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Exchange rates and oil prices: A multivariate stochastic volatility analysis

Liang Ding[a], Minh Vo[b,c,*]

[a] Macalester College, 1600 Grand Avenue, St. Paul, MN 555105, USA
[b] Metropolitan State University, 1501 Hennepin Avenue, Minneapolis, MN 55403, USA
[c] Deakin University, Burwood, VIC 3125, Australia

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ABSTRACT

This paper uses the multivariate stochastic volatility (MSV) and the multivariate GARCH (MGARCH) models to investigate the volatility interactions between the oil market and the foreign exchange (FX) market, in an attempt to extract information intertwined in the two for better volatility forecast. Our analysis takes into account structural breaks in the data. We find that when the markets are relatively calm (before the 2008 crisis), both oil and FX markets respond to shocks simultaneously and therefore no interaction is detected in daily data. However, during turbulent time, there is bi-directional volatility interaction between the two. In other words, innovations that hit one market also have some impact on the other at a later date and thus using such a dependence significantly improves the forecasting power of volatility models. The MSV models outperform others in fitting the data and forecasting exchange rate volatility. However, the MGARCH models do better job in forecasting oil volatility.

1. Introduction

It is well-known that oil prices and the U.S. dollar exchange rates are highly correlated. Given the fact that oil is quoted in U.S. dollars, it is natural to hypothesize that exchange rates drive oil prices. More specifically, other things equal, when the U.S. dollar depreciates, oil-exporting countries would raise oil prices in order to stabilize the purchasing power of their (U.S. dollar) export revenues in terms of their (predominately) euro-denominated imports. This is equivalent to a reduction in supply or a leftward shift in the supply curve. On the demand side, the U.S. dollar depreciation makes oil less expensive for consumers in other countries (in local currency), thereby increasing their crude oil demand. Both effects, the reduction in supply and the increase in demand, cause an increase in oil prices denominated in U.S. dollars. The exchange-rate-to-oil-price causality relationship is supported by the empirical evidence found in Zhang, Fan, Tsai, and Wei (2008), Krichene (2005), and Yousefi and Wirjanto (2004).

From the other perspective, exchange rates are believed to be determined by expected future fundamental conditions, among which oil is surely an important factor. Increasing oil prices lead to stronger economies for oil-exporters and higher production costs for oil-importers, hence it would cause the appreciation of oil-exporter currencies relative to those of oil-importers. So, it is likely that the causality runs from oil prices to the exchange rate. Benassy-Querea, Mignonb, and Penot (2007), Coudert, Mignon, and Penot (2007), Chen and Chen (2007), Ayadi (2005), Chaudhuri and Daniel (1998) and Krugman (1984) all provide evidence supporting this view.

These studies, despite their mixed implications, tend to suggest that oil prices and exchange rates probably both contain information that can affect each other. Accordingly, Chen, Rogoff, and Rossi (2008) and Groen and Pesenti (2010) use exchange rates to obtain a better forecast of oil (and other commodities) prices, while Amano and Norden (1998) improve the exchange rate forecast by including oil price in the model.

The current literature that examines the relationship between oil price and exchange rate, as cited above, mainly focuses on their returns. As noted by Clark (1973), Tauchen and Pitts (1983), and Ross (1989), the volatility of an asset is also related to the rate of information flow across interacted markets. So the link between the oil and the FX markets should appear not only in return but also in volatility. Examining the volatility interaction between exchange rates and oil prices can shed light on the direction of the causality relationship from a new perspective. Furthermore, if a significant connection does exist, extracting and using the information intertwined in both markets would improve forecasts of exchange rate and oil volatilities, which are critical in many areas of modern finance.

Instead of focusing on the relationship between exchange rate and oil returns, which many papers cited above have investigated,

* Corresponding author at: Metropolitan State University, 1501 Hennepin Avenue, Minneapolis, MN 55403, USA. Tel.: +1 612 659 7305; fax: +1 612 659 7268.
E-mail address: minh.vo@metrostate.edu (M. Vo).

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this paper first examines how volatilities of oil and FX markets interact. Second, the paper attempts to extract information intertwined in the two markets, if detected, for a better forecast of the exchange rate and oil volatilities.

Using the optimal lag-length algorithm with the Bayesian information criterion (BIC), we posit a bivariate model of vector of autoregression VAR(1) with stochastic volatility for the joint processes governing the returns of various exchange rates and the oil prices. We model the stochastic volatility using both the multivariate stochastic volatility (MSV) and the conditional correlation multivariate GARCH (CC-MGARCH) models. We fit two variants of the models: the constant conditional correlation and the dynamic conditional correlation to identify the better one for the data. For comprehensive discussion of the MSV and the CC-MGARCH, the reader is referred to Asai, McAleer, and Yu (2006) and Bauwens, Laurent, and Rombouts (2006). To estimate the MSV models, we use the Bayesian Markov chain Monte Carlo (MCMC) method by Jacquier, Polson, and Rossi (1994) and Kim, Shephard, and Chib (1998). For the CC-MGARCH models, we use the maximum likelihood methods proposed by Bollerslev (1990) and Engle (2002). To investigate whether structural breaks have occurred in the time series, we use the generalized M-fluctuation framework suggested by Kuan and Hornik (1995). Our tests indicate that there is a break in all variance series. Using the dynamic programming algorithm suggested by Bai and Perron (2003), we identify the break date around September 11, 2008 when Lehman Brothers was about to collapse. Thus, to account for structural break, we divide the data into two sub-samples with the break point and use each to fit the models under investigation.

Our empirical results show that during normal time (before the 2008 financial crisis,) markets are quite efficient. Shocks to one market are quickly transmitted and incorporated into the price in the other market. Therefore, we cannot detect the volatility interaction between the two markets in daily data. However, during the peak time of the financial crisis in 2008, it seems that information from one market is incorporated into the price in the other market more slowly. Therefore, we can detect the bidirectional interaction in volatility between the two markets in daily data during this period. Our result is consistent with the findings of Razgallah and Smimou (2011) that the interaction between the two markets is more significant during volatile periods. This nonlinearity of the relationship can be attributed to inefficient information incorporation during market chaos. During the periods when there is significant interaction between the markets, information in one market is useful in forecasting volatility in the other. As a result, we find that the multivariate models outperform the univariate counterparts in terms of forecasting. In addition, we show that the MSV models do a better job in forecasting FX volatility but the MGARCH models forecast better oil volatility.

The remainder of the paper is organized as follows. Section 2 describes the data set and conducts a preliminary analysis of the data. Section 3 discusses the models. Section 4 explains estimation methodology. Section 5 presents estimation results. Section 6 checks the models in terms of goodness-of-fit and evaluates their forecast power, and Section 7 concludes the paper.

2. Data and preliminary analysis

The data set used in this study consists of a daily oil spot price time series and nine daily time series of exchange rates against the U.S. dollar, including the Canadian dollar (CAD), the Norwegian krone (NOK), the euro (EUR), the Indian rupee (INR), the Japanese yen (JPY), the Singaporean dollar (SGD), the Brazilian real (BZR), the Mexican peso (MXP), and the U.S. dollar trade weighted index (USX). The Canadian dollar, Norwegian krone and Mexican peso are chosen to represent the exchange rates of oil exporting countries. The euro and the Japanese yen are to represent developed oil importers. The Indian rupee and Brazilian real are the representatives of developing oil consumers. The Singapore dollar can be considered as neutral currency that is neither major oil importer nor exporter. The dollar index provides more general sense to the results. Other major oil exporters (such as the Middle East countries) and importers (such as China) are not included because their exchange rates are either pegged to the U.S. dollar or strictly managed.

All series span from July 28, 2004 to October 28, 2009 for a total of 1312 observations. The sample from July 28, 2004 to September 28, 2009 is used for estimation while the rest is for out-of-sample forecast evaluation. The daily oil spot price time series is based on the West Texas Intermediate (WTI) crude oil. It is obtained from the historical database of the U.S. Department of Energy. The daily exchange rate data are extracted from the Federal Reserve Bank of St. Louis website. Despite their different quoting habits, all the exchange rates are converted to foreign currency per U.S. dollar. The daily return r_t, in percent, for each series p_t is approximated by $r_t = 100[\ln(p_t) - \ln(p_{t-1})]$.

Table 1 presents a wide range of descriptive statistics for all time series. All series have small mean. Their standard deviations are much greater than the means in absolute value, indicating that the means are not significantly different from zero. This is consistent with common knowledge that financial time series at this frequency usually follow a random walk. Except for USD/INR and USD/MXP, other exchange rate series have negative skewness. The excess kurtosis for each is significantly positive, indicating that they have heavy tails relative to the normal distribution, which is also typical in these financial data. Both Kolmogorov–Smirnov and Jarque–Bera tests reject the null hypothesis that the return distributions are normal at 5% level of significance. Ljung–Box

Table 1
Descriptive statistics for oil and exchange rate return time series.

Statistics	Oil	USD/CAD	USD/NOK	USD/EUR	USD/INR	USX	USD/YEN	USD/SGD	USD/BZR	USD/MXP
Mean (%)	.0468	−.0164	−.0164	.0154	.0014	−.0100	−.0151	−.0158	−.0424	.0108
Std. Dev.(%)	2.7988	.7292	.7076	.6543	.4939	.3516	.7147	.3405	1.1669	.7270
Skewness	−.0272	−.2161	−.1332	.3476	.1912	−.2926	−.7974	−.2447	−.0104	1.0156
Excess Kurtosis	4.2081	5.6297	4.8113	4.4708	10.6512	5.1187	5.3049	4.9432	12.9657	22.4909
Kolmogorov–Smirnov	.0664[*]	.0633[*]	.0691[*]	.0679[*]	.1135[*]	.0688[*]	.0625[*]	.063[*]	.0941[*]	.1208[*]
Jarque–Bera	973.30[*]	1750[*]	1275.64	1124[*]	6234[*]	1458[*]	1685[*]	1355[*]	9224[*]	27974[*]
ARCH(1)	74.37[*]	21.88[*]	16.90[*]	3.45	31.84[*]	29.04[*]	10.25[*]	6.23[*]	261.78[*]	438.90[*]
ARCH(6)	229.63[*]	137.62[*]	86.80[*]	118.39[*]	84.69[*]	211.45[*]	61.65[*]	59.10[*]	503.62[*]	553.37[*]
ARCH(12)	243.12[*]	230.55[*]	162.43[*]	166.47[*]	90.29[*]	211.45[*]	113.65[*]	125.63[*]	555.50[*]	579.87[*]
Ljung–Box (6)	57.46[*]	17.99[*]	16.43[*]	23.31[*]	18.48[*]	29.60[*]	18.08[*]	16.99[*]	20.26[*]	39.66[*]
Ljung–Box (12)	93.31[*]	55.24[*]	78.02[*]	61.97[*]	40.07[*]	59.33[*]	25.14[*]	35.59[*]	44.08[*]	60.21[*]
Ljung–Box (6) on sq. returns	417.91[*]	293.37[*]	178.45[*]	247.1[*]	71.85[*]	376.3[*]	96.50[*]	79.78[*]	1320[*]	1687[*]
Ljung–Box (12) on sq. returns	853.58[*]	561.56[*]	411.88[*]	416.6[*]	201.4[*]	776.7[*]	165[*]	216.4[*]	1870[*]	1894[*]

[*] Significance at the 5% level.

portmanteau tests on return and squared return series up to 6 and 12 lags indicate a high serial correlation in the first and second moments. The high autocorrelation in the second moment may be due to changing conditional volatility over time. The ARCH tests at 1, 6 and 12 lags reject the null hypothesis of homoscedasticity in the data at 5% significance level. Thus, the GARCH and SV framework is appropriate to model the volatility of these time series.

Figs. 1 and 2 graph the return and their distributions of some selected time series. We observe that the return displays volatility clustering, another typical feature for high frequency financial data. That is to say, large changes tend to follow large changes, and small changes tend to follow small changes. However, the sign of the change from one period to the next is unpredictable. The QQ-plots against the normal distribution show that the distribution has heavier tail than normal.

In short, the preliminary analysis shows that the data exhibit heavy tails, autocorrelation and heteroscedasticity. This suggests the importance of using the stochastic volatility framework to model conditional volatility of return in all series.

3. The model

Based on the preliminary analysis in the previous section and using the optimal lag-length algorithm with the Bayesian information criterion (BIC), we posit the bivariate $VAR(1)$ with stochastic volatility models for the joint processes governing the exchange rates and the oil returns. For comparison, we use both the bivariate SV and the bivariate GARCH(1,1) to model the joint volatility process. Two variants of the multivariate volatility models, the constant conditional correlation and the dynamic conditional correlation, are used to fit the data. This section discusses the model and the estimation methodology.

Let $\mathbf{r}_t = (\, r_t^e \quad r_t^o \,)'$ be the vector of returns of exchange rate and oil at time t. $\mathbf{r}_t$ is modeled as a vector process consisting of a deterministic part $\mathbf{m}_t$ and a stochastic part $\mathbf{e}_t$:

$$\mathbf{r}_t = \mathbf{m}_t + \mathrm{e}_t. \tag{1}$$

The deterministic part $\mathbf{m}_t$ is specified as a vector autoregressive process of order one $VAR(1)$:

$$\mathbf{m}_t = \mathbf{C} + \mathbf{B}\mathbf{r}_{t-1},$$

where $\mathbf{C} = [\, c_e \quad c_o \,]'$ and $\mathbf{B}$ is a (2×2) matrix with $\{\mathbf{B}\}_{ij} = b_{ij}$ for $i, j = e, o$. The subscripts e and o stand for exchange rate and oil, respectively. The stochastic part $\mathbf{e}_t = [\, e_t^e \quad e_t^o \,]'$ involves the bivariate stochastic volatility element of the model and depends on the model being used.

3.1. The multivariate stochastic volatility model (MSV)

The MSV model can be specified as follows:

$$\mathbf{e}_t = \Sigma_t \boldsymbol{\epsilon}_t, \tag{2}$$

where ϵ_t, given Σ_t, is bivariate and normally distributed with mean $E(\mathbf{z}_t|\Sigma_t) = 0_{2\times1}$ and variance

$$var(\boldsymbol{\epsilon}_t|\Sigma_t) = \mathbf{Z}_t = \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix}.$$

The correlation coefficient between e_t^e and e_t^o, ρ_t, can be either constant or time-varying, depending on the model. When ρ_t is a constant, we have the constant conditional correlation MSV (CCC-MSV) model. When it is time-varying, we have the dynamic conditional correlation (DCC-MSV) model. Σ_t is a (2×2) diagonal matrix of standard deviation with the following form:

$$\boldsymbol{\Sigma}_t = \begin{bmatrix} \exp\left(\dfrac{h_t^e}{2}\right) & 0 \\ 0 & \exp\left(\dfrac{h_t^o}{2}\right) \end{bmatrix},$$

where h_t^e, h_t^o are conditional log-variances at time t of r_t^e, r_t^o, respectively.

3.1.1. The constant conditional correlation MSV model

The constant conditional correlation multivariate stochastic volatility (CCC-MSV) model can be specified as follows:

$$\mathbf{e}_t = \Sigma_t \boldsymbol{\epsilon}_t, \tag{3}$$

$$\boldsymbol{\epsilon}_t|\boldsymbol{\Sigma}_t \sim N\left(0_{2\times1}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right) \tag{4}$$

$$\mathbf{h}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}(\mathbf{h}_t - \boldsymbol{\mu}) + \boldsymbol{\eta}_t, \tag{5}$$

where $\mathbf{h}_t = [\, h_t^e \quad h_t^o \,]'$ is the vector of log-variances of r_t^e and r_t^o, $\boldsymbol{\mu} = [\, \mu_e \quad \mu_o \,]'$ is the unconditional mean vector of $\mathbf{h}_t$. $\boldsymbol{\Phi} = \{\phi_{ij}\}$, for $i, j = e, o$, is a 2×2 matrix, representing the persistence and interaction between markets. The error term $\boldsymbol{\eta}_t = [\, \eta_t^e \quad \eta_t^o \,]'$ in the volatility equation is a bivariate normal random variable, $\boldsymbol{\eta}_t \sim N(0_{2\times1}, diag(\sigma_e^2, \sigma_o^2))$.

According to (3)–(5), the stochastic parts of the return series of exchange rate and oil are jointly normally distributed with variance-covariance matrix $\boldsymbol{\Sigma}_t \mathbf{Z}_t \boldsymbol{\Sigma}_t'$. The correlation between the two series are assumed to be a constant ρ. Eq. (5) models the time-varying cross-dependent volatilities of the two series. In this equation, the diagonal parameters of the matrix $\boldsymbol{\Phi}$ (ϕ_{ee} and ϕ_{oo}) capture the persistence of the volatility in each market while the off-diagonal parameters, ϕ_{eo} and ϕ_{oe}, measure the dependence of the conditional volatility of the FX market on the oil market and vice versa. If ϕ_{eo} (ϕ_{oe}) is different from zero, oil (FX) volatility may Granger-cause FX (oil) volatility.

3.1.2. The dynamic conditional correlation MSV

When the correlation between the two returns is allowed to vary over time, we get the dynamic conditional correlation MSV model (DCC-MSV). The DCC-MSV model has the following form:

$$\begin{aligned} &\mathbf{e}_t = \Sigma_t \boldsymbol{\epsilon}_t, \\ &\mathbf{h}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}(\mathbf{h}_t - \boldsymbol{\mu}) + \boldsymbol{\eta}_t, \\ &\boldsymbol{\epsilon}_t|\boldsymbol{\Sigma}_t \sim N\left(0_{2\times1}, \begin{bmatrix} 1 & \rho_t \\ \rho_t & 1 \end{bmatrix}\right), \\ &\rho_t = \frac{\exp(q_t) - 1}{\exp(q_t) + 1}, \\ &q_t = \omega + \beta(q_{t-1} - \omega) + \sigma_q z_t, \quad z_t \overset{iid}{\sim} N(0, 1). \end{aligned} \tag{6}$$

This model captures the dynamic correlation between the two markets. To constrain ρ_t to the interval $[-1, 1]$, we use the Fisher transformation as suggested by Christodoulakis and Satchell (2002), in which ρ_t is a function of q_t which follows an AR(1) stochastic process.

3.2. The multivariate GARCH model with conditional correlations

In addition to MSV, we also use the MGARCH model to fit our data. The bivariate GARCH(1,1) model of exchange rate and oil returns has the following form:

$$\begin{aligned} &\mathbf{r}_t = \mathbf{m}_t + \mathbf{e}_t, \\ &\mathbf{e}_t = \mathbf{H}_t^{1/2} \boldsymbol{\epsilon}_t, \end{aligned}$$

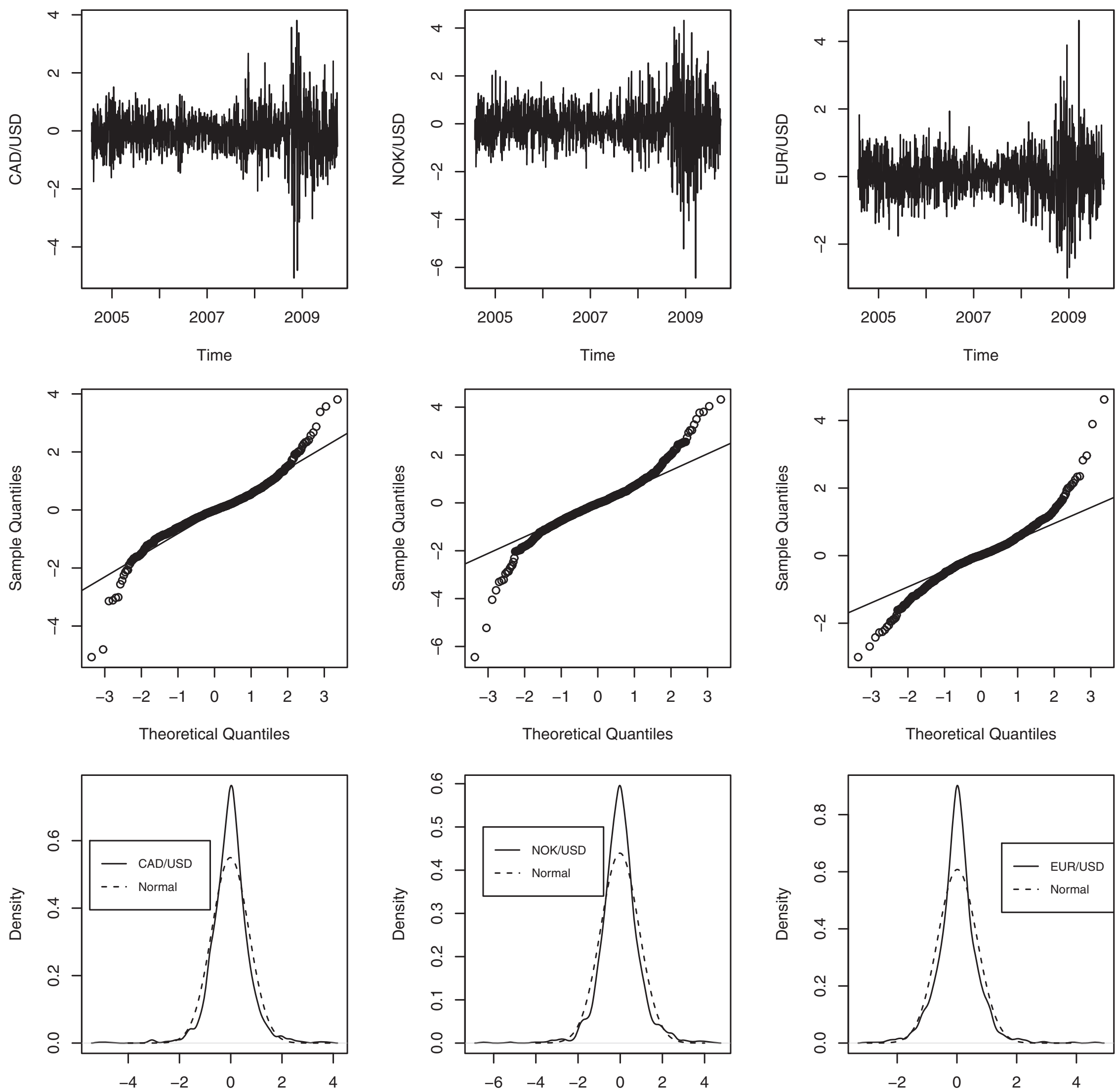

Fig. 1. Exchange rate returns and their distributions for CAD, NOK and EUR.

where $\mathbf{H}_t^{1/2}$ is a 2×2 positive definite matrix and ϵ_t is a 2×1 random vector with the following first and second moments:

$$E(\boldsymbol{\epsilon}_t) = 0_{2\times1},$$
$$Var(\boldsymbol{\epsilon}_t) = I_2,$$

where I_2 is the identity matrix of order 2.

Thus, given the information at time $t-1$, the conditional variance-covariance matrix of $\mathbf{r}_t$ is:

$$Var_{t-1}(\mathbf{r}_t) = Var(\mathbf{e}_t) = \mathbf{H}_t^{1/2} Var_{t-1}(\boldsymbol{\epsilon}_t)(\mathbf{H}_t^{1/2})' = \mathbf{H}_t.$$

The specifications of $\mathbf{H}_t$ depend on the specific MGARCH-type model. We examine the conditional correlation multivariate GARCH models in this paper.

3.2.1. The constant conditional correlation multivariate GARCH (CCC-MGARCH)

The constant conditional correlation multivariate GARCH(1,1) model of Bollerslev (1990) and Jeantheau (1998) has the following form:

$$\mathbf{H}_t = D_t R D_t, \tag{7}$$

$$D_t = diag(h_{e,t}^{1/2}, h_{o,t}^{1/2}),$$

where $h_{e,t}$ and $h_{o,t}$ can be considered as any univariate GARCH model of the two variables. R is their constant correlation matrix which is a symmetric positive definite matrix with 1's on the diagonal.

3.2.2. The dynamic conditional correlation multivariate GARCH (DCC-MGARCH)

The DCC-MGARCH model of Engle (2002) is similar to the CCC-MGARCH model but it allows the correlation matrix R in (7) to vary over time. Specifically, R_t can be specified as follows for the bivariate case:

$$R_t = (Q_t \odot I_2)^{-1/2} Q_t (Q_t \odot I_2)^{-1/2}$$

where $\odot$ is the Hadamard product of two identically sized matrices, computed by element-by-element multiplication, I_2 is the identity matrix of order 2, Q_t is a 2×2 symmetric positive definite matrix, given by

$$Q_t = (1-\alpha-\beta)\overline{Q} + \alpha \mathbf{e}_{t-1}\mathbf{e}_{t-1}' + \beta Q_{t-1},$$

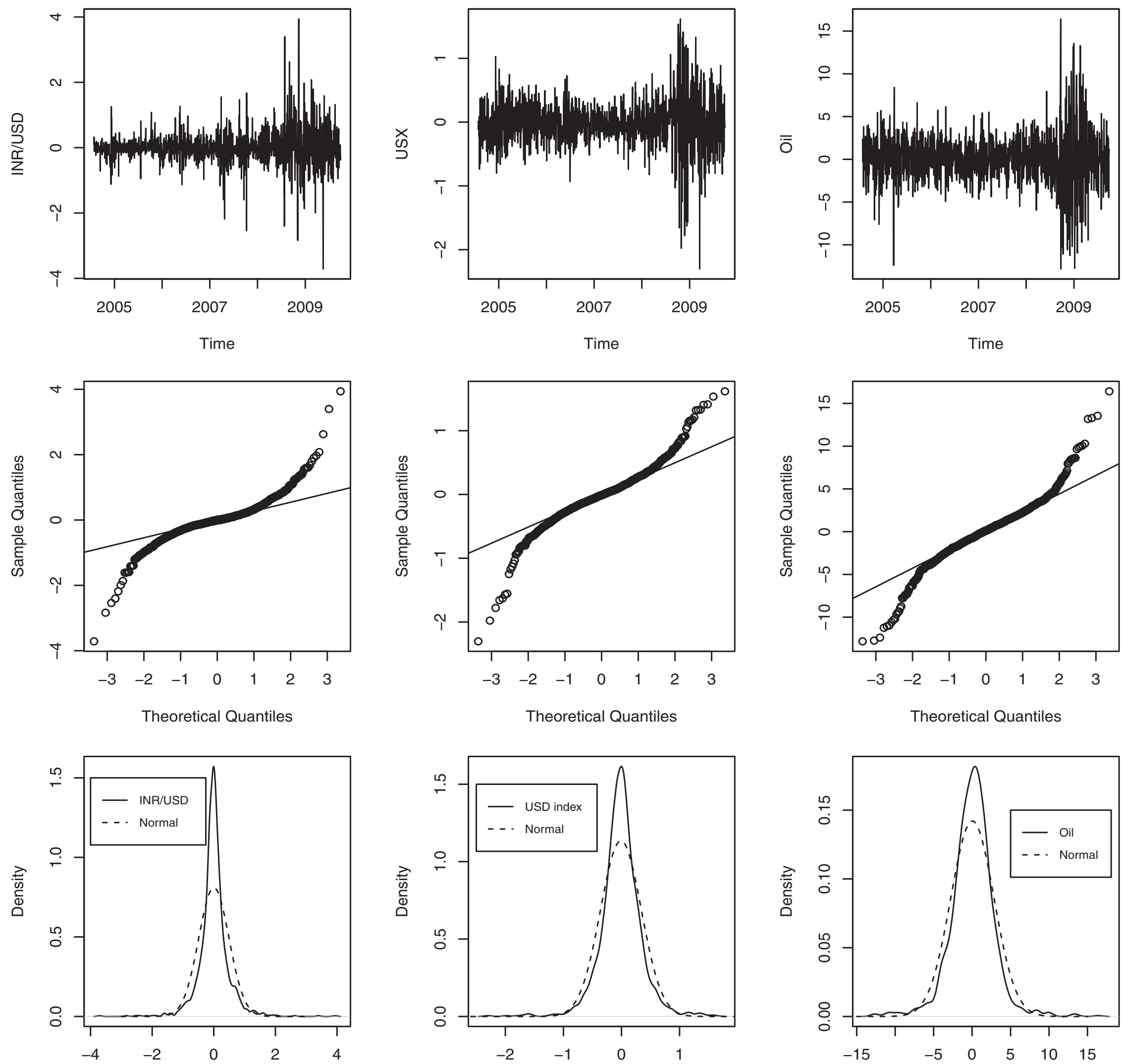

Fig. 2. Exchange rate returns and their distributions for INR, USX and oil.

where $\overline{Q}$ is the unconditional variance-covariance matrix of $\mathbf{e}_t$, and α and β are non-negative scalar parameters satisfying the stationary condition: $\alpha + \beta < 1$.

4. Estimation methodology

We use a two-step estimation method. First, we estimate the mean (Eq. (1)) and extract the residuals. In the second step, we use the residuals to estimate the volatility. To test for structural break in volatility, which is the squared residual, we use the generalized M-fluctuation framework suggested by Kuan and Hornik (1995). The test involves constructing an empirical fluctuation process, governed by a functional central limit theorem, which captures fluctuation over time. The process is then aggregated to a scalar test statistic. To identify the dates of breaks, we employ the dynamic programming algorithm as suggested by Bai and Perron (2003). Table 2 presents the test result. We see that there is a break in all volatility series occurring around September 11, 2008 when Lehman Brothers was about to collapse (Lehman Brothers filed for Chapter 11 bankruptcy protection on September 15, 2008).

To account for structural break in variance, we separate the sample into two sub-samples using the break point on September 11, 2008. We then estimate the variance models for each sub-sample.

4.1. MSV models

4.1.1. The algorithm

Due to the latent structure of variance, SV-type models are difficult to estimate since they do not have closed-form likelihood function. There are several estimation methods proposed in the literature: the generalized method of moments (GMM) by Andersen and Sorensen (1996), Melino and Turnbull (1990), and Sorensen (2000), the quasi maximum likelihood (QML) of Harvey, Ruiz, and Shephard (1994), the efficient method of moments by

Table 2
M-fluctuation test in unconditional variance and break dates.

Time series	Statistics	*p*-Value	Break date
USD/CAD	4.4318	(.0000)	09/26/2008
USD/EUR	4.4397	(.0000)	09/08/2008
USD/INR	3.8701	(.0000)	07/21/2008
USP/MXP	4.1238	(.0000)	10/02/2008
USD/BZR	3.9105	(.0000)	09/03/2008
USD/SGD	4.0335	(.0000)	08/05/2008
USX	5.0651	(.0000)	09/10/2008
USD/NOK	4.7678	(.0000)	09/09/2008
USD/JPY	3.5545	(.0000)	09/09/2008
Oil	5.1890	(.0000)	09/11/2008

Gallant, Hsieh, and Tauchen (1997), the simulated maximum likelihood by Danielsson (1994), Durbin and Koopman (1997), and Sandmann and Koopman (1998), and the Bayesian MCMC methods by Jacquier et al. (1994), and Kim et al. (1998). According to Jacquier et al. (1994), the MCMC method is superior to both the QML and the GMM. While most classical methods rely on asymptotic arguments to make inference, MCMC uses exact posterior distributions of parameters. As a result, it yields more accurate results. For this reason, we use the Bayesian MCMC to estimate our MSV models. The reader is referred to Geweke (2005), Gammerman (1997), and Johannes and Polson (2006) for a comprehensive discussion of Bayesian inference and the MCMC method.

Bayesian estimation is based on the Bayes' theorem, which states that the posterior joint distribution function of the parameters is proportional to the product of their prior distribution function and the likelihood function of the data. Let $\mathbf{\Psi}$ be the vector of parameters to be estimated, $\mathbf{H}_t$ be the vector of latent log volatility, and $\mathbf{r}_t$ be the vector of observed data (the returns of FX and oil). Bayesian inference is then based on the joint posterior distribution of unobservables ($\mathbf{\Psi}$ $\mathbf{H}_t$) given the data $\mathbf{r}_t$. Let $f(\cdot)$ be the probability density function. Using the Bayes' theorem, we get:

$$f(\mathbf{\Psi}, \mathbf{H}_t|\mathbf{r}_t) \propto f(\mathbf{r}_t|\mathbf{\Psi}, \mathbf{H}_t) f(\mathbf{H}_t|\mathbf{\Psi}) f(\mathbf{\Psi}).$$

Thus, the posterior distribution function is a balance between the prior belief, $f(\mathbf{H}_t|\mathbf{\Psi})f(\mathbf{\Psi})$, and the likelihood of the data, $L(\mathbf{\Psi}, \mathbf{H}_t|\mathbf{r}_t) \equiv f(\mathbf{r}_t|\mathbf{\Psi}, \mathbf{H}_t)$. We make inference by evaluating its moments. For simple and low-dimensional problems, the posterior distribution function may have tractable forms and so evaluating its moments is a simple task. However, in complex and higher-dimensional problems, such as the MSV model in this paper, the posterior distribution functions often do not have familiar functional forms and we have to rely on simulation to make inferences.

Markov chain Monte Carlo (MCMC) is a computational method to generate random samples from a given distribution to make inference. It was first introduced by Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953) and subsequently generalized by Hastings (1970). As the name indicates, it consists of two steps: Markov chain and Monte Carlo. The Markov chain step involves constructing Markov chains in the parameter space. Under some mild regularity conditions (see Tierney, 1994) the chains asymptotically converges to the equilibrium joint posterior distribution. In the Monte Carlo step, Monte Carlo simulation is used to make inference about parameters and state variables (volatility) from the chains.

An MCMC algorithm called Gibbs sampler, introduced by Geman and Geman (1984), is based on the Clifford–Hammersley theorem which states that a joint distribution can be characterized by its complete conditional distributions. In this paper, we use Gibbs sampler to generate Markov chains. To ensure that the algorithm converges to the equilibrium distribution, we use the method suggested in Ntzoufras (2009). We run three chains with different starting points. The convergence is reached when the trace plots of different chains mix or intertwine.

4.1.2. Estimation

The CCC-MSV model can be rewritten element wise as follows:

$$e_t^e = \exp\left(\frac{h_t^e}{2}\right)\epsilon_t^e, \tag{8}$$

$$e_t^o = \exp\left(\frac{h_t^o}{2}\right)\epsilon_t^o, \tag{9}$$

$$\rho = cov(\epsilon_t^e, \epsilon_t^o), \tag{10}$$

$$h_{t+1}^e = \mu_e + \phi_{ee}(h_t^e - \mu_e) + \phi_{eo}(h_t^o - \mu_o) + \eta_t^e, \quad \eta_t^e \overset{iid}{\sim} N(0, \sigma_e^2), \tag{11}$$

$$h_{t+1}^o = \mu_o + \phi_{oe}(h_t^e - \mu_e) + \phi_{oo}(h_t^o - \mu_o) + \eta_t^o, \quad \eta_t^o \overset{iid}{\sim} N(0, \sigma_o^2), \tag{12}$$

To estimate this system, we employ the Bayesian MCMC method with Gibbs sampling algorithm. To ensure that $\mathbf{h}_t$ is stationary, we constrain the persistent coefficients ϕ_{ee} and ϕ_{oo} to the interval $(-1, 1)$ by setting $\phi_{ii} = 2\phi_{ii}^* - 1$ for $i = e, o$, where ϕ_{ii}^* has beta prior.

In the same manner, the DCC-MSV model can be written element wise as follows:

$$\begin{aligned}
e_t^e &= \exp\left(\frac{h_t^e}{2}\right)\epsilon_t^e \\
e_t^o &= \exp\left(\frac{h_t^o}{2}\right)\epsilon_t^o \\
\rho_t &= cov(\epsilon_t^e, \epsilon_t^o) = \frac{\exp(q_t) - 1}{\exp(q_t) + 1}, \\
q_t &= \omega + \beta(q_{t-1} - \omega) + \sigma_q z_t, \quad z_t \overset{iid}{\sim} N(0, 1), \\
h_{t+1}^e &= \mu_e + \phi_{ee}(h_t^e - \mu_e) + \phi_{eo}(h_t^o - \mu_o) + \eta_t^e, \quad \eta_t^e \overset{iid}{\sim} N(0, \sigma_e^2), \\
h_{t+1}^o &= \mu_o + \phi_{oe}(h_t^e - \mu_s) + \phi_{oo}(h_t^o - \mu_o) + \eta_t^o, \quad \eta_t^o \overset{iid}{\sim} N(0, \sigma_o^2).
\end{aligned}$$

The estimation strategy used in the constant correlation MSV model is also applied to this model.

4.2. CC-MGARCH models

We use maximum likelihood to estimate the CC-MGARCH models. Let $\mathcal{F}_{t-1}$ be the σ – field generated by all the available information up to time $t-1$, then:

$$\mathbf{e}_t|\mathcal{F}_{t-1} \sim N(0, \mathbf{H}_t).$$

4.2.1. CCC-MGARCH model

Given the CCC-MGARCH model:

$$\begin{aligned}
&\mathbf{e}_t|\mathcal{F}_{t-1} \sim N(0, \mathbf{H}_t) \\
&\mathbf{H}_t = D_t R D_t,
\end{aligned}$$

let $\mathbf{\theta}$ be the vector of all parameters in the model, the log likelihood function can be written as

$$\begin{aligned}
L(\mathbf{\theta}) &= -\frac{1}{2}\sum_{t=1}^{T}(N\ \ln(2\pi) + \ln|\mathbf{H}_t| + \mathbf{e}_t'\mathbf{H}_t^{-1}\mathbf{e}_t) \\
&= -\frac{1}{2}\sum_{t=1}^{T}(N\ \ln(2\pi) + \ln|D_t R D_t| + \mathbf{e}_t'(D_t R D_t)^{-1}\mathbf{e}_t) \\
&= -\frac{1}{2}\sum_{t=1}^{T}(N\ \ln(2\pi) + \ln|R| + 2\ \ln|D_t| + z_t' R^{-1} z_t)
\end{aligned}$$

where N is the number of time series, T is the sample size and $z_t = D_t^{-1} e_t$ is the standardized residual.

4.2.2. DCC-MGARCH model

Given the DCC-MGARCH model:

$$\begin{aligned}
&\mathbf{e}_t|\mathcal{F}_{t-1} \sim N(0, \mathbf{H}_t), \\
&\mathbf{H}_t = D_t R_t D_t.
\end{aligned}$$

Let $\mathbf{\theta}$ and $\mathbf{\phi}$ be the vectors of parameters in D_t and R_t, respectively. The log likelihood for this estimator can be written as:

$$L(\mathbf{\theta}, \mathbf{\phi}) = -\frac{1}{2}\sum_{t=1}^{T}(N\ \ln(2\pi) + \ln|\mathbf{H}_t| + \mathbf{e}_t'\mathbf{H}_t^{-1}\mathbf{e}_t)$$

Table 3
Posterior estimates of CCC-MSV model for exchange rate and oil returns for the first sub-sample.

Parameter	Stat.	USD/CAD	USD/NOK	USX	USD/EUR	USD/INR	USD/BZR	USD/SGD	USD/MXP	USD/JPY
μ_e	Mean	−1.2905	−.8906	−2.7775	−1.3290	−2.7558	−.8152	−2.7379	−2.0446	−1.1993
	95%CI	(−1.61,.99)	(−1.05,−.73)	(−3.10,−2.42)	(−1.64,−.94)	(−3.13,−2.37)	(−1.23,−.45)	(−3.04,−2.47)	(−2.27,−1.82)	(−1.46,−.94)
ϕ_{ee}	Mean	.9780	.7457	.9742	.9714	.8830	.9567	.9282	.9447	.9410
	95%CI	(.94,.99)	(.34,.93)	(.94,.99)	(.93,.99)	(.82,.93)	(.92,.98)	(.78,.98)	(.89,.98)	(.83,.98)
ϕ_{eo}	Mean	.0225	.1161	.0142	.0251	−.0340	.0054	.0532	.0032	.0347
	95%CI	(0,.05)	(−.01,.47)	(−.01,.04)	(−.01,.07)	(−.18,.10)	(−.05,.08)	(0,.19)	(−.03,.04)	(−.03,.17)
σ_e	Mean	.0941	.2222	.1119	.1001	.6271	.2449	.1724	.1594	.1559
	95%CI	(.04,.17)	(.13,.45)	(.06,.18)	(.05,.14)	(.49,.76)	(.19,.30)	(.09,.34)	(.08,.23)	(.10,.27)
ρ	Mean	−.2416	−.2484	−.2389	.1934	.0283	−.1149	−.1182	−.1093	−.0206
	95%CI	(−.19,−.18)	(−.30,−.19)	(−.29,−.17)	(.13,.25)	(−.09,.03)	(−.17,−.05)	(−.18,−.05)	(−.17,−.04)	(−.08,.04)
μ_o	Mean	1.3706	1.3771	1.3719	1.4091	1.3815	1.3895	1.3849	1.3620	1.3609
	95%CI	(1.20,1.52)	(1.18,1.55)	(1.18,1.55)	(1.22,1.64)	(1.20,1.57)	(1.21,1.56)	(1.18,1.58)	(1.19,1.55)	(1.15,1.52)
ϕ_{oo}	Mean	.8691	.8376	.9071	.8451	.9412	.9225	.9367	.9177	.9139
	95%CI	(.65,.95)	(.61,.95)	(.76,.97)	(.48,.95)	(.87,.97)	(.82,.97)	(.85,.97)	(.79,.96)	(.82,.97)
ϕ_{oe}	Mean	.0156	.1714	.0278	.0446	−.0063	−.0097	.0095	−.0118	.0125
	95%CI	(−.02,.07)	(0,.44)	(−.01,.10)	(−.02,.17)	(−.01,.01)	(−.03,0)	(−.02,.05)	(−.05,.02)	(−.02,.05)
σ_o	Mean	.2000	.1491	.1583	.2016	.1336	.1588	.1323	.1776	.1620
	95%CI	(.12,.33)	(.07,.25)	(.08,.25)	(.11,.35)	(.07,.23)	(.09,.26)	(.09,.18)	(.11,.32)	(.10,.25)
DIC		5806	6282	4271	5740	4426	6455	4432	5172	6066

Table 4
Posterior estimates of CCC-MSV model for exchange rate and oil returns for the second sub-sample.

Parameter	Stat.	USD/CAD	USD/NOK	USX	USD/EUR	USD/INR	USD/BZR	USD/SGD	USD/MXP	USD/JPY
μ_e	Mean	.6698	1.1533	−.7629	.4859	−.4418	1.8370	−.9051	.7861	.1830
	95%CI	(.15,1.14)	(.79,1.58)	(−1.31,−.26)	(.08,.80)	(−1.02,.12)	(.93,2.65)	(−1.51,−.42)	(−.07,1.93)	(−.26,.76)
ϕ_{ee}	Mean	.8691	.8086	.8233	.7420	.5081	.9545	.6913	.8584	.5993
	95%CI	(.62,.98)	(.51,.97)	(.58,.95)	(.43,.96)	(.18,.75)	(.84,.99)	(.36,.93)	(.71,.96)	(.35,.80)
ϕ_{eo}	Mean	.0653	.1247	.1347	.1912	.2767	.0470	.2450	.1324	.1566
	95%CI	(0,.19)	(.01,.30)	(.02,.33)	(.03,.41)	(.08,.54)	(0,.14)	(.04,.53)	(.01,.3)	(0,.37)
σ_e	Mean	.2261	.1189	.1127	.1253	.6382	.2332	.2526	.4097	.6042
	95%CI	(.09,.44)	(.04,.30)	(.01,.34)	(.03,.29)	(.35,.92)	(.12,.43)	(.05,.64)	(.02,.64)	(.37,.88)
ρ	Mean	−.4985	−.5274	−.4966	.4237	−.3350	−.4968	−.4414	−.4220	.3386
	95%CI	(−.58,−.40)	(−.61,−.43)	(−.59,−.38)	(.31,.52)	(−.44,−.22)	(−.59,−.39)	(−.54,−.33)	(−.51,−.31)	(.21,.45)
μ_o	Mean	3.7753	3.4798	3.3706	3.5770	3.4247	3.7772	3.5493	3.5484	3.6714
	95%CI	(2.86,4.56)	(2.90,4.38)	(2.67,3.95)	(3.06,4.07)	(2.61,4.25)	(3.06,4.59)	(2.82,4.13)	(2.76,4.53)	(3.01,4.44)
ϕ_{oo}	Mean	.9388	.8279	.8313	.8580	.9506	.9516	.8130	.9760	.9774
	95%CI	(.83,.98)	(.55,.98)	(.56,.98)	(.65,.98)	(.85,.99)	(.83,.99)	(.64,.95)	(.93,.99)	(.94,.99)
ϕ_{oe}	Mean	.1131	.2817	.2320	.1886	.0745	.0410	.2296	.0197	.0560
	95%CI	(.02,.28)	(.01,.79)	(.01,.66)	(.01,.50)	(.01,.19)	(0,.13)	(.04,.48)	(0,.05)	(0,.13)
σ_o	Mean	.1256	.1559	.1735	.1240	.1084	.1663	.1220	.1255	.0874
	95%CI	(.05,.36)	(.04,.30)	(.07,.40)	(.02,.22)	(.03,.22)	(.08,.34)	(.04,.30)	(.07,.23)	(.03,.15)
DIC		2157	2277	1806	2118	1935	2306	1739	2169	2098

Table 5
Posterior estimates of DCC-MSV model for exchange rate and oil returns for the first sub-sample.

Parameter	Stat.	USD/CAD	USD/NOK	USX	USD/EUR	USD/INR	USD/BZR	USD/SGD	USD/MXP	USD/JPY
μ_e	Mean	−1.2583	−.9144	−2.3985	−1.3724	−2.7629	−.7992	−2.7488	−1.9941	−1.1909
	95%CI	(−1.60,−.82)	(−1.07,−.74)	(−2.75,−2.06)	(−1.75,−1.02)	(−3.15,−2.37)	(−1.14,−.45)	(−2.98,−2.49)	(−2.16,−1.8)	(−1.48,−.94)
ϕ_{ee}	Mean	.9803	.7305	.9403	.9563	.8859	.9546	.9191	.9127	.9200
	95%CI	(.95,.99)	(.29,.99)	(.80,.98)	(.81,.99)	(.82,.93)	(.91,.98)	(.73,.97)	(.82,.96)	(.61,.99)
ϕ_{eo}	Mean	.0106	.1863	.0725	.0320	−.0235	.0216	.0383	.0102	.0471
	95%CI	(−.02,.05)	(−.01,.56)	(0,.18)	(−.01,.16)	(−.16,.14)	(−.04,.08)	(−.01,.09)	(−.04,.05)	(−.03,.38)
σ_e	Mean	.1044	.1755	.0708	.1233	.6192	.2545	.2127	.2047	.1713
	95%CI	(.07,.14)	(.05,.48)	(.04,.09)	(.07,.27)	(.45,.78)	(.19,.33)	(.13,.42)	(.14,.31)	(.08,.37)
μ_o	Mean	1.3913	1.3162	1.4665	1.3882	1.3975	1.3488	1.3636	1.3591	1.3587
	95%CI	(1.22,1.56)	(1.06,1.52)	(1.26,1.69)	(1.18,1.57)	(1.21,1.67)	(1.19,1.50)	(1.18,1.52)	(1.17,1.55)	(1.2,1.51)
ϕ_{oo}	Mean	.8879	.8766	.8391	.8577	.9118	.8666	.9219	.9137	.8633
	95%CI	(.75,.96)	(.75,.95)	(.62,.95)	(.70,.94)	(.76,.98)	(.69,.95)	(.85,.96)	(.82,.96)	(.68,.95)
ϕ_{oe}	Mean	.0076	.1106	.0652	.0448	−.0022	−.0253	−.0034	−.0131	.0131
	95%CI	(−.04,.06)	(0,.28)	(−.10,.27)	(0,.11)	(−.01,.01)	(−.06,0)	(−.03,.03)	(−.1,.04)	(−.05,.11)
σ_o	Mean	.1950	.1145	.1311	.2116	.1719	.2247	.1708	.1876	.2044
	95%CI	(.09,.31)	(.03,.22)	(.05,.28)	(.14,.34)	(.09,.29)	(.12,.33)	(.12,.23)	(.13,.25)	(.12,.28)
ω	Mean	−.5455	−.4679	−.5220	.4082	−.0854	−.2456	−.2373	−.2468	−.0299
	95%CI	(−.64,−.39)	(−.57,−.35)	(−.68,−.37)	(.24,.56)	(−.25,.06)	(−.37,−.12)	(−.38,−.08)	(−.38,−.08)	(−.19,.12)
β	Mean	.7859	.82671	.8140	.9251	.7921	.7485	.7826	.8046	.7475
	95%CI	(.45,.95)	(.60,.93)	(.61,.91)	(.86,.96)	(.62,.90)	(.5,.94)	(.52,.93)	(.56,.93)	(.47,.9)
σ_q	Mean	.1583	.1276	.0839	.1267	.2126	.2312	.2936	.2009	.3398
	95%CI	(.03,.46)	(.07,.22)	(.04,.19)	(.06,.19)	(.13,.34)	(.12,.45)	(.16,.48)	(.11,.31)	(.21,.47)
DIC		5785	6287	4426	5703	4388	6399	4344	5135	5992

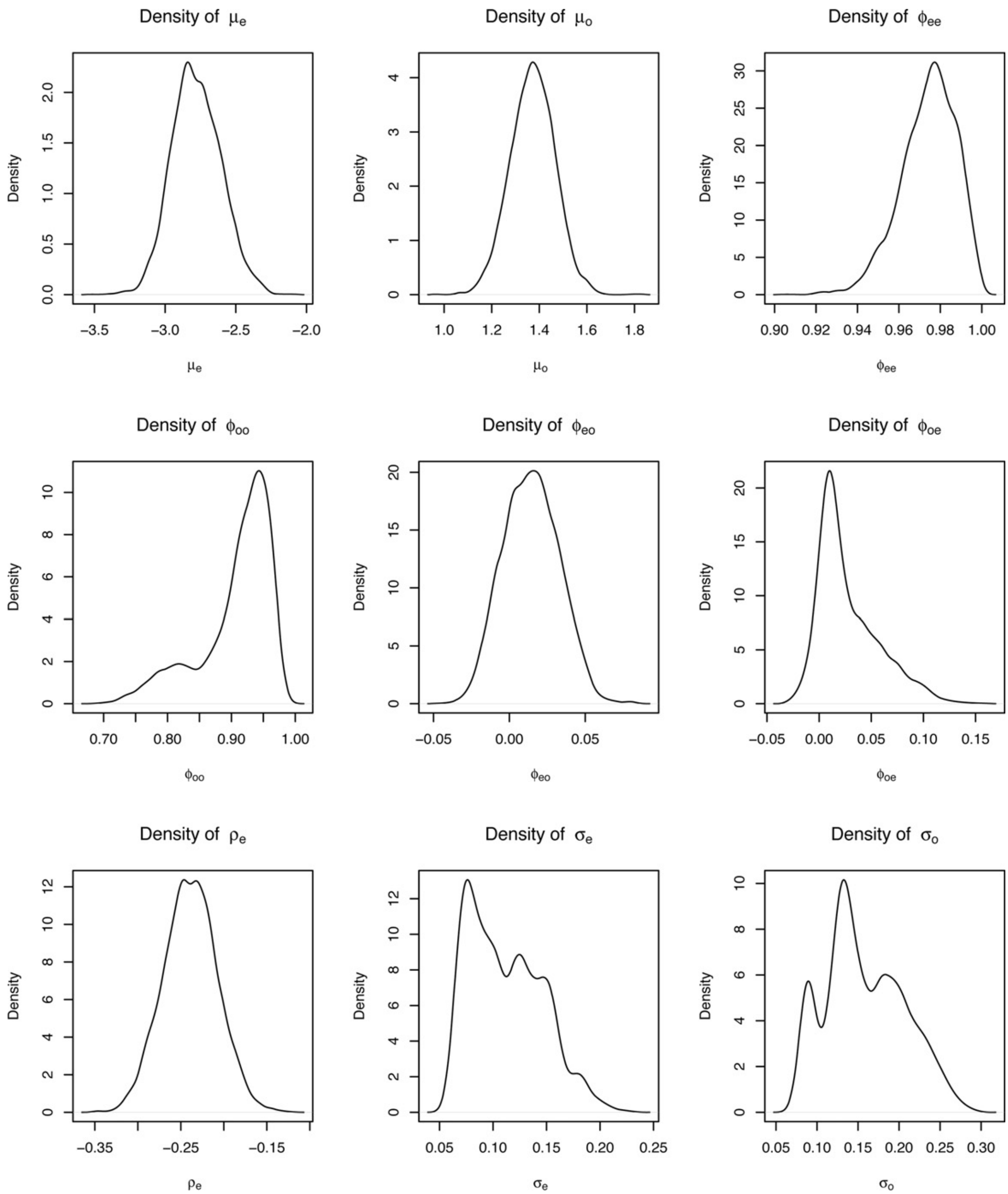

Fig. 3. Posterior densities of parameter estimates for CCC-MSV for the first sub-sample of the USD index.

$$= -\frac{1}{2}\sum_{t=1}^{T}(N\ \ln(2\pi) + \ln|D_tR_tD_t| + \mathbf{e}_t'D_t^{-1}R_t^{-1}D_t^{-1}\mathbf{e}_t)$$

$$= -\frac{1}{2}\sum_{t=1}^{T}(N\ \ln(2\pi) + 2\ \ln|D_t| + \ln|R_t| + z_t'R_t^{-1}z_t)$$

$$= -\frac{1}{2}\sum_{t=1}^{T}(N\ \ln(2\pi) + 2\ \ln|D_t| + e_t'D_t^{-1}D_t^{-1}e_t - z_t'z_t + \ln|R_t| + z_t'R_t^{-1}z_t), \tag{13}$$

where N is the number of time series, T is the sample size and $z_t = D_t^{-1}e_t$ is the standardized residual.

$L(\boldsymbol{\theta}, \boldsymbol{\phi})$ can be rewritten as a sum of a volatility component $L_v(\boldsymbol{\theta})$ and a correlation component $L_c(\boldsymbol{\theta}, \boldsymbol{\phi})$:

$$L(\boldsymbol{\theta}, \boldsymbol{\phi}) = L_v(\boldsymbol{\theta}) + L_c(\boldsymbol{\theta}, \boldsymbol{\phi})$$

where

$$L_v(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{t=1}^{T}(N\ \ln(2\pi) + 2\ \ln|D_t| + e_t'D_t^{-2}e_t)$$

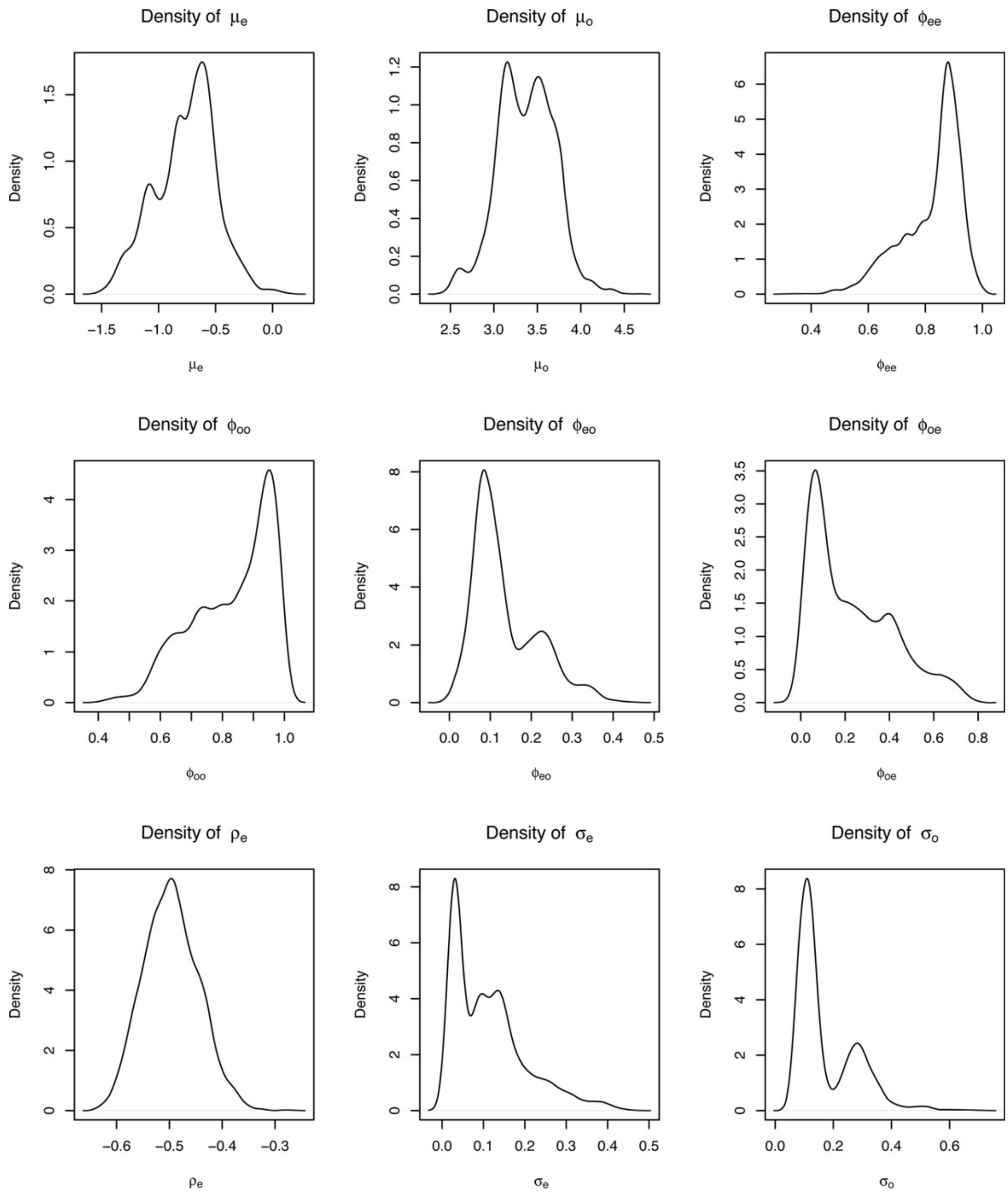

Fig. 4. Posterior densities of parameter estimates for CCC-MSV for the second sub-sample of the USD index.

is the sum of individual GARCH likelihoods, and

$$L_c(\boldsymbol{\theta}, \boldsymbol{\phi}) = -\frac{1}{2}\sum_{t=1}^{T}(\ln|R_t| + z_t' R_t^{-1} z_t - z_t' z_t)$$

is the likelihood for correlations.

We use two-step estimation methodology proposed by Engle (2002). In the first step, we estimate $\boldsymbol{\theta}$

$$\hat{\theta} = arg\max[L_v(\boldsymbol{\theta})]$$

and then we take $\hat{\theta}$ as given in the second step to estimate $\boldsymbol{\phi}$:

$$\hat{\phi} = arg\max[L_c(\hat{\theta}, \boldsymbol{\phi})]$$

5. Empirical results

5.1. CCC-MSV model

Table 3 reports the estimates of the following CCC-MSV model for the first sub-sample.

$$e_t^e = \exp\left(\frac{h_t^e}{2}\right)\epsilon_t^e,$$

$$e_t^o = \exp\left(\frac{h_t^o}{2}\right)\epsilon_t^o,$$

$$\rho = cov(\epsilon_t^e, \epsilon_t^o)$$

$$h_{t+1}^e = \mu_e + \phi_{ee}(h_t^e - \mu_e) + \phi_{eo}(h_t^o - \mu_o) + \eta_t^e, \quad \eta_t^e \overset{iid}{\sim} N(0, \sigma_e^2),$$

$$h_{t+1}^o = \mu_o + \phi_{oe}(h_t^e - \mu_s) + \phi_{oo}(h_t^o - \mu_o) + \eta_t^o, \quad \eta_t^o \overset{iid}{\sim} N(0, \sigma_o^2).$$

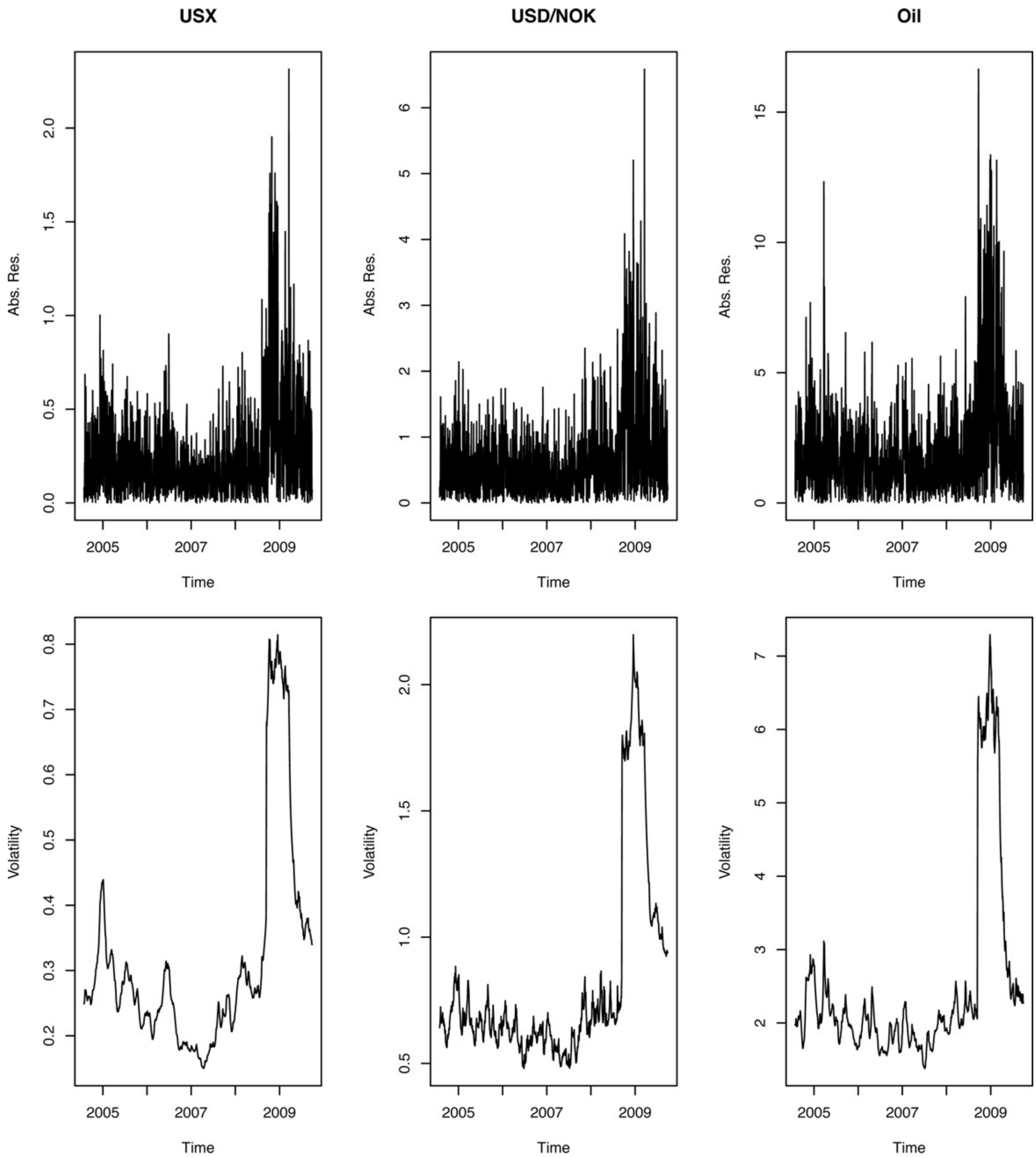

Fig. 5. Estimated volatilities of the USD index, USD/NOK and oil using CCC-MSV model.

We observe that both exchange rate and oil volatilities are very persistent. The persistent coefficient for exchange rates ϕ_{ee} ranges from.7457 for USD/NOK to.9780 for USD/CAD. For oil, ϕ_{oo} varies from.8376 to.9412. These results are consistent with the fact that exchange rate and oil price volatilities are often clustered. In most cases, the transmission coefficients (ϕ_{eo} and ϕ_{oe}) are not significantly different from zero (the 95% confidence interval does not contain 0.) Thus, when the markets are calm, information in one market is quickly transmitted and incorporated into the other market and therefore there is no evidence of volatility interaction found in daily data. However, this is not the case during the crisis. This could be attributed to inefficient information incorporation during market chaos.

In most cases, the contemporaneous correlation ρ between oil and exchange rates is significantly negative. This result is consistent with empirical findings in other studies that dollar value and oil price are overall inversely correlated.

Table 4 reports the estimates for the sub-sample after the break point. Both time series seem to be less persistent than the previous case. The coefficient ϕ_{ee} ranges from.5081 to.9545, while ϕ_{oo} ranges from.8130 to.9774. Unlike the other sub-sample, both ϕ_{eo} and ϕ_{oe} are significantly positive in almost all cases, indicating that there is bidirectional interaction in volatility between the two markets. Shocks to volatility in one market are also transmitted to the other one at a later time. Other things equal, a shock that increases oil price volatility by 10% will increase the volatilities of USD index by approximately 1.35%. On the other direction, a shock that increases USX volatility by 10% will increase the volatility in the oil market by 2.32%, other things equal.

Fig. 3 shows the posterior densities of parameters estimated in the model for the cases of the USD index before the break and Fig. 4 shows those after the break.

Fig. 5 shows the volatility estimates of the USD index, USD/NOK and oil. To the naked eyes it seems that the estimates reflect well the

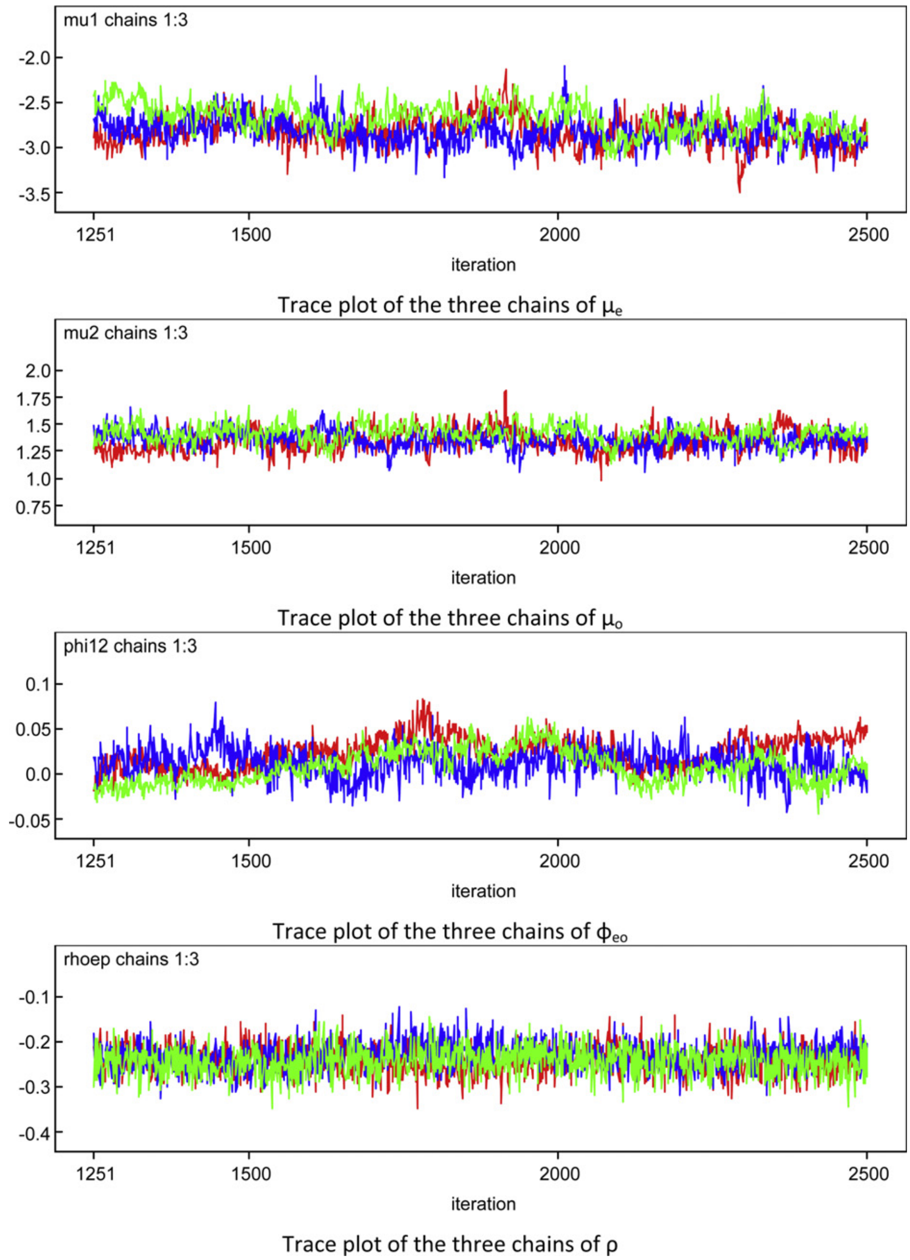

Fig. 6. Trace plots of the Markov chains of some parameters of the CCC-MSV for the USD index.

variations of the residuals. The graph clearly shows the structural break around the date Lehman Brothers collapsed.

Fig. 6 graphs the trace plots of some Markov chains. We observe that the three chains of each parameter cross and mix with one another, indicating that they converge to their equilibrium distribution.

The detected bidirectional causality relationship is not surprising. Literature has proposed various potential mechanisms connecting exchange rate and oil price. From exchange rate to oil price, purchasing power channel argues that the depreciation of the USD reduces purchasing power of oil exporters so that they want to raise price for compensation. The local price channel suggests that the U.S. dollar depreciation makes oil less expensive for consumers in non-U.S. dollar regions (in local currency), thereby increasing their crude oil demand, which eventually causes adjustments in the oil price denominated in U.S. dollars. On the opposite direction, oil prices affect the terms of trade, consumption and savings, as well as the current account balance. Therefore, they affect the supply and demand for dollars through real factors and portfolio preferences.

The insignificant interaction before the financial crisis can be attributed to the finding of Chan, Tse, and Williams (2011). They use daily data and find that commodity/currency relationships exist contemporaneously but fail to exhibit lead-lag behavior in either direction. Their results as well as ours indicate that when the markets are relatively calm, both oil and FX markets respond to information shocks simultaneously. Such an efficient information incorporation process can be affected when the markets are highly volatile, in which the significant interaction may be detected, as displayed by our results.

Table 6
Posterior estimates of DCC-MSV model for exchange rate and oil returns for the second sub-sample.

Parameter	Stat.	USD/CAD	USD/NOK	USX	USD/EUR	USD/INR	USD/BZR	USD/SGD	USD/MXP	USD/JPY
μ_e	Mean	.6461	1.2296	−.7557	.4538	−.3892	1.8299	−1.1082	.8708	.0947
	95%CI	(.12,1.21)	(.67,1.55)	(−1.46,−.20)	(−.18,.83)	(−.86,22)	(1.02,2.73)	(−1.69,−.61)	(−.01,1.78)	(−.37,.6)
ϕ_{ee}	Mean	.8071	.7580	.7878	.8610	.6872	.9471	.8407	.8661	.6065
	95%CI	(.53,.97)	(.41,.95)	(.51,.95)	(.59,.99)	(.32,.93)	(.83,.99)	(.59,.99)	(.71,.96)	(.32,.84)
ϕ_{eo}	Mean	.1213	.1470	.1489	.1128	.1903	.0539	.1305	.1170	.1599
	95%CI	(0,.32)	(.02,.36)	(0,.47)	(.01,.30)	(.02,.45)	(0,.16)	(.01,.29)	(.02,.27)	(0,.39)
σ_e	Mean	.2618	.2156	.1894	.0998	.3763	.2510	.0933	.3942	.6439
	95%CI	(.07,.73)	(.06,.53)	(.03,.40)	(.02,.34)	(.05,.83)	(.14,.48)	(.04,.21)	(.17,.63)	(.37,.94)
μ_o	Mean	3.5515	3.7210	3.3595	3.4374	3.6053	3.7275	3.2780	3.6186	3.4360
	95%CI	(2.85,4.23)	(2.92,4.23)	(2.52,4.16)	(2.55,3.93)	(2.99,4.31)	(3.07,4.54)	(2.58,3.90)	(2.7,4.52)	(2.54,4.22)
ϕ_{oo}	Mean	.9238	.9147	.8912	.8191	.9267	.9727	.7711	.9769	.9722
	95%CI	(.80,.98)	(.76,.99)	(.71,.98)	(.39,.98)	(.68,.99)	(.93,.99)	(.43,.97)	(.92,.99)	(.92,.99)
ϕ_{oe}	Mean	.1194	.1426	.1517	.2210	.0992	.0258	.2633	.0197	.0503
	95%CI	(.03,.28)	(.01,.38)	(0,.48)	(.01,.77)	(−.01,.35)	(0,.05)	(.02,.70)	(0,.06)	(0,.1)
σ_o	Mean	.0988	.0842	.1430	.1800	.0884	.1142	.2405	.1382	.1053
	95%CI	(.03,.19)	(.02,.18)	(.02,.38)	(.03,.36)	(.03,.15)	(.06,.18)	(.10,.50)	(.07,.25)	(.04,.27)
ω	Mean	−1.1089	−1.2703	−1.3681	.9027	−.9740	−1.0289	−1.0085	−.9737	.8228
	95%CI	(−1.40,−.73)	(−1.80,−.75)	(−2.12−.78)	(.39,1.42)	(−1.39,−.51)	(−1.3,−.74)	(−1.38,−.66)	(−1.19,−.77)	(.32,1.24)
β	Mean	.8190	.87078	.9087	.9261	.8108	.8342	.7995	.8259	.8403
	95%CI	(.56,.96)	(.59,.98)	(.67,.99)	(.75,.99)	(.51,.97)	(.54,.97)	(.54,.96)	(.6,.96)	(.55,.99)
σ_q	Mean	.2240	.3596	.2301	.2033	.3014	.1323	.3677	.1147	.2990
	95%CI	(.02,.71)	(.10,.98)	(.08,.39)	(.08,.45)	(.09,.62)	(.05,.23)	(.14,.74)	(.05,.28)	(.04,.93)
DIC		2149	2251	1804	2093	1976	2302	1719	2164	2073

5.2. DCC-MSV model

Table 5 reports the estimates of the first sub-sample for the following DCC-MSV model:

$$
\begin{aligned}
e_t^e &= \exp\left(\frac{h_t^e}{2}\right)\epsilon_t^e,\\
e_t^o &= \exp\left(\frac{h_t^o}{2}\right)\epsilon_t^o,\\
\rho_t &= cov(\epsilon_t^e,\epsilon_t^o) = \frac{\exp(q_t)-1}{\exp(q_t)+1},\\
q_t &= \omega + \beta(q_{t-1}-\omega) + \sigma_q z_t, \quad z_t \overset{iid}{\sim} N(0,1),\\
h_{t+1}^e &= \mu_e + \phi_{ee}(h_t^e-\mu_e) + \phi_{eo}(h_t^o-\mu_o) + \eta_t^e, \quad \eta_t^e \overset{iid}{\sim} N(0,\sigma_e^2),\\
h_{t+1}^o &= \mu_o + \phi_{oe}(h_t^e-\mu_e) + \phi_{oo}(h_t^o-\mu_o) + \eta_t^o, \quad \eta_t^o \overset{iid}{\sim} N(0,\sigma_o^2).
\end{aligned}
$$

The parameter estimates are quite close to the ones in the CCC-MSV. The volatility in both market is very persistent. The transmission coefficient ϕ_{eo} from the oil to the FX market and ϕ_{oe} from the FX to the oil market are not significantly different from zero.

Table 6 reports the estimates for the second sub-sample. The transmission coefficient ϕ_{eo} is significantly positive in all cases while ϕ_{oe} is significant in eight out of nine cases, indicating the volatility interaction between the two markets. We also observe that in eight out of nine cases, the DIC (deviance information criterion) is smaller in the DCC-MSV than the CCC-MSV, suggesting that the former fits the data better. Nevertheless, which model is more appropriate depends on its forecasting power. We will assess the forecasting performance of each model later in the paper. Figs. 7 and 8 show the posterior densities of parameters estimated for the case of the USD index in both sub-samples

Fig. 9 shows the estimated volatilities of USX, USD/NOK, oil and the correlation coefficients between FX and oil for DCC-MSV model. As in the CCC-MSV model, we see that the estimated volatilities reflect quite well the variation of the residuals. The correlation between the two markets is time-varying. It is more erratic after Lehman Brothers collapsed.

5.3. CCC-MGARCH model

Tables 7 and 8 report the parameter estimates of both sub-samples for following CCC-MGARCH(1,1) model, written in element wise form:

$$
\begin{aligned}
e_{e,t} &= \sqrt{h_{e,t}}\epsilon_{e,t},\\
e_{o,t} &= \sqrt{h_{o,t}}\epsilon_{o,t},\\
\rho &= cov(\epsilon_{e,t},\epsilon_{o,t})
\end{aligned}
$$
$$
\begin{bmatrix} h_{e,t}\\ h_{o,t}\end{bmatrix} = \begin{bmatrix} c_1\\ c_2\end{bmatrix} + \begin{bmatrix} a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix} e_{e,t-1}^2\\ e_{o,t-1}^2\end{bmatrix} + \begin{bmatrix} b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}\begin{bmatrix} h_{e,t-1}\\ h_{o,t-1}\end{bmatrix}.
$$

Parameters $a_{ii}, b_{ii}, i=1,2$ represent the persistence of the volatility, while a_{ij}, b_{ij}, $i, j=1, 2$ and $i \neq j$, are transmission coefficients, representing the impact of volatility in one market on the other. We observe that as in the MSV models, in majority of cases, volatilities are very persistent (the sum $a_{ii}+b_{ii}$ is close to unity), once again confirming the volatility clustering in these financial time series.

Unlike the MSV models, the CCC-MGARCH does not discover much volatility interaction between the two markets. The transmission coefficients a_{12} and b_{12}, from the oil market to the FX market, are not significantly different from zero for all cases in both sub-samples, suggesting that volatility in the oil market does not have significant impacts on the FX market. However, the coefficient b_{21}, from the FX to the oil market is significantly positive six out of nine cases in the first sub-sample, suggesting that shocks to FX markets are transmitted to oil market. However, when the financial crisis broke out, this is not the case. It seems that the CCC-MGARCH and CCC-MSV are quite inconsistent in terms of the direction of the volatility causality. A further evaluation of the model selection later would help us to judge which one is more reliable.

The estimated correlations between exchange rates and oil prices are comparable to those in the CCC-MSV. The pattern that rising oil price is usually associated with depreciation of the dollar seems to be robust to both MSV and MGARCH frameworks.

5.4. DCC-MGARCH model

Tables 9 and 10 report the estimates of the DCC-MGARCH(1,1) model for the two sub-samples. This model is similar to the CCC-MGARCH(1,1), except that the coefficient ρ between $\epsilon_{e,t}$ and $\epsilon_{o,t}$

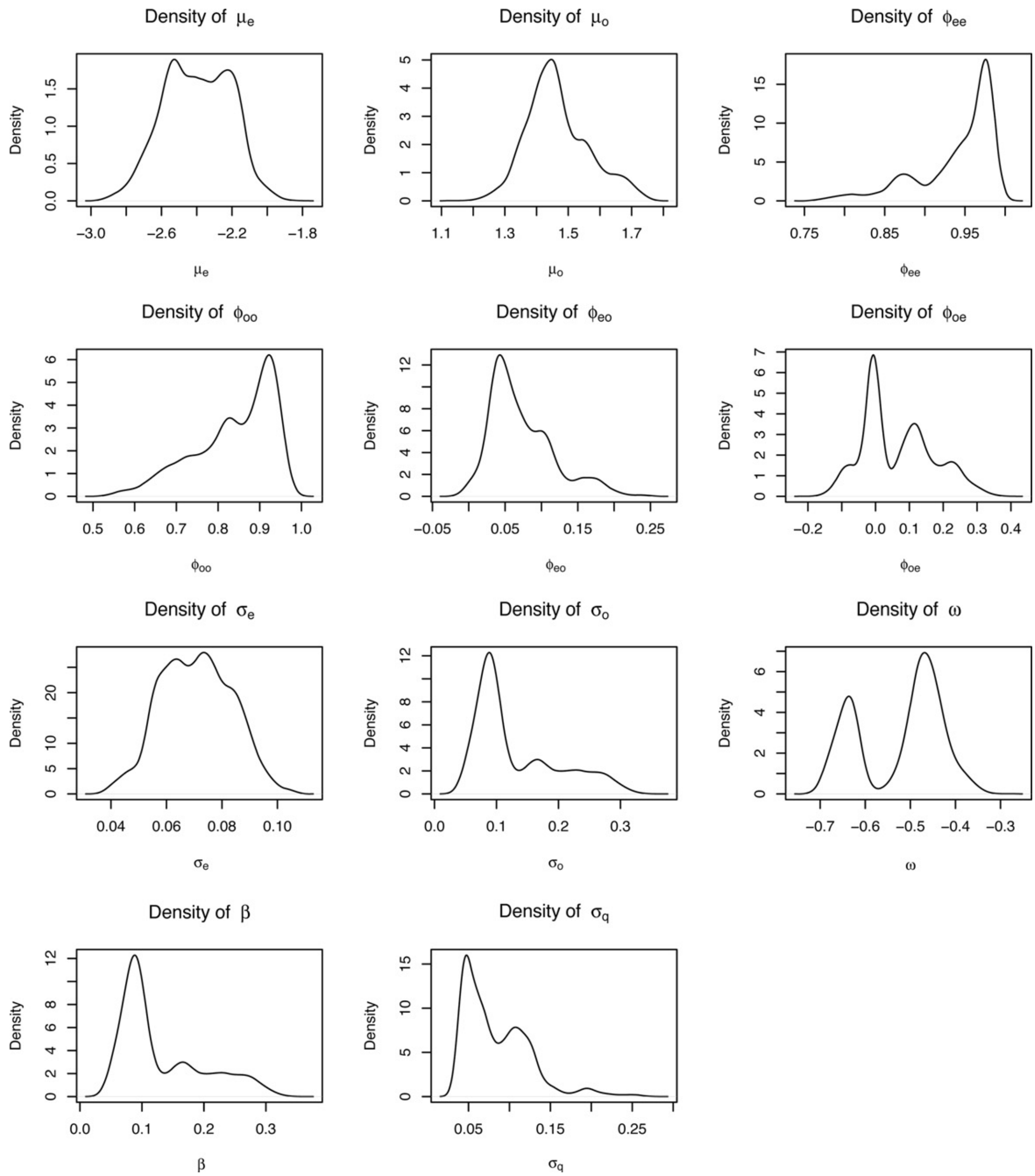

Fig. 7. Posterior densities of parameter estimates for DCC-MSV for the first sub-sample of the USD index.

is not a constant but time-varying. The result is similar to that in the CCC-MGARCH model. However, DCC-MGARCH(1,1) detects a stronger volatility causality from the FX to the oil market. This could be the reason that leads to its superior forecasting of oil volatility as we will see later.

Fig. 10 graphs the dynamic correlations of some exchange rates and oil.

6. Comparing models

6.1. Model checking

In this section, we check the goodness-of-fit of each model and compare them. To check the goodness-of-fit, we test whether the volatility in each market, indicated by each model, is sufficient in explaining the stylized facts, such as heavy tail, volatility clustering. In particular, we test if the residuals, having been standardized by the corresponding volatility in the variance equations, are standard normal. Tables 11 and 12 present the test results.

Table 11 reports various statistics of the standardized residual for CCC-MSV and DCC-MSV models for both sub-samples. We observe that for all cases, the mean in each market is close to zero while the standard deviation is close to one. Furthermore, the excess kurtosis for both markets in both models are small, indicating that the thick tail has been removed. We use both the Jarque–Bera and the Kolmogorov–Smirnov methods to test for normality. In majority of the cases, the two tests cannot reject the null hypothesis of normality at 5% level of significance. Thus, both

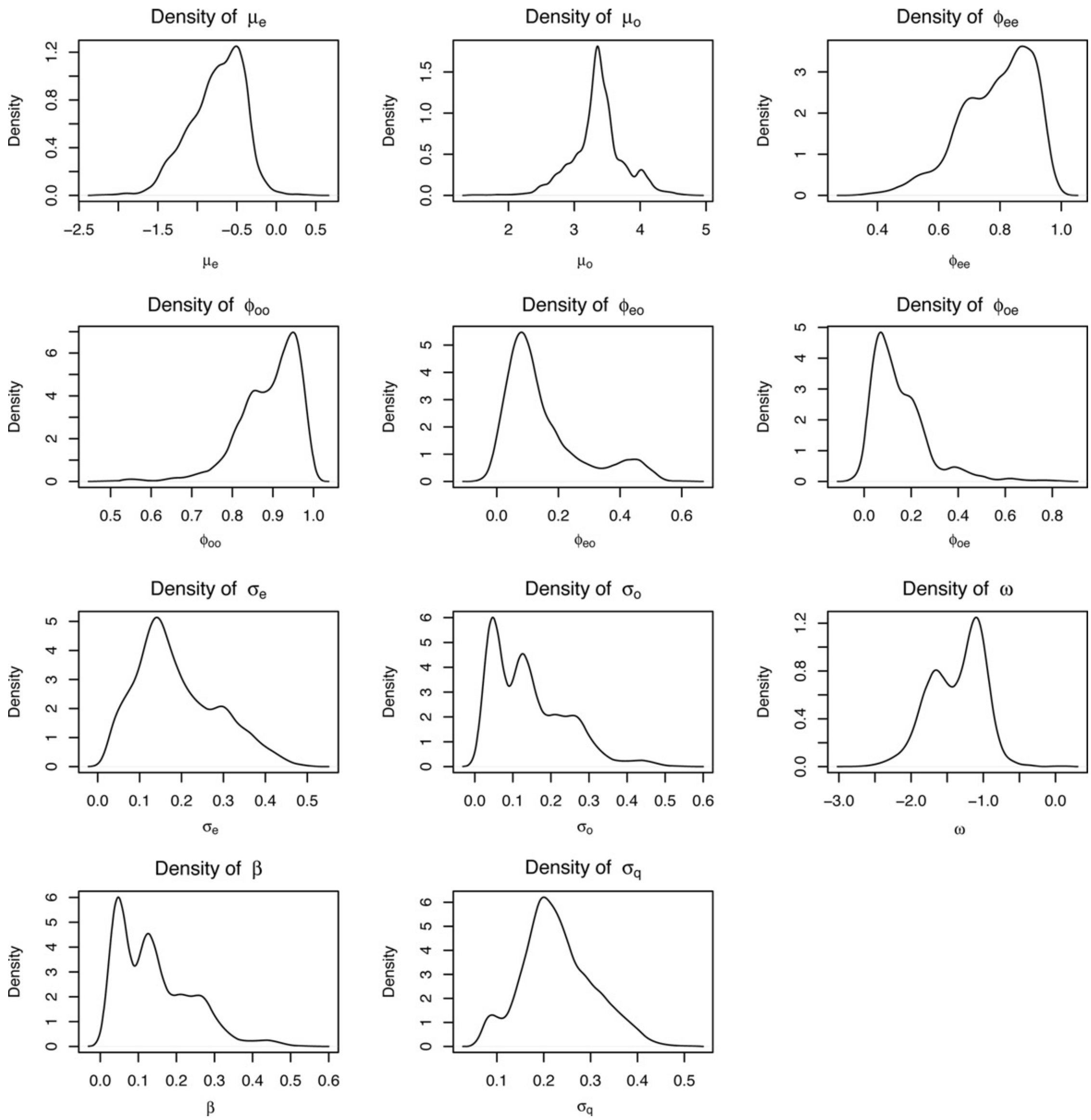

Fig. 8. Posterior densities of parameter estimates for DCC-MSV for the second sub-sample of the USD index.

CCC-MSV and DCC-MSV models do quite well in modeling heteroscedasticity in the data.

To compare CCC-MSV and DCC-MSV, in terms of goodness-of-fit, we use the deviance information criterion (DIC) proposed by Spiegelhalter, Best, Carlin, and der Linde (2002) which is a Bayesian version of the Akaike Information Criterion (AIC) (Akaike, 1973) and closely related to the Bayesian (or Schwarz) Information Criterion (BIC) (Schwarz, 1978). Like the AIC and the BIC, it trades off the model adequacy, measured by the log-likelihood, against the model complexity, measured by the number of free parameters. The smaller the DIC, the better the model. For a discussion of the use of the DIC to compare SV models, the reader is referred to Berg, Myer, and Yu (2004). We see that in all cases, the two models have similar DIC, although the DCC-MSV model has slightly lower DIC than the CCC-MSV model in seven out of nine cases. Thus, it is hard to tell which model is better for the data. Later in the paper, we will revisit the issue by comparing their forecasting performance.

Table 12 reports similar statistics for CCC-MGARCH and DCC-MGARCH models. Although the tail thickness (kurtosis) has been reduced, it is still thicker than in the cases of MSV models. The Jarque–Bera and the Kolmogorov–Smirnov tests reject the null hypothesis of normality at 5% level of significance more often than in the MSV case. Thus, it seems that MSV models outperform MGARCH counterparts in terms of goodness-of-fit.

6.2. Forecast performance

To evaluate the forecasting power of a volatility model, we first need realized volatility. There are a number of ways in literature to obtain realized volatility (see, for example Akgiray, 1989; Ding, Granger, & Engle, 1993; Merton, 1980; Perry, 1982.) In this paper, we use the approach of Merton (1980) and Perry (1982) by simply using the square of daily return as a proxy of realized daily return.

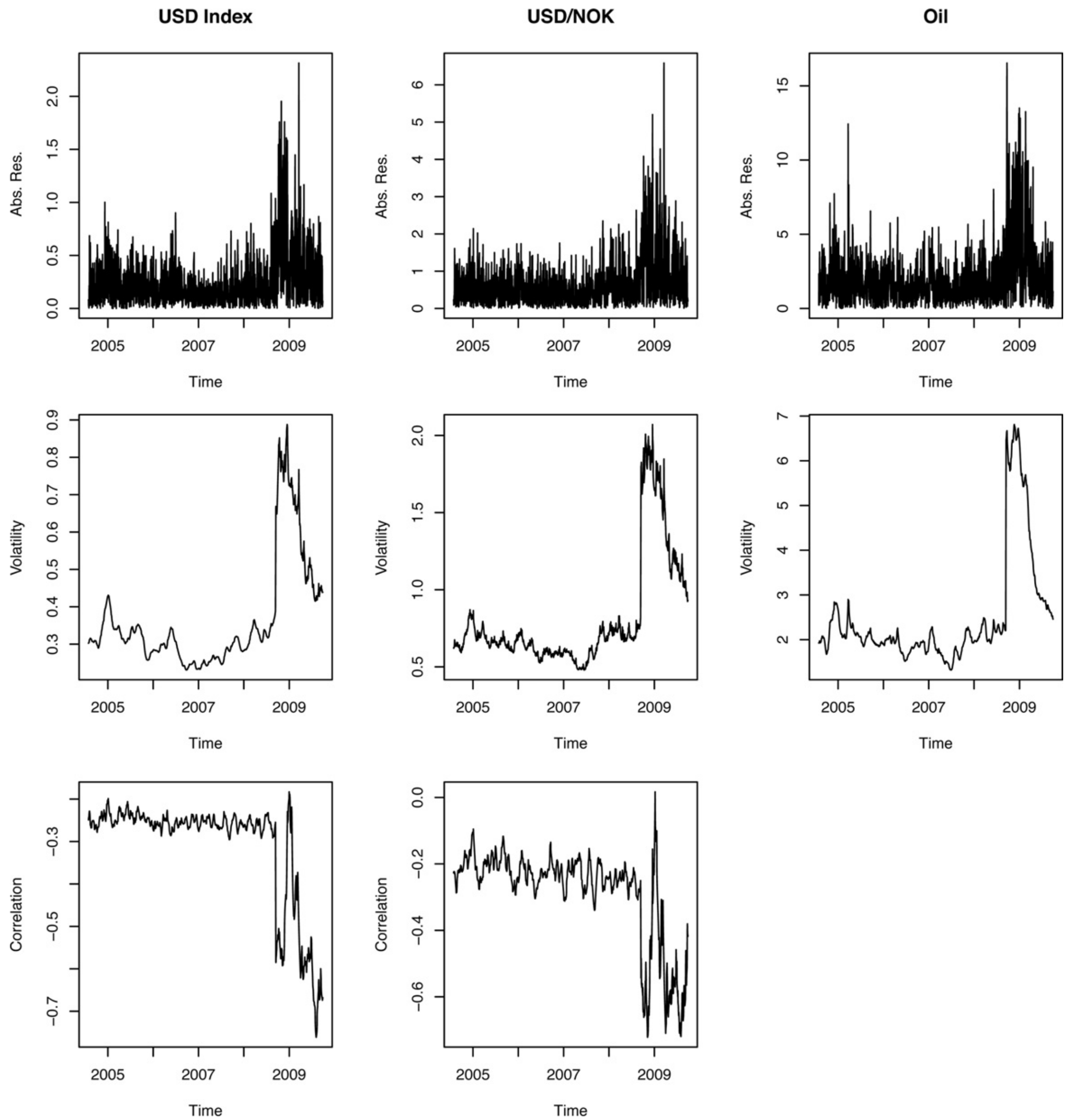

Fig. 9. Estimated volatilities of the USD index, USD/NOK and oil using DCC-MSV model.

To evaluate the forecast performance, we use two popular metrics: the mean absolute error (MAE) and the root mean square error (RMSE). They are given by:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|\hat{\sigma}_i^2 - \sigma_i^2\right|,$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\sigma}_i^2 - \sigma_i^2\right)^2},$$

where $\hat{\sigma}_i^2$ is the forecast volatility of date i indicated by the model, σ_i^2 is the realized volatility at date i. N is the number of observations in the sample. The lower the metrics, the better is the model in terms of forecast performance.

To conduct the forecast experiment, we use the estimates for each model in the previous section to forecast volatilities for the next three weeks and calculate the two metrics. To see how multivariate models perform compared to the univariate counterparts, we also include GARCH(1,1) and the univariate stochastic volatility model in the experiment. Table 13 reports the results.

For exchange rates, using MAE metric, we see that both univariate and multivariate SV models significantly outperform the GARCH counterparts in all cases. We observe that MSV model (either DCC-MSV or CCC-MSV) outperform univariate SV in all cases, indicating that using information from oil market does improve the volatility forecast in FX markets. CCC-MSV is ranked the best in seven cases while DCC-MSV is ranked best in the other two cases. For oil, using MAE measure, we see that MGARCH models outperform all others in all cases, of which the DCC-MGARCH is the best in seven cases while the CCC-MSV is the best in the remaining two.

To test whether the CCC-MSV is the best model in terms of FX volatility forecast performance, we use sign test. The null hypothesis is CCC-MSV model does not have lower MAE of FX volatility forecast than other models. The test result is presented in Table 14.

Table 7
Estimates of CCC-MGARCH(1,1) model for the first subsample.

Parameter	USD/CAD	USD/NOK	USX	USD/EUR	USD/INR	USD/BZR	USD/SGD	USD/MXP	USD/JPY
a_{10}	.0005 (.0032)	.0001 (.0234)	.0000 (.0092)	.0000 (.1050)	.0043 (.0044)	.0035 (.0209)	.0000 (.0012)	.0092 (.0093)	.0065 (.0066)
a_{20}	.0403 (.0134)	.1244 (.0276)	.1336 (.0262)	1.4827 (.0152)	.0802 (.0382)	.0505 (.0367)	.0438 (..0128)	.0766 (.0239)	.0167 (.0192)
a_{11}	.0395 (.0005)	.0206 (.0010)	.0415 (.0002)	.0183 (.0015)	.1377 (.0001)	.1971 (.0013)	.0367 (..0001)	.0828 (.0003)	.0354 (.0008)
a_{12}	.0000 (.0014)	.0005 (.1359)	.0000 (.0111)	.0004 (.0700)	.0000 (.0008)	.0000 (.0055)	.0000 (.0005)	.0000 (.0011)	.0000 (.0022)
a_{21}	.0000 (.0173)	.0000 (1.3357)	.6704 (.8720)	.0002 (.7546)	.0000 (.0462)	.0000 (.0309)	.0000 (.0287)	.0000 (.0459)	.0000 (.0336)
a_{22}	.0190 (.0658)	.0017 (.9595)	.0225 (.9690)	.0257 (6.3202)	.0220 (.1049)	.0193 (.0807)	.0232 (.0419)	.0238 (.0870)	.0167 (.0661)
b_{11}	.9512 (.1209)	.8120 (.2453)	.2549 (1.7835)	.9748 (.4794)	.8449 (.1441)	.7864 (0345)	.9294 (.4298)	.8567 (.3073)	.9407 (.1195)
b_{12}	.0005 (.1951)	.0164 (32.5130)	.0089 (65.09)	.0000 (43.31)	.0000 (.0212)	.0041 (.0409)	.0005 (1.0274)	.0000 (.5890)	.0005 (.2396)
b_{21}	.1154 (.0156)	8.9058 (.0298)	72.1191 (.0249)	10.3855 (.0338)	.0000 (.0193)	.0000 (.0142)	.4704 (.0089)	.0000 (.0205)	.1477 (.0144)
b_{22}	.9638 (.0324)	.0658 (3.3078)	.0195 (.8602)	.0116 (4.0438)	.9596 (.0440)	.9692 (.0314)	.9590 (.0222)	.9587 (.0428)	.9674 (.0317)
ρ	−.2294 (.0304)	−.2282 (.0330)	−.2286 (.0321)	.1930 (.0331)	−.0132 (.0327)	−.1102 (.0318)	−.1162 (.0307)	−.1036 (.0325)	−.0123 (.0347)
loglik	−76617	−583438	−263449	−75515	−32447	−36601	−98751	−41117	−131613

Table 8
Estimates of CCC-MGARCH(1,1) model for the second subsample.

Parameter	USD/CAD	USD/NOK	USX	USD/EUR	USD/INR	USD/BZR	USD/SGD	USD/MXP	USD/JPY
a_{10}	.0199 (.0419)	.0674 (.1239)	.0078 (.0143)	.0440 (.0771)	.2868 (2.0329)	.0269 (.0626)	.0306 (.7045)	.0216 (.0566)	.0172 (.1607)
a_{20}	.0000 (.0338)	.0000 (.0377)	.0000 (.0406)	.0000 (.0416)	.0000 (.0328)	.0000 (.0679)	.0000 (.0554)	.0000 (.1049)	.0000 (.0288)
a_{11}	.0590 (.0019)	.0000 (.0048)	.0464 (.0006)	.0000 (.0031)	.0000 (.0022)	.1405 (.0029)	.0000 (.0010)	.2382 (.0043)	.0000 (.0043)
a_{12}	.0000 (.0031)	.0000 (.0184)	.0000 (.0024)	.0000 (.0147)	.0000 (.0215)	.0001 (.0046)	.0008 (.2606)	.0000 (.0136)	.0000 (.0525)
a_{21}	.1183 (.0840)	.0000 (.1773)	.0743 (.1771)	.0000 (.3002)	.0000 (4.0566)	.0000 (.0597)	.0000 (21.82)	.0000 (.1590)	.0027 (.8315)
a_{22}	.0874 (.4156)	.0862 (.6565)	.0825 (.4043)	.0845 (.7958)	.0631 (46.4368)	.0408 (.1171)	.0671 (18.85)	.0646 (.1045)	.0958 (3.3362)
b_{11}	.8993 (.3276)	.8822 (.1313)	.8362 (1.4045)	.8123 (.4243)	.4351 (.2644)	.8267 (.1181)	.0499 (1.63)	.5688 (.2254)	.3461 (.6117)
b_{12}	.0016 (.8623)	.0089 (.9963)	.0015 (5.4968)	.0075 (3.3168)	.0039 (91.4564)	.0033 (.1603)	.0104 (585.34)	.0159 (.4016)	.0362 (16.8979)
b_{21}	.0014 (.0436)	.0000 (.0447)	.0005 (.0375)	.0000 (.0569)	.0000 (.2712)	.0815 (.0279)	.0000 (.0271)	.0000 (.0285)	13.2097 (.0796)
b_{22}	.8987 (.0456)	.9095 (.1114)	.9117 (.0777)	.9116 (.1805)	.9301 (.7069)	.9357 (.0368)	.9274 (6.9943)	.9277 (.0374)	.1406 (.8436)
ρ	−.4686 (.0540)	−.5059 (.0580)	−.4516 (.0542)	.3671 (.0652)	−.3261 (.0682)	−.4728 (.0495)	−.3863 (.0577)	−.4137 (.0515)	.3598 (.0673)
loglik	−8418	−15476	−21003	−25638	−4301	−9199	−36190	−6493	−183608

Table 9
Estimates of DCC-MGARCH(1,1) model for the first subsample.

Parameter	USD/CAD	USD/NOK	USX	USD/EUR	USD/INR	USD/BZR	USD/SGD	USD/MXP	USD/JPY
a_{10}	.0020 (.0341)	.0249 (.1175)	.0003 (.1343)	.0000 (.0103)	.0004 (.0360)	.0200 (.0612)	.0012 (.2330)	.0102 (.0273)	.0566 (.2826)
a_{20}	1.6921 (.0167)	.0187 (.0285)	4.090 (.0363)	.3795 (.0093)	3.7626 (.0378)	1.0817 (.0379)	4.1685 (.0522)	2.0401 (.0282)	3.2696 (.0335)
a_{11}	.0386 (.0010)	.0010 (.0032)	.0799 (.0004)	.0209 (.0011)	.1356 (.0002)	.2009 (.0020)	.0739 (.0003)	.0975 (.0006)	.0520 (.0029)
a_{12}	.0000 (.0108)	.0017 (.1875)	.0000 (.0479)	.0002 (.0074)	.0000 (.0094)	.0000 (.0160)	.0000 (.0553)	.0000 (.0070)	.0007 (.1146)
a_{21}	.5827 (.0237)	.1952 (1.9559)	1.862 (1.269)	.0076 (.0726)	.0003 (.0584)	.0000 (.0316)	.0003 (.4316)	.0512 (.0553)	1.0179 (.8321)
a_{22}	.0564 (1.4770)	.0034 (.3115)	.0155 (16.34)	.0444 (.4039)	.0321 (16.256)	.0526 (.8779)	.0140 (32.27)	.0628 (1.834)	.0022 (2.118)
b_{11}	.9547 (.4902)	.0462 (.2658)	.1452 (1.814)	.9727 (.3865)	.8412 (.3217)	.7839 (.1044)	.2215 (.7457)	.8168 (.8815)	.2421 (.4820)
b_{12}	.0000 (1.4955)	.0905 (13.67)	.0116 (76.52)	.0001 (3.212)	.0010 (12.25)	.0001 (.1769)	.0120 (11.60)	.0006 (2.584)	.0434 (6.099)
b_{21}	.7150 (.0391)	.3585 (.0197)	.9303 (.0378)	2.4446 (.0380)	2.9473 (.0370)	.0001 (.0412)	1.0307 (.0340)	.8214 (.0405)	2.097 (.0267)
b_{22}	.4621 (.4377)	.9359 (1.343)	.0011 (4.858)	.7236 (.2897)	.0130 (4.122)	.6993 (.2276)	.0054 (7.301)	.4432 (.4509)	.0000 (.6188)
α	.0107 (.0211)	.0086 (.0061)	.0063 (.0068)	.0059 (.0047)	.0000 (.0089)	.0058 (.0054)	.0051 (.0058)	.0131 (.0143)	.0000 (.0084)
β	.8325 (.4316)	.9878 (.0119)	.9898 (.0133)	.9905 (.0106)	.9876 (9196)	.9921 (.0083)	.9899 (.0136)	.9301 (.1058)	.9855 (2115)
loglik	3059	3527	1588	2961	2160	3765	1760	2431	3358

Table 10
Estimates of DCC-MGARCH(1,1) model for the second subsample.

Parameter	USD/CAD	USD/NOK	USX	USD/EUR	USD/INR	USD/BZR	USD/SGD	USD/MXP	USD/JPY
a_{10}	.0201 (.0409)	.1156 (.2840)	.0011 (.0072)	.0168 (.0317)	.0003 (.0071)	.0000 (.0631)	.0001 (.0047)	.0263 (.0688)	.0691 (.1674)
a_{20}	.0017 (.0364)	.0049 (.0436)	.1205 (.0304)	.0192 (.0383)	.0151 (.0235)	.5590 (.0554)	.0508 (.0421)	.0548 (.1039)	.0295 (.0325)
a_{11}	.0602 (.0015)	.0040 (.7409)	.0031 (.0008)	.0000 (.0028)	.0000 (.0008)	.0501 (.0053)	.0002 (.0005)	.1695 (.0034)	.0076 (.0036)
a_{12}	.0000 (.0036)	.0000 (.0717)	.0000 (.0061)	.0000 (.0161)	.0000 (.0018)	.0031 (.0330)	.0000 (.0045)	.0000 (.0145)	.0000 (.0205)
a_{21}	.1515 (.0922)	.0000 (.0065)	.7726 (.3013)	.0064 (.2810)	.0079 (.0628)	.0030 (.2308)	.1012 (.3193)	.0102 (.1854)	.0083 (.4692)
a_{22}	.0455 (.3085)	.0496 (1.4605)	.0557 (.1951)	.0588 (.3415)	.0507 (.1849)	.0817 (.8141)	.0519 (.1236)	.0596 (.1360)	.0703 (1.505)
b_{11}	.8987 (.3477)	.6911 (.1685)	.8192 (1.1913)	.8366 (.4126)	.9719 (.3423)	.9160 (.2933)	.8266 (1.6671)	.6192 (.2441)	.7902 (.2608)
b_{12}	.0017 (.8798)	.0275 (4.0916)	.0030 (10.82)	.0076 (3.124)	.0006 (1.166)	.0002 (2.128)	.0022 (13.12)	.0157 (.5854)	.0072 (4.499)
b_{21}	.0713 (.0267)	.1749 (.0605)	.8949 (.0552)	.3733 (.0506)	.0886 (.0308)	1.6101 (.0662)	.5483 (.0502)	.1148 (.0282)	.1882 (.0489)
b_{22}	.9318 (.0382)	.9243 (.4313)	.9012 (.2512)	.9146 (.1934)	.9395 (.0517)	.6610 (.3396)	.9293 (.2229)	.9209 (.0500)	.9117 (.2032)
α	.0000 (.0142)	.0282 (.0262)	.0309 (.0352)	.0473 (.0366)	.0000 (.0118)	.0000 (.0135)	.0974 (.0663)	.0000 (.0170)	.0014 (.0149)
β	.9794 8216	.9071 (.1206)	.8982 (.1588)	.9101 (.0860)	.9767 (8037)	.9851 (7121)	.3668 (.7169)	.9815 (2894)	.9698 (.4622)
loglik	1487	1601	1120	1419	1288	1655	1046	1515	1454

Table 11
MSV model checking.

Parameter	USD/CAD		USD/NOK		USX		USD/EUR		USD/INR	
	FX	Oil	FX	Oil	FX	Oil	FX	Oil	FX	Oil
CCC-MSV – first subsample										
Mean	−.0128	.0305	−.0082	.0353	−.0238	.0330	.0112	.0330	−.0295	.0336
Std	.9861	.9865	.9992	.9901	.9691	.9909	.9843	.9850	.9910	.9832
Skewness	−.0425	−.1315	.1431	−.1121	.0125	−.1167	.0057	−.1138	.0314	−.0947
Excess Kurtosis	−.0858	.0130	.1185	.0087	.0408	.0402	.0934	.0040	.2700	.0744
Kolmogorov–Smirnov	.0205	.0217	.0199	.021	.0274	.021	.0274	.0226	.0208	.0183
(*p*-Value)	(.3711)	(.2848)	(.4118)	(.3316)	(.0659)	(.3318)	(.0655)	(.2313)	(.3472)	(.5562)
Jarque–Bera	.5873	2.9935	4.1906	2.1738	.1210	2.4347	.4299	2.2323	3.4449	1.820
(*p*-Value)	(.7455)	(.2238)	(.1230)	(.3372)	(.9412)	(.2960)	(.8065)	(.3275)	(.1786)	(.4023)
DIC	5806		6282		4271		5740		4426	
CCC-MSV – second subsample										
Mean	−.0058	−.0067	−.0133	−.0122	−.0092	−.0121	.0188	−.0077	.0218	−.0237
Std	1.0004	.9752	.9774	.9932	.9792	.9714	.9977	.9819	1.0632	1.0030
Skewness	−.1726	−.0325	−.1191	−.0350	−.3016	−.0295	.3950	−.0093	−.0902	−.0248
Excess Kurtosis	.1561	−.3621	.4941	−.3643	.1314	−.3251	.8708	−.2627	2.0300	−.3466
Kolmogorov–Smirnov	.0433	.0333	.0385	.0351	.0771	.032	.0744	.0383	.0441	.0306
(*p*-Value)	(.2775)	(.6896)	(.4596)	(.6117)	(.0007)	(.7467)	(.0014)	(.4674)	(.2529)	(.8025)
Jarque–Bera	1.6532	1.3057	3.5521	1.3301	4.2345	1.0381	15.57	.6328	46.55	1.1741
(*p*-Value)	(.4375)	(.5205)	(.1692)	(.5142)	(.1203)	(.5950)	(.0004)	(.7287)	(.0000)	(.5559)
DIC	2157		2277		1806		2118		1935	
DCC-MSV – first subsample										
Mean	−.0105	.0330	−.0085	.0333	−.0176	.0265	.0094	.0336	−.0306	.0325
Std	.9812	.9748	.9993	1.0051	.8149	.9609	.9914	.9859	.9910	.9762
Skewness	−.0314	−.1146	.1375	−.1157	−.0030	−.1334	−.0088	−.1295	.0216	−.1175
Excess Kurtosis	−.1326	−.0447	.1884	.1991	.3707	.3305	.04909	−.0220	.2614	.0269
Kolmogorov–Smirnov	.0176	.0206	.0229	.0198	.0311	.0229	.0287	.0234	.0205	.0225
(*p*-Value)	(.6158)	(.3636)	(.2141)	(.4250)	(.0200)	(.2110)	(.0448)	(.1851)	(.3692)	(.2365)
Jarque–Bera	.8615	2.3273	4.8800	4.1115	6.1047	7.9352	.1432	2.8987	3.1542	2.4243
(*p*-Value)	(.6500)	(.3123)	(.0871)	(.1279)	(.0472	(.0189)	(.9308)	(.2347)	(.2065)	(.2975)
DIC	5785		6287		4426		5703		4388	
DCC-MSV – second subsample										
Mean	−.0009	−.0107	−.0132	−.0126	.0043	−.0149	.0178	−.0169	.0173	−.0209
Std	.9978	.9780	.9623	.9311	.8974	.9068	.9966	1.0054	.9593	.8275
Skewness	−.1764	.0022	−.2156	.0184	−.3013	.0448	.3418	−.0214	−.0812	.0261
Excess Kurtosis	.2243	−.3113	.7385	−.2770	.2737	−.2118	.5136	−.3672	1.6905	−.1131
Kolmogorov–Smirnov	.0402	.0299	.0350	.0297	.0619	.0346	.0725	.0363	.0557	.0320
(*p*-Value)	(.3911)	(.8303)	(.6164)	(.8360)	(.0179)	(.6341)	(.0021)	(.5568)	(.0499)	(.7466)
Jarque–Bera	2.0298	.9105	8.3892	.7213	4.9312	.4779	8.260	1.3177	32.47	.1185
(*p*-Value)	(.3624)	(.6342)	(.0150)	(.6972)	(.0849)	(.7874)	(.0160)	(.5174)	(.0000)	(.9424)
DIC	2149		2251		1804		2093		1976	

Parameter	USD/BZR		USD/SGD		USD/MXP		USD/JPY	
	FX	Oil	FX	Oil	FX	Oil	FX	Oil
CCC-MSV – first subsample								
Mean	−.0556	.0334	−.0200	.0316	−.0815	.0376	.0414	.0312
Std	.9709	.9769	.9674	.9782	.9963	.9844	.9885	.9842
Skewness	.2591	−.1237	−.1093	−.1097	.2584	−.1070	−.3324	−.1045
Excess Kurtosis	−.4432	.0863	.4559	.0902	−.2809	−.0423	.3222	.0161
Kolmogorov–Smirnov	.0365	.0211	.0177	.0206	.0345	.0207	.0282	.0229
(*p*-Value)	(.0025)	(.3255)	(.6028)	(.3569)	(.0058)	(.3528)	(.0525)	(.2121)
Jarque–Bera	19.78	2.9985	11.233	2.4708	14.74	2.028	23.64	1.9020
(*p*-Value)	(.0000)	(.2232)	(.0036)	(.2907)	(.0000)	(.3625)	(.0000)	(.3863)
DIC	6455		4432		5172		6066	
CCC-MSV – second subsample								
Mean	−.0430	−.0080	−.0035	−.0102	.0450	−.0175	−.0625	−.0073
Std	.9760	.9580	.9958	.9739	.9717	.9710	1.0381	.9833
Skewness	.0148	.0076	−.4191	−.0134	.0745	.0554	−.3738	−.0231
Excess Kurtosis	−.5021	−.4320	.6706	−.3323	−.3035	−.3157	.8928	−.4102
Kolmogorov–Smirnov	.029	.0266	.0466	.0322	.0305	.0354	.0492	.0317
(*p*-Value)	(.8583)	(.926)	(.1851)	(.7408)	(.8082)	(.5955)	(.1317)	(.7587)
Jarque–Bera	2.5250	1.8352	12.94	1.0558	1.1047	1.0728	15.29	1.6654
(*p*-Value)	(.2829)	(.3994)	(.0015)	(.5898)	(.5755)	(.5848)	(.0000)	(.4348)
DIC	2306		1739		2169		2098	
DCC-MSV – first subsample								
Mean	−.0555	.0364	−.0199	.0344	−.0795	.0386	.0417	.0334
Std	.9688	.9883	.9807	.9806	.9676	.9815	.9783	.9877
Skewness	.2565	−.1230	−.0687	−.1120	.2571	.−0961	−.3026	−.1113
Excess Kurtosis	−.4504	−.0160	.2394	−.0352	−.3365	−.0684	.2881	−.0112
Kolmogorov–Smirnov	.0388	.0254	.0189	.0224	.0374	.0224	.0281	.0249
(*p*-Value)	(.0009)	(.1118)	(.4953)	(.2421)	(.0017)	(.2396)	(.0536)	(.1275)
Jarque–Bera	19.83	2.6095	3.4038	2.1985	16.08	1.7574	19.47	2.1336
(*p*-Value)	(.0000)	(.2712)	(.1823)	(.3331)	(.0003)	(.4153)	(.0000)	(.3440)

Table 11 (*Continued.*)

Parameter	USD/BZR		USD/SGD		USD/MXP		USD/JPY	
	FX	Oil	FX	Oil	FX	Oil	FX	Oil
DIC	6399		4344		5135		5992	
DCC-MSV – second subsample								
Mean	−.0437	−.0046	.0006	−.0131	.0429	−.0120	−.0647	−.0100
Std	.9816	1.0042	.9885	.9649	.9642	.9883	1.0186	.9632
Skewness	.0140	.0082	−.3957	−.0066	.0691	.0487	−.3538	−.0102
Excess Kurtosis	−.4979	−.3463	.6746	−.3824	−.3381	−.3325	.6837	−.3689
Kolmogorov–Smirnov	.0310	.0269	.0483	.0320	.0261	.0366	.0507	.0318
(*p*-Value)	(.7884)	(.9206)	(.1485)	(.7471)	(.9383)	(.5412)	(.1074)	(.7578)
Jarque–Bera	2.4798	1.1477	12.1757	1.4172	1.2969	1.1533	10.94	1.3153
(*p*-Value)	(.2894)	(.5633)	(.0022)	(.4923)	(.5228)	(.5617)	(.0041)	(.5180)
DIC	2302		1719		2164		2073	

Table 12
MGARCH model checking.

Parameter	USD/CAD		USD/NOK		USX		USD/EUR		USD/INR	
	FX	Oil	FX	Oil	FX	Oil	FX	Oil	FX	Oil
CCC-MGARCH – first subsample										
Mean	−.0150	.0228	−.0163	.0244	−.0257	.0242	.0155	.0266	−.0273	.0235
Std	1.0018	1.0010	1.0039	.9978	1.0263	.9662	1.0048	1.0002	.9967	1.0002
Skewness	−.0647	−.2551	.1643	−.2178	.0489	−.1438	.0203	−.1883	−.1933	−.2891
Excess Kurtosis	.4003	1.4041	.4597	1.1717	.7217	.8549	.5285	1.0905	5.2721	1.7930
Kolmogorov–Smirnov	.0284	.0296	.0315	.0297	.0405	.035	.042	.0322	.0797	.0329
(*p*-Value)	(.3787)	(.3288)	(.2595)	(.3231)	(.0683)	(.1608)	(.0532)	(.2363)	(.0000)	(.2143)
Jarque–Bera	7.8282	96.8666	13.99	67.91	23.22	35.48	12.3736	57.8994	1206	153.81
(*p*-Value)	(.0199)	(.0000)	(.0009)	(.0000)	(.0000)	(.0000)	(.0020)	(.0000)	(.0000)	(.0000)
CCC-MGARCH – second subsample										
Mean	−.0034	−.0179	.0102	−.0209	00727	−.0205	.0124	−.0206	.0245	−.0236
Std	1.0002	1.0169	.9992	1.0120	.9957	1.0117	.9965	1.0084	1.0049	1.0151
Skewness	−.2569	.1111	−.1389	.1004	−.4027	.1165	.4149	.0947	−.2262	.0849
Excess Kurtosis	.6811	−.0670	.8578	−.1237	.5345	−.0916	.7714	−.1167	4.3799	−.1558
Kolmogorov–Smirnov	.0662	.036	.0455	.0416	.0687	.0397	.0804	.0442	.0775	.0344
(*p*-Value)	(.2974)	(.9409)	(.7592)	(.8469)	(.2578)	(.8846)	(.1208)	(.7896)	(.1472)	(.9594)
Jarque–Bera	7.0477	.4673	7.9290	.4552	8.8584	.5355	12.1958	.4024	180.79	.4160
(*p*-Value)	(.0294)	(.7916)	(.0189)	(.7964)	(.0119)	(.7650)	(.0022)	(.8177)	(.0000)	(.8121)
DCC-MGARCH – first subsample										
Mean	−.0163	.0235	−.0087	.0252	−.0178	.0249	.0108	.0264	−.0240	.0250
Std	.9990	1.0030	.9956	1.0018	.9992	1.0021	1.0025	.9961	.9999	.9972
Skewness	−.0655	−.2634	.1587	.2172	.0614	.2054	.0124	−.2457	−.1947	−.2460
Excess Kurtosis	.4089	1.3892	.4547	1.2128	.9744	1.5335	.5241	1.2344	5.2891	1.8327
Kolmogorov–Smirnov	.0283	.03	.0293	.0312	.0453	.0317	.0393	.0319	.0786	.0329
(*p*-Value)	(.3809)	(.3123)	(.3386)	(.2686)	(.0292)	(.2518)	(.0826)	(.2446)	(.0000)	(.2146)
Jarque–Bera	8.1535	95.79	13.4770	72.11	42.05	109.3	12.1281	76.66	1214	156
(*p*-Value)	(.0169)	(.0000)	(.0011)	(.0000)	(.0000)	(.0000)	(.0023)	(.0000)	(.0000)	(.0000)
DCC-MGARCH – second subsample										
Mean	.0049	−.0229	−.0137	−.0283	−.0030	−.0250	.0240	−.0278	.0136	−.0296
Std	.9946	1.0209	.9956	1.0018	.9993	1.0187	.9988	1.0161	1.0239	1.0260
Skewness	−.1416	.0946	−.1392	.0800	−.2870	.1136	.3820	.0586	−.1693	.0461
Excess Kurtosis	.6175	.0467	.6673	.0134	.2968	.1040	.7325	.0019	3.6782	−.0279
Kolmogorov–Smirnov	.0511	.0389	.0364	.0372	.0771	.0385	.0684	.041	.0694	.0377
(*p*-Value)	(.5064)	(.8260)	(.882)	(.8642)	(.0910)	(.8352)	(.175)	(.7741)	(.1632)	(.8528)
Jarque–Bera	5.3939	.4465	6.0934	.2956	4.737	.7443	12.681	.1578	151.9	.0934
(*p*-Value)	(.0674)	(.7999)	(.0475)	(.8625)	(.0936)	(.6892)	(.0017)	(.9241)	(.0000)	(.9543)

Parameter	USD/BZR		USD/SGD		USD/MXP		USD/JPY	
	FX	Oil	FX	Oil	FX	Oil	FX	Oil
CCC-MGARCH before break								
Mean	−.0340	.0236	−.0251	.0241	−.0638	.0252	.0143	.0222
Std	.9984	.9994	.9975	.9992	.9970	1.0002	.9988	1.0011
Skewness	.4708	−.2600	−.4439	−.2882	.3927	−.2824	−.6211	−.2695
Excess Kurtosis	.7239	1.5993	3.6367	1.6713	.3838	1.7688	2.0719	1.4378
Kolmogorov–Smirnov	.0557	.0308	.0397	.0303	.0632	.0347	.0476	.0338
(*p*-Value)	(.0033)	(.2826)	(.0782)	(.3013)	(.0005)	(.1681)	(.0186)	(.1889)
Jarque–Bera	61.11	122.6	605.56	135.49	33.10	149.4	252.4	102.3
(*p*-Value)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)

Table 12 (*Continued*).

Parameter	USD/BZR		USD/SGD		USD/MXP		USD/JPY	
	FX	Oil	FX	Oil	FX	Oil	FX	Oil
CCC-MGARCH after break								
Mean	.0066	−.0157	.0057	−.0226	.0633	−.0258	−.0441	−.0274
Std	1.0131	1.0263	1.0029	1.0075	1.0029	1.0236	1.0171	1.0115
Skewness	.2875	.1477	−.3386	.1076	.1786	.1293	−.6181	.0991
Excess Kurtosis	.4322	−.0194	1.0027	−.1092	.8425	−.0944	1.9652	−.0480
Kolmogorov–Smirnov	.0431	.0398	.0608	.0346	.062	.0419	.0734	.034
(*p*-Value)	(.814)	(.8812)	(.3995)	(.9571)	(.3746)	(.8403)	(.1931)	(.9637)
Jarque–Bera	4.9796	.8012	13.9836	.4863	8.1435	.6549	50.59	.3640
(*p*-Value)	(.0829)	(.6698)	(.0000)	(.7841)	(.0170)	(.7207)	(.0000)	(.8335)
DCC-MGARCH before break								
Mean	−.0305	.0246	−.0134	.0247	−.0666	.0268	.0258	.0216
Std	.9993	.9967	1.0002	1.0013	1.0006	.9937	1.0002	1.0008
Skewness	.4728	−.2976	−.4412	−.2127	.3842	−.2715	−.4619	−.2632
Excess Kurtosis	.7342	1.702	4.6448	1.6006	.3847	1.5521	1.7359	1.7439
Kolmogorov–Smirnov	.0533	.0325	.0376	.0336	.0631	.0358	.0555	.036
(*p*-Value)	(.0057)	(.2265)	(.1082)	(.1963)	(.0005)	(.1425)	(.0034)	(.1383)
Jarque–Bera	62.10	140.9	965.3	118.9	31.98	117.2	167.4	143.8
(*p*-Value)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)	(.0000)
DCC-MGARCH after break								
Mean	−.0147	−.0089	−.0040	−.0285	.0562	−.0288	−.0624	−.0246
Std	1.0379	1.0079	1.0180	1.0323	1.0005	1.0203	1.0010	1.0190
Skewness	.2485	.1024	−.3257	.0649	.3455	.1025	−.5500	.0360
Excess Kurtosis	.5886	.2358	.8344	−.0083	1.1735	.0707	2.0038	−.0948
Kolmogorov–Smirnov	.0384	.0321	.0543	.0436	.0543	.0419	.0849	.039
(*p*-Value)	(.8389)	(.9519)	(.4267)	(.7066)	(.4263)	(.7507)	(.0472)	(.8234)
Jarque–Bera	6.8257	1.197	12.74	.1875	20.98	.5574	58.45	.1137
(*p*-Value)	(.0329)	(.5495)	(.0017)	(.9104)	(.0000)	(.7567)	(.0000)	(.9447)

Table 13
Model forecast performance.

FX	Model	FX				Oil			
		MAE		RMSE		MAE		RMSE	
		Value	Rank	Value	Rank	Value	Rank	Value	Rank
USD/CAD	DCC-MSV	.5331	1	.7789	3	6.9062	6	8.8525	5
	CCC-MSV	.5339	2	.7783	2	6.4557	4	8.7221	4
	SV	.5527	3	.7778	1	6.8218	5	8.9209	6
	DCC-MGARCH	1.0923	5	1.3509	5	5.1254	2	5.5425	1
	CCC-MGARCH	1.0100	4	1.2168	4	5.0883	1	5.6469	2
	GARCH(1,1)	1.1288	6	1.3929	6	5.7320	3	6.0316	3
USD/NOK	DCC-MSV	.6464	2	.7930	2	6.1100	5	8.0287	4
	CCC-MSV	.6220	1	.7815	1	5.6021	3	8.0419	5
	SV	.6782	3	.8072	3	6.8218	6	8.9209	6
	DCC-MGARCH	1.2822	6	1.4776	5	4.8126	1	5.2398	1
	CCC-MGARCH	1.2475	5	1.4812	6	5.1557	2	5.6573	2
	GARCH(1,1)	1.1573	4	1.3589	4	5.7320	4	6.0316	3
USD/MXP	DCC-MSV	.4796	2	.6028	2	6.8178	4	8.6479	4
	CCC-MSV	.4773	1	.6019	1	6.9043	6	8.6888	5
	SV	.5248	3	.6210	3	6.8218	5	8.9209	6
	DCC-MGARCH	.9801	5	1.1071	5	4.7953	1	5.2699	1
	CCC-MGARCH	.8057	4	.9472	4	5.0522	2	5.5700	2
	GARCH(1,1)	1.3554	6	1.4779	6	5.7320	3	6.0316	3
USD/BZR	DCC-MSV	.5276	2	.6311	2	6.3586	4	8.6691	4
	CCC-MSV	.4716	1	.6127	1	6.5108	5	8.6725	5
	SV	.5754	3	.6553	3	6.8218	6	8.9209	6
	DCC-MGARCH	.8803	4	1.0588	4	4.3365	1	4.8238	1
	CCC-MGARCH	1.0471	6	1.2465	6	4.6545	2	5.2074	2
	GARCH(1,1)	.9153	5	1.0972	5	5.7320	3	6.0316	3

Table 13 (*Continued*)

FX	Model	FX				Oil			
		MAE		RMSE		MAE		RMSE	
		Value	Rank	Value	Rank	Value	Rank	Value	Rank
USD/JPY	DCC-MSV	.5680	2	.6740	2	6.4805	4	7.8535	5
	CCC-MSV	.5648	1	.6727	1	6.1045	3	7.5873	4
	SV	.6849	6	.7518	4	6.8218	6	8.9209	6
	DCC-MGARCH	.6673	5	.7625	5	4.9359	1	5.3432	1
	CCC-MGARCH	.6414	4	.8244	6	6.7993	5	7.2164	3
	GARCH(1,1)	.6295	3	.7223	3	5.7320	2	6.0316	2
USD/INR	DCC-MSV	.2674	2	.4185	2	8.1506	6	9.9144	6
	CCC-MSV	.2546	1	.4271	3	6.1690	4	8.8319	4
	SV	.3065	3	.4183	1	6.8218	5	8.9209	5
	DCC-MGARCH	.5833	4	.6903	4	4.4888	1	4.9357	1
	CCC-MGARCH	.6694	5	.7949	5	5.1327	2	5.6585	2
	GARCH(1,1)	.7900	6	.9265	6	5.7320	3	6.0316	3
USD/SGD	DCC-MSV	.1470	2	.2180	2	6.5686	5	8.5301	5
	CCC-MSV	.1460	1	.2196	3	6.1999	4	8.4301	4
	SV	.1486	3	.2166	1	6.8218	6	8.9209	6
	DCC-MGARCH	.3414	6	.4367	6	4.6055	1	5.0588	1
	CCC-MGARCH	.2643	4	.3515	4	5.1902	2	5.7099	2
	GARCH(1,1)	.3308	5	.4205	5	5.7320	3	6.0316	3
USD/Index	DCC-MSV	.1419	3	.1592	3	7.7202	6	9.0946	6
	CCC-MSV	.1089	1	.1446	1	5.6129	3	8.0055	4
	SV	.1243	2	.1495	2	6.8218	5	8.9209	5
	DCC-MGARCH	.3435	6	.4326	6	5.3288	2	5.7241	1
	CCC-MGARCH	.3204	4	.4025	4	5.2496	1	5.7510	2
	GARCH(1,1)	.3262	5	.4056	5	5.7320	4	6.0316	3
USD/EUR	DCC-MSV	.1687	1	.1932	1	5.1307	2	7.7249	5
	CCC-MSV	.1877	2	.2041	2	5.4073	4	7.7041	4
	SV	.1927	3	.2150	3	6.8218	6	8.9209	6
	DCC-MGARCH	.4404	5	.6071	6	4.6953	1	5.1461	1
	CCC-MGARCH	.5523	6	.7016	4	5.2660	3	5.7837	2
	GARCH(1,1)	.4147	4	.5647	4	5.7320	5	6.0316	3

Table 14
Sign tests for out-of-sample forecast performance.

	DCC-MSV	CCC-MSV	SV	CCC-MGARCH	DCC-MGARCH	GARCH(1,1)
Null hypothesis: CCC-MSV model does not have lower mean average error of FX volatility forecast than other models						
Statistics	2	–	0	0	0	0
p-Value	(.08984)	–	(.0019)	(.0019)	(.0019)	(.0019)
Null hypothesis: DCC-MGARCH model does not have lower mean average error of oil volatility forecast than other models						
Statistics	0	0	0	2	–	0
p-Value	(.0019)	(.0019)	(.0019)	(.08984)	–	(.0019)

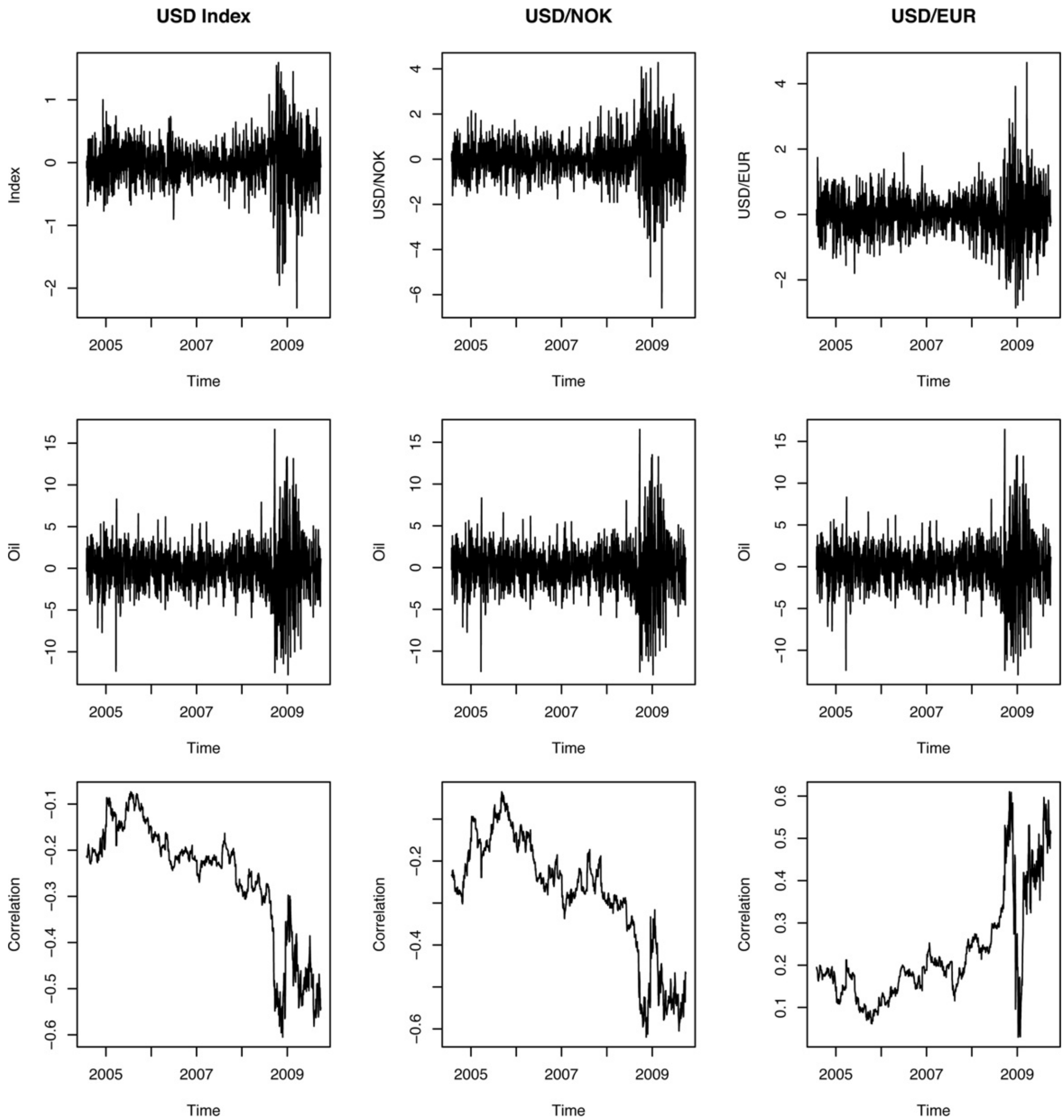

Fig. 10. Estimated time-varying correlation between oil and USD index, USD/NOK and USD/EUR.

There is no evidence that the CCC-MSV model outperforms the DCC-MSV. However, with .2% level of significance, the CCC-MSV outperforms others. For oil volatility forecast, Table 14 confirms that there is no evidence that the DCC-GARCH model outperforms the CCC-GARCH counterpart but with .2% level of significance, the DCC-GARCH outperforms others.

7. Conclusion

This paper studies the volatility interaction between the oil market and the FX market in an attempt to extract information intertwined in the two for better volatility forecasting. We employ the multivariate stochastic volatility as well as the multivariate GARCH framework, allowing the volatility in one market to Granger-cause that in the other. To account for structural breaks in the series, we first identify the break point and then use it to divide the time series into sub-samples and model each separately. We show that during normal time the market is quite efficient in processing information. Both oil and FX markets respond to shocks almost simultaneously. However, in turbulent time, there is bidirectional volatility interaction between the two. In other words, innovations that hit one market also have some impact on the other at a later date and thus using such a dependence significantly improves the forecasting power of volatility models. As a result, the MSV and the MGARCH outperform their univariate counterparts in forecasting volatility in turbulent times. In addition, we find that the MSV models outperform the MGARCH ones in forecasting FX volatility. However, the MGARCH models do a better job in forecasting oil volatility.

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References

Akaike, H. (1973). Information theory and an extension of the maximum likelihood principle. In B. N. Petrov, & F. Csaki (Eds.), *Proceedings of the second international symposium on information theory* Budapest, (pp. 267–281). Akademiai Kiado.

Akgiray, V. (1989). Conditional heteroscedasticity in time series of stock returns: Evidence and forecast. *Journal of Business*, *62*, 55–80.

Amano, R. A., & Norden, S. (1998). Oil prices and the rise and fall of the US real exchange rate. *Journal of International Money and Finance*, *17*, 299–316.

Andersen, T., & Sorensen, B. (1996). GMM estimation of a stochastic volatility model: A Monte Carlo study. *Journal of Business and Economic Statistics*, *14*, 329–352.

Asai, M., McAleer, M., & Yu, J. (2006). Multivariate stochastic volatility: A review. *Econometric Review*, *25*, 145–175.

Ayadi, F. O. (2005). Oil price fluctuations and the Nigerian economy. *OPEC Review*, *29*, 199–217.

Bai, J., & Perron, P. (2003). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics*, *18*, 1–22.

Bauwens, L., Laurent, S., & Rombouts, J. V. K. (2006). Multivariate GARCH models: A survey. *Journal of Applied Econometrics*, *21*, 79–109.

Benassy-Querea, A., Mignonb, V., & Penot, A. (2007). China and the relationship between the oil price and the dollar. *Energy Policy*, *35*, 5795–5805.

Berg, A., Myer, R., & Yu, J. (2004). Deviance information criterion for comparing stochastic volatility models. *Journal of Business and Economic Statistics*, *22*, 107–120.

Bollerslev, T. (1990). Modeling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *Review of Economics and Statistics*, *72*, 498–505.

Chan, K., Tse, Y., & Williams, M. (2011). The relationship between commodity prices and currency exchange rates: Evidence from the futures markets. *NBER Book Commodity Prices and Markets, East Asia Seminar on Economics*, *20*.

Chaudhuri, K., & Daniel, B. C. (1998). Long-run equilibrium real exchange rates and oil prices. *Economics Letters*, *58*, 231–238.

Chen, S. S., & Chen, H. C. (2007). Oil prices and real exchange rates. *Energy Economics*, *29*, 390–404.

Chen, Y. C., Rogoff, K., & Rossi, B. (2008). Can exchange rates forecast commodity prices? NBER Working Paper No. 13901.

Christodoulakis, G. A., & Satchell, S. E. (2002). Correlated ARCH (CorrARCH): Modelling the time-varying conditional correlation between financial asset returns. *European Journal of Operational Research*, *139*, 351–370.

Clark, P. (1973). A subordinate stochastic process model with finite variance for speculative prices. *Econometrica*, *41*, 135–155.

Coudert, V., Mignon, V., & Penot, A. (2007). Oil price and the dollar. *Energy Studies Review*, *15*, 48–65.

Danielsson, J. (1994). Stochastic volatility in asset prices: Estimation with simulated maximum likelihood. *Journal of Econometrics*, *64*, 375–400.

Ding, Z., Granger, C. W. J., & Engle, R. F. (1993). A long memory of stock returns and a new model. *Journal of Empirical Finance*, *1*, 83–106.

Durbin, J., & Koopman, S. (1997). Monte Carlo maximum likelihood estimation for non-Gaussian state space models. *Biometrika*, *84*, 669–684.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics*, *20*(3), 339–350.

Gallant, A. R., Hsieh, D., & Tauchen, G. (1997). Estimation of stochastic volatility models with diagnostics. *Journal of Econometrics*, *81*, 159–192.

Gammerman, D. (1997). *Markov Chain Monte Carlo: Stochastic simulation for Bayesian inference*. Chapman & Hall.

Geman, S., & Geman, D. (1984). Stochastic relaxation, Gibbs distributions, and the Bayesian restoration images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, *6*, 721–741.

Geweke, J. (2005). *Contemporary Bayesian econometrics and statistics*. Wiley & Sons.

Groen, J. J. J., & Pesenti, P. A. (2010). Commodity prices, commodity currencies and global economic developments. NBER Working Paper 15743.

Harvey, A. C., Ruiz, E., & Shephard, N. (1994). Multivariate stochastic variance models. *Review of Economic Studies*, *61*, 247–264.

Hastings, W. K. (1970). Monte Carlo sampling methods using Markov chains and their applications. *Biometrika*, *57*(1), 97–109.

Jacquier, E., Polson, N. G., & Rossi, P. E. (1994). Bayesian analysis of stochastic volatility models. *Journal of Business and Economic Statistics*, *12*, 371–389.

Jeantheau, T. (1998). Strong consistency of estimators for multivariate ARCH models. *Econometric Theory*, *14*, 70–86.

Johannes, M., & Polson, N. (2006). MCMC methods for continuous-time financial econometrics. In Y. Aït-Sahalia, & L. Hansen (Eds.), *Handbook of financial econometrics*. Elsevier.

Kim, S., Shephard, N., & Chib, S. (1998). Stochastic volatility: Likelihood inference and comparison with ARCH models. *Review of Economic Studies*, *65*, 363–393.

Krichene, N. (2005). A simultaneous equations model for world crude oil and natural gas markets. IMF Working Paper.

Krugman, P. R. (1984). Oil shocks and exchange rate dynamics. In J. A. Frankel (Ed.), *Exchange Rates and International Macroeconomics.*. University of Chicago Press.

Kuan, C. M., & Hornik, K. (1995). The generalized fluctuation test: A unifying view. *Econometric Review*, *14*, 135–161.

Melino, A., & Turnbull, S. M. (1990). Pricing foreign currency options with stochastic volatility. *Journal of Econometrics*, *45*, 239–265.

Merton, R. (1980). On estimating the expected return on the market: An explanatory investigation. *Journal of Financial Economics*, *8*, 323–361.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. (1953). Equations of state calculations by fast computing machines. *Journal of Chemical Physics*, *21*(6), 1087–1092.

Ntzoufras, I. (2009). *Bayesian modeling using WINBUGS*. Wiley & Sons.

Perry, P. (1982). The time-variance relationship of security returns: Implications for the return-generating stochastic process. *Journal of Finance*, *37*, 857–870.

Razgallah, B., & Smimou, K. (2011). Oil prices and the greenback: It takes two to tango. *Applied Financial Economics*, *21*, 519–528.

Ross, S. (1989). Information and volatility: The no-arbitrage martingale approach to timing and resolution irrelevancy. *Journal of Finance*, *44*, 1–17.

Sandmann, G., & Koopman, S. J. (1998). Estimation of stochastic volatility models via Monte Carlo maximum likelihood. *Journal of Econometrics*, *87*, 271–301.

Schwarz, G. (1978). Estimating the dimension of a model. *Annals of Statistics*, *6*, 461–464.

Sorensen, M. (2000). Prediction-based estimating functions. *Econometrics Journal*, *3*, 123–147.

Spiegelhalter, D. J., Best, N. G., Carlin, B. P., & der Linde, A. V. (2002). Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society*, *64*, 583–639.

Tauchen, G., & Pitts, M. (1983). The price variability-volume relationship on speculative markets. *Econometrica*, *51*, 485–505.

Tierney, L. (1994). Markov chains for exploring posterior distributions. *Annals of Statistics*, *22*(4), 1701–1762.

Yousefi, A., & Wirjanto, T. S. (2004). The empirical role of the exchange rate on the crude-oil price formation. *Energy Economics*, *26*, 783–799.

Zhang, Y. J., Fan, Y., Tsai, H. T., & Wei, Y. M. (2008). Spillover effect of US dollar exchange rate on oil prices. *Journal of Policy Modeling*, *30*, 973–991.