# Dependence of Exchange Rate Volatility on Commodity Prices
# Example of Russian Ruble

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**Abstract**

This paper examines the relationship between volatility of of the Russian Ruble / US Dollar exchange rate and volatility of the Brent Oil price. For this we use a classic GARCH model, a tree-GARCH model and a DCC-GARCH model. We also test for structural breaks. We show that at times of decreasing oil price returns, the volatility of the Russian Ruble / US Dollar exchange rate is less persistent and the conditional correlation between Russian Ruble / US Dollar exchange rate returns and Brent oil price returns is more persistent than in normal market conditions.

# Contents

# 1 Introduction

Exchange rate levels tend to be associated with the price of goods and services a country exports. In the case of Russia, which relies heavily on the export of oil, with over 50% of Russias export being oil or gas products. Oil as a non-renewable commodity plays an important role in Russias economy. Over the last 20 year the oil price fluctuated a lot, depending on the demand as well as on the supply of oil by the main oil producing countries. Russia and other oil reliant economies try to stabilizes their governments budgets by increasing their oil exports. Russia furthermore suffered from several crisis over the last two decades, political and economic.

In case of an increase in the oil price the currency of an oil-dependent country like Russia, will appreciate, and vice versa. Since prices are difficult to predict, it is usually easier to predict volatilities. A similar relationship could therefore be present in the second order moments. Therefore, we investigate the relationship between volatility of the Russian Ruble / US-Dollar exchange rate and the volatility of the Brent Oil price.

We inspect the relationship by firstly estimating GARCH-models for oil and the RUB/USD. We build a Tree-GARCH for the RUB/USD splitting the sample with the oil returns and other economic variables into different regimes Audrino and Bühlmann (2001). The goal is to estimate the dependence between oil prices and RUB/USD on the univariate level. In the last step we estimate a DCC-GARCH based on the Tree-GARCH Audrino and Trojani (2011) to investigate the dependence of RUB/USD and oil prices.

In section 2 we describe the time series data that we used in our analysis. In section 3 we talk about the methods implemented. In particular, subsection 3.1 describes the implementation of ARIMA models and choosing the best specification; subsection 3.2 contains estimation and selection of the univariate GARCH - a classic GARCH model as well as a tree-GARCH model; subsection 3.3 discusses the DCC-GARCH model. In section 4 we present the results and make a conclusion.

# 2 Data

For the analysis we used daily RUB/USD exchange rates provided by the Central Bank of Russia and Brent oil prices from the Federal Reserve Bank of St Louis. For splitting of the Tree-GARCH, Russian government bond yields, US-Treasuries (10-Year and 3-Month), Russian riskfree rates, Russian government bond zero coupon yield curve, RUB/EUR exchange rate, SP500 and MOEX indices were used. All datasets are from the 07.01.2000 until the 01.05.2020. The exchange rates as well the oil-prices were transformed to daily log-returns. Following the conventions, data during weekends is not included. Due to bank holidays, the data is incomplete on a few days per year. These days were excluded as well. Due to severe outliers in the oil and RUB/USD time series (as shown in figure 1 and 2) we excluded the first percentile and the 99th. percentile. The subsequent figure 1 and 2 visualize the auto-correlation-function for the log returns of oil and RUB/USD series. There are some violations of the bounds for the RUB/USD series of log returns. In the next Chapter we conclude that there is a MA(1) process for the RUB/USD log returns.

| Data Description | Source |
|---|---|
| Brent Oil Prices | Federal Reserve Bank of St. Louis |
| RUB/USD FX | Central Bank of Russia |
| Long-Term Government Bond Yields for Russia | Federal Reserve Bank of St. Louis |
| 10-Year Treasury Constant Maturity Rate | Federal Reserve Bank of St. Louis |
| 3-Month Treasury Constant Maturity Rate | Federal Reserve Bank of St. Louis |
| Riskfree rates Russian Federation | Bank of Russia |
| Russian Government Bond Zero Coupon Yield Curve | Bank of Russia |
| RUB/EUR FX | Central Bank of Russia |
| SP500 | Thomson-Reuters Datastream |
| MOEX | Thomson-Reuters Datastream |

Table 1: Data Sources

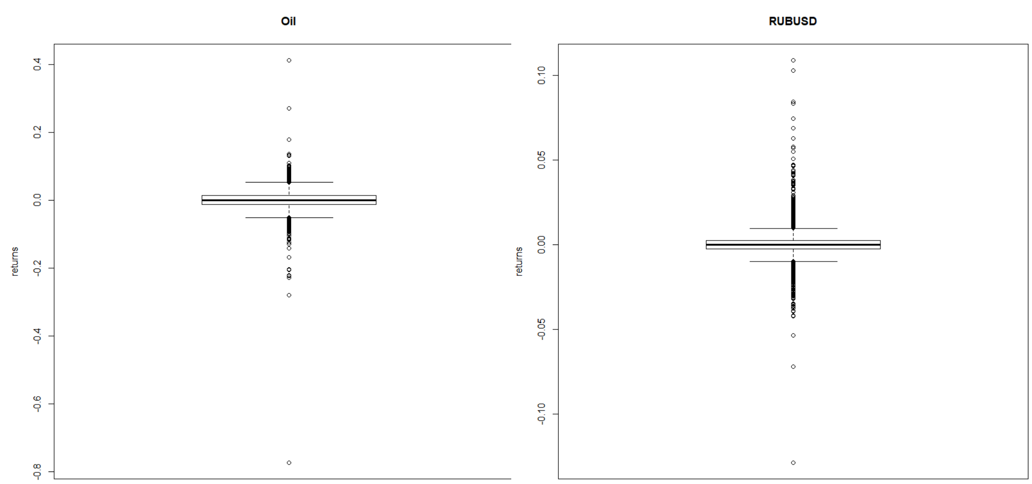


Figure 1: Boxplots of oil (left) and RUB/USD (right) returns

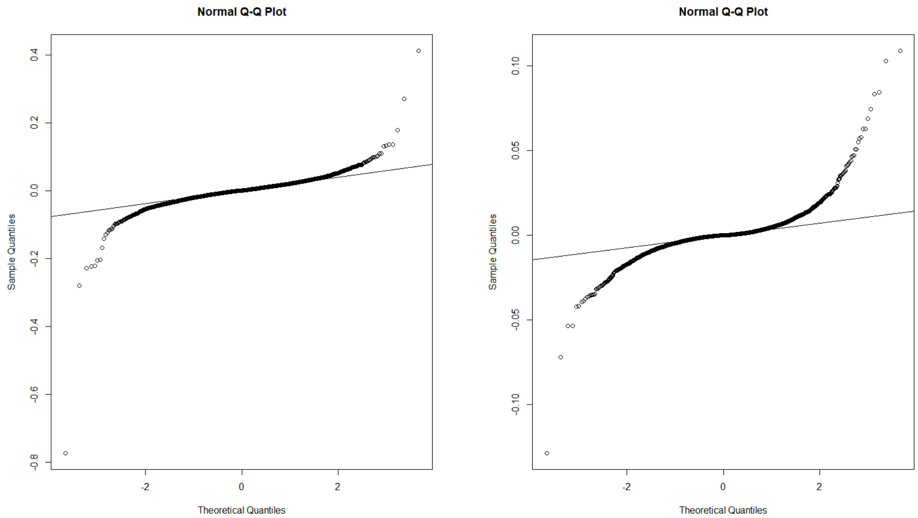


Figure 2: QQ-Plots of oil (left) and RUB/USD (right) returns

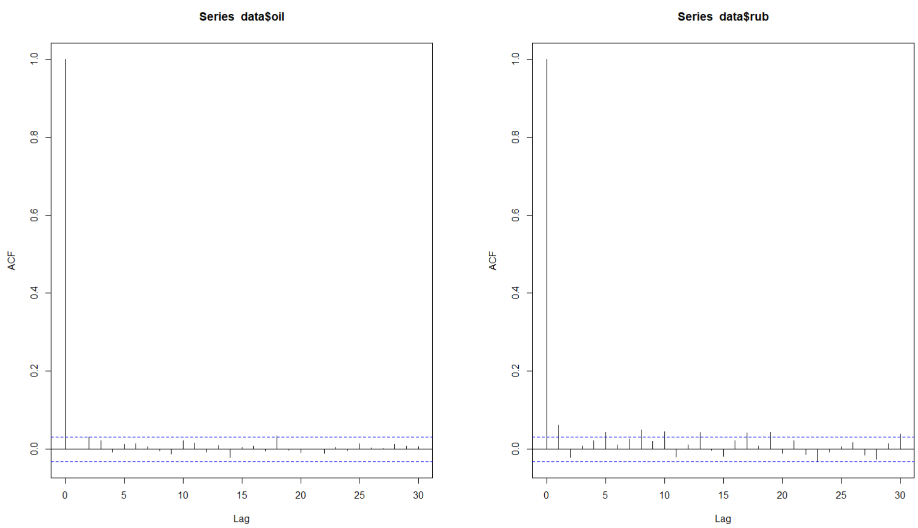


Figure 3: ACF plots of series of oil (left) and RUB/USD (right) returns

# 3 Model Estimation

In the following chapter we first estimate ARIMA models that fit the Oil and RUB/USD series the best. In the second step we use the resulting errors form the ARIMA to fit several possible univariate GARCH-models to the two time series. There we also estimate a tree-GARCH, which is splitting the RUB/USD series based on the variables introduced before. We estimate DCC-models based on the different univariate GARCH-models for RUB/USD and Oil.

## 3.1 ARIMA

An ARIMA$(p, d, q)$ model is defined as:

$$\Delta^d y_t = c + \sum_{i=1}^{p} \alpha_i \Delta^d y_{t-i} + \sum_{j=1}^{q} \beta_j \epsilon_{t-j}$$

, where $\epsilon_t$ is a linear innovation process of $y_t$. $\Delta^d$ defines the order of differencing that is needed to make the $\{y_t\}$ series stationary, where $\Delta^n = \Delta\left(\Delta^{n-1}\right)$. That is, $\Delta^1 = y_t - y_{t-1}$, $\Delta^2 = (y_t - y_{t-1}) - (y_{t-1} - y_{t-2}) = y_t - 2y_{t-1} + y_{t-2}$ and so on.

Fitting of an ARIMA model starts with estimating the value $d$ first to determine whether the series is already stationary or needs to be differenced first. This can be done by performing a Dickey-Fuller test for unit root on the series of interest.

To perform a Dickey-Fuller test, a regression

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \epsilon_t$$

is fit and a null hypothesis of $\alpha_1 = 1$ (unit root) is tested against $\alpha_1 < 1$ (stationarity). Then, the test statistic is calculated as $DF = \dfrac{\hat{\alpha_1} - 1}{s.e.(\hat{\alpha_1})}$, which has its own Dickey-Fuller critical values.

| | $\hat{\alpha_1}$ | s.e.$(\hat{\alpha_1})$ | $DF$ | 1% crit.value | result |
|---|---|---|---|---|---|
| Oil | 0.00065 | 0.01623 | $-61.574$ | $-3.434$ | reject $H_0$ |
| RUB/USD | 0.06119 | 0.01610 | $-57.952$ | | reject $H_0$ |

Table 2: Dickey-Fuller test results for oil and RUB/USD series

For both series the D-F test yielded a $d = 0$ on a 1% significance level, which is in line with our expectations, since the series are already return series.

To get the coefficient estimates of a given ARIMA model, the following log-likelihood function needs to be maximized:

$$LL(\theta) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{n}\epsilon_t^2$$

, where $\theta$ is a set of parameters to be estimated and $\epsilon_t$ is an error term obtained from an ARIMA model. In particular, $\epsilon_1 = y_1 - c$. Our ARIMA function has the following procedure: if both $p \leq 1$ and $q \leq 1$, then $\epsilon_t = y_t - c - \sum_{i=1}^{p} \alpha_i y_{t-i} - \sum_{j=1}^{q} \beta_j \epsilon_{t-j}, \forall t > 1$. If $p > 1$, $q > 1$ and $p < q$, then

$$\epsilon_t = \begin{cases} y_t - c - \sum_{i=1}^{t-1} \alpha_i y_{t-i} - \sum_{j=1}^{t-1} \beta_j \epsilon_{t-j}, \forall t \in [2, p] \\ y_t - c - \sum_{i=1}^{p} \alpha_i y_{t-i} - \sum_{j=1}^{t-1} \beta_j \epsilon_{t-j}, \forall t \in [p+1, q] \\ y_t - c - \sum_{i=1}^{p} \alpha_i y_{t-i} - \sum_{j=1}^{q} \beta_j \epsilon_{t-j}, \forall t > q \end{cases}$$

For all other possible cases the specification is respectively similar.

After maximizing the log-likelihood for a given set of parameters $(p, d, q)$, the Bayesian Information Criterion is calculated as

$$BIC = -\frac{2}{T} LL(\theta) + \frac{p+q+1}{T} \log(T)$$

This procedure of estimating an ARIMA model and calculating a BIC for it is repeated for a grid of different values of $p$ and $q$ (in particular, values of $0, 1, 2, 3$ for each were tested). The resulting models are compared based on their BIC value and a model with the lowest value of BIC is chosen as an optimal.

| $d = 0$ | | p | | | |
|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 |
| q | 0 | $-18270.72$ | $-18262.47$ | $-17444.25$ | $-17884.21$ |
| | 1 | $-18262.48$ | $-18254.27$ | $-17737.94$ | $-17769.22$ |
| | 2 | $-17071.03$ | $-17614.50$ | $-17656.06$ | $-17810.88$ |
| | 3 | $-17877.88$ | $-17604.96$ | $-17790.65$ | $-16753.55$ |

Table 3: BIC values of different ARIMA models for oil time series

| $d = 0$ | | p | | | |
|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 |
| q | 0 | $-27665.10$ | $-27670.46$ | $-26692.44$ | $-26728.22$ |
| | 1 | $-27671.18$ | $-27561.24$ | $-27006.11$ | $-27114.11$ |
| | 2 | $-26497.54$ | $-26958.33$ | $-26852.31$ | $-27182.03$ |
| | 3 | $-27054.68$ | $-27003.20$ | $-27089.60$ | $-26733.96$ |

Table 4: BIC values of different ARIMA models for RUB/USD time series

Tables 3 and 4 display the BIC values of all the 16 ARIMA specifications tried for both series. Therefore, for the oil series, a model with lowest BIC

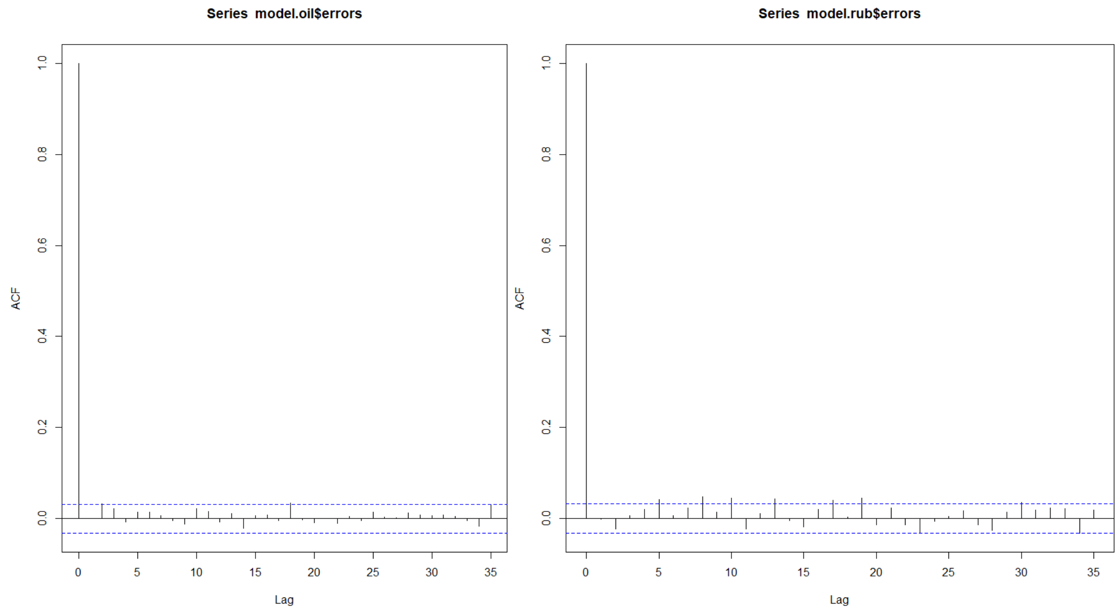


Figure 4: ACF plots of the ARIMA error series of oil (left) and RUB/USD (right) returns

appeared to be ARIMA$(0, 0, 0)$, while for the RUB/USD series that was an ARIMA$(0, 0, 1)$.

The resulting models specifications are presented in table 5.

| | $p$ | $d$ | $q$ | $c$ | $\beta_1$ | $LL$ | BIC |
|---|---|---|---|---|---|---|---|
| Oil | 0 | 0 | 0 | 0.00028 | $--$ | 9139.479 | $-18270.72$ |
| RUB/USD | 0 | 0 | 1 | 0.00011678 | 0.06285 | 13843.83 | $-27671.18$ |

Table 5: Selected ARIMA specifications

### 3.1.1 Structural breaks

The selected ARIMA models were tested for presence of structural breaks. For each quarter of a year, the returns dataset was split into two subsamples - before the first day of that quarter and after. Then, for each of those splits, on each of the resulting subsamples an ARIMA model was estimated using the procedure described in the previous section. The two resulting models are then compared to the initial model estimated on the whole data period via an LR test. In the LR test, we take as a restricted model the same model for the entire period, and as an unrestricted model two different models for each of the subsamples.

In particular, the unrestricted model is then represented as follows:

$$y_t = \begin{cases} c_1 + \sum_{i=1}^{p} \alpha_{1i} y_{t-i} + \sum_{j=1}^{q} \beta_{1j} \epsilon_{t-j}, & \text{if } t < t^* \\ c_2 + \sum_{i=1}^{p} \alpha_{2i} y_{t-i} + \sum_{j=1}^{q} \beta_{2j} \epsilon_{t-j}, & \text{if } t \geq t^* \end{cases}$$

, where $t^*$ is the time point of split. The LR test statistic is then calculated as

$$LR = -2(LL_{UR} - LL_R) \sim \chi^2(n)$$

, where $LL_{UR}$ is the maximized log-likelihood of the unrestricted model, $LL_R$ is the maximized log-likelihood of the restricted model, $n$ is the degrees of freedom equal to the number of restrictions. The structural breaks are identified iteratively, i.e. if a structural break is found, then the identical test is run on the two identified sub-subsamples until no more significant structural breaks are identified.

For the ARIMA model, no statistically significant structural brakes on 1% significance level were identified.

## 3.2 Univariate GARCH

In this section the squared residuals of the ARIMA processes are analysed and used to estimate the conditional variances for oil and RUB/USD series.

### 3.2.1 Structural Breaks

The auto-correlation plots for the squared residuals of the ARIMA in 5 show significant lags up to high orders, especially for RUB/USD. This either indicates a long-memory process or the presence of structural breaks.

As in section 3.1, possible structural breaks are iteratively identified via a likelihood ratio test using a GJR-GARCH(1,1) (because it is identified as best full-sample model in section 3.2.5). One structural break at the start of 2008 right before the financial crisis and one at the start of 2005 is identified. April 2014 is also indicated for oil, but not significant for RUB/USD at a 5% level[1]. For simplicity we therefore do not treat this as a structural break. Figure 6 depict the cumulative returns of the oil and RUB/USD series. From the plot the named above structural breaks for the oil series can also be seen visually; especially those in 2008 and in 2014 are highly pronounced.

The Lagrange multiplier test for conditional homoscedasticity is not rejected only soundly rejected for the third timeframe identified from 2008 to 2020. Before, no consistent evidence for conditional heteroskedasticity is found. Therefore, we exclude the time before 2008 from the following analysis. The optimal ARIMA-models are reestimated using only the timeframe from 2008 to 2020.

Table 6

[1] This indication could be connected to the implemented economic sanctions against Russia due to the Crimea.

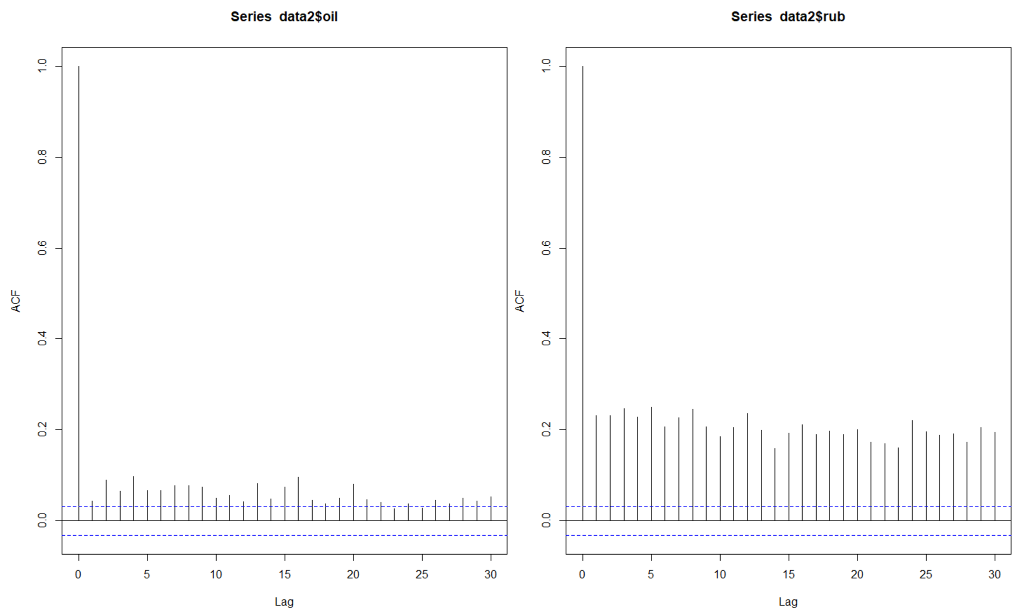


Figure 5: ACF plots of squared series of oil and RUB/USD returns

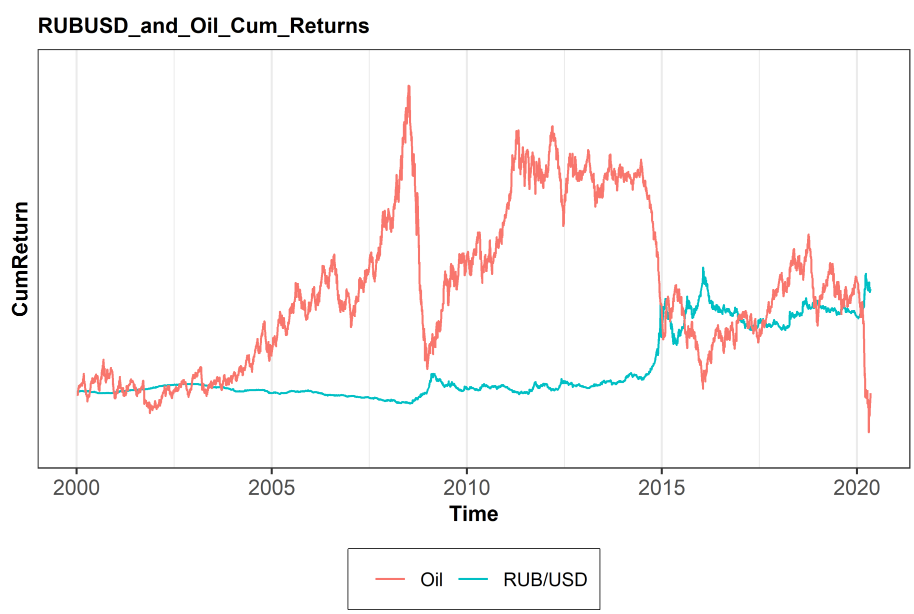


Figure 6: Cumulative returns of oil and RUB/USD series

| Timeframe | Asset | 1lag(s) | 5lag(s) | 10lag(s) | 20lag(s) | 40lag(s) |
|---|---|---|---|---|---|---|
| 2000-01-01/2004-12-31 | RUB/USD | 13.92* | 17.75* | 24.03* | 40.40 | 45.92 |
| 2000-01-01/2004-12-31 | oil | 0.02 | 5.09 | 6.12 | 15.14 | 29.18 |
| 2005-01-01/2007-12-31 | RUB/USD | 1.10 | 8.29 | 13.29 | 30.68 | 43.09 |
| 2005-01-01/2007-12-31 | oil | 1.14 | 4.23 | 6.15 | 12.80 | 33.46 |
| 2008-01-01/2020-04-01 | RUB/USD | 45.67* | 166.85* | 205.63* | 229.21* | 264.68* |
| 2008-01-01/2020-04-01 | oil | 13.23* | 98.76* | 137.94* | 185.84* | 199.77* |

Table 6: LM test for conditional homoscedasticity for identified timeframes. *denotes rejection of the null hypothesis at the 1% confidence level.

### 3.2.2 Asymmetries

In the following section we analyze another stylized fact, that is common in financial data, before estimating the GARCH-models: We investigate whether past positive returns have a different effect on the current volatility. In essence, we are interested to see if the volatility increases differently if the exchange rate of the Ruble appreciated or depreciated and the price of oil increased or decreased. To uncover the asymmetric leverage effect we estimated the auto-correlation of positive innovations and of negative innovations. The results are shown in table 7 for oil and in figure 8 for RUB/USD. We can infer from these results that the oil time series shows the characteristics of asymmetric leverage effects. As expected the RUB/USD time series does not show a significant asymmetric effect. This can be explained by the symmetry of FX markets.

| | 1lag(s) | 2lag(s) | 5lag(s) | 10lag(s) | 20lag(s) | 40lag(s) |
|---|---|---|---|---|---|---|
| corr $\epsilon^+$ | -0.054* | -0.002 | -0.022 | -0.015 | 0.005 | -0.021 |
| corr $\epsilon^-$ | 0.062* | 0.038 | 0.047* | 0.027 | 0.021 | -0.011 |

Table 7: Asymmetries for oil - auto-correlation of positive and negative returns. *denotes rejection of the null hypothesis at the 5% confidence level.

| | 1lag(s) | 2lag(s) | 5lag(s) | 10lag(s) | 20lag(s) | 40lag(s) |
|---|---|---|---|---|---|---|
| corr $\epsilon^+$ | 0.003 | 0.012 | 0.079* | 0.068* | 0.006 | -0.003 |
| corr $\epsilon^-$ | -0.013 | -0.058* | -0.012 | 0.001 | -0.037 | -0.011 |

Table 8: Asymmetries for RUB/USD - auto-correlation of positive and negative residuals ($\epsilon$). *denotes rejection of the null hypothesis at the 5% confidence level.

### 3.2.3 Classic GARCH

The conditional volatility is estimated with classic GARCH-models. For all upcoming models and their evaluation, the ARIMA-parameters are regarded as fixed. Due to an indication of fat tails for returns, we test 2 distributional assumptions: normal and student-t-distributed. A GJR-GARCH specification is tested to account for asymmetries.

To estimate the GARCH-parameters of a given specification, we maximize the log-likelihood for the assumed distribution. [2] [3] All possible model specifications with lags up to order 3 are estimated.

#### 3.2.4 Tree-GARCH

The Tree-GARCH method suggested by Audrino and Bühlmann (2001) uses a different GARCH-model for partitions of the data. The subsamples are determined by using observable (past) data as threshholds for partitioning the observations.

In this application, the Tree-GARCH offers the opportunity to investigate whether variables that are not part of the GARCH-models itself impact volatility of a series and its correlation structure (see subsection ??). Therefore we build a Tree-GARCH model for RUB/USD exchange rate using past returns, squared returns of RUB/USD and oil as well as Russian an US equity indices and government bond yields as splitting variables.

The partitioning algorithm implemented tests 8 splits for each splitting variable based on empirical quantiles and chooses the variable and threshhold value that maximizes the total log-likelihood improvement of the 2 submodels compared to the GARCH-model of the parent node. Instead of splitting until no improvement in the log-likelihood is reached, we use 3 splits to reduce the computational effort. The same model specification of a parsimonious GARCH(1,1) model with t-distributed innovations is used.

| | 1st split | threshhold | below/above | 2nd/3rd split | threshhold | below/above |
|---|---|---|---|---|---|---|
| regime 1 | return oil | -1.0801 | below | US-3M riskfree | 2.00 | below |
| regime 2 | return oil | -1.0801 | below | US-3M riskfree | 2.00 | above |
| regime 3 | return oil | -1.0801 | above | RUS-5y gov. bond | 7.480000 | below |
| regime 4 | return oil | -1.0801 | above | RUS-5y gov. bond | 7.48 | above |

Table 9: Splits tree before pruning. All variables are observations from the previous day and given in percent.

Table 9 shows that the past 25% quantile of oil return splits the volatility regime / GARCH model for RUB/USD. The second splits occur with respect to the level of interest rates. This preresult shows that exchange rate volatility is associated with past values of oil and interest rates. Since interest rate differences are a key driver of exchange rates (covered and uncovered interest rate parity), also the second split result is economically plausible.

[2] The log-likelihood loss function for t-distributed errors is adjusted to avoid convergence issues with low degrees of freedom and computational issues with with degrees of freedom.

[3] The sample variance is used as starting value.

To avoid overfitting, the tree is pruned by selecting the model with the minimum AIC. Here, the optimal subtree only uses the first split, resulting in one quarter of the observations in the first subsample with oil-returns lower than -1.08% and 3 quarters of the observations in the other subsample.

#### 3.2.5 Univariate Model Coefficients and Selection

Now the estimated GARCH-models for the full sample and the Tree-Garch models are compared.

| | model | $\alpha_1$ | $\beta_1$ | (d.f. of t)) |
|---|---|---|---|---|
| 1 | Garch(1,1) normal | 0.068 | 0.916 | / |
| 2 | Garch(1,1) t | 0.078 | 0.909 | 9.998 |
| 3 | GJR-Garch(1,1) t | 0.100 | 0.915 | 9.998 |
| 4 | Subsample 1 -Tree-Garch | 0.131 | 0.800 | 9.998 |
| 5 | Subsample 2 -Tree-Garch | 0.076 | 0.909 | 9.976 |

Table 10: Coefficient comparison - univariate GARCH-models for RUB/USD

Table 10 shows that the the selection of models for RUB/USD are all close to non-stationary (GJR-GARCH is stationary due to the threshhold coefficient). Comparing the Tree-GARCH submodels, the AR component of the negative oil return subsample 1 is lower indicating that volatility in RUB is less persistent in case of oil price drops. The t-distributions degrees of freedom does not differ in subsamples.

| | | Garch(1,1) normal | Garch(1,1) t | GJR-Garch(1,1) t | Tree-Garch(1,1) t |
|---|---|---|---|---|---|
| 1 | logLik | 7881.96 | 7910.92 | 7918.36 | 7937.27 |
| 2 | AIC | -15755.91 | -15811.84 | -15824.71 | -15854.54 |

Table 11: Likelihood comparison - univariate GARCH-models for RUB/USD

Table 11 shows the log-likelihoods and AICs for the estimated models. The models are evaluated using the AIC which penalizes for additional parameters - using the BIC did not change the model rankings. Changing only the distributional assumption improves the loglikehood by 65. Even if asymmetric effects were not as strong for RUB/USD, adding the threshhold still improves the model in terms of AIC. The Tree-GARCH model is optimal with respect to the AIC. Models with higher order lags for the full sample were consiered, but yielded worse AICs.

For the upcoming DCC analysis, the Tree-GARCH and the best full-sample model for RUB/USD and oil is used (both a GJR-GARCH(1,1).

To evaluate forecasting performance of the models, we conduct one-day-ahead forecasts in figures 7, 8, 9 compared to volatility proxys.

Basic Minzer-Zarnowitz regressions with robust standard errors of the squared daily returns on the volatility forecasts are reported in 12. The null hypothesis

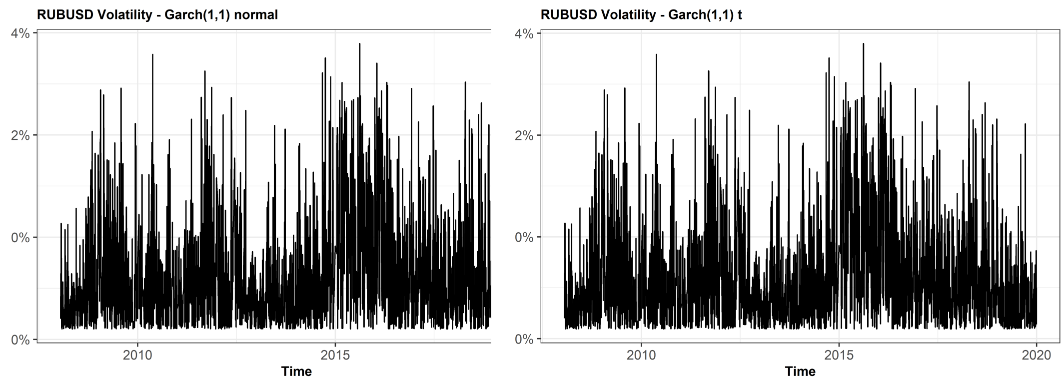


Figure 8: Forecasts for RUB/USD

of $\alpha = 0$ and $\beta = 1$ is rejected by all models, the coefficient of beta is not close to one. $R^2$ are almost 0. Most likely, these results come from a coding error and cannot be used to evaluate the models.

| | model | $\alpha$ | $\beta$ | R^2 MZ |
|---|---|---|---|---|
| 1 | Garch(1,1) normal | 0.000* | 0.145* | 0.021 |
| 2 | Garch(1,1) t | 0.000* | 0.145* | 0.021 |
| 3 | GJR-Garch(1,1) t | 0.000* | 0.143* | 0.020 |
| 4 | Tree-Garch(1,1) t | 0.000* | 0.172* | 0.027 |

Table 12: MZ - Regression with squared return proxy. *denote significant difference from 0 for $\alpha$ or 1 for $\beta$.

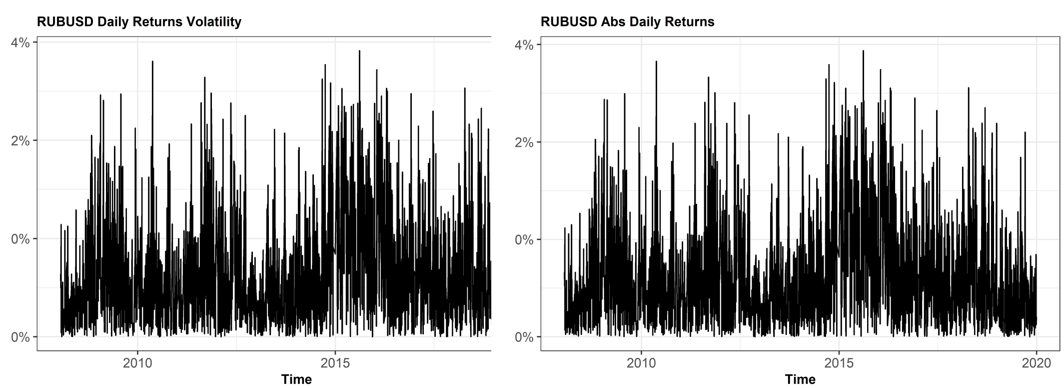


Figure 7: Volatility Proxys and Forecasts for RUB/USD

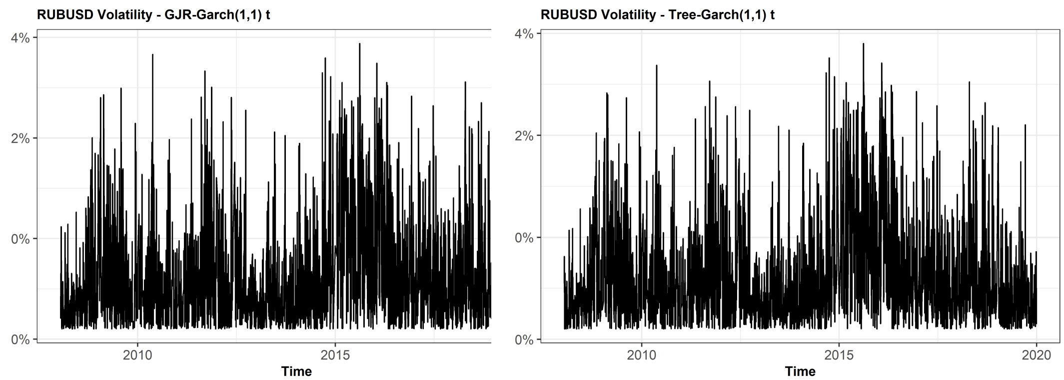


Figure 9: Forecasts for RUB/USD

### 3.3 DCC-GARCH

Since we are interested in the dependence of exchange rate volatility and the oil price, a multivariate GARCH-model would allow us to improve the predictions. We used the resulting GJR-GARCH-models for RUB/USD and Oil to estimate a dynamic conditional correlations GARCH (DCC-GARCH). The second model, we estimated, is a DCC-Tree-GARCH for the two subsamples that resulted from the splitting of the RUB/USD series. After estimating the univariate GARCH-parameters and the conditional volatilities we estimated the joint $\alpha 1$ and $\beta 1$ and the degree of freedom for the multivariate t-distribution. To estimate the GARCH-parameters of a given specification, we maximize the log-likelihood for the assumed multivariate t-distribution. For this analysis only DCC order of (1,1) is considered.
As shown in the following two tables, the AIC as well as the Likelihood is better in the DCC-Tree-GARCH. In table 10 the estimated joint parameters for the DCC-GARCH and the two subsamples are shown. Subsample 1 refers to a the subsample if the series if the oil returns fall below -1%. The joint GARCH parameters defer from Subsample 1 and Subsample 2.

| | DCC-GARCH(1,1) | DCC-Tree-GARCH(1,1) |
|---|---|---|
| logLik | 10714.13392 | 11045.85608 |
| AIC | -21402.26784 | -22065.71217 |

Table 13: Likelihood comparison - DCC-GARCH models

We apply the Diebold-Mariano and West (DMW) test for pair-wise comparison for the one-step-ahead in-sample prediction with MSE and MAE as loss function. We use the realized variance as proxy. Therefore, we can conclude that the DCC-Tree-Garch outperforms the DCC-GARCH for $\alpha = 0.05$, as the test statistic is larger than the critical value of 1.96.

Our DCC-Tree-GARCH gives two significantly different estimates for the

| | model | joint-$\alpha_1$ | joint-$\beta_1$ |
|---|---|---|---|
| 1 | DCC-GARCH(1,1) | 0.01645 | 0.26405 |
| 2 | Subsample 1 -Tree-Garch(1,1) | 0.00000 | 0.80647 |
| 3 | Subsample 2 -Tree-Garch(1,1) | 0.02288 | 0.27678 |

Table 14: Coefficient comparison - DCC-GARCH-models for RUB/USD and Oil

| | DCC-GARCH | DCC-Tree-GARCH | DMW | p |
|---|---|---|---|---|
| MSE | 0.1120 | 0.0437 | 48.0395 | 0.0000 |
| MEA | 0.0269 | 0.0061 | 72.0525 | 0.0000 |

Table 15: Diebold-Mariano and West (DMW) test for dcc-GARCH and dcc-Tree-GARCH

joint $\alpha_1$ and the joint $\beta_1$, depending on the subsample. The subsample with oil return lower the -1% has a $\alpha_1$ of 0 which turns the DCC-GARCH into CCC-GARCH. Furthermore the $\beta_1$ parameter is higher in subsample 1 in comparison to subsample 2. The interpretation is that in times of declining oil prices the moving-average component of the DCC diminishes and only the auto-regressive component stays.
Furthermore, comparing the DCC-Tree-GARCH to the standard DCC-GARCH we can conclude that the DCC-Tree-GARCH fits the data better than the standard DCC-GARCH, since it has a better AIC.

# 4 Conclusion

We analysed the dependence of volatility of the Ruble-USD on the oil price development. For univariate volatility of the exchange rate, the Tree-GARCH model used the past oil price level to partition the sample. The regime for lower oil prices has less persistence in volatility dynamics indicating more market uncertainty. The Tree-GARCH is the optimal model with respect to the AIC compared to other full-sample GARCHs tested. This indicates that volatility estimation of the RUB/USD can be improved by adding oil price information.

We show that at times of decreasing oil price returns, the conditional correlation between RUB/USD exchange rate returns and Brent oil price returns is stronger and persistent; the DCC model converges into a CCC model.

To conclude, the Ruble / USD exchange rate has a higher dependency on the oil price volatility for negative oil price movements than in normal market conditions.

# References


Audrino, F. and Bühlmann, P. (2001). Tree-structured generalized autoregressive conditional heteroscedastic models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 63(4):727–744.

Audrino, F. and Trojani, F. (2011). A general multivariate threshold garch model with dynamic conditional correlations. *Journal of Business & Economic Statistics*, 29(1):138–149.