Statistics 568 Bayesian Analysis

Spring 2021

Homework 3

Due: Wed 02/10/21 @ 11:59pm rutgers.instructure.com/courses/120689

Problem 1. BDA Chapter 3, Exercise 11.

Problem 2. BDA Chapter 3, Exercise 14.

Problem 3. Suppose that (y_1, \dots, y_5) are independent samples from a Cauchy distribution with unknown center θ and known scale 1:

$$p(y_i | \theta) \propto 1/(1 + (y_i - \theta)^2).$$

Assume that the prior distribution for θ is uniform on [0,1]. Given the observations $(y_1, \dots, y_5) = (-2, -1, 0, 1.5, 2.5)$:

- 1. Plot the posterior density (you can use the curve () function in R).
- 2. Determine the derivative and the second derivative of the log posterior density.
- 3. Find the posterior mode of θ by iteratively solving the equation determined by setting the derivative of the log-likelihood to zero.
- 4. Construct the normal approximation based on the second derivative of the log posterior density at the mode. Plot the approximate normal density and compare to the exact density computed in the first part.