§6.5 Exercícios

1. Calcule as seguintes integrais, ao longo das curvas C, orientadas positivamente.

a) $\oint_C y^2dx + x^2dy$; C é a fronteira do quadrado $D = [-1,1] \times [-1,1]$.

b) $\oint_C (3x^2 + y)dx + 4y^2dy$; C é a fronteira do triângulo de vértices (0,0), (1,0) e (0,2).

c) $\oint_C (e^x - 3y)dx + (e^y - 6x)dy$; C é a elipse de equação $x^2 + 4y^2 = 1$.

d) $\oint_C x^{-1}e^y dx + (e^y lnx + 2x)dy$; C é a fronteira da região limitada por $x = y^4 + 1$ e $x = 2$.

e) $\oint_C (2xy - x^2)dx + (x - y^2)dy$; C é a fronteira da região limitada por $y = x^2$ e $y^2 = x$.

f) $\oint_C (x + y)dx + (y - x)dy$; C é a circunferência $x^2 + y^2 - 2ax = 0$.

g) $\oint_C (2x - y^3)dx - xydy$; C é a fronteira da região limitada pelas curvas $x^2 + y^2 = 4$ e $x^2 + y^2 = 9$.

h) $\oint_C (2x - y)dx + (x + 3y)dy$; C é a fronteira do pentágono de vértices (0,0), (0,2), (1,3), (2,2) e (2,0).

2. Seja C uma curva fechada, orientada positivamente, limitando uma região do plano xy, de área A. Verifique que, se a_1, a_2, a_3, b_1, b_2 e b_3 são constantes reais, então

$$\oint_C (a_1x + a_2y + a_3)dx + (b_1x + b_2y + b_3)dy = (b_1 - a_2)A.$$

3. Determine $\oint_C (x^2 + y + 2xy^3)dx + (5x + 3x^2y^2 + y)dy$, onde C é a união das curvas C_1, C_2 e C_3 dadas por C_1: $x^2+y^2 = 1$, $y \geq 0$; C_2: $x+y+1 = 0$, $-1 \leq x \leq 0$; C_3: $x - y - 1 = 0$, $0 \leq x \leq 1$. Especifique a orientação escolhida.

$y = -x - 1$

4. a) Mostre que a área de uma região fechada e limitada D do plano xy pode ser obtida através da seguinte integral de linha:

$$\text{área } (D) = \oint_{\partial D} xdy.$$

b) Use a) para calcular a área da região limitada pelo eixo y, pelas retas $y = 1$, $y = 3$, e pela curva $x = y^2$.

5. Seja $F = (F_1, F_2)$ um campo vetorial de classe C^1 no $\mathbb{R}^2$, exceto em (0,0), tal que $\frac{\partial F_2}{\partial x}(x,y) = \frac{\partial F_1}{\partial y}(x,y) + 4$ para todo $(x,y) \neq (0,0)$. Sabendo que $\oint_\gamma F_1 dx + F_2 dy = 6\pi$, onde γ é a circunferência $x^2 + y^2 = 1$, orientada no sentido anti-horário, calcule $\oint_C F_1 dx + F_2 dy$, onde C é a elipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$, orientada no sentido anti-horário.

6. Seja $F(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} + 3x \right)$ um campo vetorial em $\mathbb{R}^2$. Calcule a integral de linha do campo F ao longo das curvas C_1 e C_2, orientadas no sentido anti-horário, onde:

a) C_1 é a circunferência de equação $x^2 + y^2 = 4$.

b) C_2 é a fronteira do retângulo $R = \{(x,y) \in \mathbb{R}^2 | -\pi \leq x \leq \pi \ , \ -3 \leq y \leq 3\}$.

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(1) a) $\oint_C y^2\,dx + x^2\,dy$ $\quad$ C: fronteira do quadrado $D = [-1,1] \times [-1,1]$

• $F_1(x,y) = y^2 \qquad \frac{\partial F_1}{\partial y} = 2y$

• $F_2(x,y) = x^2 \qquad \frac{\partial F_2}{\partial x} = 2x$

$$\oint_C y^2\,dx + x^2\,dy = \iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\,dx\,dy =$$

$$= \int_{-1}^{1}\int_{-1}^{1} 2x - 2y\,dx\,dy = \int_{-1}^{1} \left[x^2 - 2xy\right]_{-1}^{1} dy =$$

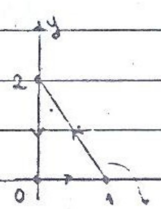
$$= \int_{-1}^{1} \cancel{1} - 2y - \cancel{1} - 2y\,dy = \int_{-1}^{1} -4y\,dy =$$

$$= \left[-2y^2\right]_{-1}^{1} = -2 + 2 = \boxed{0_{//}}$$

b) $\oint_C (3x^2+y)\,dx + 4y^2\,dy$ $\quad$ C: fronteira do triângulo de vértices $(0,0)$, $(1,0)$ e $(0,2)$

• $F_1(x,y) = 3x^2 + y \qquad \frac{\partial F_1}{\partial y} = 1$

• $F_2(x,y) = 4y^2 \qquad \frac{\partial F_2}{\partial x} = 0$



$0 \leq x \leq 1$

$0 \leq y \leq -2x+2$

$y = -2x+2$

$$\iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\,dx\,dy = \int_0^1\int_0^{-2x+2} -1\,dy\,dx = \int_0^1 \left[-y\right]_0^{-2x+2} dx =$$

$$= \int_0^1 2x - 2\,dx = \left[x^2 - 2x\right]_0^1 = 1 - 2 = \boxed{-1_{//}}$$

c) $\oint_C (e^x - 3y)\,dx + (e^y - 6x)\,dy$ C: elipse $x^2 + 4y^2 = 1$

$\frac{x^2}{1} + \frac{y^2}{1/4} = 1$

• elipse:

$$\begin{cases} x = r\cos\theta \\ y = \frac{1}{2}\, r\,\text{sen}\theta \end{cases} \qquad \frac{(r\cos\theta)^2 + \cancel{\tfrac{1}{4}}(r\,\text{sen}\theta)^2}{1/4} = 1 \qquad 0 \le \theta \le 2\pi$$

$r^2 = 1 \therefore r = 1$ $\quad 0 \le r \le 1$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \cos\theta & -r\,\text{sen}\theta \\ \tfrac{1}{2}\text{sen}\theta & \tfrac{1}{2}\, r\cos\theta \end{vmatrix} = \frac{1}{2}\, r\cos^2\theta + \frac{1}{2}\, r\,\text{sen}^2\theta = \frac{r}{2}$$

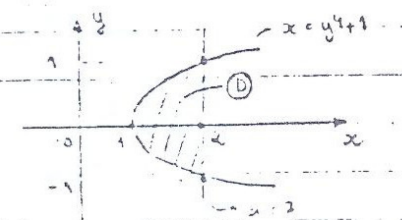
• $F_1(x,y) = e^x - 3y$ $\qquad \frac{\partial F_1}{\partial y} = -3$

• $F_2(x,y) = e^y - 6x$ $\qquad \frac{\partial F_2}{\partial x} = -6$

$$\iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dx\,dy = \int_0^{2\pi}\int_0^1 -3\cdot\frac{r}{2}\, dr\, d\theta =$$

$$= -\frac{3}{2}\int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^1 d\theta = -\frac{3}{2}\cdot\frac{1}{2}\int_0^{2\pi} 1\, d\theta = -\frac{3}{4}\left[\theta\right]_0^{2\pi} = \boxed{\frac{3\pi}{2}}$$

d) $\oint_C x^{-1}\, e^y\, dx + (e^y \ln x + 2x)\, dy$ C: $x = y^4 + 1$ e $x = 2$



$-1 \le y \le 1$

$y^4 + 1 \le x \le 2$

• $F_1(x,y) = x^{-1}\cdot e^y$ $\quad \frac{\partial F_1}{\partial y} = \frac{e^y}{x}$

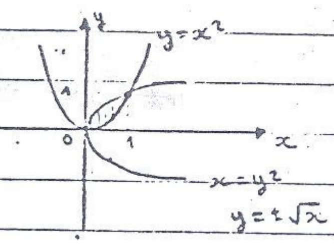
• $F_2(x,y) = e^y \cdot \ln x + 2x$ $\quad \frac{\partial F_2}{\partial x} = \frac{e^y}{x} + 2$

$$\iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\,dx\,dy = \int_{-1}^{1}\int_{y^4+1}^{2} \cancel{\frac{e^y}{x}} + 2 - \cancel{\frac{e^y}{x}}\,dx\,dy =$$

$$= 2\cdot\int_{0}^{1}\int_{y^4+1}^{2} 2\,dx\,dy = 2\cdot\int_{0}^{1} 4-2y^4-2\,dy = 2\int_{0}^{1} 2-2y^4\,dy =$$

$$= 2\cdot\left[2y - \frac{2y^5}{5}\right]_0^1 = 2\cdot\left(\frac{2}{1_5} - \frac{2}{5_1}\right) = 2\cdot\frac{8}{5} = \boxed{\frac{16}{5}}\,,,$$

e) $\oint_C (2xy - x^2)\,dx + (x-y^2)\,dy$ $\quad$ C: $y = x^2$ e $y^2 = x$



interseção:

$y = x^2 \Rightarrow (x^2)^2 = x$

$x^4 - x = 0$

$x(x^3-1) = 0$

$x = 0 \quad x = 1$

$0 \le x \le 1 \quad x^2 \le y \le \sqrt{x}$

• $F_1(x,y) = 2xy - x^2$ $\quad \frac{\partial F_1}{\partial y} = 2x$

• $F_2(x,y) = x - y^2$ $\quad \frac{\partial F_2}{\partial x} = 1$

$$\iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\,dx\,dy = \int_0^1\int_{x^2}^{\sqrt{x}} 1-2x\,dy\,dx =$$

$$= \int_0^1 \left[y - 2xy\right]_{x^2}^{\sqrt{x}} dx = \int_0^1 x^{1/2} - 2x^{3/2} - x^2 + 2x^3\,dx =$$

$$= \left[\frac{2}{3} x^{3/2} - \frac{4}{5} x^{5/2} - \frac{x^3}{3} + \frac{x^4}{2} \right]_0^1 =$$

$$= \frac{2}{3} - \frac{4}{5} - \frac{1}{3} + \frac{1}{2} = \frac{1}{3_{/15}} - \frac{4}{5_{/6}} + \frac{1}{2_{/10}} = \frac{15 - 24 + 10}{30} = \boxed{\frac{1}{30}}$$

f) $\oint_C (x+y)\,dx + (y-x)\,dy$ $\qquad$ $C: x^2 + y^2 - 2ax = 0$

$(x-a)^2 + y^2 = a^2$

$$\begin{cases} x = r\cos\theta + a \\ y = r\sin\theta \end{cases}$$

$(r\cos\theta + \cancel{a} - \cancel{a})^2 + (r\sin\theta)^2 = a^2$.

$r^2\cos^2\theta + r^2\sin^2\theta = a^2$

$r^2 = a^2 \therefore r = a$

$0 \le r \le a$ $\quad$ $0 \le \theta \le 2\pi$

- $F_1(x,y) = x + y$ $\qquad \frac{\partial F_1}{\partial y} = 1$
- $F_2(x,y) = y - x$ $\qquad \frac{\partial F_2}{\partial x} = -1$

$$\iint_R \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dx\,dy = \int_0^{2\pi}\int_0^a -2 \cdot r\,dr\,d\theta =$$

$$= -2 \int_0^{2\pi} \left[\frac{r^2}{2} \right]_0^a d\theta = -\cancel{2} \cdot \frac{1}{\cancel{2}} \int_0^{2\pi} a^2\,d\theta = \boxed{-2\pi a^2}$$

g) $\oint_C (2x - y^3)\,dx - xy\,dy$ $\qquad$ $C: x^2 + y^2 = 4$ e $x^2 + y^2 = 9$

$2 \le r \le 3 \qquad 0 \le \theta \le 2\pi$

- $F_1(x,y) = 2x - y^3$ $\qquad \frac{\partial F_1}{\partial y} = -3y^2$ $\qquad \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

• $F_2(x,y) = -xy$ $\qquad \dfrac{\partial F_2}{\partial x} = -y$

$$\iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dx\,dy = \iint_D -y + 3y^2\, dx\,dy =$$

$$= \int_0^{2\pi}\int_2^3 (-r\sin\theta + 3r^2\sin^2\theta).r\, dr\, d\theta =$$

$$= \int_0^{2\pi}\int_2^3 -r^2\sin\theta + 3r^3\sin^2\theta\, dr\, d\theta =$$

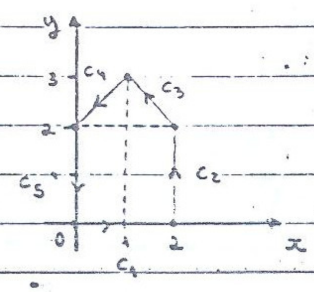
$$= \int_0^{2\pi} \left[-\frac{r^3}{3}\sin\theta + \frac{3r^4}{4}\sin^2\theta\right]_2^3 d\theta =$$

$$= \int_0^{2\pi} -\frac{27}{3}\sin\theta + \frac{243}{4}\sin^2\theta + \frac{8}{3}\sin\theta - \frac{48}{4}\sin^2\theta\, d\theta$$

$$= \int_0^{2\pi} -\frac{19}{3}\sin\theta + \frac{195}{4}\sin^2\theta\, d\theta$$

$$= \cancel{\left[\frac{19}{3}\cos\theta\right]_0^{2\pi}} + \frac{195}{4}\cdot\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi} = \frac{195}{4}\cdot\frac{2\pi}{\cancel{2}} = \boxed{\frac{195\pi}{4}}$$

h) $\oint_C (2x-y)\,dx + (x+3y)\,dy$ $\qquad$ C: pentágono de vértices: (0,0), (0,2), (1,3), (2,2), (2,0)



• $F_1(x,y) = 2x - y$ $\qquad \dfrac{\partial F_1}{\partial y} = -1$

• $F_2(x,y) = x + 3y$ $\qquad \dfrac{\partial F_2}{\partial x} = 1$

$$\iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dx\, dy = \iint_D 1 - (-1)\; dx\, dy = 2 . \iint_D dx\, dy =$$

$$= 2 . \text{ área de } D = 2 . \left(\underbrace{\frac{2.1}{2}}_{A_\triangle} + \underbrace{2.2}_{A_\square} \right) = 2.5 = \boxed{10}$$

② $\oint_C (a_1 x + a_2 y + a_3)\, dx + (b_1 x + b_2 y + b_3)\, dy = (b_1 - a_2) . A$

$F_1(x,y) = a_1 x + a_2 y + a_3 \qquad \frac{\partial F_1}{\partial y} = a_2$

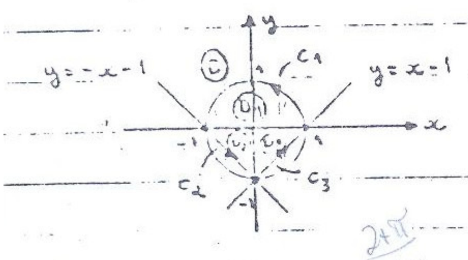
$F_2(x,y) = b_1 x + b_2 y + b_3 \qquad \frac{\partial F_2}{\partial x} = b_1$

$$\iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dx\, dy = \iint_D b_1 - a_2 \; dx\, dy =$$ sendo b_1 e a_2 = cte

$$= (b_1 - a_2) \iint_D dx\, dy = \boxed{(b_1 - a_2) . A(D)}$$ c.q.d.

③ $\oint_C (x^2 + y + 2xy^3)\, dx + (5x + 3x^2y^2 + y)\, dy \qquad C = C_1 \cup C_2 \cup C_3$

$$\begin{cases} C_1: x^2 + y^2 = 1 , & y \geq 0 \\ C_2: x + y + 1 = 0 , & -1 \leq x \leq 0 \qquad y = -x - 1 \\ C_3: x - y - 1 = 0 , & 0 \leq x \leq 1 \qquad y = x - 1 \end{cases}$$



$\frac{2+\pi}{2}$

• $F_1(x,y) = x^2 + y + 2xy^3$

$\frac{\partial F_1}{\partial y} = 1 + 6xy^2$

• $F_2(x,y) = 5x + 3x^2y^2 + y$

$\frac{\partial F_2}{\partial x} = 5 + 6xy^2$

$$\int_C F \cdot dr = \int_{C_1} F \cdot dr + \int_{C_2} F \cdot dr + \int_{C_3} F \cdot dr$$

$$\int_C F \cdot dr = \iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, dx\,dy = \iint_D 5 + \cancel{6xy^2} - 1 - \cancel{6xy^2} \, dx\,dy =$$

$$= \iint_D 4 \, dx\,dy = 4 \cdot \iint_D dx\,dy$$

$\underline{D_1}$: $x^2 + y^2 = 1$ $\quad \begin{cases} x = r\cos\theta & 0 \le r \le 1 \\ y = r\sin\theta & 0 \le \theta \le \pi \end{cases}$

$$\int_{C_1} F \cdot dr = 4 \cdot \int_0^{\pi} \int_0^1 r \, dr\,d\theta = 4 \cdot \int_0^{\pi} \left[\frac{r^2}{2} \right]_0^1 d\theta =$$

$$= 4 \cdot \int_0^{\pi} \frac{1}{2} \, d\theta = \frac{\cancel{4}^{2} \cdot 1}{\cancel{2}_{1}} \left[\theta \right]_0^{\pi} = 2\pi$$

$\underline{D_2}$: $-1 \le x \le 0 \qquad -x-1 \le y \le 0$

$$\int_{C_2} F \cdot dr = 4 \cdot \int_{-1}^{0} \int_{-x-1}^{0} dy\,dx = 4 \cdot \int_{-1}^{0} x + 1 \, dx =$$

$$= 4 \cdot \left[\frac{x^2}{2} + x \right]_{-1}^{0} = 4 \cdot \left(-\frac{1}{2} + 1 \right) = \frac{\cancel{4}^{2} \cdot 1}{\cancel{2}_{1}} = 2$$

$\underline{D_3}$: $0 \le x \le 1 \qquad x-1 \le y \le 0$

$$\int_{C_3} F \cdot dr = 4 \int_0^1 \int_{x-1}^{0} dy\,dx = 4 \cdot \int_0^1 -x + 1 \, dx = 4 \cdot \left[-\frac{x^2}{2} + x \right]_0^1 =$$

$$= 4 \cdot \left(-\frac{1}{2} + 1 \right) = \frac{\cancel{4}^{2} \cdot 1}{\cancel{2}_{1}} = 2$$

$$\int_C F \cdot dr = 2\pi + 2 + 2 = \boxed{2\pi + 4}$$ no sentido anti-horário

④ a) $\text{área}(D) = \oint_{\partial D} x \, dy$

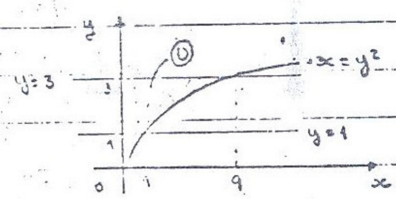
$F_1(x,y) = 0 \qquad \dfrac{\partial F_1}{\partial y} = 0$

$F_2(x,y) = x \qquad \dfrac{\partial F_2}{\partial x} = 1$

$$\oint_{\partial D} x \, dy = \iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \, dx\,dy = \iint_D 1 - 0 \, dx\,dy = \iint_D dx\,dy =$$

área (D), c.q.d.

b) C: $y = 1$, $y = 3$, $x = y^2$ e eixo y



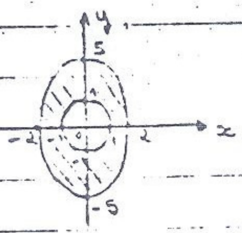
$1 \leq y \leq 3$

$0 \leq x \leq y^2$

$$A = \iint_D dx\,dy = \int_1^3 \int_0^{y^2} dx\,dy = \int_1^3 y^2 \, dy = \left[\frac{y^3}{3}\right]_1^3 = \frac{27}{3} - \frac{1}{3} = \boxed{\frac{26}{3}}$$

⑤ $\dfrac{\partial F_2}{\partial x}(x,y) = \dfrac{\partial F_1}{\partial y}(x,y) + 4 \qquad \forall (x,y) \neq (0,0)$

- $\oint_\gamma F_1 \, dx + F_2 \, dy = 6\pi \qquad \gamma: x^2 + y^2 = 1$ sentido anti-horário
- $\oint_C F_1 \, dx + F_2 \, dy = ?$ $\qquad C: \dfrac{x^2}{4} + \dfrac{y^2}{25} = 1$ sentido anti-horário



$$\frac{\partial F_2}{\partial x}(x,y) - \frac{\partial F_1}{\partial y}(x,y) = 4$$

D: A elipse − A circunf.

$$\iint_D \underbrace{\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}}_{=4} \, dx\,dy = \oint_C F_1 dx + F_2 dy - \oint_\gamma F_1 dx + F_2 dy$$

$= 6\pi$

$$\oint_C F_1 dx + F_2 dy = \underbrace{\iint_D 4\,dx\,dy}_{\circledast_1} + \overbrace{\oint_\gamma F_1 dx + F_2 dy}^{=6\pi} \quad (**)$$

$$\circledast_1 \quad 4 \cdot \iint_D dx\,dy = 4 \cdot \text{área}(D) = 4 \cdot 9\pi = 36\pi$$

$$\text{área}(D) = \underbrace{\text{área elipse}}_{\circledast_2} - \overbrace{\text{área circunf.}}^{=\pi \cdot r^2 = \pi} = 10\pi - \pi = 9\pi$$

elipse: $\begin{cases} x = 2r\cos\theta \\ y = 5r\sin\theta \end{cases}$ $\quad \dfrac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} 2\cos\theta & -2r\sin\theta \\ 5\sin\theta & 5r\cos\theta \end{vmatrix} =$

$0 \le r \le 1 \quad 0 \le \theta \le 2\pi$ $\quad = 10r\cos^2\theta + 10r\sin^2\theta = 10r$

$$\circledast_2 \int_0^{2\pi}\int_0^1 10r\,dr\,d\theta = \int_0^{2\pi} \left[\frac{10r^2}{2}\right]_0^1 d\theta = \int_0^{2\pi} 5\,d\theta = 10\pi$$ área da elipse

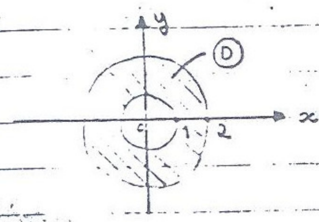
$$(**) \quad \oint_C F_1 dx + F_2 dy = 36\pi + 6\pi = \boxed{42\pi}$$

(6) $F(x,y) = \left(\dfrac{-y}{x^2+y^2}, \dfrac{x}{x^2+y^2} + 3x \right)$

a) $C_1: \; x^2 + y^2 = 4$ sentido anti-horário

• $F_1(x,y) = \dfrac{-y}{x^2+y^2}$ $\qquad \dfrac{\partial F_1}{\partial y} = \dfrac{\overbrace{-1.(x^2+y^2)+y.2y}^{=-x^2-y^2+2y^2}}{(x^2+y^2)^2} = \dfrac{-x^2+y^2}{(x^2+y^2)^2}$

• $F_2(x,y) = \dfrac{x}{x^2+y^2} + 3x$ $\qquad \dfrac{\partial F_2}{\partial x} = \dfrac{\overbrace{1.(x^2+y^2)-x.2x}^{=x^2+y^2-2x^2}}{(x^2+y^2)^2} + 3 = \dfrac{-x^2+y^2}{(x^2+y^2)^2} + 3$



$$\text{área}(D) = \pi.2^2 - \pi.1^2 = 4\pi - \pi = 3\pi$$

$\gamma: x^2+y^2=1 \qquad C_1: x^2+y^2=4$

$$\iint_D \overbrace{\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}}^{=3}\, dx\,dy = \oint_{C_1} F.dr - \oint_\gamma F.dr$$

$$\oint_{C_1} F.dr = \underbrace{\iint_D 3\, dx\,dy}_{=3.\,\text{área}(D) = 9\pi} + \oint_\gamma F.dr \quad \circledast \qquad (**)$$

$\circledast \quad \oint_\gamma F.dr = \qquad \gamma: x^2+y^2=1 \qquad \sigma(t) = (\cos t, \sin t)$

$\sigma'(t) = (-\sin t, \cos t)$

$0 \leq t \leq 2\pi$

$$= \int_0^{2\pi} F(\sigma(t)).\sigma'(t)\, dt = \qquad F(\sigma(t)) = \left(\frac{-\sin t}{\sin^2 t+\cos^2 t}, \frac{\cos t}{\sin^2 t+\cos^2 t} + 3\cos t\right)$$

$F(\sigma(t)) = (-\sin t, \cos t + 3\cos t)$

$$= \int_0^{2\pi} \underbrace{(-\sin t, \cos t + 3\cos t)}_{F(\sigma(t))} . \underbrace{(-\sin t, \cos t)}_{\sigma'(t)}\, dt =$$

$$= \int_0^{2\pi} (-\sin t).(-\sin t) + \cos t.\cos t + 3\cos t\,\cos t\, dt =$$

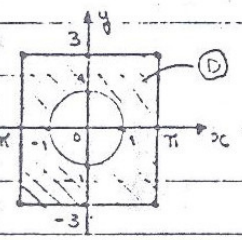
$$= \int_0^{2\pi} \underbrace{\operatorname{sen}^2 t}_{=1-\cos^2 t} + 4\cos^2 t \, dt = \int_0^{2\pi} 1 + 3\cos^2 t \, dt =$$

$$= [\theta]_0^{2\pi} + \frac{3}{2}\int_0^{2\pi} 1 + \cos 2\theta \, dt =$$

$$= 2\pi + \frac{3}{2}\left[\theta + \frac{\operatorname{sen} 2\theta}{2}\right]_0^{2\pi} = 2\pi + 3\pi = 5\pi$$

(**) $\oint_C F \cdot dr = 9\pi + 5\pi = \boxed{14\pi_{//}}$

b) $C_2: \; R = \{(x,y) \in \mathbb{R}^2 \mid -\pi \leq x \leq \pi, \; -3 \leq y \leq 3\}$



$$\text{área}(D) = 2\pi \cdot 6 - \pi \cdot 1^2$$
$$= 12\pi - \pi = 11\pi$$

$\gamma: x^2 + y^2 = 1$ $\qquad$ C_2: retângulo

$$\iint_D \overbrace{\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}}^{=3} dx\,dy = \oint_{C_2} F \cdot dr - \oint_\gamma F \cdot dr$$

$$\oint_{C_2} F \cdot dr = \underbrace{3 \iint_D dx\,dy}_{=3.\,\text{área}(D) = 33\pi} + \overbrace{\oint_\gamma F \cdot dr}^{=5\pi} = 33\pi + 5\pi = \boxed{38\pi_{//}}$$