Statistics 568 Bayesian Analysis Spring 2021

Homework 6

Due: Wed 03/03/21 @ 11:59pm rutgers.instructure.com/courses/120689

Problem 1. Rejection sampling for Gaussian tail. Consider rejection sampling from the (unormalized) target distribution

$$q(\theta) = \phi(\theta)\,\mathbf{1}\{\theta > c\},$$

where ϕ is the standard normal density, and $c > 0$. Consider using an Exponential distribution with parameter λ as the proposal distribution.

1. Derive that the smallest covering constant needed for the $Exp(\lambda)$ proposal is

$$M^* = \frac{\exp\left\{\left(\lambda^2 - 2\lambda c\right)/2\right\}}{\sqrt{2\pi}\lambda\left(1 - \Phi(c)\right)}.$$

2. Prove that using the $Exp(\lambda)$ proposal with the above covering constant M^*, the best acceptance rate of the rejection sampler is $1/M^*$.

3. Show that the best choice of λ that meets the above acceptance rate is

$$\lambda^* = \left(c + \sqrt{c^2+4}\right)/2.$$

4. Graphically depict the target distribution and their corresponding best proposal distribution, properly scaled by the corresponding M^* values, for $c = 0, 1, 2$ (that is, you will produce three plots, compactly displayed). Report the best acceptance rates in these three situations.

Note. You may either think of the target distribution as $N(0,1)$ truncated on $[c,\infty)$, and cover it with the scaled $Exp(\lambda)$ shifted to the right by c. Alternatively, you can also think of the target as the $N(-c,1)$ distribution truncated on $[0,\infty)$, and cover it with the scaled $Exp(\lambda)$ starting at zero. These constitute identical sampling tasks. The math of the latter may be easier.

Problem 2. BDA Chapter 10, Exercise 6.

Problem 3. BDA Chapter 10, Exercise 7.