Decision Trees and Random Forests

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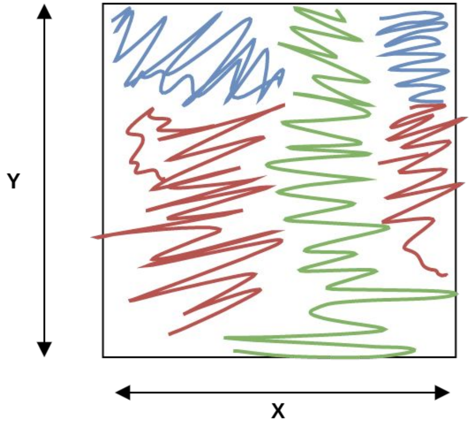

Perfect segmentation

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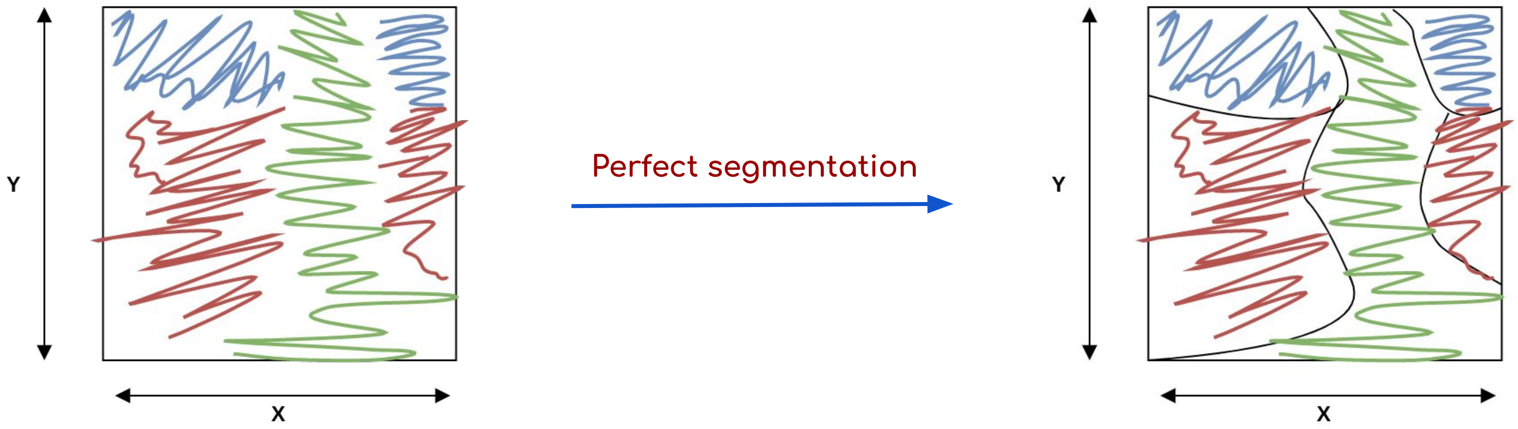

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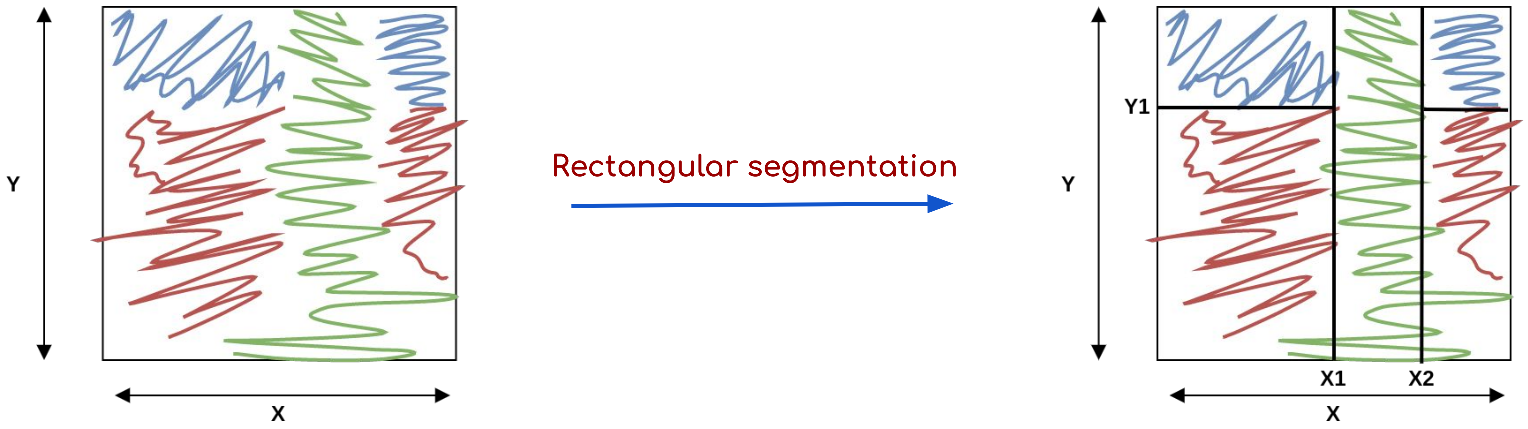

Decision Trees

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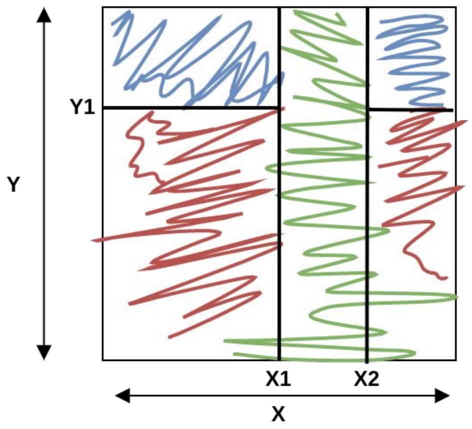
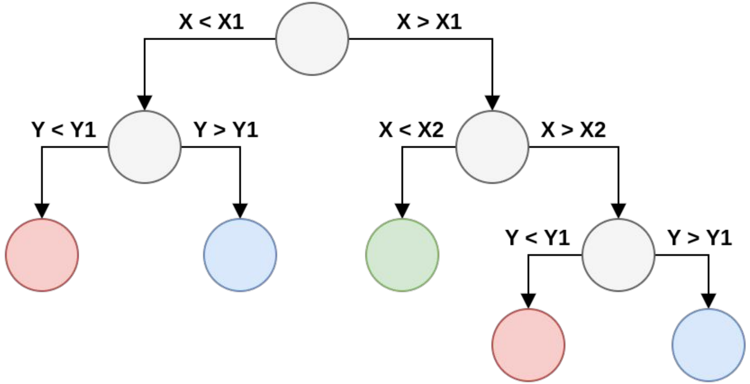
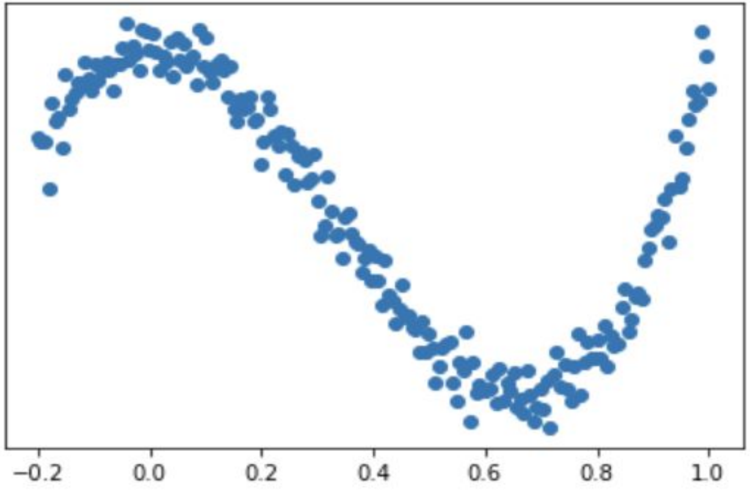


Equivalent Decision Tree



Regression example



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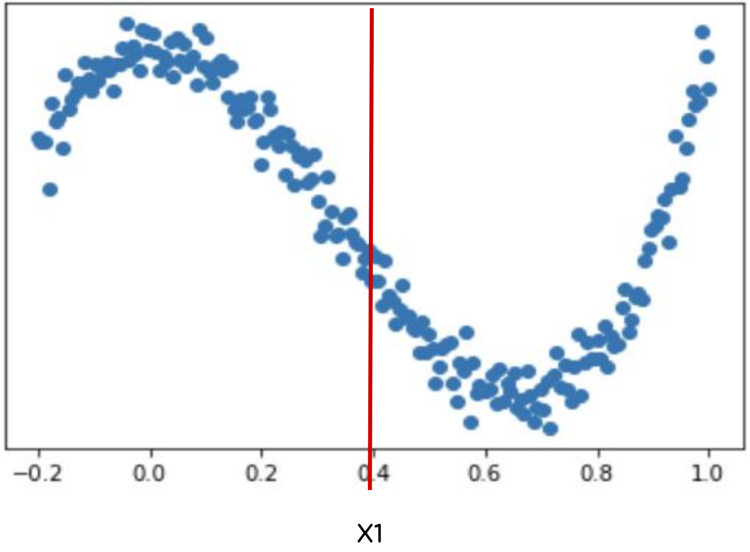

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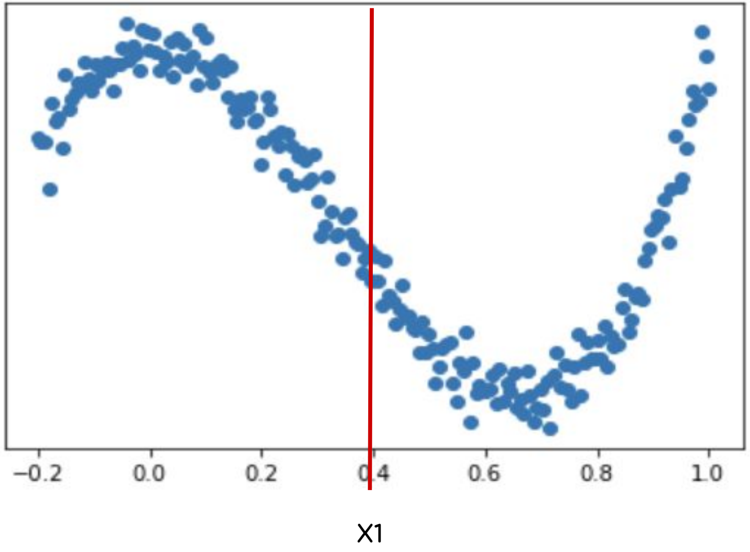

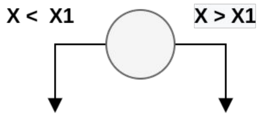

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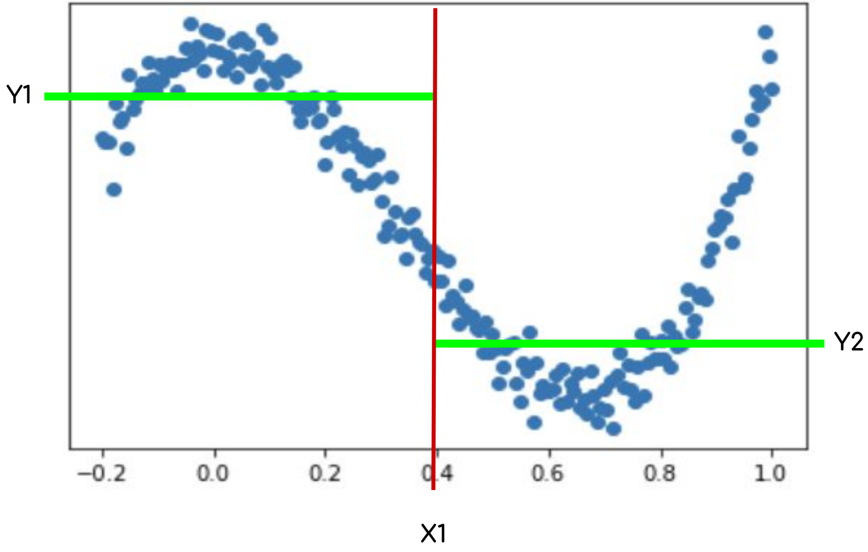

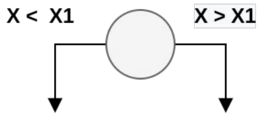

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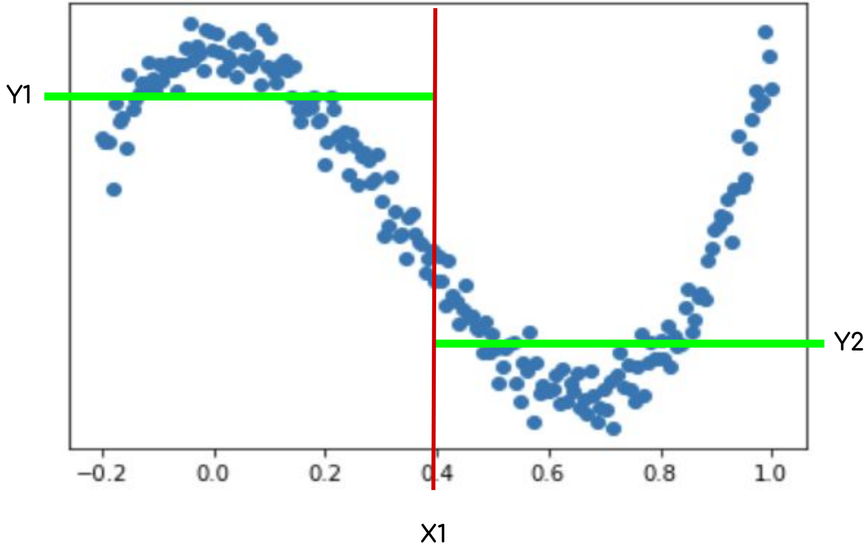

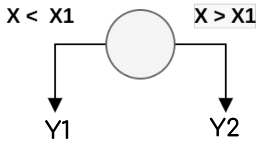

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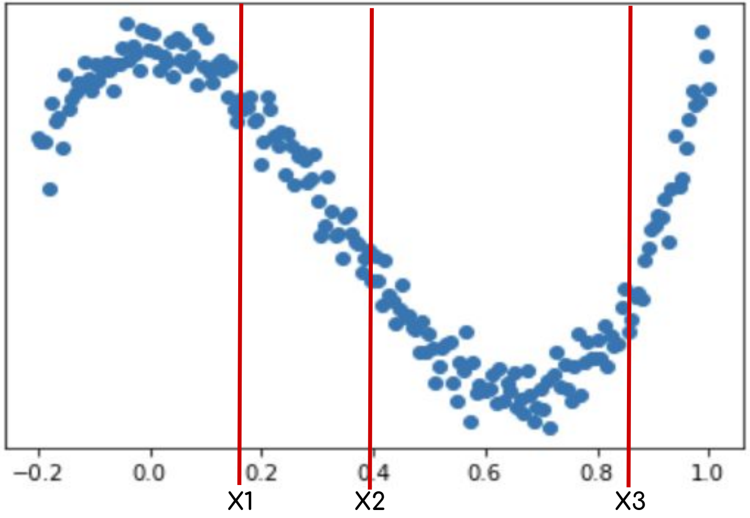

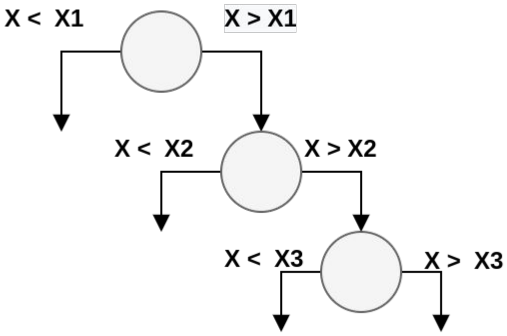

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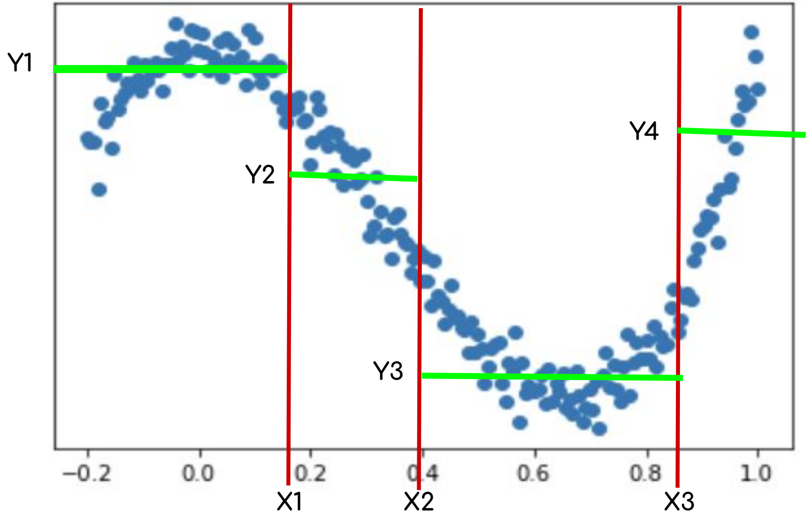

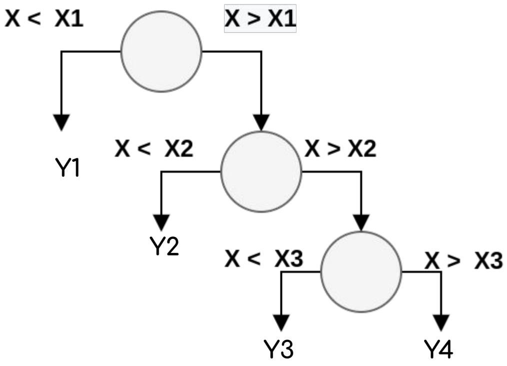

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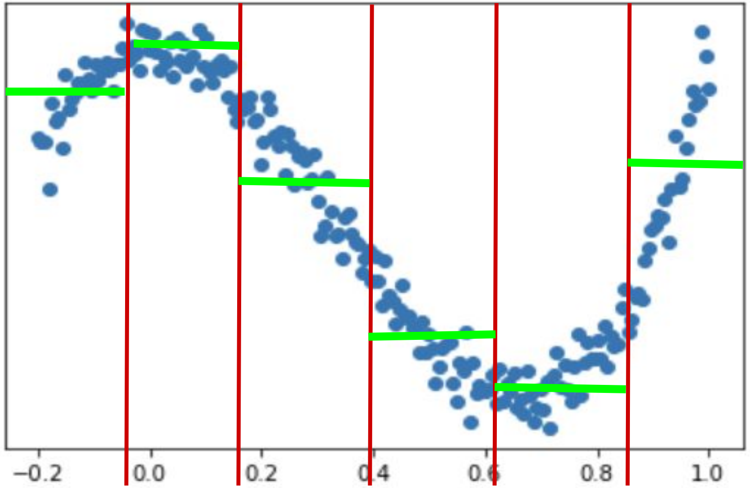

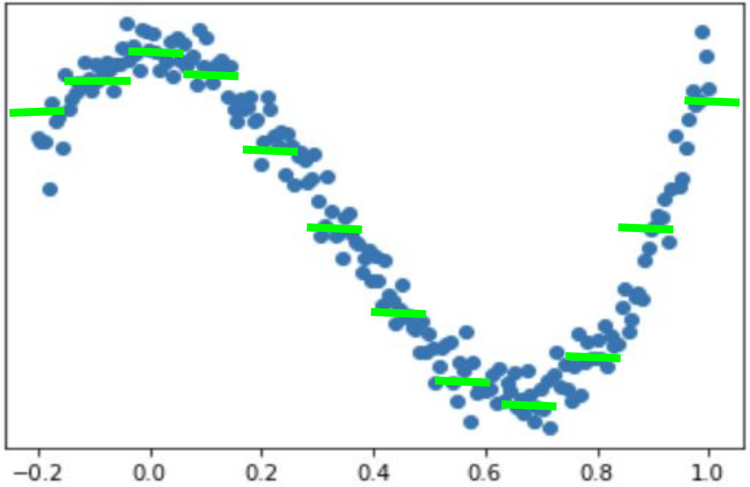

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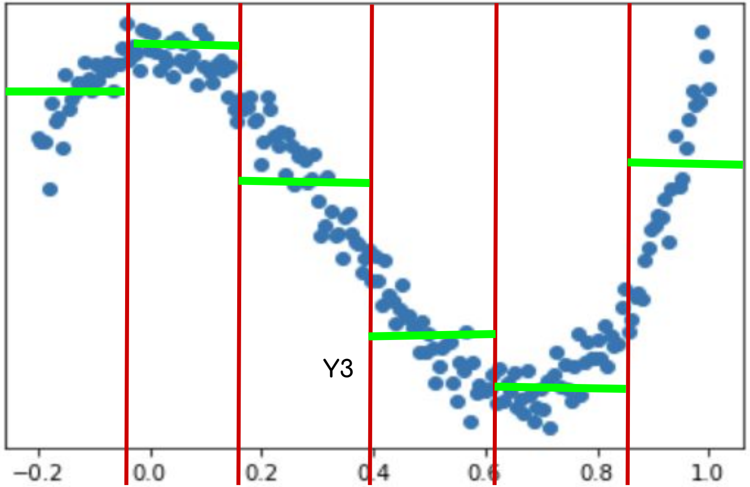
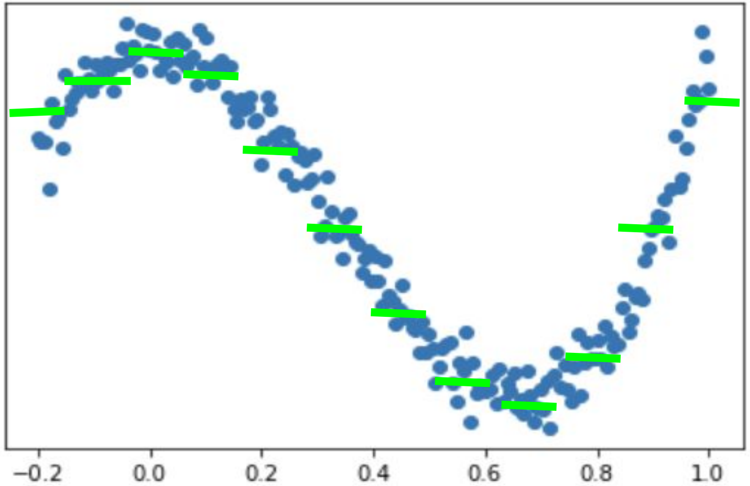


1. Same thing for **high dimensional data** (you can segment your space using different **features**)
2. Same thing for **classification** (use **majority class** instead of the **mean**)



Building decision trees

- Decision trees are generally built only to a maximum depth and not until there is only one data point inside leaf nodes. We prefer moderately deep trees than very deep trees. Why?

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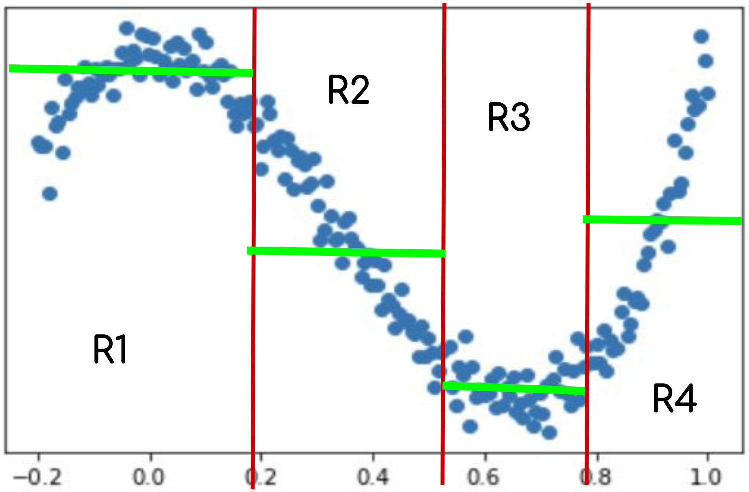
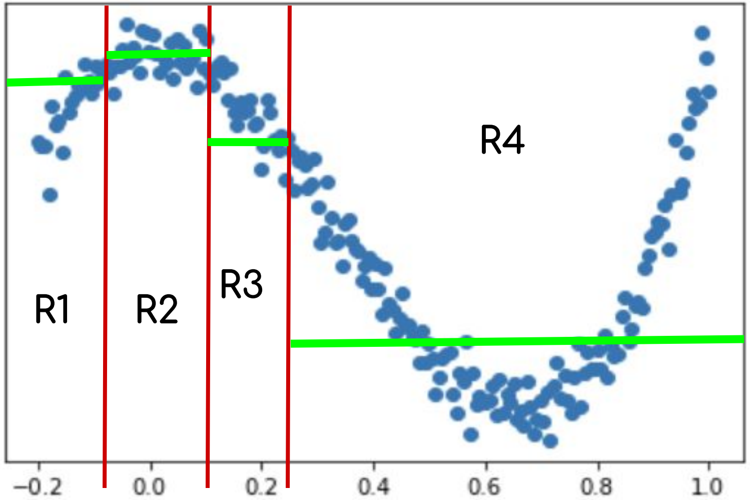
- We have a limited number of splits. What features we choose to split on? And in what order? How to split a given feature?

Building trees: example

Split on some feature: What boundary?



Tree 1



Tree 2

Building trees: example

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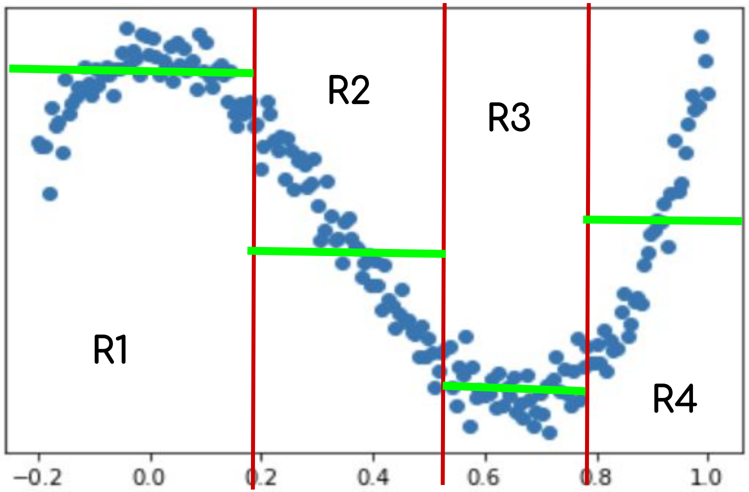

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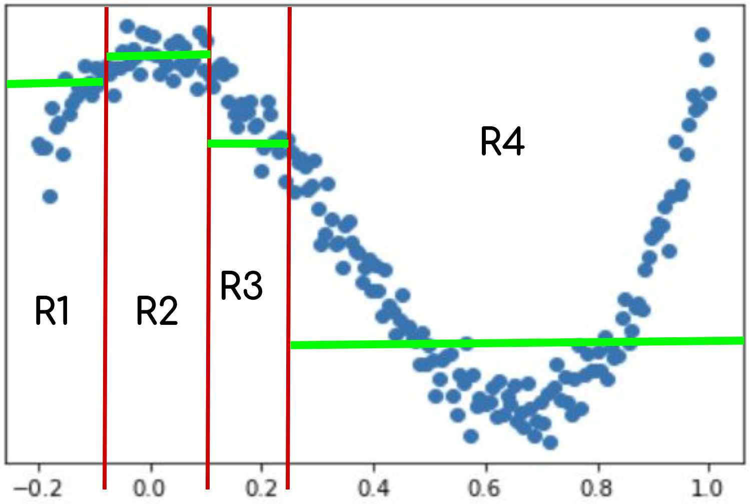

Tree 2

Cost(Tree) = $\sum$Cost(Region^i) s.t. Cost(Region^i) = MSE(Region^i) = MSE(y^j-m^i) where m^i is the mean of Region^i

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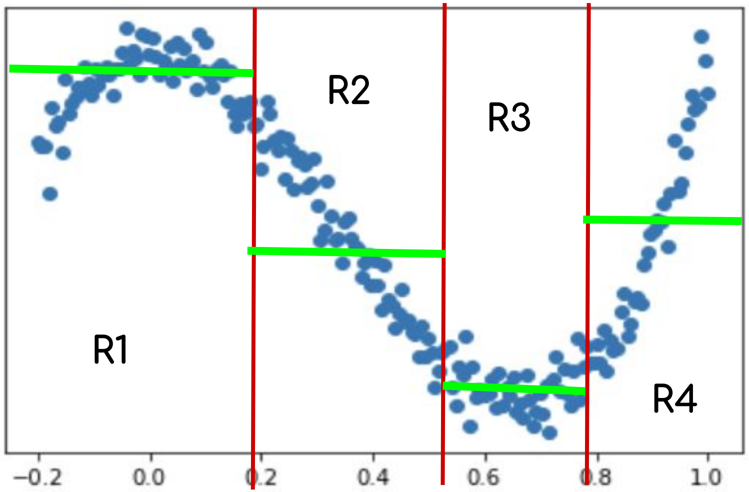

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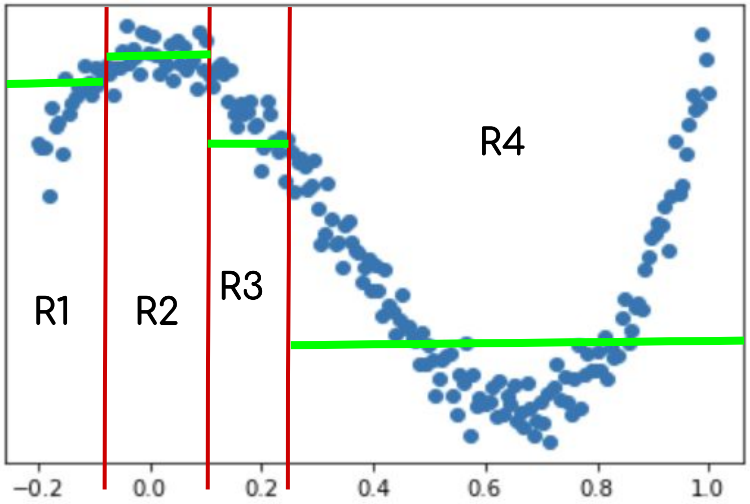

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Multiple features:In what order?

a	b	c	CLASS
0	0	0	NEG
0	0	1	NEG
0	1	0	POS
0	1	1	POS
1	0	0	NEG
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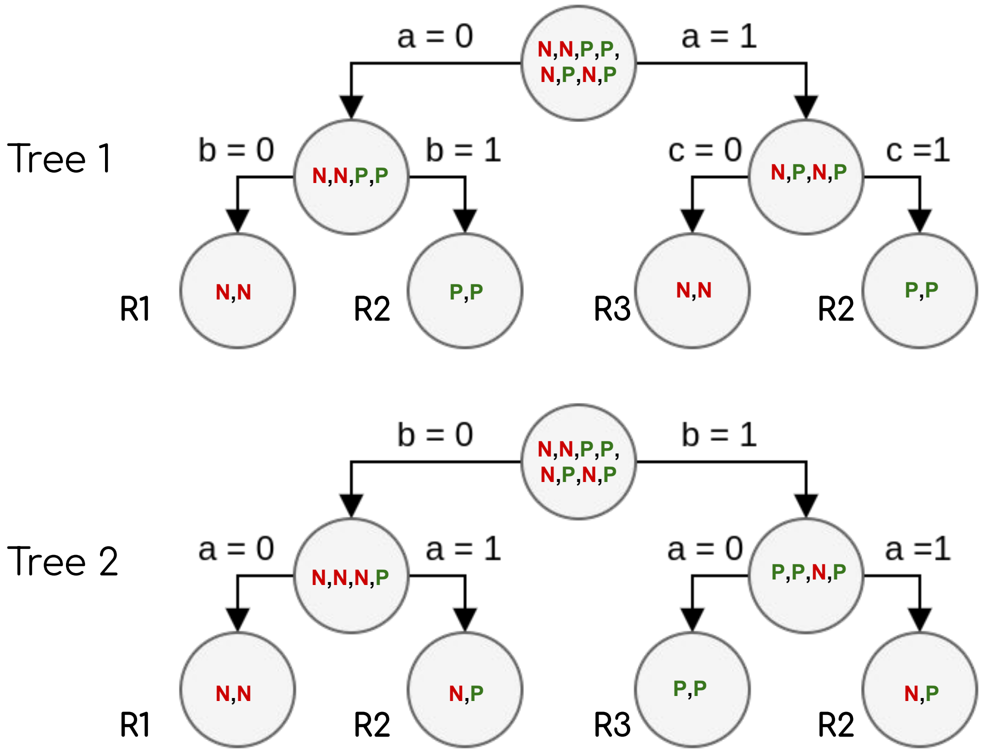

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a = 0

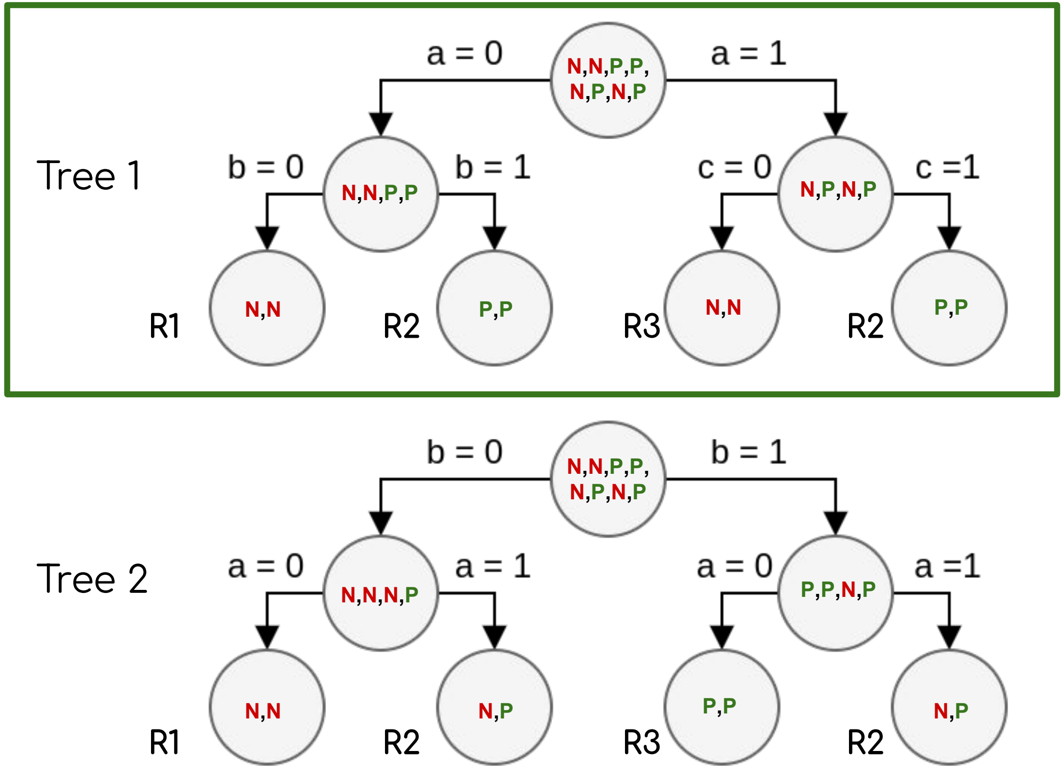
b = 0

b = 1

c = 0

c = 1

a = 1



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Building trees: example

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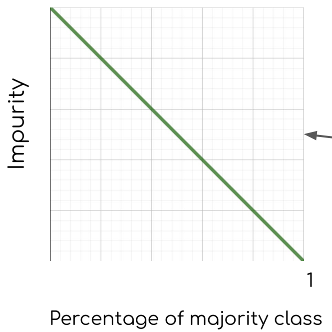
P,P,P,P Is purer than P,P,P,N Is purer than P,P,N,N

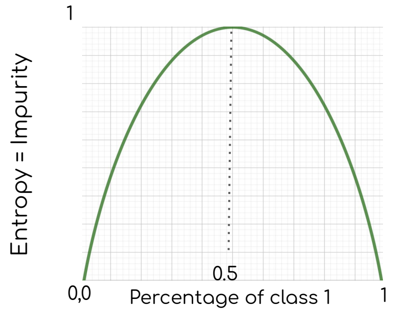
How to quantify Impurity?

First Idea: 1 - percentage of the majority class



The more the majority class is major the smaller the impurity

Second Idea: Entropy



The more the classes are equally probable, the more entropy (impurity) is high and vice versa

Entropy = -∑log(percentage(C^j)) x percentage(C^j)

Building trees: Algorithm

- **Inputs:**
  - Depth of the tree: K
  - Learning data points

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What can we do?

Optimize greedily!

Building trees: Algorithm

- **Inputs:**
  - Depth of the tree: K
  - Learning data points

A. **Algorithm:**
   a. For each internal node with depth = $d < K$:
      i. Choose the feature f s.t when splitted it decreases the loss the most.
      ii. Split on f and create two binary nodes and increment their depth to $d + 1$

Feasible

Building trees: Algorithm

Greedy algorithm:

1. Simple and efficient
2. BUT we are not guaranteed to build the optimal tree
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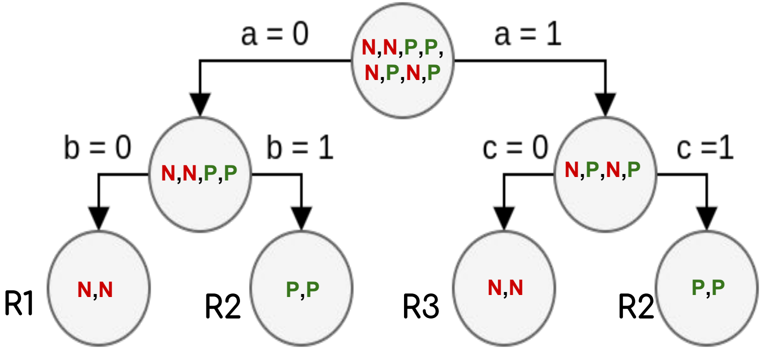

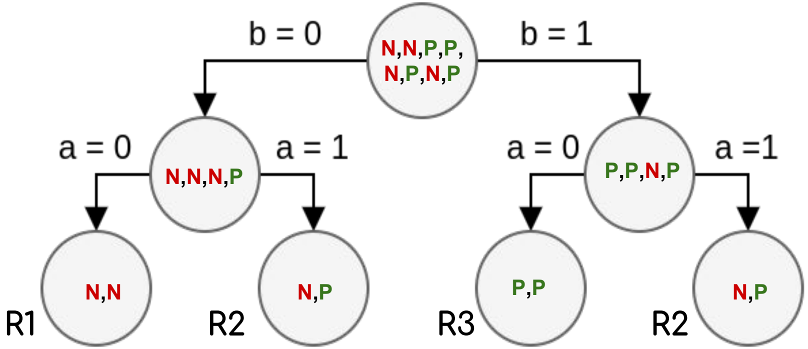

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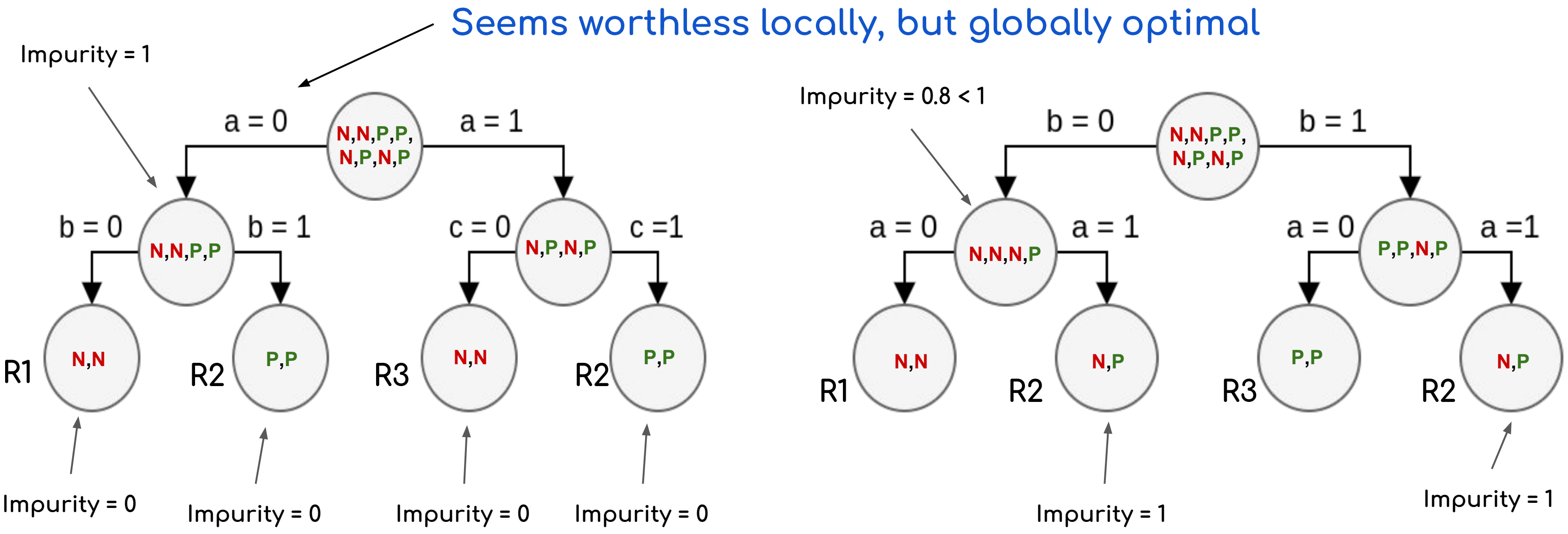

Take away points:

- Decision trees are simple
- Decision trees are interpretable (their behavior can be explained using splitting rules) and mimic rule based human decision making (trees can be easily visualized)
- Decision trees are built using a greedy algorithm which usually works well in practice.
- Decision trees are a flexible model: Small bias (fits non linear relationships) but a big variance (a small change in data might result in a large change of the final tree i.e overfit easily)

Decision trees are overfit easily (high variance)! How to remediate this problem?

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Bootstrapping: A sampling technique where for each data set with n points, sample n points uniformly with replacement (i.e a given data point might be sampled multiple times)

Thoeretically, Bagging (Bootstrap aggregating), can be used on any prediction model.
We show here a pseudo algorithm on bagging with regression decision trees:

- Inputs:
  - Number of trees B
  - Depth of a tree

A. Algorithm:
   a. For b in range(B):
      i. Bootstrap a data set D^b of size n
      ii. Fit a decision tree T^b on D^b
      iii. Add T^b to the already built trees

B. Prediction:
   a. Take a input *i*
   b. For each b compute $T^b(i)$
   c. Return the average of all predictions

Bagging and Random Forests

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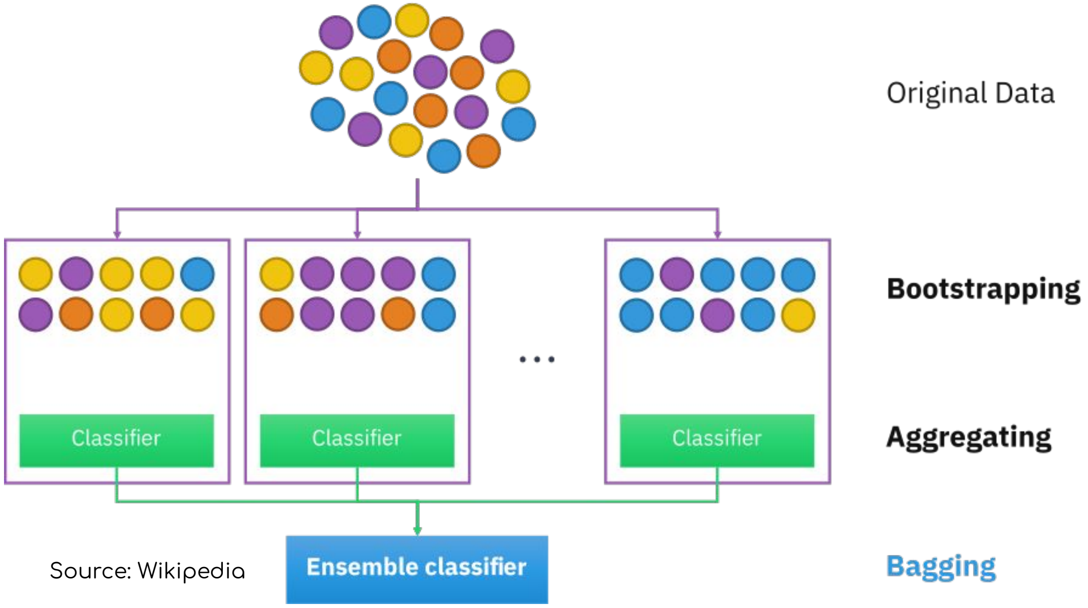

Source: Wikipedia

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=> random 1/3 of data is not used to construct each tree i.e each data point is not used in 1/3 of trees

=> This remaining data can be used as a validation subset to approximate test errors
=> No need for cross validation

Take away points:

- Increasing the total number of trees will not cause overfitting, therefore one can increase the number of trees until the error settles

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- Individual trees can be built to deep levels, we don’t care about individual overfitting since this effect is overridden by averaging

- If there are strong features, all trees will use them in first levels => Trees will have similar behavior (correlated) => We don’t gain much by averaging => Use Random Forests

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=> Generally we build a tree using m randomly chosen features (where $m < p$ the total number of features)

=> In practice we use $m = \sqrt{p}$

=> When stronger features are not chosen, other features has more chances to be explored => trees are different and uncorrelated

Take away points:

- Random forests is just a bagging of decision trees but trees are built on a subset of features only.
- When $m = p$, random forests = bagging