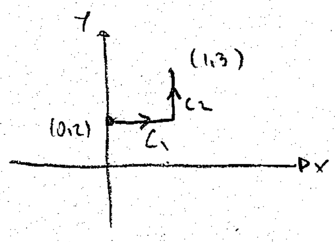
$$\int_0^1 \cancel{\left(0, -\frac{t^2}{2}\right)\cdot(1,0)dt} + \int_0^1 \left(t^2 - t, -\frac{1}{2} + 2t\right)\cdot(0,1)dt = \int_0^1 \left(-\frac{1}{2} + 2t\right)dt = \left(-\frac{t}{2} + t^2\right)\Big|_0^1$$

0

$$= \frac{1}{2} \;//$$

2) b) $\begin{cases} P_y = -\dfrac{3x^2}{y^2} \\ Q_x = -\dfrac{3x^2}{y^2} \end{cases} \to$ independem do caminho

$$F = \left(\frac{3x^2}{y}, -\frac{x^3}{y^2}\right)$$



C_1: $y = 2$; $x = t \to 0 \le t \le 1$

$\gamma(t) = (t, 2) \to \gamma'(t) = (1, 0)$

C_2: $x = 1$; $y = t \to 2 \le t \le 3$

$\gamma(t) = (1, t) \to \gamma'(t) = (0, 1)$

$$\int_0^1 \left(\frac{3t^2}{2}, -\frac{t^3}{4}\right)\cdot(1,0)dt + \int_2^3 \left(\frac{3}{t}, -\frac{1}{t^2}\right)\cdot(0,1)dt = \int_0^1 \frac{3t^2}{2}dt - \int_2^3 \frac{1}{t^2}dt =$$

$$\cancel{\frac{1}{2}} + \left(\frac{1}{3} - \cancel{\frac{1}{2}}\right) = \frac{1}{3} \;//$$

$$t^{-2} = \frac{t^{-1}}{-1}$$

3) a) $V(x,y) = (\overbrace{2xy^3 - y^2\cos x}^{P}, \overbrace{1 - 2y\,\text{sen}\,x + 3x^2y^2}^{Q})$

$$\begin{cases} P_y = 2xy^3 - y^2\cos x \\ Q_x = 1 - 2y\,\text{sen}\,x + 3x^2y^2 \end{cases}$$

$f(x,y) = \int P_y dx = x^2y^3 - y^2\,\text{sen}\,x + A(y)$

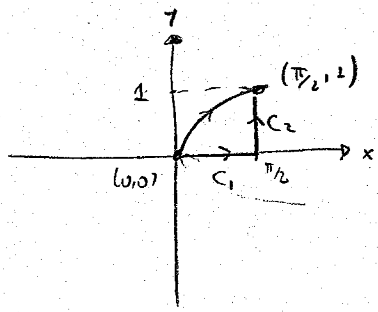
$f(x,y) = \int Q_x dy = y - y^2\,\text{sen}\,x + x^2y^3 + B(x)$

$\rightarrow \begin{cases} A(y) = y \\ B(x) = 0 \end{cases}$

$f(x,y) = x^2y^3 - y^2\,\text{sen}\,x + y$

b) $P_y = 6xy - y^2\,\text{sen}\,x$

$Q_x = -y^2\,\text{sen}\,x + 6xy$ $\rightarrow$ independe do caminho



$C: \; 2x = \pi y^2$ de $P_1(0,0)$ a $P_2(\pi/2, 1)$

$C_1: \; y = 0 \rightarrow x = t \rightarrow 0 \le t \le \pi/2$

$\sigma_1(t) = (t, 0) \rightarrow \sigma_1'(t) = (1, 0)$

$C_2: \; y = t \rightarrow 0 \le y \le 1$

$x = \pi/2$

$\sigma_2(t) = (\pi/2, t) \rightarrow \sigma_2'(t) = (0, 1)$

$$\int_0^{\frac{\pi}{2}} \cancel{F(\sigma_1(t)) \cdot \sigma_1'(t)}^{\,0} + \int_0^1 \left(\pi t^3, 1 - 2t + \frac{3\pi^2}{4} t^2\right) \cdot (0,1)\, dt = 1 - 1 + \frac{\pi^2}{4} = \frac{\pi^2}{4}$$

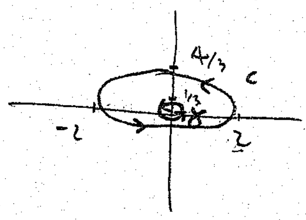
$\frac{\pi^2 t^3}{4}$

4) $\oint_C \frac{yx^2\,dx - x^3\,dy}{(x^2+y^2)^2}$ com $C: \frac{x^2}{9} + \left(y-\frac{1}{3}\right)^2 = 1$

$$P_y = \frac{x^2(x^2+y^2)^{\cancel{2}} - 2y^2x^2 \cdot 2\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{4}}} = \frac{x^2(x^2+y^2) - 4x^2y^2}{(x^2+y^2)} = \frac{x^4-3x^2y^2}{(x^2+y^2)}$$

$$Q_x = -\left[\frac{3x^2(x^2+y^2)^{\cancel{2}} - 2x^4 \cdot 2\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{4}}}\right] = \frac{-3x^2(x^2+y^2)+4x^4}{(x^2+y^2)} = \frac{x^4-3x^2y^2}{(x^2+y^2)}$$

independe do caminho



Restrições $(x,y) = (0,0)$

$\gamma: x^2+y^2 = a^2$

$\sigma(t) = (a\cos t, a \operatorname{sen} t)$

$\sigma'(t) = (-a\operatorname{sen} t, a\cos t)$

$$\oint_C F\,ds = \oint_{C\cup\gamma^-} F\,ds + \oint_\gamma \bar{F}\,d\bar{s}$$

$$F(\sigma(t)) = \left(\frac{+a^3\cos^2 t \operatorname{sen} t}{a^4}, \frac{-a^3\cos^3 t}{a^4}\right)$$

$$\oint_{C\cup\gamma^-} F\,ds = 0 + \oint_\gamma \bar{F}\,d\bar{s} \Rightarrow \oint_C F\,ds = \oint_\gamma \bar{F}\,d\bar{s}$$

$$\oint_\gamma \bar{F}\,d\bar{s} = \int_0^{2\pi} \left(\frac{+a^3\cos^2 t \operatorname{sen} t}{a^4}, \frac{-a^3\cos^3 t}{a^4}\right)(-a\operatorname{sen} t, a\cos t) = \int_0^{2\pi}\left(\frac{-\cancel{a^4}\cos^2 t \operatorname{sen}^2 t}{a^4} - \frac{\cancel{a^4}\cos^3 t\cos t}{\cancel{a^4}}\right)dt$$

$$= \int_0^{2\pi} -\cancel{\cos^4 t} - \cos^2 t(1-\cancel{\cos^2 t}) = -\int_0^{2\pi}\cos^2 t\,dt = -\int_0^{2\pi}\frac{1+\cos 2t}{2} = -\pi$$

5) $\displaystyle\oint_C \underbrace{\frac{x+y}{x^2+y^2}}_{P}\,dx + \underbrace{\frac{y-x}{x^2+y^2}}_{Q}\,dy$

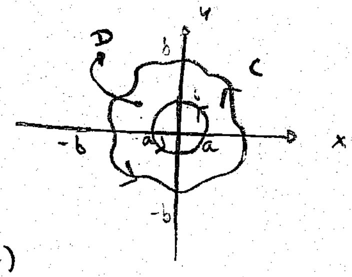
$$P_y = \frac{(x^2+y^2) - 2y(x+y)}{(x^2+y^2)^2} \quad \rightarrow \quad Q_x - P_y = \frac{-\cancel{2(x^2+y^2)} + \cancel{2(x^2+y^2)}}{(x^2+y^2)^2} = 0 \Rightarrow \oint_C Pdx+Qdy = \iint$$

$$Q_x = \frac{-(x^2+y^2) - 2x(y-x)}{(x^2+y^2)^2}$$

↳ independe do caminho

Restrição : $(x,y) \neq (0,0)$

Se C é uma curva qlqr, então :



(I) ANTI-HORÁRIO (C)

$$\boxed{\oint_C Pdx + Qdy = \oint_{C\cup\gamma} Pdx+Qdy + \int_\gamma Pdx+Qdy} \quad (A)$$

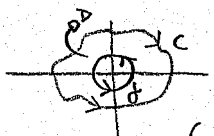
$$\oint_{C\cup\gamma^-} Pdx+Qdy = \iint_D (\cancel{Q_x - P_y})^{0}\,dxdy = 0 \Rightarrow \oint_C Pdx+Qdy = \oint_\gamma Pdx+Qdy \quad \text{onde } \gamma \Rightarrow x^2+y^2=a^2$$

então : $\begin{cases} x = a\cos t \\ y = a\,\text{sen}t \end{cases} \rightarrow \begin{array}{l} \sigma(t) = (a\cos t, a\,\text{sen}t) \\ \sigma'(t) = (-a\,\text{sen}t, a\cos t) \end{array} \rightarrow F(\sigma(t)) = \left(\frac{\cancel{a}(\cos t + \text{sen}t)}{a^2}, \frac{\cancel{a}(\text{sen}t - \cos t)}{a^2}\right)$

$$\oint_\gamma Pdx+Qdy = \int_0^{2\pi} F(\sigma(t))\cdot\sigma'(t)\,dt = \int_0^{2\pi} \left(\frac{\cos t + \text{sen}t}{a}, \frac{\text{sen}t - \cos t}{a}\right)\left(-a\,\text{sen}t, a\cos t\right)$$

$$= \int_0^{2\pi} \left[-\cancel{\text{sen}t\cos t} - \text{sen}^2 t + \cancel{\text{sen}t\cos t} - \cos^2 t\right] dt = \int_0^{2\pi} -1\,dt = -t\Big|_0^{2\pi} = -2\pi$$

(II) HORÁRIO (C) ⟶



Já sei que : $\displaystyle\oint_C Pdx+Qdy = \oint_\gamma Pdx+Qdy \Longrightarrow \oint_{C^-} Pdx+Qdy = -\oint_\gamma Pdx+Qdy = -(-2\pi) = 2\pi$

i) a) $F_1(x,y) = \operatorname{sen} x + 4xy$

$F_2(x,y) = 2x^2 - \cos y$

$$\frac{\partial F_1}{\partial y} = 4x \quad \rightarrow \quad \frac{\partial F_1}{\partial y} - \frac{\partial F_2}{\partial x} = 0 \rightarrow \text{independe do caminho}$$

$$\frac{\partial F_2}{\partial x} = 4x$$

$$\oint_C F_1 dx + F_2 dy = \iint_D \cancel{\left(\frac{\partial F_1}{\partial y} - \frac{\partial F_2}{\partial x}\right)}^{0} dx dy = 0$$

b) $F_1(x,y) = \dfrac{y}{x^2+y^2} \rightarrow \dfrac{\partial F_1}{\partial y} = \dfrac{(x^2+y^2) - 2y^2}{(x^2+y^2)^2} = \dfrac{x^2-y^2}{(x^2+y^2)^2}$

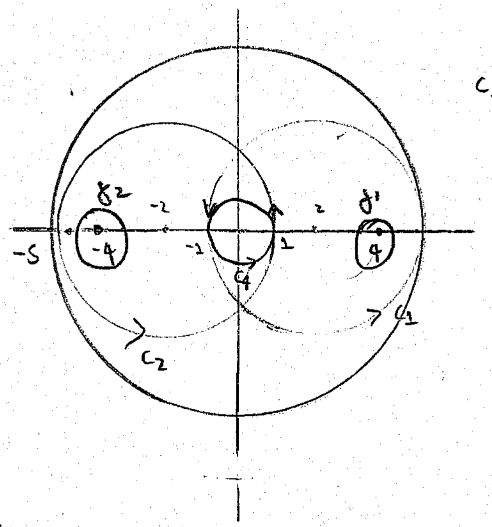
$F_2(x,y) = \dfrac{-x}{x^2+y^2} \qquad \dfrac{\partial F_2}{\partial x} = \dfrac{-(x^2+y^2) + 2x^2}{(x^2+y^2)^2} = \dfrac{x^2-y^2}{(x^2+y^2)^2}$

$\rightarrow \dfrac{\partial F_2}{\partial x} - \dfrac{\partial F_1}{\partial y} = 0$ q

$$\oint_C F_1 dx + F_2 dy = \iint_D 0 \, dx dy = 0$$

7)

$$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$$



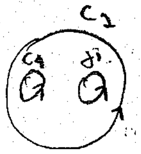
$\oint_{C_1} = 11$

$\oint_{C_2} = 9$

$\oint_{C_3} = 13$

$\oint_{C_4} = ?$

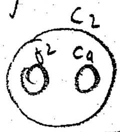
Olhando C_1:



$$\oint_{C_1} = \underbrace{\oint_{C_1 \cup \gamma_1^{-1} \cup C_4^{-1}}}_{0} + \oint_{C_4} + \oint_{\gamma_1} \Rightarrow \boxed{11 = \oint_{C_4} + \oint_{\gamma_2}} \rightarrow \oint_{\gamma_1} = 11 - \oint_{C_4}$$

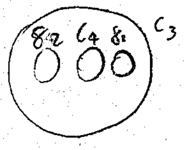
$\left(\iint \frac{\partial F_1}{\partial x} - \frac{\partial F_2}{\partial y} = 0 \right)$

Olhando C_2:



$$\ldots \Rightarrow \boxed{9 = \oint_{C_4} + \oint_{\gamma_2}} \rightarrow \oint_{\gamma_2} = 9 - \oint_{C_4}$$

Olhando C_3:



$$\oint_{C_3} = \underbrace{\oint_{C_3 \cup C_4 \cup \gamma_2 \cup \gamma_1}}_{0} + \oint_{C_4} + \oint_{\gamma_2} + \oint_{\gamma_1} \Rightarrow 13 = \cancel{\oint_{C_4}} + 9 - \oint_{C_4} + 11 - \cancel{\oint_{C_4}}$$

$$13 - 9 - 11 = -\oint_{C_4}$$

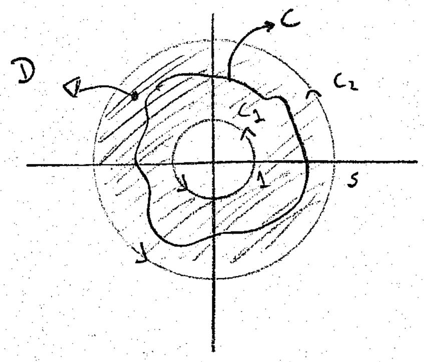
$$\boxed{\oint_{C_4} = 7}$$

3) $\left|\oint_{C_3}\right|$ (direita) $= \underbrace{\cancel{\oint_{C_3 \cup C_2^-}}}_{0} + \oint_{C_2} = \left|\oint_{C_2}\right| \rightarrow$

$\left|\oint_{C_3}\right|$ (esquerda) $= \underbrace{\cancel{\oint_{C_3 \cup C_1^-}}}_{0}$ (esquerda) $+ \left|\oint_{C_1}\right|$

$$\oint_{C_3} = \oint_{C_3\,(\text{direita})} + \oint_{C_3\,(\text{esquerda})} = -\oint_{C_2} + \oint_{C_1} = -15+12 = -3$$

4)



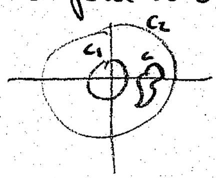
$$\oint_{C_1} = \oint_{C_2} = 2\pi$$

$$\oint_C = \underbrace{\cancel{\int_{C \cup C_1^-}}}_{0} + \oint_{C_1} \rightarrow \oint_C = \oint_{C_1}$$

C em anti-horário : $\oint_C = \oint_{C_1} = \oint_{C_2} = 2\pi$

C em horário : $\oint_C = \oint_{C_1} = \oint_{C_2} = -2\pi$

C fora do C_1



$$\oint_{C_2} = \underbrace{\cancel{\int_{C_2 \cup C_1^- \cup C^-}}}_{0} + \oint_{C_1} + \oint_C \rightarrow \cancel{\oint_{C_2}} = \cancel{\oint_{C_1}} + \oint_C$$

$$\boxed{\oint_C = 0}$$

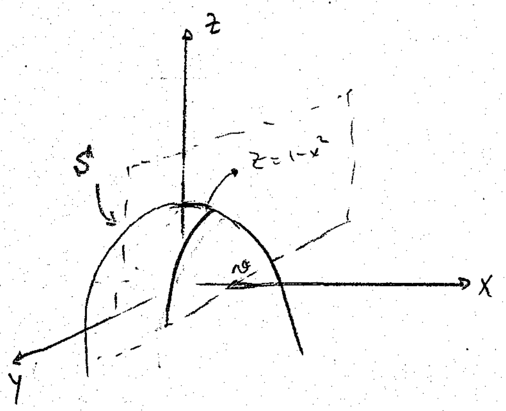
7.2 1) $\begin{cases} x = v\cos u \\ y = v\,\text{sen}\,u \\ z = 1 - v^2 \end{cases} \rightarrow \varphi(u,v) = (v\cos u, v\,\text{sen}\,u, 1-v^2)$

a) $x^2+y^2 = v^2 \rightarrow x^2+y^2 = 1-z \rightarrow z = 1-(x^2+y^2) \rightarrow$ parabolóide deslocado em z

$\begin{cases} \dfrac{\partial\varphi}{\partial u} = (-v\,\text{sen}\,u, v\cos u, 0) \\ \dfrac{\partial\varphi}{\partial v} = (\cos u, \text{sen}\,u, -2v) \end{cases}$

$N(u,v) = \dfrac{\partial\varphi}{\partial u} \times \dfrac{\partial\varphi}{\partial v} = \begin{vmatrix} i & j & k \\ -v\,\text{sen}\,u & v\cos u & 0 \\ \cos u & \text{sen}\,u & -2v \end{vmatrix} = (-2v^2\cos u, -2v^2\,\text{sen}\,u, -v)$

para $v = $ temos que N é nulo o que não pode, então S não é regular em (0,0,1)

b) 

$\varphi(u_0,v) = (v\cos u_0, v\,\text{sen}\,u_0, 1-v^2)$

$u_0 = 0 \rightarrow \varphi(0,v) = (v, 0, 1-v^2)$

$x = v$

$z = 1-v^2 \rightarrow z = 1-x^2$

(d) c) $\underset{v}{u=0} \rightarrow \varphi(0,v) \rightarrow \dfrac{\partial\varphi(0,v)}{\partial u} = (-v\,\text{sen}\,u, v\cos u, 0) \xrightarrow{\varphi(0,1)} (0,1,0)$

Vc trocou as respostas pq derivou em relação à variável trocada

(c) d) $\underset{v=1}{u} \rightarrow \varphi(u,1) \rightarrow \dfrac{\partial\varphi}{\partial v}(0,1) = (1,0,-2)$

Reta normal:

$\cancel{(x-1)} = -2t \rightarrow \boxed{x = 1-2t}$

$\boxed{y=0}$

$z = -t \rightarrow \boxed{z=-t}$

$t \in \mathbb{R}$

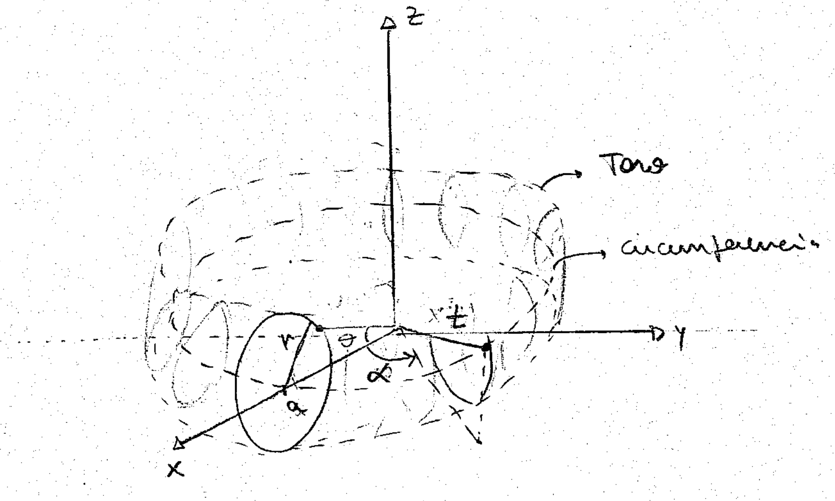
e) $N(u,v) = (-2v^2\cos u, -2v^2\,\text{sen}\,u, -v) \xrightarrow{\varphi(0,1)} (-2,0,-1)$

Plano tangente: $N(0,1) \cdot (x-1, y-0, z-0) = (-2,0,-1)(x-1,y,z) = 0$

para $\varphi(0,1) \rightarrow \begin{matrix} u=0 \\ v=1 \end{matrix} \rightarrow \begin{cases} x=1 \\ y=0 \\ z=0 \end{cases}$

Eq. do plano tg: $-2x+2-z=0 \rightarrow \boxed{2x+z-2=0}$

2) $C: (x-a)^2 + z^2 = r^2 \;;\; 0 < r < a$

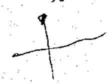


a) $C: \begin{cases} x = r\cos\theta + a \\ z = r\,\text{sen}\,\theta \end{cases}$

olhando para a circunferencia que forma o toro:

$S: \varphi(\theta,\alpha) = ((r\cos\theta + a)\cos\alpha,\ (r\cos\theta + a)\,\text{sen}\,\alpha,\ r\,\text{sen}\,\theta)$

$\cos\theta = 0$
$\text{sen}\,\theta = 1$



b) $\dfrac{\partial\varphi(\theta,\alpha)}{\partial\theta} = (-r\,\text{sen}\,\theta\cos\alpha,\ -r\,\text{sen}\,\theta\,\text{sen}\,\alpha,\ r\cos\theta)$

$\dfrac{\partial\varphi(\theta,\alpha)}{\partial\alpha} = (-(r\cos\theta + a)\,\text{sen}\,\alpha,\ (r\cos\theta + a)\cos\alpha,\ 0)$

$$\frac{\partial\varphi}{\partial\theta} \times \frac{\partial\varphi}{\partial\alpha} = \begin{vmatrix} i & j & k \\ -r\,\text{sen}\,\theta\cos\alpha & -r\,\text{sen}\,\theta\,\text{sen}\,\alpha & r\cos\theta \\ -(r\cos\theta+a)\,\text{sen}\,\alpha & (r\cos\theta+a)\cos\alpha & 0 \end{vmatrix}$$

$= ((r\cos\theta + a)\,r\cos\theta\cos\alpha,\ -(r\cos\theta + a)\,r\cos\theta\,\text{sen}\,\alpha,\ -(r\cos\theta + a)\,r\,\text{sen}\,\theta)$

$= (r\cos\theta + a)\,(r\cos\theta\cos\alpha,\ -r\cos\theta\,\text{sen}\,\alpha,\ -r\,\text{sen}\,\theta)$ //

c) para $\theta = \frac{\pi}{2} \rightarrow (-r\cos\alpha,\ -r\,\text{sen}\,\alpha,\ 0)$ → não fica nulo nunca!

para $\theta = 0 \rightarrow (0, 0, r)$

... ...

$\Rightarrow$ S é regular.

3) $\varphi_1(u,v) = (u, v, 0)$; $\varphi_2(u,v) = (u^3, v^3, 0)$

a) S_1 : $x = u$; $y = v$; $z = 0$

S_2 : $x = u^3$; $y = v^3$; $z = 0$

$$\frac{\partial \varphi_1}{\partial u} \times \frac{\partial \varphi_1}{\partial v} = (1,0,0) \times (0,1,0) = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0,0,1)$$

$N_1(u,v) = (0,0,1)$ → normal à S_1 ⟹ S_1 = plano xy
↳ normal ao plano xy

$$\frac{\partial \varphi_2}{\partial u} \times \frac{\partial \varphi_2}{\partial v} = (3u^2, 0, 0) \times (0, 3v^2, 0) = \begin{vmatrix} i & j & k \\ 3u^2 & 0 & 0 \\ 0 & 3v^2 & 0 \end{vmatrix} = (0, 0, 9u^2v^2)$$

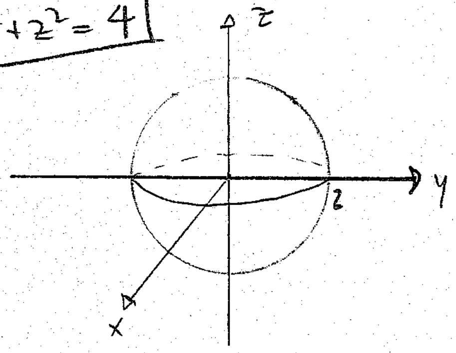
$N_2(u,v) = (0,0,9u^2v^2)$ → normal à S_2 ⟹ S_2 = plano xy
↳ normal ao plano xy

b) S_1 : $N_1(u,v) = (0,0,1)$ → regular $\forall (x,y) \in D$

S_2 : $N_2(u,v) = (0,0,9u^2v^2)$ → ñ regular para $(0,v)$, $(u,0)$ ou $(0,0)$

c) Não, pois as derivadas parciais não existem em $(0,0,0)$ não sendo possível nem achar o plano tangente nesse ponto.

1) $\boxed{x^2+y^2+z^2=4}$



no ponto $(1,1,\sqrt{2})$:

$x = 2\,\text{sen}\phi\cos\theta$ | $y = 2\,\text{sen}\phi\,\text{sen}\theta$ | $z = 2\cos\phi$

$1 = 2\,\text{sen}\phi\cos\theta$ | $1 = 2\,\text{sen}\phi\,\text{sen}\theta$ | $\sqrt{2} = 2\cos\phi$

$\cos\theta = \dfrac{1}{2\,\text{sen}\phi}$ | $\cos\theta = \text{sen}\theta \Rightarrow \theta = \frac{\pi}{4}$ | $\phi = \frac{\pi}{4}$

(I)

a) $\varphi(\phi,\theta) = (2\,\text{sen}\phi\cos\theta,\ 2\,\text{sen}\phi\,\text{sen}\theta,\ 2\cos\phi)$; $0 \le \phi \le \pi$ e $0 \le \theta \le 2\pi$

$$\frac{\partial\varphi}{\partial\phi} = (2\cos\phi\cos\theta,\ 2\cos\phi\,\text{sen}\theta,\ -2\,\text{sen}\phi)$$

$$\frac{\partial\varphi}{\partial\theta} = (-2\,\text{sen}\phi\,\text{sen}\theta,\ 2\,\text{sen}\phi\cos\theta,\ 0)$$

$$\frac{\partial\varphi}{\partial\phi} \times \frac{\partial\varphi}{\partial\theta} = \begin{vmatrix} i & j & k \\ 2\cos\phi\cos\theta & 2\cos\phi\,\text{sen}\theta & -2\,\text{sen}\phi \\ -2\,\text{sen}\phi\,\text{sen}\theta & 2\,\text{sen}\phi\cos\theta & 0 \end{vmatrix} = N(\phi,\theta)$$

$$N(\phi,\theta) = (4\,\text{sen}^2\phi\cos\theta,\ 4\,\text{sen}^2\phi\,\text{sen}\theta,\ 4\,\text{sen}\phi\cos\phi\cos^2\theta + 4\,\text{sen}\phi\cos\phi\,\text{sen}^2\theta)$$

$$N(\phi,\theta) = (4\,\text{sen}^2\phi\cos\theta,\ 4\,\text{sen}^2\phi\,\text{sen}\theta,\ 2\,\text{sen}(2\phi)) \quad (II)$$

(I) em (II): $N(\pi/4, \pi/4) = (\sqrt{2}, \sqrt{2}, 2)$

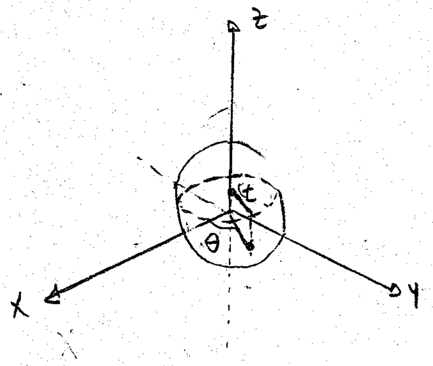
$N(\pi/4, \pi/4)\cdot(x-1, y-1, z-\sqrt{2}) = 0 \Rightarrow (\sqrt{2}, \sqrt{2}, 2)(x-1, y-1, z-\sqrt{2}) = 0$

$\sqrt{2}x - \sqrt{2} + \sqrt{2}y - \sqrt{2} + 2z - 2\sqrt{2} = 0$

$\sqrt{2}(x+y) + 2z - 4\sqrt{2} = 0 \Rightarrow \boxed{x + y + \sqrt{2}z = 4}$ plano tg em $(1,1,\sqrt{2})$

b) $x^2+y^2+z^2 = F(x,y,z) = K \to$

A esfera será uma superfície de nível para $K = 4$



A revolução da curva $x^2+z^2=4$ em torno do z dará a superfície $x^2+y^2+z^2=4$

$C: \ \sigma(\phi) = (x(\phi), z(\phi))$

$\sigma(\phi) = (2\cos\phi, 2\,\mathrm{sen}\,\phi) \qquad 0 \le \phi \le 2\pi$

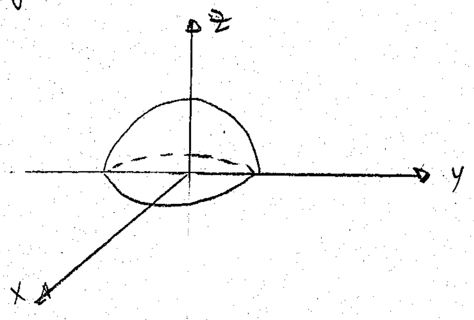
$S: \ \varphi(\phi,\theta) = (2\cos\phi\cos\theta, 2\cos\phi\,\mathrm{sen}\,\theta, 2\,\mathrm{sen}\,\phi) \ ; \quad 0 \le \theta \le 2\pi$

$\to$ Plano tg : $\quad x+y+\sqrt{2}z = 4$

(mesma parametrização)

c) $g(x,y) = \sqrt{4-x^2-y^2}$

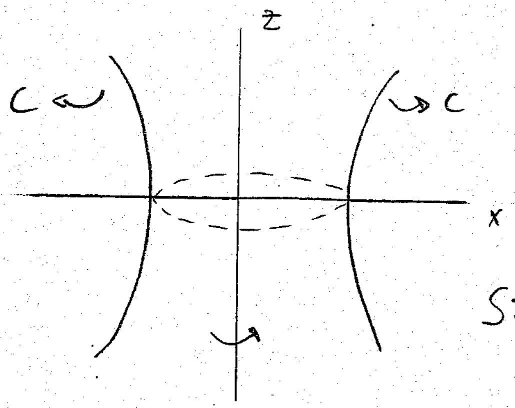
Plano tg $\to \quad x+y+\sqrt{2}z = 0$



D:

5) $x^2+y^2-z^2=1 \to S$

S é gerado pela revolução de $x^2-z^2=1$ (c) em torno do eixo z



$C: \gamma(\phi) = (\sec\phi, \operatorname{tg}\phi)$

$S: \varphi(\phi,\theta) = (\sec\phi\cos\theta, \sec\phi\operatorname{sen}\theta, \operatorname{tg}\phi)$

b) $$\frac{\partial\varphi}{\partial\phi} = (\operatorname{sen}\phi\cos\phi\cos\theta, \operatorname{sen}\phi\cos\phi\operatorname{sen}\theta, \sec^2\phi) = \left(\frac{\operatorname{sen}2\phi}{2}\cos\theta, \frac{\operatorname{sen}2\phi}{2}\operatorname{sen}\theta, \frac{1}{\cos^2\phi}\right)$$

$$\frac{\partial\varphi}{\partial\theta} = (-\sec\phi\operatorname{sen}\theta, \sec\phi\cos\theta, 0) = \left(-\frac{\operatorname{sen}\theta}{\cos\phi}, \frac{\cos\theta}{\cos\phi}, 0\right)$$

$$\frac{\partial\varphi}{\partial\phi} \times \frac{\partial\varphi}{\partial\theta} = \begin{vmatrix} i & j & k \\ \frac{\operatorname{sen}2\phi}{2}\cos\theta & \frac{\operatorname{sen}2\phi}{2}\operatorname{sen}\theta & \frac{1}{\cos^2\phi} \\ -\frac{\operatorname{sen}\theta}{\cos\phi} & \frac{\cos\theta}{\cos\phi} & 0 \end{vmatrix} = N(\phi,\theta)$$

$$N(\phi,\theta) = \left(-\frac{\cos\theta}{\cos^3\phi}, -\frac{\operatorname{sen}\theta}{\cos^3\phi}, \frac{\cancel{\operatorname{sen}2\phi}^{\operatorname{sen}\phi}\cos^2\theta}{\cancel{2\cos\phi}} + \frac{\cancel{\operatorname{sen}2\phi}^{\operatorname{sen}\phi}\operatorname{sen}^2\theta}{\cancel{2\cos\phi}}\right)$$

$$N(\phi,\theta) = \left(-\frac{\cos\theta}{\cos^3\phi}, -\frac{\operatorname{sen}\theta}{\cos^3\phi}, \operatorname{sen}\phi\right) //$$

c) Para $(x_0,y_0,0)$: $\varphi(\phi,\theta) = (\sec\phi\cos\theta, \sec\phi\operatorname{sen}\theta, \operatorname{tg}\phi)$

$\Rightarrow x_0 = \sec\phi\cos\theta$, $x_0 = \frac{\cos\theta}{\cos\phi}$ | $y_0 = \sec\phi\operatorname{sen}\theta$, $y_0 = \frac{\operatorname{sen}\theta}{\cos\phi}$ | $\operatorname{tg}\phi = 0$, $\boxed{\phi = 0}$

$\boxed{x_0 = \cos\theta \quad y_0 = \operatorname{sen}\theta \quad \phi = 0} \to \boxed{x_0^2+y_0^2=1}$ (I)

$\Rightarrow \varphi(0,\theta) = (\cos\theta, \operatorname{sen}\theta, 0) = (x_0,y_0,0)$

$\Rightarrow N(\phi,\theta) = N(0,\theta) = (-\cos\theta, -\operatorname{sen}\theta, 0) = (-x_0,-y_0,0)$

$N(0,\theta)\cdot(x-x_0, y-y_0, z-0) = 0 \Rightarrow (-x_0,-y_0,0)\cdot(x-x_0,y-y_0,z) = 0$

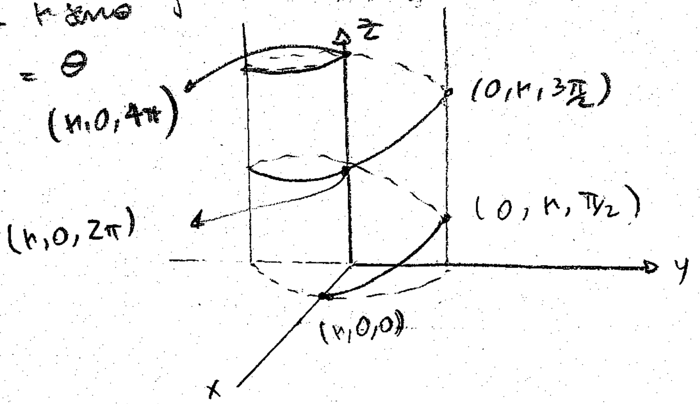
$\boxed{x_0^2 - x_0x + y_0^2 - y_0y = 0}$ (II) y_0

(I) e (II): $1 - x_0x - y_0y = 0 \Rightarrow \boxed{x_0x + y_0y = 1} \to$ Equação do plano tg em $(x_0,y_0,0)$

6) $\varphi(r,\theta) = (r\cos\theta, r\,\text{sen}\theta, \theta) \to 0 \leq r \leq 1$ e $0 \leq \theta \leq 4\pi$ (23)

$x(r,\theta) = r\cos\theta$ \
$y(r,\theta) = r\,\text{sen}\theta$ $\Big\} \to x(r,\theta)^2 + y(r,\theta)^2 = 1 \to$ circunferência em xy \
$z(r,\theta) = \theta$

hélice circular



b) $\dfrac{\partial\varphi}{\partial r} = (\cos\theta, \text{sen}\theta, 0)$

$\dfrac{\partial\varphi}{\partial\theta} = (-r\,\text{sen}\theta, r\cos\theta, 1)$

$$\frac{\partial\varphi}{\partial r} \times \frac{\partial\varphi}{\partial\theta} = \begin{vmatrix} i & j & k \\ \cos\theta & \text{sen}\theta & 0 \\ -r\,\text{sen}\theta & r\cos\theta & 1 \end{vmatrix} = (\text{sen}\theta, -\cos\theta, r)$$

$N(r,\theta) = (\text{sen}\theta, -\cos\theta, r) \to$

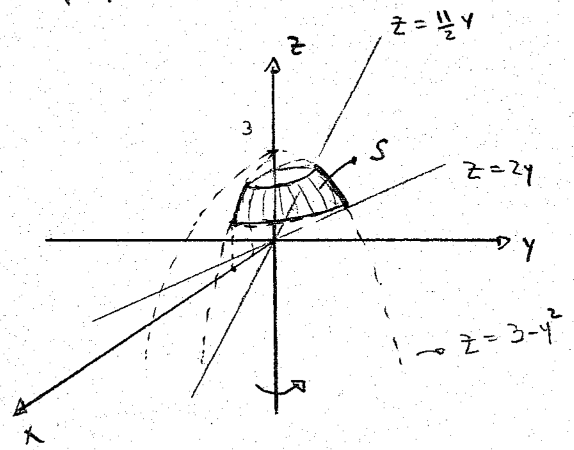
c) é Regular $\forall (r,\theta) \in D$

7.4

1) $\gamma: z = 3 - y^2 \rightarrow$ arco de parábola entre as semi-retas $z = 2y$ e $z = \frac{11}{2}y$ com $y \geq 0$

$S \rightarrow$ superfície de revolução de γ em torno de z :

a)



$y \geq 0 \rightarrow t \geq 0$

γ: $z \geq 2y \rightarrow z \geq 2t \rightarrow 3 - t^2 \geq 2t$ (I)

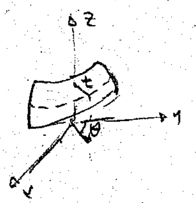
$z \leq \frac{11}{2}y \rightarrow z \leq \frac{11}{2}t \rightarrow 3 - t^2 \leq \frac{11}{2}t$ (II)

(I) : $t^2 + 2t - 3 \leq 0 \rightarrow \cancel{-3 \leq} t \leq 1$

(II) : $t^2 + \frac{11}{2}t - 3 \geq 0 \rightarrow \cancel{t \leq -6}$ ou $t \geq \frac{1}{2}$

$\Downarrow$

$\frac{1}{2} \leq t \leq 1$; $0 \leq \theta \leq 2\pi$



$\sigma(t) = (y(t), z(t)) \Rightarrow \sigma(t) = (t, 3 - t^2)$

$\downarrow$

Revolução gerará $S \Rightarrow$

$$\boxed{\varphi(t,\theta) = (t \operatorname{sen}\theta, t\cos\theta, 3 - t^2)}$$

OBS: Já que a curva "γ" foi definida no plano zy, qnd girarmos p/ formar S, o ângulo θ será com y

b) $\frac{\partial\varphi}{\partial\theta} = (t\cos\theta, -t\operatorname{sen}\theta, 0)$

$\frac{\partial\varphi}{\partial t} = (\operatorname{sen}\theta, \cos\theta, -2t)$

$$\Rightarrow \frac{\partial\varphi}{\partial\theta} \times \frac{\partial\varphi}{\partial t} = \begin{vmatrix} i & j & k \\ t\cos\theta & -t\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \cos\theta & -2t \end{vmatrix} = N(t,\theta)$$

$$N(t,\theta) = (2t^2\operatorname{sen}\theta, 2t^2\cos\theta, t) \Rightarrow |N(t,\theta)| = \sqrt{4t^4 + t^2} = t\sqrt{4t^2+1}$$

$$A(S) = \int_0^{2\pi} d\theta \int_{1/2}^{1} t\sqrt{4t^2+1}\, dt = \frac{2\pi}{8}\int_{1/2}^{1} \sqrt{4t^2+1}\, d[4t^2+1] = \frac{\cancel{2}}{3}(4t^2+1)^{3/2} \cdot \frac{\cancel{2}\pi}{\cancel{8}_{2}}\Bigg|_{1/2}^{1}$$

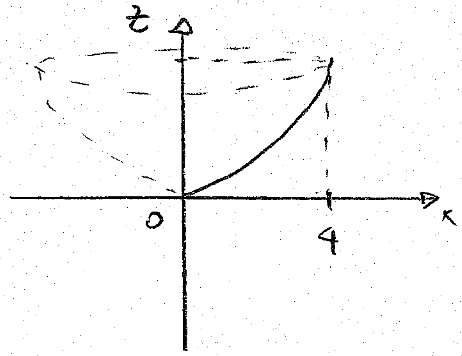
$$= \frac{\pi}{6}\left(5^{3/2} - 2^{3/2}\right) = \frac{\pi}{6}\left(5\sqrt{5} - 2\sqrt{2}\right) //$$

2) $C: z = x^2$ → em termos de z

$0 \leq x \leq 4$

(24)

a)



$C: \gamma(t) = (x(t), z(t))$

$\gamma(t) = (t, t^2)$

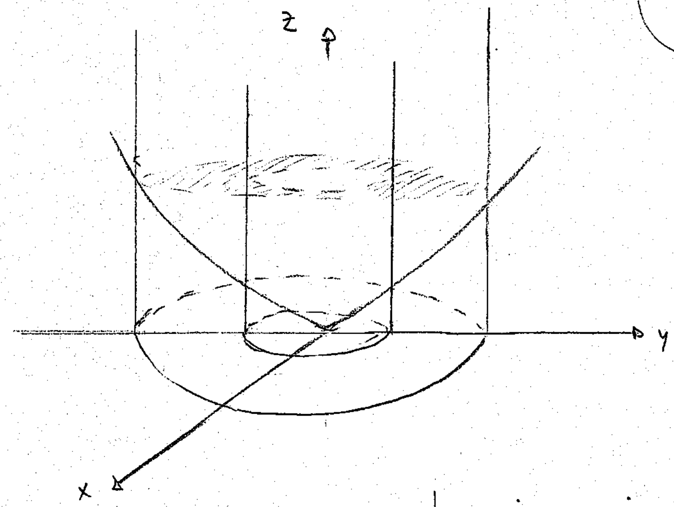
$(0 \leq t \leq 4)$

$S: \varphi(t,\theta) = (t\cos\theta, t\,\text{sen}\,\theta, t^2)$ →

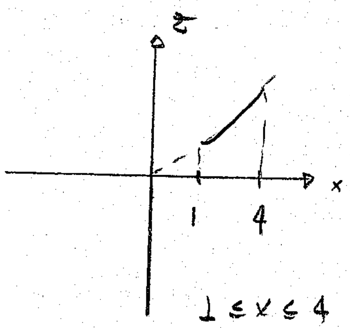
$(0 \leq \theta \leq 2\pi)$

OBS: Agora, como a curva C foi parametrizada no plano xz, será a partir deste plano que irá girar, ou seja, o ângulo θ é com $\underline{x}$

b)



⇒



$1 \leq x \leq 4$

$\boxed{1 \leq t \leq 4}$

$\boxed{0 \leq \theta \leq 2\pi}$

$$N(t,\theta) = \frac{\partial\varphi}{\partial t} \times \frac{\partial\varphi}{\partial\theta} = \begin{vmatrix} i & j & k \\ \cos\theta & \text{sen}\,\theta & 2t \\ -t\,\text{sen}\,\theta & t\cos\theta & 0 \end{vmatrix} = (-2t^2\cos\theta, -2t^2\,\text{sen}\,\theta, t)$$

$$|N(t,\theta)| = \sqrt{4t^4 + t^2} = t\sqrt{4t^2+1}$$

$$A(S) = \int_0^{2\pi} d\theta \int_1^4 t\sqrt{4t^2+1}\,dt = \frac{2\pi}{8}\int_1^4 \sqrt{4t^2+1}\,d[4t^2+1] = \frac{2\pi}{8}\cdot(4t^2+1)^{3/2}\Big|_1^4 =$$

$$= \frac{\pi}{6}\left(17\sqrt{17} - 5\sqrt{5}\right)_4$$

3) $C: z = 1 - x^2$ $\qquad$ $1 - t^2 \geq 0 \rightarrow t^2 \leq 1 \rightarrow -1 \leq t \leq 1$

$\begin{cases} y \geq 0 \\ z \geq 0 \end{cases}$ e $z \leq 1 - y^2$ $\qquad$ interseção: $1 - y^2 = 1 - x^2 \rightarrow |x| = |y|$

$$\varphi(x,y) = (x, y, f(x,y)) = (x, y, 1-x^2)$$

$$\frac{\partial \varphi}{\partial x} = (1, 0, -2x) \rightarrow \frac{\partial \varphi}{\partial x} \times \frac{\partial \varphi}{\partial y} = \begin{vmatrix} i & j & k \\ 1 & 0 & -2x \\ 0 & 1 & 0 \end{vmatrix} = (2x, 0, 1)$$

$$\frac{\partial \varphi}{\partial y} = (0, 1, 0)$$

$$N(x,y) = (2x, 0, 1) \rightarrow |N(x,y)| = \sqrt{4x^2 + 1}$$

Interseção: $1 - y^2 = 1 - x^2 \Rightarrow |x| = |y|$

$\Rightarrow 1 - x^2 \geq 0 \rightarrow -1 \leq x \leq 1 \longrightarrow 0 \leq x \leq 1$ ou $0 \leq y \leq 1$

$D_1: 0 \leq x \leq 1 \rightarrow 0 \leq y \leq x$

$D_2: 0 \leq y \leq 1 \rightarrow 0 \leq x \leq y$

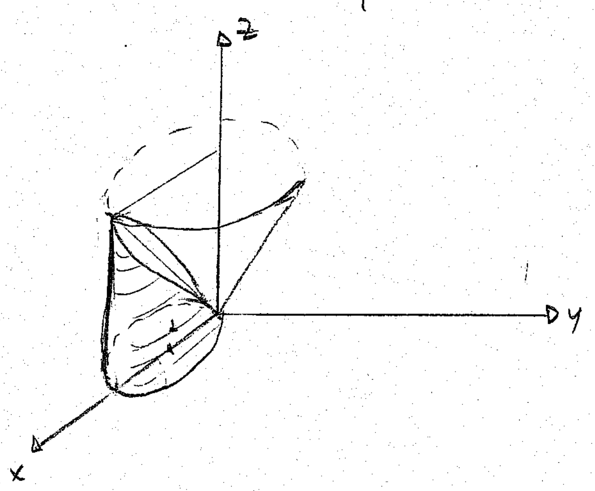
$$A_1 = \int_0^1 dx \int_0^x \sqrt{4x^2+1}\, dy = \int_0^1 x\sqrt{4x^2+1}\, dx = \frac{1}{8} \cdot \frac{2}{3} (4x^2+1)^{3/2} \Big|_0^1 = \frac{5\sqrt{5} - 1}{12}$$

$$A_2 = \int_0^1 dy \int_0^y \sqrt{4y^2+1}\, dx = \frac{5\sqrt{5} - 1}{12}$$

$$A_T = A_1 + A_2 = \frac{5\sqrt{5} - 1}{6}$$

$$P = \frac{(5\sqrt{5} - 1) A}{6} \quad //$$

4) a) $x^2 + y^2 = 2x$ $\begin{cases} z = 0 \\ z = \sqrt{x^2+y^2} \end{cases}$



Parametrizando a circunferencia que forma o cilindro:

$(x-1)^2 + y^2 = 1 \Rightarrow \begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$

$r = 2\cos\theta$

$\Rightarrow \begin{cases} x = 2\cos^2\theta \\ y = \text{sen}\,2\theta \end{cases}$

$\sigma(\theta) = (2\cos^2\theta, \text{sen}\,2\theta, 0)$

Parametrizando S: $\varphi(\theta, t) = (2\cos^2\theta, \text{sen}\,2\theta, t)$

$\Rightarrow \begin{cases} -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \\ 0 \leq t \leq 2\cos\theta \end{cases}$

$\begin{cases} \dfrac{\partial\varphi}{\partial\theta} = (-2\,\text{sen}\,2\theta, 2\cos 2\theta, 0) \\ \\ \dfrac{\partial\varphi}{\partial t} = (0, 0, 1) \end{cases}$

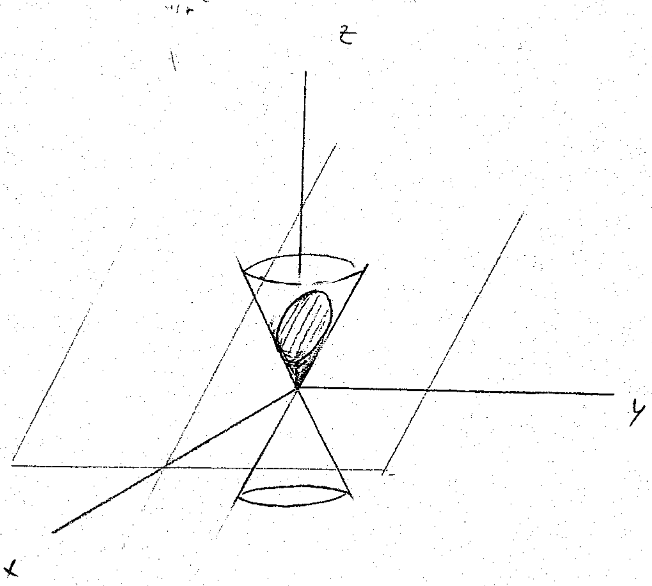
$\Rightarrow \dfrac{\partial\varphi}{\partial\theta} \times \dfrac{\partial\varphi}{\partial t} = \begin{vmatrix} i & j & k \\ -2\,\text{sen}\,2\theta & 2\cos 2\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = N(\theta, t)$

$N(\theta, t) = (2\,\text{sen}\,2\theta, 2\cos 2\theta, 0)$

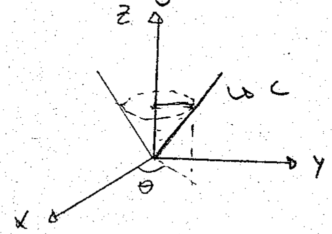
$|N(\theta, t)| = 2$

$$A(S) = \int_{-\pi/2}^{\pi/2} d\theta \int_0^{2\cos\theta} 2\, dt = \int_{-\pi/2}^{\pi/2} 4\cos\theta\, dt = 4\,\text{sen}\,\theta \Big|_{-\pi/2}^{\pi/2} = 8 \,//$$

b) $z^2 = x^2 + y^2$ $\begin{cases} z = 0 \\ z = \frac{3-x}{2} \end{cases}$



Parametrizando o cone:



$C: \sigma(t) = (0, t, t)$

$S: \varphi(t,\theta) = (t\cos\theta, t\sin\theta, |t|)$ $\quad z^2 = x^2+y^2 = 0 + t^2 \to z = \sqrt{t^2} = |t|$

$\Rightarrow 0 \leq \theta \leq 2\pi$

$0 \leq t \leq \frac{3 - t\cos\theta}{2}$

$\downarrow$

$2t \leq 3 - t\cos\theta$

$t(2+\cos\theta) \leq 3$

$t \leq \frac{3}{2+\cos\theta} \Rightarrow 0 \leq t \leq \frac{3}{2+\cos\theta}$

$$\Rightarrow \frac{\partial\varphi}{\partial t} \times \frac{\partial\varphi}{\partial\theta} = \begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & 1 \\ -t\sin\theta & t\cos\theta & 0 \end{vmatrix} = (-t\cos\theta, -t\sin\theta, t)$$

$|N(t,\theta)| = t\sqrt{2}$

$$A(S) = \int_0^{2\pi} d\theta \int_0^{\frac{3}{2+\cos\theta}} t\sqrt{2} = \frac{\sqrt{2}}{2}\int_0^{2\pi} d\theta \, [t^2]_0^{\frac{3}{2+\cos\theta}} = \frac{9\sqrt{2}}{2}\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2} = \text{Resolve aí...}$$

OU

$\varphi(x,y) = (x, y, \sqrt{x^2+y^2})$

$\frac{\partial\varphi}{\partial x} = \left(1, 0, \frac{x}{\sqrt{x^2+y^2}}\right)$ $\quad \Rightarrow \frac{\partial\varphi}{\partial x} \times \frac{\partial\varphi}{\partial y} = \begin{pmatrix} 1 \\ 0 \\ \frac{x}{\sqrt{x^2+y^2}} \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ \frac{y}{\sqrt{x^2+y^2}} \end{pmatrix} = \left(-\frac{x}{\sqrt{x^2+y^2}}, -\frac{y}{\sqrt{x^2+y^2}}, 1\right)$

$\frac{\partial\varphi}{\partial y} = \left(0, 1, \frac{y}{\sqrt{x^2+y^2}}\right)$ $\quad |N(x,y)| = 2$

OUTRA FORMA DE FAZER:

b)

Parametrização: $\varphi(x,y) = (x, y, \sqrt{x^2+y^2})$

$$\frac{\partial \varphi}{\partial x} = \left(1, 0, \frac{x}{\sqrt{x^2+y^2}}\right) \qquad \frac{\partial \varphi}{\partial y} = \left(0, 1, \frac{y}{\sqrt{x^2+y^2}}\right)$$

$$\Rightarrow N(x,y) = \frac{\partial \varphi}{\partial x} \times \frac{\partial \varphi}{\partial y} = \begin{vmatrix} i & j & k \\ 1 & 0 & \frac{x}{\sqrt{x^2+y^2}} \\ 0 & 1 & \frac{y}{\sqrt{x^2+y^2}} \end{vmatrix} = \left(\frac{-x}{\sqrt{x^2+y^2}}, \frac{-y}{\sqrt{x^2+y^2}}, 1\right)$$

$$|N(x,y)| = \sqrt{2}$$

$$A(S) = \iint_D \sqrt{2}\,dx\,dy = \sqrt{2}\iint_D dx\,dy = \sqrt{2}\cdot A(D)$$

Domínio (Projeção da Intersecção): $\frac{3-x}{2} = \sqrt{x^2+y^2} \rightarrow \frac{(3-x)^2}{4} = x^2+y^2$ $\left(\begin{array}{l} 3-x \geq 0 \\ \boxed{x \leq 3} \end{array}\right.$

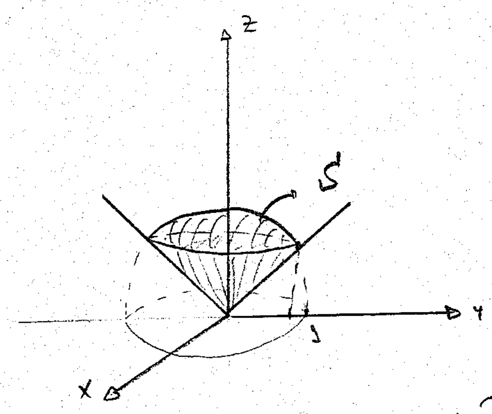
$$\rightarrow \frac{9}{4} - \frac{6x}{4} + \frac{x^2}{4} = x^2+y^2 \rightarrow \frac{3x^2}{4} + \frac{6x}{4} - \frac{9}{4} + y^2 = 0$$

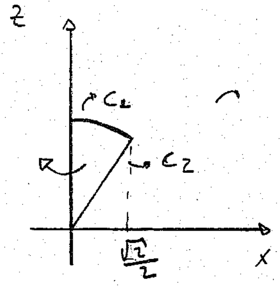
$$\rightarrow x^2 + 2x - 3 + \frac{4y^2}{3} = 0 \rightarrow (x+1)^2 + \frac{4y^2}{3} = 4 \rightarrow \boxed{\frac{(x+1)^2}{4} + \frac{y^2}{3} = 1} \rightarrow D$$

$$A(D) = \pi a b = \pi \cdot 2\sqrt{3} = 2\pi\sqrt{3}$$

$$A(S) = \sqrt{2}\cdot 2\pi\sqrt{3} = 2\pi\sqrt{6}$$

c)





interseção p/ y=

$$\begin{cases} x^2+z^2=1 \\ z^2 = x^2 \end{cases}$$

$2x^2 = 1$

$z = x = \pm\frac{\sqrt{2}}{2}$

$$C_1: \; x^2+z^2 = 1 \;\longrightarrow\; \sigma_1(\alpha) = (\underbrace{\cos\alpha}_{x(\alpha)}, \underbrace{\operatorname{sen}\alpha}_{z(\alpha)}) \;\rightarrow\; \pi/4 \le \alpha \le \pi/2$$

$$C_2: \; z = x \;\longrightarrow\; \sigma_2(t) = (\underbrace{t}_{x(t)}, \underbrace{t}_{z(t)}) \;\rightarrow\; 0 \le t \le \frac{\sqrt{2}}{2}$$

Por Revolução de C_1 e C_2 (em torno de z!)

$$\begin{cases} \varphi_1(\alpha,\theta) = (\cos\alpha\cos\theta, \cos\alpha\operatorname{sen}\theta, \cos\alpha) \\ \varphi_2(t,\theta) = (t\cos\theta, t\operatorname{sen}\theta, t) \end{cases} \rightarrow 0 \le \theta \le 2\pi$$

$$\frac{\partial\varphi_1}{\partial\alpha} = (-\operatorname{sen}\alpha\cos\theta, -\operatorname{sen}\alpha\operatorname{sen}\theta, \cos\alpha)$$

$$\Rightarrow N(\alpha,\theta) = \frac{\partial\varphi_1}{\partial\alpha} \times \frac{\partial\varphi_2}{\partial\theta} = \begin{vmatrix} i & j & k \\ -\operatorname{sen}\alpha\cos\theta & -\operatorname{sen}\alpha\operatorname{sen}\theta & \cos\alpha \\ -\cos\alpha\operatorname{sen}\theta & \cos\alpha\cos\theta & 0 \end{vmatrix}$$

$$\frac{\partial\varphi_1}{\partial\theta} = (-\cos\alpha\operatorname{sen}\theta, \cos\alpha\cos\theta, 0)$$

$$\Rightarrow N(\alpha,\theta) = (-\cos^2\alpha\cos\theta, -\cos^2\alpha\cdot\operatorname{sen}\theta, \operatorname{sen}\alpha\cos\alpha) \Rightarrow |N(\alpha,\theta)| = \sqrt{\cos^4\alpha + \cos^2\alpha\operatorname{sen}^2\alpha}$$

$$\Rightarrow |N(\alpha,\theta)| = |\cos\alpha| = \cos\alpha$$

$$\frac{\partial\varphi_2}{\partial t} = (\cos\theta, \operatorname{sen}\theta, 1) \Rightarrow N(t,\theta) = \frac{\partial\varphi_2}{\partial t} \times \frac{\partial\varphi_2}{\partial\theta} = \begin{vmatrix} i & j & k \\ \cos\theta & \operatorname{sen}\theta & 1 \\ -t\operatorname{sen}\theta & t\cos\theta & 0 \end{vmatrix} = (-t\cos\theta, -t\operatorname{sen}\theta, t)$$

$$\frac{\partial\varphi_2}{\partial\theta} = (-t\operatorname{sen}\theta, t\cos\theta, 0)$$

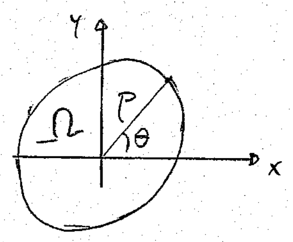
$$|N(t,\theta)| = t\sqrt{2}$$

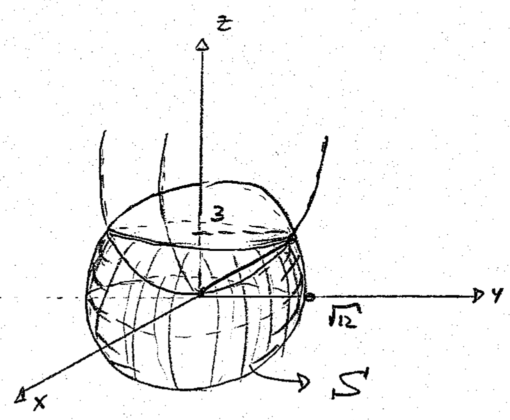
$$\therefore A(S_1) = \int_0^{2\pi} d\theta \int_{\pi/4}^{\pi/2} \cos\alpha \, d\alpha = 2\pi \operatorname{sen}\alpha \Big|_{\pi/4}^{\pi/2} = 2\pi\left(1 - \frac{\sqrt{2}}{2}\right)$$

$$\therefore A(S_2) = \int_0^{2\pi} d\theta \int_0^{\sqrt{2}/2} t\sqrt{2} = \frac{2\pi\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\sqrt{2}\pi}{2}$$

$$A(S) = A(S_1) + A(S_2) = -2\pi\frac{\sqrt{2}}{2} + \pi\frac{\sqrt{2}}{2} + 2\pi = -\frac{\pi\sqrt{2}}{2} - 2\pi = \pi\frac{(4-\sqrt{2})}{2} \; //$$

d) $\begin{cases} x^2+y^2+z^2=12 \\ z = x^2+y^2 \end{cases}$

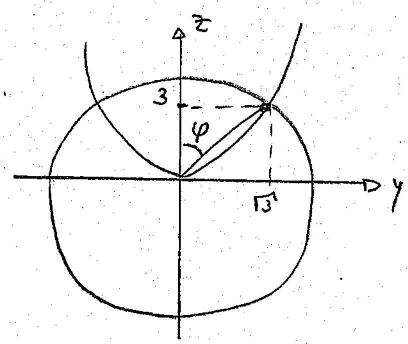




$\begin{cases} x = \rho \operatorname{sen}\varphi \cos\theta \\ y = \rho \operatorname{sen}\varphi \operatorname{sen}\theta \\ z = \rho \cos\varphi \end{cases}$

Interseção: $z^2+z=12$

$z^2+z-12=0 \rightarrow \boxed{z=3}$
$\cancel{z=-4}$



para $x=0$ e $z=3$

$3 = y^2 \rightarrow y = \pm\sqrt{3}$

$\rightarrow \frac{\pi}{6} \leq \varphi \leq \pi$

$\rightarrow 0 \leq \theta \leq 2\pi$

$z = \rho\cos\varphi$

$3 = \rho \cos\frac{\pi}{6}$

$3 = \rho\frac{\sqrt{3}}{2}$

$\rho = \frac{6}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3}$

$\Phi(\varphi,\theta) = \left(2\sqrt{3}\operatorname{sen}\varphi\cos\theta,\ 2\sqrt{3}\operatorname{sen}\varphi\operatorname{sen}\theta,\ 2\sqrt{3}\cos\varphi\right)$

$\begin{cases} \dfrac{\partial\Phi}{\partial\varphi} = \left(2\sqrt{3}\cos\varphi\cos\theta,\ 2\sqrt{3}\cos\varphi\operatorname{sen}\theta,\ -2\sqrt{3}\operatorname{sen}\varphi\right) \\ \dfrac{\partial\Phi}{\partial\theta} = \left(-2\sqrt{3}\operatorname{sen}\varphi\operatorname{sen}\theta,\ 2\sqrt{3}\operatorname{sen}\varphi\cos\theta,\ 0\right) \end{cases}$

$$\frac{\partial\Phi}{\partial\varphi}\times\frac{\partial\Phi}{\partial\theta} = \begin{vmatrix} i & j & k \\ 2\sqrt{3}\cos\varphi\cos\theta & 2\sqrt{3}\cos\varphi\operatorname{sen}\theta & -2\sqrt{3}\operatorname{sen}\varphi \\ -2\sqrt{3}\operatorname{sen}\varphi\operatorname{sen}\theta & 2\sqrt{3}\operatorname{sen}\varphi\cos\theta & 0 \end{vmatrix} = \left(12\operatorname{sen}^2\varphi\cos\theta,\ 12\operatorname{sen}^2\varphi\operatorname{sen}\theta,\ 12\operatorname{sen}\varphi\cos\varphi\right)$$
$$= N(\varphi,\theta)$$

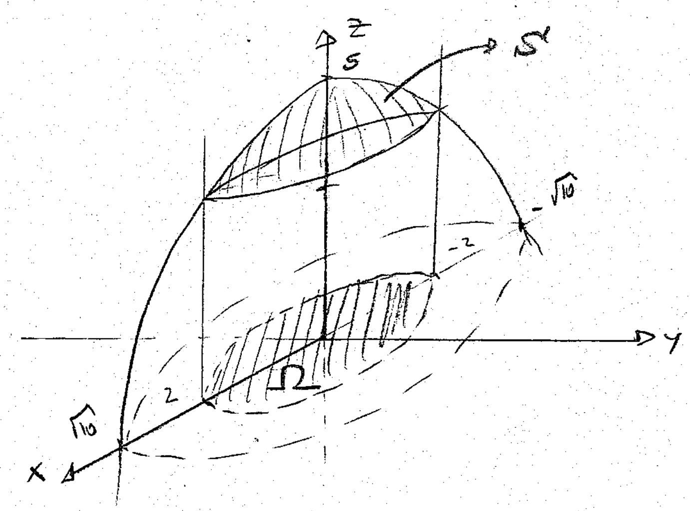
$\|N(\varphi,\theta)\| = \sqrt{144\operatorname{sen}^4\varphi + 144\operatorname{sen}^2\varphi\cos^2\varphi} = 12\sqrt{\operatorname{sen}^2\varphi(\operatorname{sen}^2\varphi+\cos^2\varphi)} = 12|\operatorname{sen}\varphi| \rightarrow \frac{\pi}{6}\leq\varphi\leq\pi$

$\|N(\varphi,\theta)\| = 12\operatorname{sen}\varphi$

$$A(S) = \iint_\Omega 12\operatorname{sen}\varphi\, d\varphi\, d\theta = \int_0^{2\pi} d\theta \int_{\pi/6}^{\pi} 12\operatorname{sen}\varphi\, d\varphi = -24\pi\cos\varphi\Big|_{\pi/6}^{\pi} = -24\pi\left(-1-\frac{\sqrt{3}}{2}\right) = 24\pi\left(\frac{\sqrt{3}}{2}+1\right)$$

$$A(S) = 12\pi(\sqrt{3}+2) = 6\pi(\sqrt{12}+4)$$

f) $z = 5 - \frac{x^2}{2} - y^2$



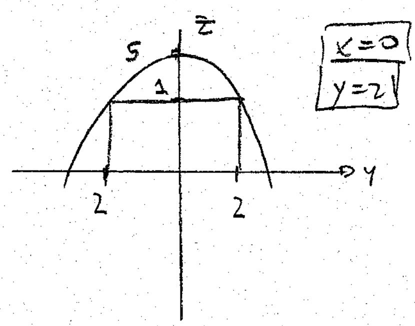
$x^2 + 4y^2 = 4 \rightarrow \frac{x^2}{4} + y^2 = 1$

$z = 0 \rightarrow 5 = \frac{x^2}{2} + y^2$

$\rightarrow \frac{x^2}{10} + \frac{y^2}{5} = 1$

Interseção: (Própio cilindro)

$\Rightarrow \Omega \rightarrow \frac{x^2}{4} + y^2 = 1$



para $x = 0$ e $y = 2 \rightarrow z = 5 - 4 \rightarrow z = 1$

$\varphi(x,y) = \left(x, y, 5 - \frac{x^2}{2} - y^2\right)$

$\frac{\partial \varphi}{\partial x} = (1, 0, -x)$ $\Rightarrow N(x,y) = \frac{\partial \varphi}{\partial x} \times \frac{\partial \varphi}{\partial y} = \begin{vmatrix} i & j & k \\ 1 & 0 & -x \\ 0 & 1 & -2y \end{vmatrix} = (x, 2y, 1)$

$\frac{\partial \varphi}{\partial y} = (0, 1, -2y)$

$|N(x,y)| = \sqrt{x^2 + 4y^2 + 1}$

Se o $|N(x,y)|$ fosse constante, eu poderia achar $c.\iint_\Omega dxdy$ fazendo $\iint_\Omega dxdy = \pi ab$ 4

$\frac{(2r\cos\theta)^2}{4} + r^2 \operatorname{sen}^2\theta = 1$

$4r^2 = 4 \rightarrow \boxed{r = 1}$

$0 \leq r \leq 1$

$$A(S) = \iint_\Omega \sqrt{x^2 + 4y^2 + 1}\, dx\, dy$$

$\begin{cases} x = 2r\cos\theta \\ y = r\operatorname{sen}\theta \end{cases} \rightarrow J = \begin{vmatrix} 2\cos\theta & \operatorname{sen}\theta \\ -2r\operatorname{sen}\theta & r\cos\theta \end{vmatrix} = 2r \rightarrow |J| = 2r$

(mudei o domínio, então colocar J)

$$A(S') = \int_0^{2\pi} d\theta \int_0^1 \sqrt{4r^2+1} \cdot 2r\, dr = \frac{\pi}{2} \int_0^1 \sqrt{4r^2+1}\, d[4r^2+1] = \frac{2}{3}\frac{\pi}{2}(4r^2+1)^{3/2}\Big|_0^1$$

$$A(S') = \frac{\pi}{3}\left(5^{3/2} - 1\right) = \frac{\pi}{3}\left(5\sqrt{5} - 1\right)$$

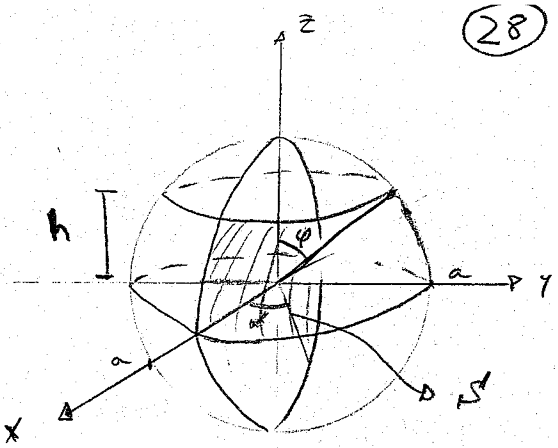
(28)

g) $\Phi(\varphi,\theta) = (a\,\text{sen}\varphi\cos\theta,\ a\,\text{sen}\varphi\,\text{sen}\theta,\ a\cos\varphi)$

$\boxed{0 \le \theta \le \alpha}$

$\cos\varphi = \frac{h}{a} \to \varphi = \arccos\frac{h}{a}$

$\boxed{\arccos\left(\frac{h}{a}\right) \le \varphi \le \frac{\pi}{2}}$



$$\frac{\partial\Phi}{\partial\varphi} = (a\cos\varphi\cos\theta,\ a\cos\varphi\,\text{sen}\theta,\ -a\,\text{sen}\varphi)$$

$$\frac{\partial\Phi}{\partial\theta} = (-a\,\text{sen}\varphi\,\text{sen}\theta,\ a\,\text{sen}\varphi\cos\theta,\ 0)$$

$$N(\varphi,\theta) = \frac{\partial\Phi}{\partial\varphi} \times \frac{\partial\Phi}{\partial\theta} = \begin{vmatrix} i & j & k \\ a\cos\varphi\cos\theta & a\cos\varphi\,\text{sen}\theta & -a\,\text{sen}\varphi \\ -a\,\text{sen}\varphi\,\text{sen}\theta & a\,\text{sen}\varphi\cos\theta & 0 \end{vmatrix} = (a^2\text{sen}^2\varphi\cos\theta,\ a^2\text{sen}^2\varphi\,\text{sen}\theta,\ a^2\text{sen}\varphi\cos\varphi)$$

$$|N(\varphi,\theta)| = \ldots = a^2\,\text{sen}\varphi$$

$$A(S) = \int_0^{\alpha} d\theta \int_{\arccos\frac{h}{a}}^{\pi/2} a^2\,\text{sen}\varphi\, d\varphi = -\alpha\, a^2\left(\cancel{\cos\frac{\pi}{2}}^{\,0} - \cos\left(\arccos\frac{h}{a}\right)\right) = a^2\alpha\frac{h}{a}$$

$$A(S) = a\alpha h \ //$$

4.6 | 1) a) $x = a \operatorname{sen}\varphi \cos\theta$

$y = a \operatorname{sen}\varphi \operatorname{sen}\theta$

$z = a \cos\varphi$

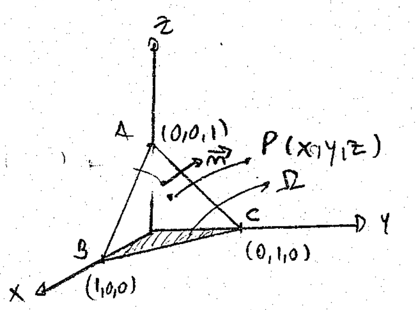
$\Phi(\varphi,\theta) = (a \operatorname{sen}\varphi\cos\theta,\ a\operatorname{sen}\varphi\operatorname{sen}\theta,\ a\cos\varphi)$

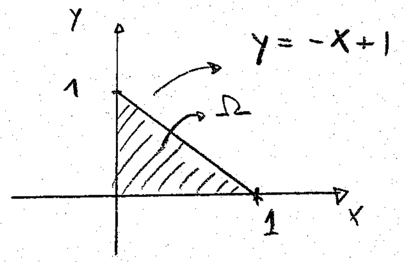
$$\frac{\partial\Phi}{\partial\varphi} \times \frac{\partial\Phi}{\partial\theta} = \begin{vmatrix} i & j & k \\ a\cos\varphi\cos\theta & a\cos\varphi\operatorname{sen}\theta & -a\operatorname{sen}\varphi \\ -a\operatorname{sen}\varphi\operatorname{sen}\theta & a\operatorname{sen}\varphi\cos\theta & 0 \end{vmatrix} = (a^2\operatorname{sen}^2\varphi\cos\theta,\ a^2\operatorname{sen}^2\varphi\operatorname{sen}\theta,\ a^2\operatorname{sen}\varphi\cos\varphi)$$

$|N(\varphi,\theta)| = a^2\operatorname{sen}\varphi$

$$A(S') = \int_0^{2\pi} d\theta \int_0^{\pi} a^2\operatorname{sen}\varphi \cdot a^2\operatorname{sen}^2\varphi\, d\varphi = 2\pi a^4 \int_0^{\pi} \operatorname{sen}^3 d\varphi = \dots = \frac{8\pi a^4}{3}$$

b) $\iint xyz\, ds$; S :





S: Plano $\Rightarrow$ $N = \vec{AB} \times \vec{BC} = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix} = (1,1,1)$

$N \cdot \vec{AP} = 0 \rightarrow (1,1,1)(x,\ y,\ z-1) = 0 \rightarrow \boxed{x+y+z=1} \rightarrow S$

$N = (1,1,1) \rightarrow |N| = \sqrt{3}$

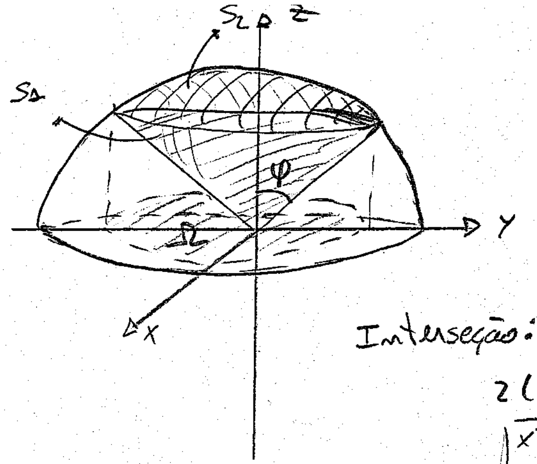
$\varphi(x,y) = (x,\ y,\ 1-x-y)$

$$\iint_S xyz\, ds = \iint_D xy(1-x-y)\cdot\sqrt{3}\, dx\,dy = \sqrt{3}\int_0^1 dx \int_0^{-x+1} (xy - x^2y - xy^2)\, dy = \sqrt{3}\int_0^1 dx \cdot \left(\frac{xy^2}{2} - \frac{x^2y^2}{2} - \frac{xy^3}{3}\right)\Bigg|_0^{-x+1}$$

$$= \sqrt{3}\int_0^1 \left(\frac{x(1-x)^2}{2} - \frac{x^2(1-x)^2}{2} - \frac{x(1-x)^3}{3}\right) dx = \dots = \frac{\sqrt{3}}{120}$$

c) $\iint (y^2+z^2)\,ds$

S: $\longrightarrow$

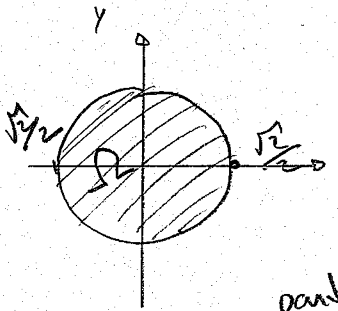


$x^2+y^2+z^2=1$
e
$z=\sqrt{x^2+y^2}$

Interseção: $x^2+y^2+x^2+y^2=1$

$2(x^2+y^2)=1$

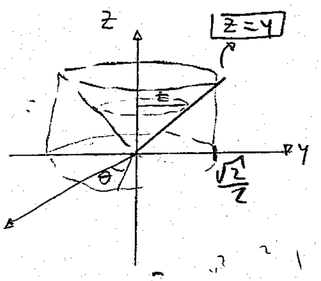
$\boxed{x^2+y^2=\frac{1}{2}} \rightsquigarrow R=\frac{\sqrt{2}}{2}$



$x=$

$$\iint (y^2+z^2)\,ds = \underbrace{\iint_{S_1}(y^2+z^2)\,ds}_{\text{parte do cone}} + \underbrace{\iint_{S_2}(y^2+z^2)\,ds}_{\text{parte da esfera}}$$

Parte do cone $\rightarrow$



$\boxed{0\le\theta\le 2\pi}$

$\gamma(t) = (\underbrace{t}_{y(t)}, \underbrace{t}_{z(t)}) \Rightarrow \boxed{\varphi_1(t,\theta) = (t\,\text{sen}\,\theta, t\cos\theta, t)}$

$\boxed{0\le t\le\frac{\sqrt{2}}{2}}$

$|N_1| = t\sqrt{2}$

Parte da esfera:

$x = \rho\,\text{sen}\,\varphi\cos\theta$
$y = \rho\,\text{sen}\,\varphi\,\text{sen}\,\theta$
$z = \rho\cos\varphi$

$\rightarrow$ para $\boxed{\rho=1} \Rightarrow \varphi_2(\varphi,\theta) = (\text{sen}\,\varphi\cos\theta, \text{sen}\,\varphi\,\text{sen}\,\theta, \cos\varphi)$

$|N_2| = \text{sen}\,\varphi$

$\boxed{0\le\varphi\le\pi/4}$

$$\iint_{S_1}(y^2+z^2)\,ds = \int_0^{2\pi}d\theta\int_0^{\sqrt{2}/2}(t^2\cos^2\theta+t^2)\cdot t\sqrt{2}\,dt = \sqrt{2}\int_0^{2\pi}d\theta\int(t^3\cos^2\theta+t^3)\,dt =$$

$$\sqrt{2}\int_0^{2\pi}d\theta\left(\frac{\cos^2\theta}{16}+\frac{1}{16}\right) = \frac{\pi}{8}\sqrt{2} + \frac{\sqrt{2}}{36}\cdot\int_0^{2\pi}((\cos 2\theta+1)\,d\theta = \frac{\pi\sqrt{2}}{8/2}+\frac{\pi\sqrt{2}}{16}+\cancel{\frac{\text{sen}\,2\theta}{4}}\Big|_0^{2\pi} = \frac{3\pi\sqrt{2}}{16}$$

$$\iint_{S_2}(\text{sen}^2\varphi\,\text{sen}^2\theta+\cos^2\varphi)\cdot\text{sen}\,\varphi\,d\varphi\,d\theta = \int_0^{2\pi}d\theta\int_0^{\pi/4}(\text{sen}^3\varphi\,\text{sen}^2\theta+\cos^2\varphi\,\text{sen}\,\varphi)\,d\varphi =$$

$$= \int_0^{2\pi} d\theta \int_0^{\pi/4} (\text{sen}^3\varphi\,\text{sen}^2\theta + \cos^2\varphi\,\text{sen}\,\varphi)\,d\varphi = \int_0^{2\pi} d\theta \int_0^{\pi/4} \left(\text{sen}\,\varphi\,(1-\cos^2\varphi)\,\text{sen}^2\theta + \cos^2\varphi\,\text{sen}\,\varphi\right) d\varphi$$

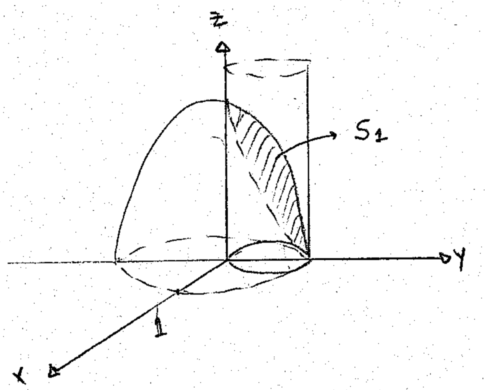
$$= \int_0^{2\pi} d\theta \int_0^{\pi/4} \text{sen}\,\varphi\,\text{sen}^2\theta - \text{sen}\,\varphi\cos^2\varphi\,\underbrace{(1-\text{sen}^2\theta)}_{\cos^2\theta}\,d\varphi$$

$$= \int_0^{2\pi} \left(\text{sen}^2\theta\,(-\cos\varphi)\Big|_0^{\pi/4} + \cos^2\theta \left[-\frac{\cos^3\varphi}{3}\right]_0^{\pi/4} \right) d\theta$$

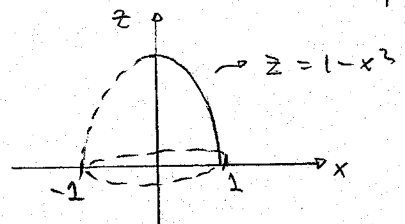
$$= \dots = \pi\left(\frac{4}{3} - \frac{7\sqrt{2}}{12}\right)$$

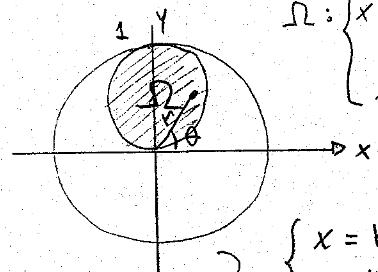
$$\iint_S (y^2+z^2)\,ds = \pi\left(\frac{4}{3} - \frac{7\sqrt{2}}{12} + \frac{3\sqrt{2}}{16}\right) \ //$$

2)



$$\begin{cases} \text{Parabolóide}: & z = 1 - x^2 - y^2 \\ \text{Cilindro}: & x^2 + (y - \frac{1}{2})^2 = \frac{1}{4} \end{cases}$$





$$\Omega: \begin{cases} x^2 + (y-\frac{1}{2})^2 \le \frac{1}{4} \\ \text{ou} \\ x^2 + y^2 \le y \end{cases}$$

(I) $\to \begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$ $\boxed{0 \le \theta \le \pi}$

$|J(r,\theta)| = r$

$\to x^2+y^2 \le y \to r^2 \le r\,\text{sen}\,\theta$

$\boxed{0 \le r \le \text{sen}\,\theta}$

Parametrizando S_1:

$\varphi(x,y) = (x, y, 1-x^2-y^2)$

$\frac{\partial\varphi}{\partial x} \times \frac{\partial\varphi}{\partial y} = (1,0,-2x) \times (0,1,-2y) = (2x, 2y, 1)$

$|N| = \sqrt{4(x^2+y^2)+1}$ usando (I)

$|N| = \sqrt{4r^2+1}$

$$(S_1) = \iint_{S_1} \frac{|xy|}{x^2+y^2}\,ds = \int_0^{\pi} d\theta \int_0^{\text{sen}\theta} \sqrt{4r^2+1}\cdot r\,\frac{|\cancel{r^2}\,\text{sen}\theta\cos\theta|}{\cancel{r^2}}\,dr$$

$$\int_0^{\pi} |\text{sen}\theta\cos\theta|\,d\theta \int_0^{\text{sen}\theta} \sqrt{4r^2+1}\cdot d[4r^2+1] = \frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{8}\,4}\int_0^{\pi} |\text{sen}2\theta|\cdot\left[(4r^2+1)^{3/2}\right]_0^{\text{sen}\theta} d\theta = \frac{1}{12}\int_0^{\pi} |\text{sen}2\theta| \left[(4\text{sen}^2\theta+1)^{3/2} - 1\right]$$

$$= \frac{1}{12}\int_0^{\pi} |\operatorname{sen}\theta\cos\theta| \cdot \left[\left(4\operatorname{sen}^2\theta+1\right)^{3/2}-1\right] d\theta$$

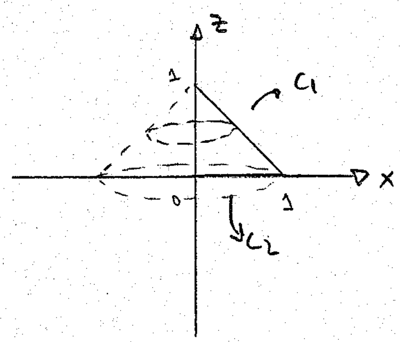
$$\begin{cases} 4\operatorname{sen}^2\theta + 1 = u \\ 8\operatorname{sen}\theta\cos\theta\, d\theta = du \end{cases}$$

$$= -\frac{1}{8}\cdot\frac{1}{12}\int_0^{\frac{\pi}{2}} \left(u^{3/2}-1\right)\cdot du + \frac{1}{8}\cdot\frac{1}{12}\int_{\pi/2}^{\pi}\left(u^{3/2}-1\right) du$$

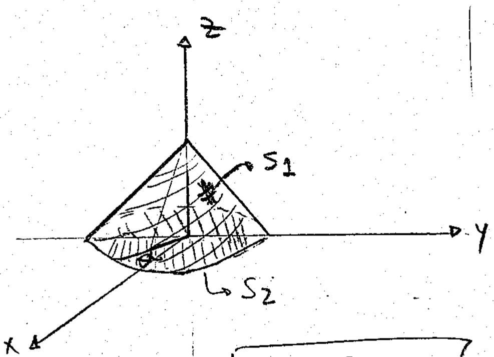
$$= \frac{1}{96}\left[\frac{2}{5}\left(1+4\operatorname{sen}^2\theta\right)^{5/2} - \left(1+4\operatorname{sen}^2\theta\right)\right]_0^{\pi/2} - \frac{1}{96}\left[\frac{2}{5}\left(1+4\operatorname{sen}^2\theta\right)^{5/2} - \left(1+4\operatorname{sen}^2\theta\right)\right]_{\pi/2}^{\pi}$$

$$= \dots = \frac{25\sqrt{5}-11}{120} \quad //$$

3)



$\Rightarrow$



$$\boxed{S = S_1 \cup S_2}$$

$\rho(x,y,z) = \sqrt{x^2+y^2} \rightsquigarrow \rho = \frac{m}{A} \to m = \rho\cdot A$

$$M(S) = \iint_S \rho(x,y,z)\, dS = \iint_S \sqrt{x^2+y^2}\, dS$$

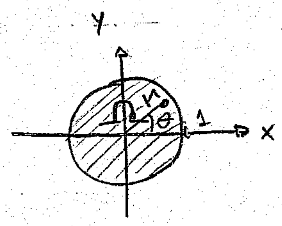
C_1: $\sigma(t) = (\underbrace{t}_{x(t)}, \underbrace{1-t}_{z(t)})$ $\quad 0 \le t \le 1 \Longrightarrow \varphi_1(t,\theta) = (t\cos\theta, t\operatorname{sen}\theta, 1-t)$

$$N_1 = \frac{\partial\varphi_1}{\partial t}\times\frac{\partial\varphi_1}{\partial\theta} = (\cos\theta, \operatorname{sen}\theta, -1)\times(-t\operatorname{sen}\theta, t\cos\theta, 0) = (t\cos\theta, t\operatorname{sen}\theta, t)$$

$0 \le \theta \le 2\pi$

$|N_1| = t\sqrt{2}$

C_2:



$\to \Omega: x^2+y^2 \le 1$ $\begin{cases} x = r\cos\theta \\ y = r\operatorname{sen}\theta \end{cases} \Rightarrow \begin{matrix} 0 \le \theta \le 2\pi \\ 0 \le r \le 1 \end{matrix} \Rightarrow |J(r,\theta)| = r$

$$\varphi_2(r,\theta) = (r\cos\theta, r\operatorname{sen}\theta, 0) \rightsquigarrow N_2 = \frac{\partial\varphi_2}{\partial r}\times\frac{\partial\varphi_2}{\partial\theta} = (\cos\theta, \operatorname{sen}\theta, 0)\times(-r\operatorname{sen}\theta, r\cos\theta, 0) = (0,0,r)$$

$|N_2| = r$

$$M_1 = \iint_{S_1} \sqrt{x^2+y^2}\, ds = \int_0^{2\pi} d\theta \int_0^1 t \cdot t\sqrt{2}\, dt = 2\pi\sqrt{2} \cdot \frac{t^3}{3}\Big|_0^1 = \frac{2\pi\sqrt{2}}{3}$$

$$M_2 = \iint_{S_2} \sqrt{x^2+y^2}\, ds = \int_0^{2\pi} d\theta \int_0^1 \overset{\rho}{r} \cdot \overset{\|N\|}{r} \cdot \cancel{\overset{|J|}{r}}\, dr = 2\pi\, \frac{r^3}{\cancel{3}}\Big|_0^1 = \frac{2\pi}{3}$$

$$M_T = M_1 + M_2 = \frac{2\pi}{3}\left(\sqrt{2}+1\right)$$

4) a) $S: \varphi(u,v) = (u, v, u^2+v^2)$ onde $u^2+v^2 \leq 1$ $\Big\}$ $f(x,y,z) = k$ (constante)

$N(u,v) = \frac{\partial\varphi}{\partial u} \times \frac{\partial\varphi}{\partial v} = (1,0,2u) \times (0,1,2v) = (-2u, 2v, 1)$

$|N(u,v)| = \sqrt{4(u^2+v^2)+1}$ $\qquad D: u^2+v^2 \leq 1$

$$M = \iint_D \sqrt{4(u^2+v^2)+1} \cdot k\, du\, dv = k \iint_D \sqrt{4(u^2+v^2)+1}\, du\, dv$$

$$x_c = \frac{\iint_D x\, \cancel{k} \sqrt{4(u^2+v^2)+1}\, du\, dv}{\cancel{k} \iint_D \sqrt{4(u^2+v^2)+1}\, du\, dv} = \frac{\iint_D u \sqrt{4(u^2+v^2)+1}\, du\, dv}{\iint_D \sqrt{4(u^2+v^2)+1}\, du\, dv}$$

$\begin{cases} u = r\cos\theta \\ v = r\,\mathrm{sen}\,\theta \end{cases} \rightarrow \begin{matrix} 0 \leq \theta \leq 2\pi \\ 0 \leq r \leq 1 \end{matrix} \rightarrow |J(r,\theta)| = r$

$$x_c = \frac{\int_0^{2\pi} d\theta \int_0^1 r^2\cos\theta \cdot \sqrt{4r^2+1}\, dr}{\int_0^{2\pi} d\theta \int_0^1 r\sqrt{4r^2+1}\, dr} = 0$$

(pois vai cair em $\left[A \,\mathrm{sen}\,\theta\right]_0^{2\pi} = 0$)

$$y_c = \frac{\int_0^{2\pi} d\theta \int_0^1 r^2\,\mathrm{sen}\,\theta \sqrt{4r^2+1}\, dr}{\int_0^{2\pi} d\theta \int_0^1 r\sqrt{4r^2+1}\, dr} = 0$$

(pois vai cair em $A\left[-\cos\theta\right]_0^{2\pi} = 0$)

$$Z_c = \frac{\int_0^{2\pi} d\theta \int_0^1 r^2 \cdot r \cdot \sqrt{4r^2+1}\, dr}{\int_0^{2\pi} d\theta \int_0^1 r\sqrt{4r^2+1}\, dr} = \frac{\int_0^1 r^3\sqrt{4r^2+1}\, dr}{\frac{1}{8}\int_0^1 (4r^2+1)^{1/2} \cdot d[4r^2+1]} =$$

OBS: $t^2 = 4r^2+1$

$\frac{t^2-1}{4} = r^2$

$\frac{2t}{4}\, dt = 2r\, dr$

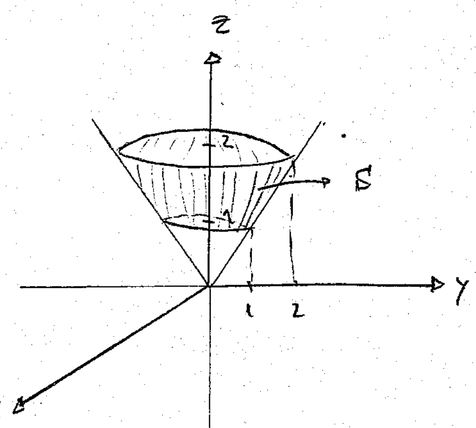
$\frac{t}{2}\, dt = d[r^2]$

$$= \frac{\frac{1}{2}\int_0^1 r^2\sqrt{4r^2+1}\; d[r^2]}{\frac{2}{3}\cdot\frac{1}{8}\cdot\left(5^{3/2}-1\right)} = \frac{1}{\cancel{2}}\cdot\frac{3}{\cancel{2}}\cdot\overset{2}{\cancel{8}}\cdot\left(5^{3/2}-1\right)^{-1}\cdot\int_0^1 \frac{t^2-1}{4}\cdot t \cdot \frac{t}{2}\, dt$$

$$= \dots = \frac{25\sqrt{5}+1}{10\left(5\sqrt{5}-1\right)}$$

$$CM = \left(0,\ 0,\ \frac{25\sqrt{5}+1}{10(5\sqrt{5}-1)}\right)$$

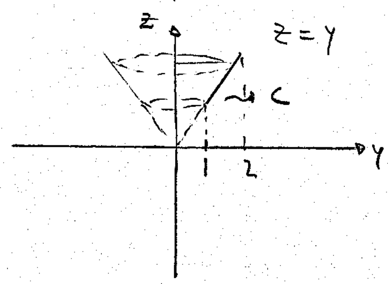
b)



$$x_c = \frac{\iint_S x\, \cancel{k}\, ds}{\iint_S \cancel{k}\, ds} = \frac{\iint_S x\, ds}{\iint_S ds}$$

$C: \; r(t) = (\underbrace{t}_{y(t)}, \underbrace{t}_{z(t)}) \rightarrow 1 \le t \le 2$

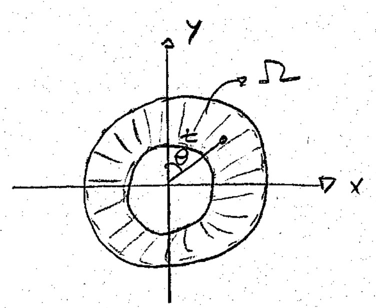
$0 \le \theta \le 2\pi$



$S: \; \varphi(t,\theta) = (t\,\text{sen}\theta,\ t\cos\theta,\ t) \rightarrow |J| = t$

$N(t,\theta) = \frac{\partial\varphi}{\partial t} \times \frac{\partial\varphi}{\partial\theta} = (\text{sen}\theta, \cos\theta, 1) \times (t\cos\theta, -t\,\text{sen}\theta, 0)$

$N(t,\theta) = (t\,\text{sen}\theta, t\cos\theta, -t) \Rightarrow |N(t,\theta)| = t\sqrt{2}$



$$x_c = \frac{\int_0^{2\pi} d\theta \int_1^2 \cancel{k}\, t\sqrt{2}\cdot t\,\text{sen}\theta\, dt}{\int_0^{2\pi} d\theta \int_1^2 \cancel{k}\, t\sqrt{2}\cdot t\, dt} = \frac{\int_0^{2\pi} \text{sen}\theta\, d\theta\, \left(\frac{8-1}{3}\right)}{2\pi\cdot\frac{(4-1)}{2}}$$

$$x_c = \frac{7}{3}\cdot\frac{1}{3\pi}\cdot\left[-\underbrace{\cancel{\cos\theta}}_{0}\right]_0^{2\pi} = -\frac{45}{56\pi}\cdot 0 = 0$$

$y_c = \dots = 0$

$$z_c = \frac{\int_0^{2\pi} d\theta \int_1^2 t \cdot t\sqrt{2}\, dt}{\int_0^{2\pi} d\theta \int_1^2 t\sqrt{2}\, dt} = \frac{\cancel{2\pi}\,\frac{(8-1)}{3}}{\cancel{2\pi}\,\frac{(4-1)}{2}} = \frac{7}{3}\cdot\frac{2}{3} = \frac{14}{9}$$

$$CM = \left(0, 0, \frac{14}{9}\right)$$

$\boxed{1.8}$

1) $F(x,y,z) = (x, y, -2z)$

$S: x^2+y^2+z^2 = 4 \longrightarrow$ Explicitando: $z = \sqrt{4-(x^2+y^2)} = f(x,y)$

N: exterior

"difícil" de achar a normal

$\Downarrow$

Parametrizar: $\begin{cases} x = \rho\,\text{sen}\varphi\cos\theta \\ y = \rho\,\text{sen}\varphi\,\text{sen}\theta \\ z = \rho\cos\varphi \end{cases}$ para $\rho = 2$ (é superfície)

$0 \le \varphi \le \pi \;/\; 0 \le \theta \le 2\pi$

$\Phi(\varphi,\theta) = (2\,\text{sen}\varphi\cos\theta, 2\,\text{sen}\varphi\,\text{sen}\theta, 2\cos\varphi) \rightarrow F = (2\,\text{sen}\varphi\cos\theta, 2\,\text{sen}\varphi\,\text{sen}\theta, -4\cos\varphi)$

$\frac{\partial\Phi}{\partial\varphi} = (2\cos\varphi\cos\theta, 2\cos\varphi\,\text{sen}\theta, -2\,\text{sen}\varphi)$

$\frac{\partial\Phi}{\partial\theta} = (-2\,\text{sen}\varphi\,\text{sen}\theta, 2\,\text{sen}\varphi\cos\theta, 0)$

$$N = \frac{\partial\Phi}{\partial\varphi} \times \frac{\partial\Phi}{\partial\theta} = \begin{vmatrix} i & j & k \\ 2\cos\varphi\cos\theta & 2\cos\varphi\,\text{sen}\theta & -2\,\text{sen}\varphi \\ -2\,\text{sen}\varphi\,\text{sen}\theta & 2\,\text{sen}\varphi\cos\theta & 0 \end{vmatrix} = (4\,\text{sen}^2\varphi\cos\theta, 4\,\text{sen}^2\varphi\,\text{sen}\theta, 4\,\text{sen}\varphi\cos\varphi)$$

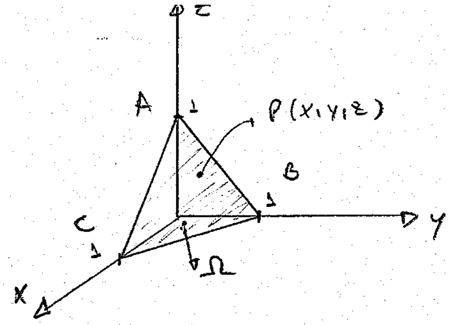
$$\iint_S (F\cdot n)\,\underbrace{ds}_{\substack{=|N|.d\Omega \\ =|N|.d\varphi d\theta}} = \int_0^{2\pi} d\theta \int_0^{\pi} (2\,\text{sen}\varphi\cos\theta, 2\,\text{sen}\varphi\,\text{sen}\theta, -4\cos\varphi)\cdot\frac{(4\,\text{sen}^2\varphi\cos\theta, 4\,\text{sen}^2\varphi\,\text{sen}\theta, 4\,\text{sen}\varphi\cos\varphi)}{|N|}\cdot|N|\,d\varphi$$

$$= \int_0^{2\pi} d\theta \int_0^{\pi} (8\,\text{sen}^3\varphi - 16\,\text{sen}\varphi\cos^2\varphi)\,d\varphi = \int_0^{2\pi} d\theta \int_0^{\pi} 8\,\text{sen}\varphi(1-\cos^2\varphi)\,d\varphi + \int_0^{2\pi} d\theta \int_0^{\pi} 16\cos^2\varphi\, d[\cos\varphi]$$

$$32\pi + 16\pi\int_0^{\pi} \cos^2\varphi\, d[\cos\varphi] + 32\pi\int_0^{\pi} \cos^2\varphi\, d[\cos\varphi] = 32\pi - \frac{32\pi}{3} - \frac{64\pi}{3} = -\cancel{\frac{96\pi}{3}} + \cancel{32\pi} = 0$$

2) $F(x,y,z) = (x,y,z)$

$S : \Delta ABC \Rightarrow$



$A(0,0,1)$
$B(0,1,0)$
$C(1,0,0)$

$\overline{AC} \times \overline{CB} = N$

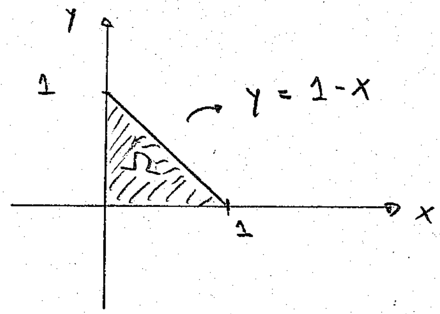
$(1,0,-1) \times (-1,1,0) = N \rightarrow N = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix} = (1,1,1)$

$N \cdot \overline{AP} = 0 \rightarrow (1,1,1) \cdot (x, y, z-1) = 0 \rightarrow \boxed{x+y+z=1} \Rightarrow S$

$z = 1-x-y = f(x,y)$

Parametrizando:

$\varphi(x,y) = (x, y, 1-x-y) \rightsquigarrow F = (x, y, 1-x-y)$



$$\iint_S (F \cdot n)\,ds = \iint_\Omega \left(F(x,y) \cdot \frac{N}{\cancel{|N|}}\right) \cdot \cancel{|N|}\, \underbrace{d\Omega}_{=dx\,dy}$$

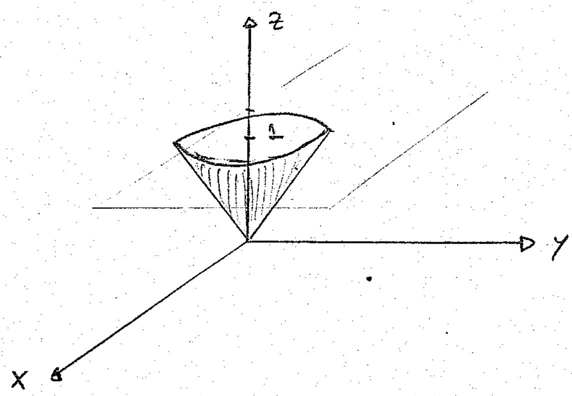
$$= \int_0^1 dx \int_0^{1-x} (x,y,1-x-y) \cdot (1,1,1)\, dy = \int_0^1 dx \int_0^{1-x} (\cancel{x}+\cancel{y}+1-\cancel{x}-\cancel{y})\, dy = \int_0^1 dx \cdot (1-x)$$

$$= -\frac{(1-x)^2}{2}\Bigg|_0^1 = \frac{1}{2} \;//$$

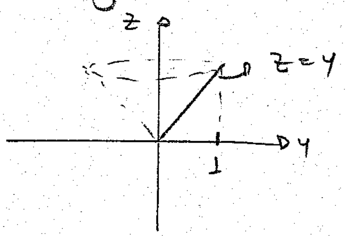
(N com componente $z>0 \Rightarrow$ não precisa mudar o sinal da resposta)

3) $F(x,y,z) = (y, z, xz)$

$S: \quad x^2 + y^2 \le z \le 1 \quad \Rightarrow$



Parametrizando o cone (Revolução)



$0 \le t \le 1$

$0 \le \theta \le 2\pi$

$\sigma(t) = (y(t), z(t)) = (t, t)$

$\varphi(t,\theta) = (t\,\text{sen}\theta, t\cos\theta, t)$

$$\frac{\partial \varphi}{\partial \theta} = (t\cos\theta, -t\,\text{sen}\theta, 0)$$

$$\frac{\partial \varphi}{\partial t} = (\text{sen}\theta, \cos\theta, 1)$$

$$\frac{\partial \varphi}{\partial t} \times \frac{\partial \varphi}{\partial \theta} = \begin{vmatrix} i & j & k \\ \text{sen}\theta & \cos\theta & 1 \\ t\cos\theta & -t\,\text{sen}\theta & 0 \end{vmatrix} = (t\,\text{sen}\theta, t\cos\theta, -t)$$

$N(t,\theta) = (t\,\text{sen}\theta, t\cos\theta, -t)$

$$\iint_S (F \cdot n)\, dS = \iint_\Omega F(t,\theta) \cdot \frac{N(t,\theta)}{\cancel{\|N(t,\theta)\|}} \cancel{\|N(t,\theta)\|}\, d\Omega = \int_0^{2\pi} d\theta \int_0^1 (t\cos\theta, -t, t^2\,\text{sen}\theta) \cdot (t\,\text{sen}\theta, t\cos\theta, -t)\, dt$$

$$= \int_0^{2\pi} d\theta \int_0^1 (t^2\,\text{sen}\theta\cos\theta - t^2\cos\theta + t^3\,\text{sen}\theta)\, dt = \int_0^{2\pi} d\theta \left[\frac{t^3}{3}\text{sen}\theta\cos\theta - \frac{t^3}{3}\cos\theta + \frac{t^4}{4}\text{sen}\theta\right]_0^1$$

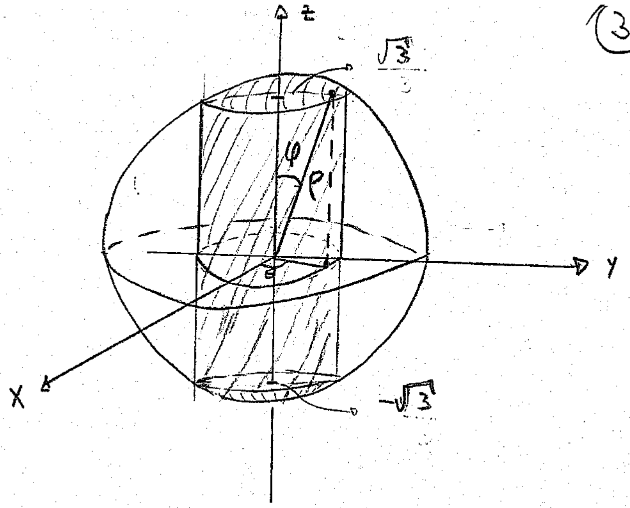
$$= \int_0^{2\pi} \left(\frac{1}{3}\text{sen}\theta\cos\theta - \frac{1}{3}\cos\theta + \frac{1}{4}\text{sen}\theta\right) d\theta = \frac{1}{3}\int_0^{2\pi} \text{sen}\theta\, d[\text{sen}\theta] - \frac{1}{3}\int_0^{2\pi} \cos\theta\, d\theta + \frac{1}{4}\int_0^{2\pi} \text{sen}\theta\, d\theta$$

$$= \cancel{\frac{\text{sen}^2\theta}{2}\Big|_0^{2\pi}}^{\,0} - \cancel{\frac{\text{sen}\theta}{3}\Big|_0^{2\pi}}^{\,0} - \frac{1}{4}\cos\theta\Big|_0^{2\pi} = -\frac{1}{4}(\cos 2\pi - \cos 0) = 0$$

4) $F(x,y,z) = (x, y, z)$

$S: \quad x^2+y^2 \leq 1 \quad e \quad x^2+y^2+z^2 \leq 4$

Interseção: $z^2 + 1 = 4 \to z = \sqrt{3}$



S_1 - Superfície lateral do cilindro para $-\frac{\sqrt{3}}{3} \leq z \leq \frac{\sqrt{3}}{3}$

$\varphi_1(\theta, t) = (2\cos\theta, 2\operatorname{sen}\theta, t)$

$\frac{\partial \varphi_1}{\partial \theta} = (-2\operatorname{sen}\theta, 2\cos\theta, 0) \implies \frac{\partial \varphi_1}{\partial \theta} \times \frac{\partial \varphi_1}{\partial t} = (2\cos\theta, +2\operatorname{sen}\theta, 0) \implies N_1$

$\frac{\partial \varphi_1}{\partial t} = (0, 0, 1)$

S_2: Cilindro $\cap$ esfera (em cima) $\longrightarrow \begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq \varphi \leq \pi/6 \end{cases}$

$\Phi_2(\varphi, \theta) = (2\operatorname{sen}\varphi\cos\theta, 2\operatorname{sen}\varphi\operatorname{sen}\theta, 2\cos\varphi)$

$\frac{\partial \Phi_2}{\partial \varphi} = (2\cos\varphi\cos\theta, 2\cos\varphi\operatorname{sen}\theta, -2\operatorname{sen}\varphi) \to \frac{\partial \Phi_2}{\partial \varphi} \times \frac{\partial \Phi_2}{\partial \theta} = (4\operatorname{sen}^2\varphi\cos\theta, 4\operatorname{sen}^2\varphi\operatorname{sen}\theta, 4\operatorname{sen}\varphi\cos\varphi) = N_2$

$\frac{\partial \Phi_2}{\partial \theta} = (-2\operatorname{sen}\varphi\operatorname{sen}\theta, 2\operatorname{sen}\varphi\cos\theta, 0)$

S_3: cilindro $\cap$ esfera (em baixo) $\longrightarrow \begin{matrix} 0 \leq \theta \leq 2\pi \\ \frac{5\pi}{6} \leq \varphi \leq \pi \end{matrix}$

$\Phi_2(\varphi,\theta) =$ IGUAL $S_2 \leadsto N_3$

$\frac{\cos^3 \pi/6}{3} = \frac{\cancel{3}\sqrt{3}}{8 \cancel{3}}$

$\frac{\operatorname{sen}^3 \pi/6}{3} = \frac{1}{24}$

$\implies \iint_S (F \cdot n)\, dS = \underbrace{\iint_{S_1} (F \cdot N_1)\, dS}_{(I)} + \iint_{S_2} (F \cdot n_2)\, dS + \iint_{S_3} (F \cdot n_3)\, dS$

$(I) = \int_0^{2\pi} d\theta \int_{-\sqrt{3}}^{\sqrt{3}} \underbrace{(2\cos\theta, 2\operatorname{sen}\theta, t) \cdot (2\cos\theta, 2\operatorname{sen}\theta, 0)}_{=4} \, dt = 8\pi(\sqrt{3}+\sqrt{3}) = 16\pi\sqrt{3}$

$(II) = \int_0^{2\pi} d\theta \int_0^{\pi/6} (2\operatorname{sen}\varphi\cos\theta, 2\operatorname{sen}\varphi\operatorname{sen}\theta, -2\cos\varphi) \cdot (4\operatorname{sen}^2\varphi\cos\theta, 4\operatorname{sen}^2\varphi\operatorname{sen}\theta, 4\operatorname{sen}\varphi\cos\varphi)\, d\varphi$

$= \int_0^{2\pi} d\theta \int_0^{\pi/6} (8\operatorname{sen}^3\varphi - 8\operatorname{sen}^2\varphi\cos\varphi)\, d\varphi = 16\pi \int_0^{\pi/6} [\operatorname{sen}\varphi(1-\cos^2\varphi) - \operatorname{sen}^2\varphi\cos\varphi]\, d\varphi = 16\pi \cdot \left(-\frac{1}{2}\right) + 16\pi \int_0^{\pi/6} \cos^2\varphi\, d[\cos\varphi] - 16\pi \int_0^{\pi/6} \operatorname{sen}^2\varphi\, d[\operatorname{sen}\varphi]$

$= -8\pi + 16\pi\left(\frac{\sqrt{3}}{8} - \frac{1}{3}\right) - \cancel{16}^{2}\pi\left(\frac{1}{\cancel{24}_{3}}\right) = -8\pi - \frac{16\pi}{3} - \frac{2\pi}{3} + 2\pi\sqrt{3}$

$\underline{III}) = \int_0^{2\pi} d\theta \int_{\frac{5\pi}{6}}^{\pi} (2\,sen\varphi\cos\theta,\ 2\,sen\varphi\,sen\theta,\ -2\,sen\varphi)\cdot(4\,sen^2\varphi\cos\theta,\ 4\,sen^2\varphi\,sen\theta,\ 4\,sen\varphi\cos\varphi)\,d\varphi$

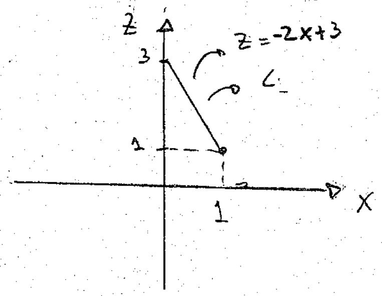
$= \dots = 16\pi\int_{\frac{5\pi}{6}}^{\pi}\left[\left(sen\varphi(1-\cos^2\varphi) - sen^2\varphi\cos\varphi\right)\right]d\varphi = 16\pi\left\{\int_{\frac{5\pi}{6}}^{\pi} sen\varphi\,d\varphi + \int_{\frac{5\pi}{6}}^{\pi}\cos^2\varphi\,d[\cos\varphi] - \int_{\frac{5\pi}{6}}^{\pi} sen^2\varphi\,d[sen\varphi]\right\}$

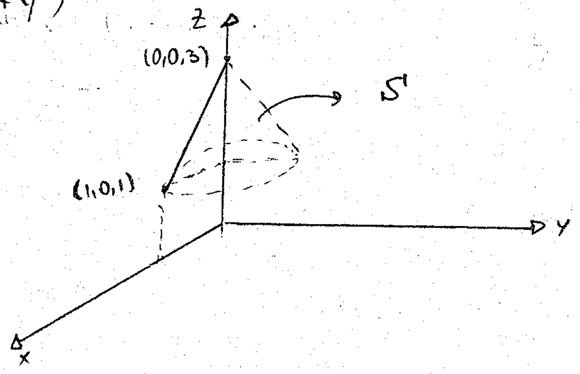
$= 16\pi\left(-0+\frac{1}{2}-\frac{1}{3}-\frac{\sqrt{3}}{8}+\frac{1}{24}\right) = 8\pi - \frac{16\pi}{3} - 2\pi\sqrt{3} + \frac{2\pi}{3}$

TOTAL: $16\pi\sqrt{3} - \cancel{8\pi} - \frac{16\pi}{3} - \cancel{\frac{2\pi}{3}} \cancel{-2\pi\sqrt{3}} + \cancel{8\pi} - \frac{16\pi}{3} \cancel{-2\pi\sqrt{3}} + \cancel{\frac{2\pi}{3}}$

$16\pi\sqrt{3} - \frac{32\pi}{3} = 16\pi\left(\sqrt{3} - \frac{2}{3}\right) = \frac{16\pi}{3}\left(3\sqrt{3}-2\right)$

7) $F(x,y,z) = (yz,\ xz,\ x^2+y^2)$





$C:\ \sigma(t) = (\underbrace{t}_{x(t)},\ \underbrace{-2t+3}_{z(t)}) \rightsquigarrow 0 \le t \le 1$

$S:\ \varphi(t,\theta) = (t\cos\theta,\ t\,sen\theta,\ -2t+3) \to 0 \le \theta \le 2\pi$

$\frac{\partial\varphi}{\partial t} = (\cos\theta,\ sen\theta,\ -2)$ $\to N(t,\theta) = \frac{\partial\varphi}{\partial t}\times\frac{\partial\varphi}{\partial\theta} = (+2t\cos\theta,\ 2t\,sen\theta,\ t)$

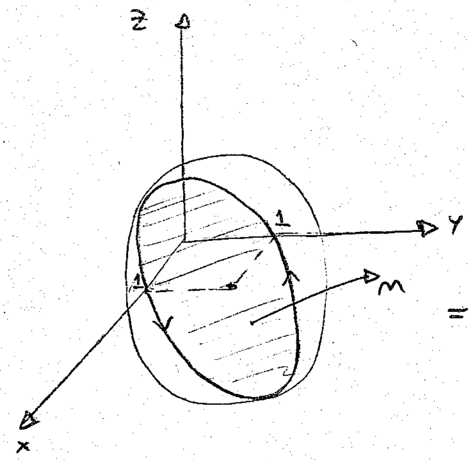
$\frac{\partial\varphi}{\partial\theta} = (-t\,sen\theta,\ t\cos\theta,\ 0)$

$\iint_S (F\cdot n)\,dS = \iint_S \left(F\cdot\frac{N}{|N|}\right)dS = \iint_D (F\cdot N)\,dD = \int_0^{2\pi} d\theta\int_0^1 \left((-2t+3)t\,sen\theta,\ (-2t+3)t\cos\theta,\ t^2\right)\cdot(+2t\cos\theta,\ 2t\,sen\theta,\ t)\,dt$

$\int d\theta \int \left[(-2t^2+3t)^2 + t^3\right] dt = \dots$

7.10

1) $(x-1)^2+(y-1)^2+z^2 = 2 \; \cap \; x+y=2 \Rightarrow C$



$F = (y, z, x)$

$\oint F \cdot dr = \oint y\,dx + z\,dy + x\,dz$

$= \iint_S (\text{rot}\,F \cdot m)\,dS \longrightarrow$ Stokes

$\text{rot}\,F = \nabla \times F = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (y, z, x)$

$\text{rot}\,F = \left(\frac{\partial x}{\partial y} - 1, -1 + \frac{\partial y}{\partial z}, \frac{\partial z}{\partial x} - 1\right)$

$\text{rot}\,F = (-1, -1, -1)$

$C \Rightarrow$ Intersecção:

$x^2 + y^2 + z^2 = 2(x+y)$

$x^2 + (2-x)^2 + z^2 = 2(\cancel{x} + 2 - \cancel{x})$

$x^2 + \cancel{4} - 4x + x^2 + z^2 = \cancel{4}$

$2x^2 - 4x + z^2 = 0 \longrightarrow x^2 - 2x + \frac{z^2}{2} = 0 \rightarrow \boxed{(x-1)^2 + \frac{z^2}{2} = 1} \; (C)$

A superfície é o "próprio" plano, ou seja, a normal de S é igual a do plano:

$x + y = 2 \rightsquigarrow m = (1, 1, 0)$

$\iint_S (-1,-1,-1) \cdot \frac{(1,1,0)}{|(1,1,0)|}\,dS = \iint_D (-1,-1,-1) \cdot (1,1,0) \cdot dx\,dz = \iint_D (-1-1)\,dx\,dz$

onde $D = \{ (x,z) \mid (x-1)^2 + \frac{z^2}{2} \leq 1 \}$

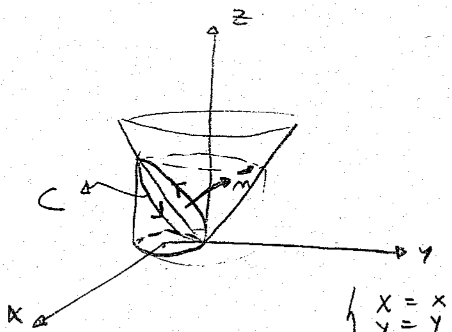
$= \iint_D -2\,dx\,dz = -2 \iint_D dx\,dz = -2 \cdot (\pi a b) = -2\pi\sqrt{2} \cdot 1 = -2\pi\sqrt{2}$

3) $F = \left(2xy,\ \left[(1-y)z + x^2 + x\right],\ \frac{x^2}{z} + e^z\right)$

$C:\ x^2 + y^2 = 1,\ z \geq 0$ e $z^2 = x^2 + (y-1)^2$

$\rightarrow\ x^2 = 1 - y^2$

$z^2 = 1 - y^2 + (y-1)^2 \longrightarrow z^2 = 2 - 2y\ (C)$



$\begin{cases} x = x \\ y = y \\ z = \sqrt{x^2 + (y-1)^2} \end{cases}$

$z = f(x,y) = \sqrt{x^2 + (y-1)^2}$

$\frac{\partial f}{\partial x} = \left(1, 0, \frac{-x}{\sqrt{x^2 + (y-1)^2}}\right)$

$\frac{\partial f}{\partial y} = \left(0, 1, \frac{y-1}{\sqrt{x^2 + (y-1)^2}}\right)$

$N = \left(\frac{-x}{\sqrt{x^2 + (y-1)^2}}, \frac{-y+1}{\sqrt{x^2 + (y-1)^2}}, 1\right)$

$$\oint_C F \cdot dr = \iint_S (\text{rot} F \cdot n)\, dS$$

$$\text{rot} F = \nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & (1-y)z + x^2 + x & \frac{x^2}{z} + e^z \end{vmatrix} = (-1+y, -x, 1)$$

$$\iint_D (\text{rot} F \cdot N)\, dx\, dy = \iint (-1+y, -x, 1) \cdot \left(\frac{-x}{\sqrt{}}, \frac{-y+1}{\sqrt{}}, 1\right) dx\, dy$$

$D = \{(x,y) \mid x^2 + y^2 \leq 1\}$

$$= \iint_D dx\, dy = A(D) = \pi R^2 = \pi$$

4) $F = (y + z^3x^2,\ 2z + 5x + y^3,\ 4y + z\cos x)$

$C:\ z = \frac{x^2}{3} + 2y^2$ e $3z + 2x = 5$

Interseção: $\frac{5}{3} - \frac{2x}{3} = \frac{x^2}{3} + 2y^2$

$\frac{x^2}{3} + \frac{2x}{3} + \frac{1}{3} + 2y^2 = 2$

$\frac{(x+1)^2}{6} + y^2 = 1$ D

$$\text{rot} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y + z^3x^2 & 2z + 5x + y^3 & 4y + z\cos x \end{vmatrix}$$

$\text{rot} F = (2, 3z^2x^2 + z\,\text{sen}\,x, 4)$

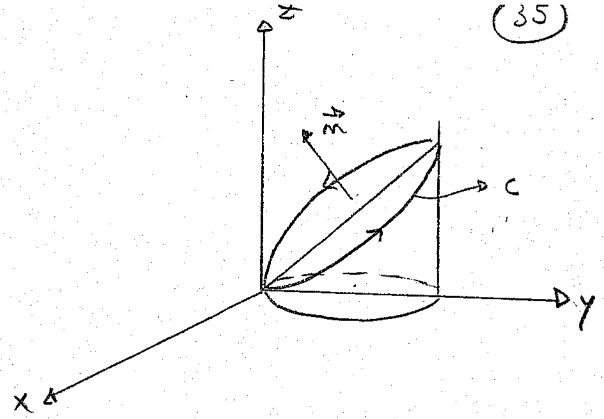
$$\oint_C F \cdot dr = \iint_S (\text{rot} F \cdot n)\, dS = \iint_D (2, 3z^2x^2 + z\,\text{sen}\,x, 4)\left(\frac{2}{3}, 0, \frac{3}{3}\right) dx\, dy$$

$$\iint_D \frac{(4+12)}{3}\, dx\, dy = \frac{16}{3} \iint_D dx\, dy = \frac{16 \cdot \pi \cdot \sqrt{6} \cdot 1}{3} = \frac{16\pi\sqrt{6}}{3}$$

6) $F = (y+z,\ z+x,\ x+y)$

C: Interseção: $y = z$ e $x^2 + y^2 = 2y$

$$\boxed{x^2 + (y-1)^2 = 1}$$



Plano $z = y = f(x,y)$

$$N = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right) = (0,-1,-1)$$

$$\oint_C F \cdot dr = \iint_S (\text{rot}F \cdot n)\, dS$$

$$= \iint_D (0,0,0)\cdot(0,-1,-1)\, dx\,dy = 0$$

$$\text{rot}F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y+z & z+x & x+y \end{vmatrix}$$

$$\text{rot}F = (0,0,0)$$

7) $\text{rot}F = (x, -2y, z)$

$$\oint_C F\cdot dr = \iint_S (\text{rot}F \cdot n)\, dS = \iint_D (\text{rot}F \cdot N)\, dx\,dy$$

$$= \int_0^{\pi/4} d\theta \int_0^{\pi/2} (a\,\text{sen}\varphi\cos\theta, -2a\,\text{sen}\varphi\,\text{sen}\theta, a\cos\varphi)\cdot N\, d\varphi$$

$$= \dots = \frac{a^3}{2}$$

Parametrização esférica:

$$\begin{cases} x = a\,\text{sen}\varphi\cos\theta \\ y = a\,\text{sen}\varphi\,\text{sen}\theta \\ z = a\cos\varphi \end{cases} \qquad \boxed{\begin{array}{c} 0 \le \varphi \le \pi/2 \\ 0 \le \theta \le \pi/4 \end{array}}$$

$$\Phi(\varphi,\theta) = (a\,\text{sen}\varphi\cos\theta,\ a\,\text{sen}\varphi\,\text{sen}\theta,\ a\cos\varphi)$$

$$N = \frac{\partial\Phi}{\partial\varphi} \times \frac{\partial\Phi}{\partial\theta} = (a^2\,\text{sen}^2\varphi\cos\theta,\ a^2\,\text{sen}^2\varphi\,\text{sen}\theta,\ a^2\,\text{sen}\varphi\cos\varphi)$$

9) $F = (y, 2x, xyz)$

$S_4 \to$ base da pirâmide

S_1, S_2 e $S_3 \to$ faces laterais da pirâmide

$S = S_1 \cup S_2 \cup S_3$

$C = \partial S$ ("bordo de S")

$$\text{rot}F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & 2x & xyz \end{vmatrix}$$

$$\text{rot}F = (xz, -yz, 1)$$

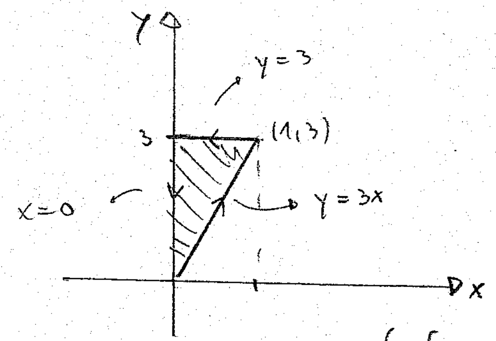
$$\iint_{S\cup S_4} (\text{rot}F \cdot n)\, dS = \iint_{S_1\cup S_2\cup S_3} (\text{rot}F\cdot n)\, dS + \iint_{S_4} (\text{rot}F \cdot n)\, dS = 0$$

$$\Rightarrow \iint_{S_1\cup S_2\cup S_3} (\text{rot}F\cdot n)\, dS = -\iint_{S_4} (\text{rot}F \cdot n)\, dS \Rightarrow \iint_{S_1\cup S_2\cup S_3} (\text{rot}F\cdot n)\, dS = \iint_{S_4} (\text{rot}F \cdot n)\, dS$$

n exterior

normal interior (para cancelar o sinal)

S_4 :



$\vec{n} = (0,0,-1) \rightarrow$ exterior
$\downarrow$
$\vec{n}' = (0,0,1) \rightarrow$ interior

$$\iint_{S_4} (\text{rot}F \cdot \vec{n}') \, ds = \iint_{S_4} (xz, -yz, 1) \cdot (0,0,1) \, ds = \iint_{S_4} ds = A(S_4) = \frac{3}{2}$$

Daí : $\displaystyle\iint_{S_1 \cup S_2 \cup S_3} (\text{rot}F \cdot n) \, ds = \frac{3}{2}$ //

0) $F = (x - x^2 z, \, yz^3 - y^2, \, x^2 y - xz)$

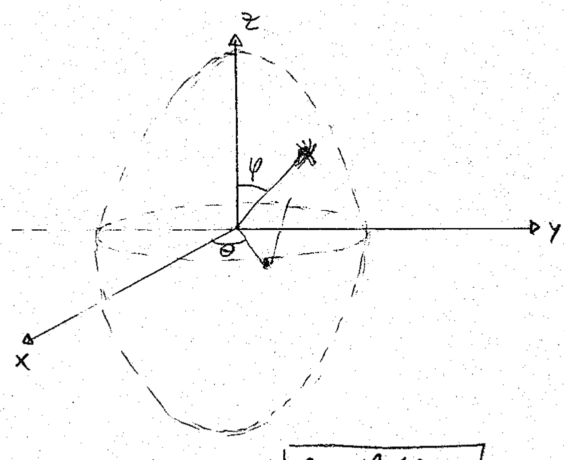
$$\text{rot}F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x - x^2 z & yz^3 - y^2 & x^2 y - xz \end{vmatrix} = \left(x^2 - 3yz^2, \, -x^2 - 2xz - z, \, 0\right)$$

S : qlqr sup. fechada no plano $xy \rightarrow \vec{n} = (0,0,1)$

$$\iint_S (\text{rot}F \cdot n) \, dS = \iint_D (x^2 - 3yz^2, -x^2 - 2xz - z, 0) \cdot (0,0,1) \, dx dy = 0 \, //$$

1) $F = (2xy, 0, -xz)$

$$\text{rot}F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy & 0 & -xz \end{vmatrix} = (0, z, -2x)$$



$S \rightarrow$ superfície do elipsóide é fechada limitada
e $F(x,y,z)$ é $C^1 \Rightarrow \displaystyle\iint_S (\text{rot}F \cdot n) \, dS = 0$
NUA
Parametrizando S :

$(\varphi,\theta) = (2 \,\text{sen}\varphi \cos\theta, \, 3 \,\text{sen}\varphi \cos\theta, \, 4 \cos\varphi) \rightarrow N(\varphi,\theta) = (12 \,\text{sen}^2\varphi \cos\theta, \, 8 \,\text{sen}^2\varphi \,\text{sen}\theta, \, 6 \,\text{sen}\varphi \cos\varphi) \Rightarrow$ $\boxed{\begin{array}{c} 0 \le \varphi \le \pi, \\ 0 \le \theta \le 2\pi \end{array}}$

$$\iint_S (\text{rot}F \cdot n) \, dS = \int_0^{2\pi} d\theta \int_0^{\pi} (0, 4\cos\varphi, -4 \,\text{sen}\varphi \cos\theta) \cdot (12 \,\text{sen}^2\varphi \cos\theta, \, 8 \,\text{sen}^2\varphi \,\text{sen}\theta, \, 6 \,\text{sen}\varphi \cos\varphi) \, d\varphi$$

$\ldots = 0$ //

12) $F = \left(2xyz + \operatorname{sen} x,\ x^2z + e^y,\ x^2y + \frac{1}{z}\right)$

$$\operatorname{rot} F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xyz + \operatorname{sen} x & x^2z + e^y & x^2y + \frac{1}{z} \end{vmatrix} = (0, 0, 0) \rightarrow \text{Campo gradiente}$$

$\downarrow$

$\vec{F}$ é conservativo

$C: \gamma(t) = (\cos^3 t, \operatorname{sen}^2 t, (t+1)^2) \rightarrow \boxed{0 \leq t \leq \pi}$

$\Rightarrow F(x,y,z)$ é C^1 em uma regiao aberta e $\operatorname{rot}\vec{F} = \vec{0} \Rightarrow \vec{F}$ é CONSERVATIVO

Entao: $\displaystyle\int_A^B F \cdot dr = f(B) - f(A)$

$\Rightarrow F = \nabla f \rightarrow \nabla f = \left(2xyz + \operatorname{sen} x,\ x^2z + e^y,\ x^2y + \frac{1}{z}\right)$

$\dfrac{\partial f}{\partial x} = 2xyz + \operatorname{sen} x \xrightarrow{\int dx} f = x^2yz - \cos x + A(y,z)$

$\dfrac{\partial f}{\partial y} = x^2z + e^y \xrightarrow{\int dy} f = x^2yz + e^y + B(x,z)$

$\dfrac{\partial f}{\partial z} = x^2y + \dfrac{1}{z} \xrightarrow{\int dz} f = x^2yz + \ln|z| + C(x,y)$

$f(x,y,z) = x^2yz - \cos x + e^y + \ln|z|$

para $t = 0 \Rightarrow \gamma(0) = (1, 0, 1) \rightarrow A = (1,0,1)$

para $t = \pi \Rightarrow \gamma(\pi) = (-1, 0, (\pi+1)^2) \rightarrow B = (-1, 0, (\pi+1)^2)$

$\displaystyle\oint_A^B F \cdot dr = f(B) - f(A) = f(-1, 0, (\pi+1)^2) - f(1,0,1) = -\cancel{\cos 1} \cancel{+1} + 2\ln(\pi+1) - \left[-\cancel{\cos 1} \cancel{+1}\right]$

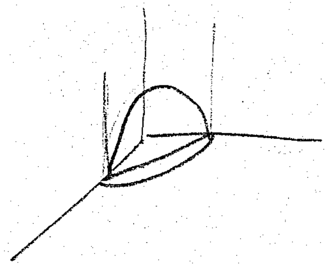
$= 2\ln(\pi+1)$ //

5) $F = (2xyz + 2x, x^2 z, x^2 y)$

$$\text{rot}F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xyz+2x & x^2 z & x^2 y \end{vmatrix} = (0,0,0) \rightarrow \vec{F} \text{ é CONSERVATIVO}$$

~~Já que C é uma curva fechada C^1 no plano onde F é CONSERVATIVO e $\forall b$ é C^1 $\Rightarrow \oint_C F \cdot dr = \iint_S (\text{rot}F \cdot n)\, ds = 0$~~

~~PROVA~~



$F = \nabla f \Rightarrow \nabla f = (2xyz + 2x, x^2 z, x^2 y)$

$\dfrac{\partial f}{\partial x} = 2xyz + 2x \xrightarrow{\int dx} f = x^2 yz + x^2 + A(y,z)$

$\dfrac{\partial f}{\partial y} = x^2 z \xrightarrow{\int dy} f = x^2 yz + B(x,z)$

$\dfrac{\partial f}{\partial z} = x^2 y \xrightarrow{\int dz} f = x^2 yz + C(x,y)$

$\Rightarrow f = x^2 yz + x^2$

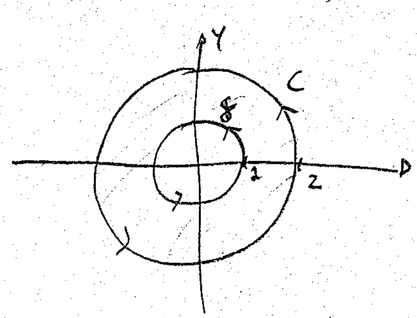
$C: \; z = \sqrt{4 - x^2 + y^2} = \sqrt{4 - x^2 - (2-x)^2} = \sqrt{\cancel{4} - 2x^2 + 4x - \cancel{4}} = \sqrt{-2x^2 + 4x}$

$A = (0,2,0)$; $B = (2,0,0)$

$\oint_A^B F \cdot dr = f(B) - f(A) = f(2,0,0) - f(0,2,0) = 4$

a) $F = \left(\dfrac{-y}{x^2+y^2}, \dfrac{x}{x^2+y^2}, z\right) \rightsquigarrow$ não está definido para $(0,0,z)$

$\text{rot}F = \ldots = (0,0,0) \rightarrow$ F NÃO é CONS. pois tem restrição



$\oint F \cdot dr = \oint_{C \cup \gamma^-} F \cdot dr + \oint_{\gamma} F \cdot dr \Rightarrow \oint_C F dr = \int_{\gamma} F \cdot dr$

$\sigma(t) = (\cos\theta, \sin\theta, 0)$

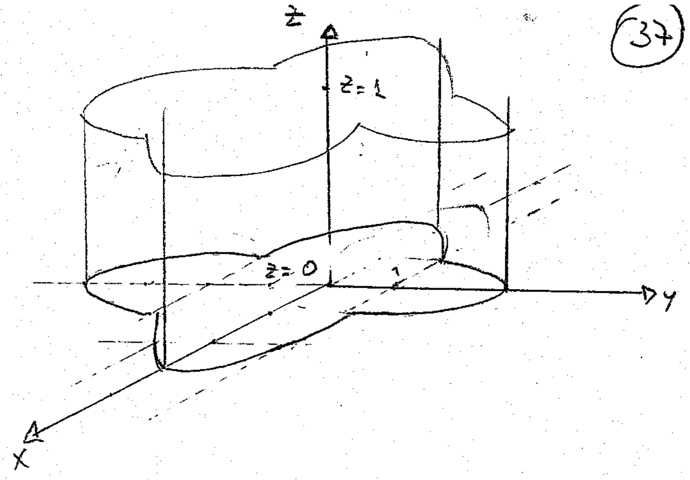
$\sigma'(t) = (-\sin\theta, \cos\theta, 0)$

$\oint_{\gamma} F \cdot dr = \iint \underbrace{(\text{rot}F \cdot n)}_{0} ds = 0$

$\oint_{\gamma} F \cdot dr = \int_0^{2\pi} \left(\dfrac{-\sin\theta}{1}, \dfrac{\cos\theta}{1}, 0\right) \cdot (-\sin\theta, \cos\theta, 0)\, d\theta = 2\pi$ //

$\oint_C F \cdot dr = \oint_{\gamma} F \cdot dr = 2\pi$ //

7.12



1) $F = \left(2x + xy^2,\ -zy,\ \frac{z^2}{2} - y^2 z\right)$

$S:$ base $\to$ $x^2 + (y-1)^2 = 4,\ y \geq 1$

$x^2 + (y+1)^2 = 4,\ y \leq -1$

$(x-2)^2 + y^2 = 1,\ x \geq 2$

$(x+2)^2 + y^2 = 1,\ x \leq -2$

$$\iint_S (F \cdot n)\, ds = \iiint_W \operatorname{div} F \, dx\,dy\,dz$$

$$\operatorname{div} F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$\operatorname{div} F = 2 + y^2 \cancel{-z} \cancel{+z} - y^2 = 2 \cancel{+y^2} \cancel{-y^2}$$

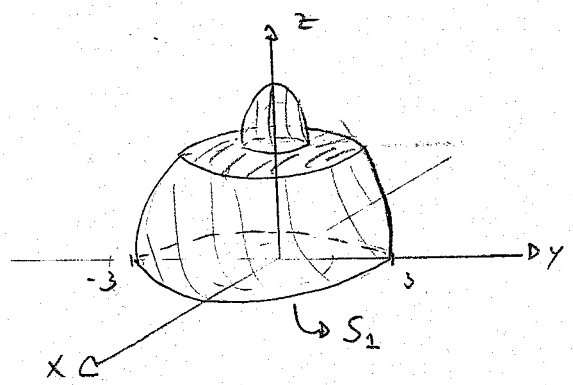
$$= \iiint_W 2\,dx\,dy\,dz$$

$$= 2\iiint_W dx\,dy\,dz = 2\int dx \int dy \int_0^1 dz = 2\iint_D dx\,dy = 2A(D) = 2(8 + \pi + 4\pi)$$

$$= 10\pi + 16$$

2) $F = (x,\ -2y + e^x \cos z,\ z + x^2)$

$$S: \begin{cases} z = 9 - (x^2+y^2)\ ; & 0 \leq z \leq 5 \\ z = 5 & ;\ 1 \leq x^2 + y^2 \leq 4 \\ z = 8 - 3(x^2+y^2)\ ; & x^2 + y^2 \leq 1 \end{cases}$$



NÃO TEM BASE $(x^2 + y^2 = 9)$

$$\iint_{S \cup S_1} (F \cdot n)\, ds = \iiint_W \operatorname{div} F \, dx\,dy\,dz = 0$$

$\operatorname{div} F = 1 - 2 + 1 = 0$

$n' = (0, 0, -1)$

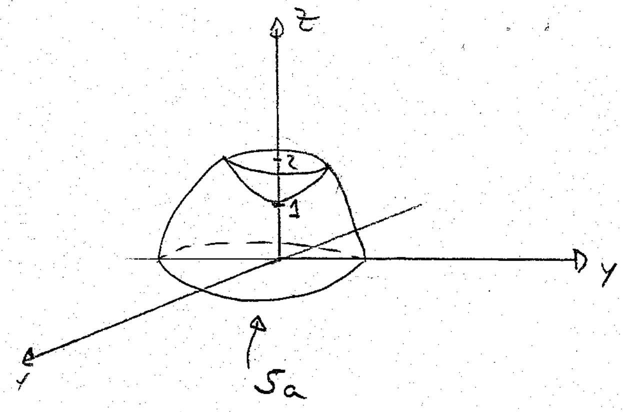
$$\iint_S (F \cdot n)\, ds + \iint_{S_1} (F \cdot n)\, ds = 0 \Rightarrow \iint_S (F \cdot n)\, ds = \iint_{S_1} (F \cdot n')\, ds$$

Parametrizando S_1: $x^2 + y^2 = 9 \Rightarrow \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\quad z = 0$

$$\iint_S (F \cdot n')\, ds = \iint_D (x, \dots, z + x^2) \cdot (0, 0, -1)\, dx\,dy = \iint_D -x^2\, dx\,dy = \iint_D -r^2\cos^2\theta \cdot r\, d\theta\, dr$$

$$= -\int_0^{2\pi} d\theta \int_0^3 r^3 \cos^2\theta\, dr = -\int_0^{2\pi} \cos^2\theta\, d\theta \cdot \frac{81}{4} = \dots = \frac{81\pi}{4}$$

1) $F = \left(e^{y} + \cos yz \, , \, -2zy + \operatorname{sen} xz \, , \, z^2 + \frac{3}{\sqrt{2}} \right)$



$S = S_1 \cup S_2$

$_1 : \; z = 4 - 2x^2 - y^2 \;\; ; \; 0 \leq z \leq 2$

$_2 : \; z = 1 + x^2 + \frac{y^2}{2} \; ; \;\; 1 \leq z \leq 2$

$S_1 : \; z = 0 \rightarrow \boxed{\frac{x^2}{2} + \frac{y^2}{4} = 1}$

$\quad\quad z = 2 \rightarrow \boxed{x^2 + \frac{y^2}{2} = 1}$

$S_2 : \; z = 1 \rightarrow \boxed{x = y = 0}$

$\quad\quad z = 2 \rightarrow \boxed{x^2 + \frac{y^2}{2} = 1}$

$\operatorname{div} F = (0 - 2z + 2z) = 0$

Fechando para usar GAUSS:

$$\iint_{S \cup S_a} (F \cdot n)\, ds = \iiint_W \operatorname{div} F \, dx\,dy\,dz = 0 \rightarrow \iint_S (F \cdot n)\, ds + \iint_{S_a} (F \cdot n)\, ds = 0$$

$n = (0,0,-1)$ – exterior

$$\iint_S (F \cdot n)\, ds = - \iint_{S_a} (F \cdot n)\, ds$$

$$\iint_{S_a} (F \cdot n)\, ds = \iint_D \left(\ldots, \ldots, z^2 + \frac{3}{\sqrt{2}} \right) \cdot (0,0,-1)\, dx\,dy = - \iint_D \left(z^2 + \frac{3}{\sqrt{2}} \right) dx\,dy \xrightarrow{z=0}$$

$$= - \iint_D \frac{3}{\sqrt{2}}\, dx\,dy = \frac{-3}{\sqrt{2}} \underset{\left(\frac{x^2}{2} + \frac{y^2}{4} = 1 \right)}{\underset{\downarrow}{A(S_a)}} = \frac{-3}{\sqrt{2}} \cdot (\pi a b) = \frac{-3 \cdot \pi \cdot \cancel{\sqrt{2}} \cdot 2}{\cancel{\sqrt{2}}} = -6\pi$$

$$\Rightarrow \iint_S (F \cdot n)\, ds = - \iint_{S_a} (F \cdot n)\, ds = 6\pi$$

5) S não tem base

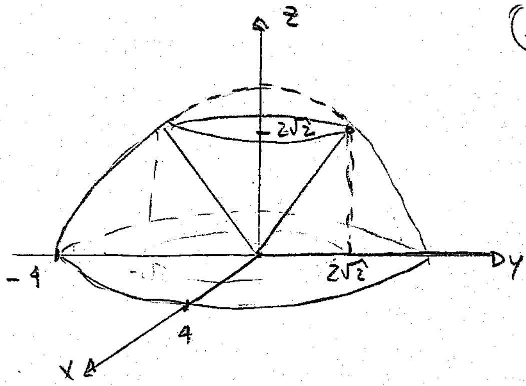
Sa → base

$F = (y^2x,\ z^2y+x,\ x^2z-5)$

$$\iint_{S\cup Sa} (F.n)\,ds = \iiint_W \mathrm{div}F.\,dxdydz$$

$$= \iiint_W (x^2+y^2+z^2)\,dxdydz \quad (\mathrm{I})$$

$$= \iint_S (F.n)\,ds + \iint_{Sa} (F.n)\,ds \quad (\mathrm{II})$$

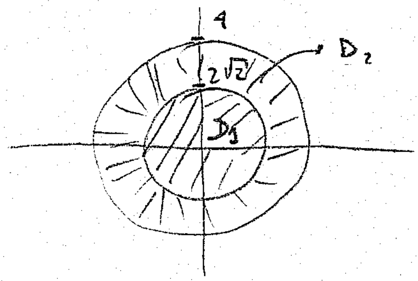


$\mathrm{div}F = y^2+z^2+x^2$

$z = \sqrt{x^2+y^2}\ ;\ (0 \le z \le 2\sqrt{2})$

$x^2+y^2+z^2=16\ ;\ (8 \le x^2+y^2 \le 16)$

$$(\mathrm{I})\ \iiint (x^2+y^2+z^2)\,dxdydz = \iint_{D_1}\left[\int_0^{\sqrt{x^2+y^2}} (x^2+y^2+z^2)\,dz\right]dxdy + \iint_{D_2}\left[\int_0^{\sqrt{16-x^2-y^2}} (x^2+y^2+z^2)\,dz\right]dxdy$$



$$= \iint_{D_1}\left[(x^2+y^2)\sqrt{x^2+y^2} + \frac{(x^2+y^2)\sqrt{x^2+y^2}}{3}\right]dxdy$$

$$+ \iint_{D_2}\left((x^2+y^2)\sqrt{16-x^2-y^2} + \frac{(16-x^2-y^2)\sqrt{16-x^2-y^2}}{3}\right)dxdy$$

$$= \int_0^{2\pi} d\theta \int_0^{2\sqrt{2}} \frac{4}{3}.r^3.r\,dr\,d\theta + \int_0^{2\pi} d\theta \int_{2\sqrt{2}}^{4}\left(r^3\sqrt{16-r^2} + \frac{r}{3}(16-r^2)^{3/2}\right)dr\,d\theta$$

$$= \ldots = \frac{1024\sqrt{2}\,\pi}{5}$$

$$\text{II)}\quad \iint_{S_a} (F\cdot n)\,dS = \iint_{D} F\cdot N\,dx\,dy = \iint_{\substack{D\\(z=0)}} (y^2x, x, -5)\cdot(0,0,-1)\,dx\,dy = 5\iint_D dx\,dy$$

$$= 5\,A(D) = 5.\pi.4^2 = 80\pi$$

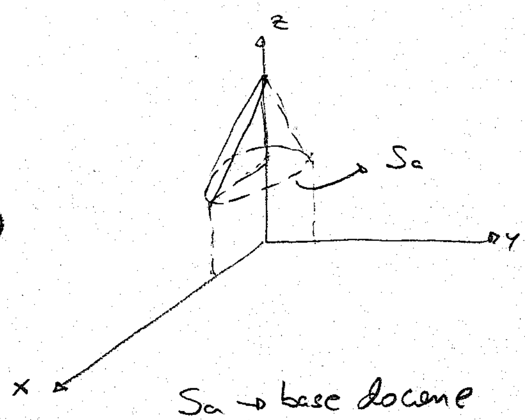
$$\iint_S (F.n) = \iiint \operatorname{div}F\,dx\,dy\,dz - \iint_{S_a}(F.n)\,dS = \frac{1024\pi\sqrt{2}}{5} - 80\pi = \frac{1024\pi\sqrt{2} - 400\pi}{5} \;//$$

$$8)\quad \iint_S (\operatorname{rot}F.n)\,dS = \iiint \operatorname{div}(\operatorname{rot}F)\,dx\,dy\,dz = 0 \;//$$

$$\operatorname{rot}F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & 0 & -xz \end{vmatrix} = (0, z, -2x)$$

$$\operatorname{div}(\operatorname{rot}F) = 0 + 0 + 0 = 0$$

$$9)\quad F = \left(-\frac{y^3}{3} + ze^x,\ \frac{x^3}{3} - \cos yz,\ xy\right)$$



$$\operatorname{rot}F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -\frac{y^3}{3} + ze^x & \frac{x^3}{3} - \cos yz & xy \end{vmatrix} = (x - y\operatorname{sen} yz,\ e^x - y,\ x^2 + y^2)$$

Sa → base do cone

S → sup. do cone

$S_a \to x^2 + y^2 \leq 1$

$$\operatorname{div}(\operatorname{rot}F) = 1 - 1 + 0 = 0$$

GAUSS: $\displaystyle\iint_{S\cup S_a} (\operatorname{rot}F.n)\,dS = \iiint_W \underbrace{\operatorname{div}(\operatorname{rot}F)}_{0}\,dx\,dy\,dz = 0$

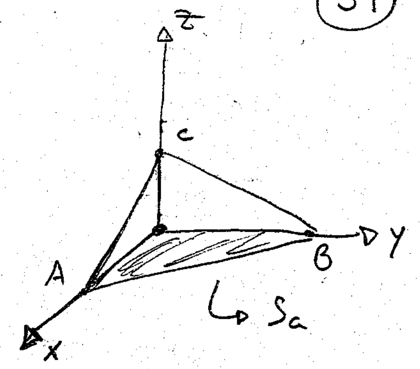
$$\iint_S (\operatorname{rot}F.n)\,dS = -\iint_{S_a} (\operatorname{rot}F.n)\,dS = -\iint_D \operatorname{rot}F.N\,dx\,dy = -\iint_D (x - y\operatorname{sen}yz, e^x - y, x^2+y^2)\cdot(0,0,-1)\,dx\,dy$$

(usando a normal interior)

$$\iint_D (x^2+y^2)\,dx\,dy = \int_0^{2\pi} d\theta \int_0^1 r^2.\,r\,dr\,d\theta = \frac{2\pi}{4} = \pi/2$$

$\boxed{S_a \to \text{base}}$

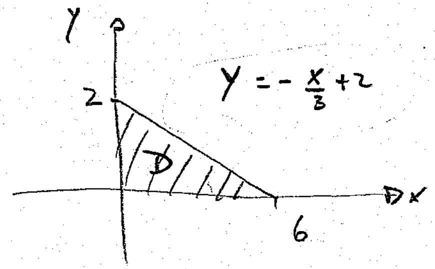
10) $F = (3y+z,\ x+4z,\ 2y+x)$



$$\text{rot}\,F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3y+z & x+4z & 2y+x \end{vmatrix} = (-2, 0, -2)$$

$\text{div}(\text{rot}\,F) = 0$

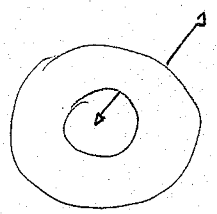
$$\iint_{S \cup S_a} (\text{rot}F \cdot n)\,ds = \iiint_W \text{div}(\text{rot}F) \cdot n\,dx\,dy\,dz = 0$$



$$\iint_S (\text{rot}F \cdot n)\,ds = -\iint_{S_a} (\text{rot}F \cdot n)\,ds$$

$$= -\iint_D (-2,0,-2) \cdot (0,0,-1)\,.\,dx\,dy = -2 \iint dx\,dy = 2 \cdot \frac{6 \cdot \cancel{2}}{\cancel{2}} = -12$$

11) $F = \left(\frac{x}{x^2+y^2+z^2},\ \frac{y}{x^2+y^2+z^2},\ \frac{z}{x^2+y^2+z^2} \right)$



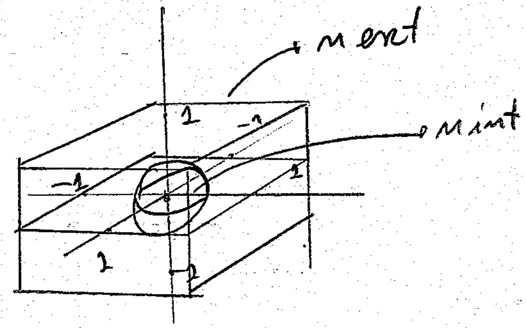
$\text{div}\,F = \frac{1}{x^2+y^2+z^2}$

a) $W \Rightarrow x^2+y^2+z^2 = a^2 \quad a < b$

$x^2+y^2+z^2 = b^2$

$$\iint_S (F \cdot n)\,ds = \iiint_W \text{div}F\,.\,dx\,dy\,dz$$

$$= \iiint_W \frac{1}{x^2+y^2+z^2}\,dx\,dy\,dz = \int_0^{2\pi} d\theta \int_0^{\pi} d\varphi \int_a^b \frac{1}{\cancel{r^2}} \cdot \cancel{r^2} \operatorname{sen}\varphi\,.\,dr = \dots = 4\pi(b-a)$$



b) $$\iint_S (F \cdot n)\,ds = \iint_{S \cup \delta} (F \cdot n)\,ds - \iint_{\delta} (F \cdot n)\,ds$$

$$\iint_{S \cup \delta} (F \cdot n)\,ds = \iiint_{S \cup \delta} \text{div}F\,.\,dx\,dy\,dz = 0$$

$$\iint (F \cdot n)\,ds = -\iint_{\delta} (F \cdot n)\,ds$$

$$\int_\gamma\int (\bar{r}, n)\, ds \longrightarrow \quad \Phi(\theta,\varphi) = (a\,\text{sen}\,\varphi\cos\theta,\ a\,\text{sen}\,\varphi\,\text{sen}\,\theta,\ a\cos\varphi)$$

$$N(\theta,\varphi) = (-a^2\,\text{sen}^2\varphi\cos\theta,\ -a^2\,\text{sen}^2\varphi\,\text{sen}\,\theta,\ -a^2\,\text{sen}\,\varphi\cos\varphi)$$

$$-\int_\gamma\int (\bar{r}\cdot n)\, ds = -\int_0^{2\pi} d\theta \int_0^{\pi} -\frac{(\cancel{a}\,\text{sen}\,\varphi\cos\theta,\ \cancel{a}\,\text{sen}\,\varphi\,\text{sen}\,\theta,\ \cancel{a}\cos\varphi)}{\cancel{a}^{3}} \cdot (a^2\,\text{sen}^2\varphi\cos\theta,\ a^2\,\text{sen}^2\varphi\,\text{sen}\,\theta,\ a^2\,\text{sen}\,\varphi\cos\varphi)\, \cancel{d}\,\text{sen}\,\varphi$$

$$= \dots = 4\pi \quad //$$