STAT1005 Week 1 Cheat Sheet

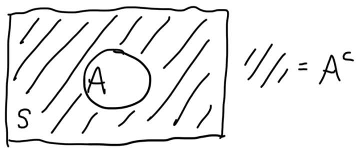
De Mere's Problem

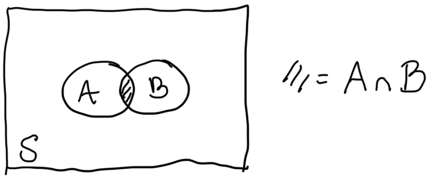
- Is the problem where De Mere believed in a gambling game where you throw two pairs of six sided dice 24 times, if you always bet on double six's you would make money.
    - In throwing a pair of dice you have 36 (6x6) possible outcomes {(1,1), (1,2), ..., (6,5), (6,6)}
    - The probability of throwing a pair of 6's is 1/36, so the probability of not throwing a pair of 6's is 35/36
    - The probability of throwing a pair of 6's from 24 throws is (35/36)^24 = 0.5086
    - So the probability of at least one double six is 1 - 0.5086 = 0.4914
    - Therefore he would lose money

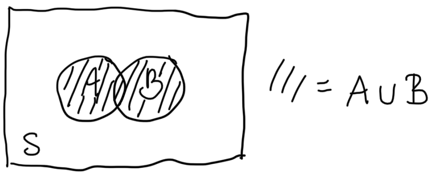
Sample Space's and events

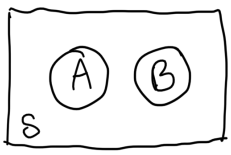
- The sample space is all possible outcomes, denoted by S
    - So for a single product that is either defective (D) or not (N), the sample space is S = {D, N}
- An event is a subset of outcomes within the sample space S.
    - An event is simple if it consists of one outcome, and compound if it consists of more than one outcome

Operations in Set Theory

- The complement of an event A, is the set of all outcomes in S that are not denoted in A, denoted as A^c (or A')
    - 

- The intersection of two events A and B, is the event consisting of all outcomes that are in both A and B, denoted as $A \cap B$
    - 

- The union of two event A and B, is the event consisting of all outcomes that are either in A, B or both, denoted as $A \cup B$
    - 

- When A and B events have no outcomes in common, they are said to be disjoint or mutually exclusive events. This is denoted as $A \cap B = \emptyset$, where $\emptyset$ denotes the event consisting of no outcomes (a null or empty event).
    - 

De Morgan's Laws

- $(A \cup B)^c = A^c \cap B^c$
- $(A \cap B)^c = A^c \cup B^c$

Naive Definition of Probability

- Is based on the assumption that all outcomes are equally likely
  - Let A be an event for an experiment with a finite sample space S. The naive probability of A is:
    - $P\ \ (A) = \frac{|A|}{|S|} = \frac{\text{number of outcomes favourable to A}}{\text{total number of outcomes in S}}$
      - (We use |A| to denote the size of A (number of outcomes in A))
- The naive definition wont work when the outcomes are not equally likely to occur
- The naive model is used when:
  - There is symmetry:
    - Things are equally likely (coin with equal sides, a deck of cards with equally sized/shaped cards
  - Outcomes are equally likely by design:
    - A simple random sample of n people from a very large population ensures that all subsets of size n are equally likely
  - It is a null model:
    - It is assumed the naive definition applies to see what predictions it yields, then compare to observed data to assess whether a hypothesis of equally likely outcomes is tenable, for things like a contingency table