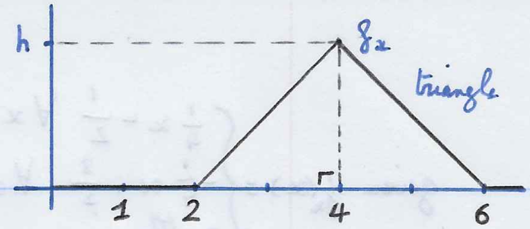

given the following distribution :

0) find h.

1) What is $E(X)$ when X is a R.V. with the previous density function?

2) Same with $V[X]$

3) compute a) $P(3 \leq X \leq 6)$, b) $P(1 \leq X \leq 4)$

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0. given X a continuous r.v. as per the graph, its density function f_x has the following property :

$$\int_{-\infty}^{+\infty} f_x(x)\,dx = 1 \text{ with } X(\Omega) = [2;6] \Rightarrow \int_2^6 f_x(x)\,dx = 1$$

The curve of f_x representing a triangle w/ respect to the x-coordinate we have

$$\text{area of triangle} = \frac{\text{height} \times \text{lenght}}{2} = 1$$

w/ lenght $= 6 - 2 = 4$

$\Rightarrow \quad 4/2 \times h = 1 \quad \Rightarrow \quad \boxed{h = 1/2}$

1. we define $E[X] = \int_{-\infty}^{+\infty} x\, f_x(x)\,dx = \int_2^6 x\, f_x(x)\,dx$

step 1. find f_x on $[2,4]$ and $[4;6]$ such as

$$f_x(x) = \begin{cases} ax + b & \forall x \in [2;4] \\ a'x + b' & \forall x \in [4,6] \\ 0 & \text{otherwise} \end{cases}$$

consider $y = ax + b$ and $y' = a'x + b'$

$$\begin{cases} 4a + b = 1/2 \\ 2a + b = 0 \end{cases} \quad \text{and} \quad \begin{cases} 4a' + b' = 1/2 \\ 6a' + b' = 0 \end{cases}$$

$$\begin{cases} 2a = 1/2 \\ b = -2a \end{cases} \qquad \begin{cases} -2a' = 1/2 \\ b' = -6a' \end{cases}$$

$$\Rightarrow y = \frac{1}{4}x - \frac{1}{2} \qquad \Rightarrow y' = -\frac{1}{4}x + \frac{3}{2}$$

step 2. find $E[X]$

given $f_x(x) = \begin{cases} \frac{1}{4}x - \frac{1}{2} & \forall x \in [2,4] \\ -\frac{1}{4}x + \frac{3}{2} & \forall x \in [4;6] \\ 0 \text{ otherwise} \end{cases}$ and $E(X) = \int_2^6 x . f_x(x)\,dx$

$$E[X] = \int_2^4 \frac{1}{4}x^2\,dx - \int_2^4 \frac{1}{2}x\,dx - \int_4^6 \frac{1}{4}x^2\,dx + \int_4^6 \frac{3}{2}x\,dx$$

$$= \left[1/12 . x^3\right]_2^4 - \left[1/4\,x^2\right]_2^4 - \left[1/12\,x^3\right]_4^6 + \left[3/4\,x^2\right]_4^6$$

$$= \underbrace{\frac{4^3}{12} - \frac{2^3}{12}}_{\frac{14}{3}} \underbrace{- \frac{4^2}{4} + \frac{2^2}{4}}_{-3} \underbrace{- \frac{6^3}{12} + \frac{4^3}{12}}_{-\frac{38}{3}} + \underbrace{\frac{3.6^2}{4} - \frac{3.4^2}{4}}_{15}$$

casio magic

$$= \frac{14}{3} \quad -3 \quad -\frac{38}{3} \quad + 15$$

$$\boxed{E[X] = 4}$$

2. we define $E[X^2] = \int_{-\infty}^{+\infty} x^2 . f_x(x)\,dx$, $\Rightarrow E[X^2] = \int_2^6 x^2 . f_x(x)\,dx$

$$E[X^2] = \int_2^4 \frac{1}{4}x^3\,dx - \int_2^4 \frac{1}{2}x^2\,dx - \int_4^6 \frac{1}{4}x^3\,dx + \int_4^6 \frac{3}{2}x^2\,dx$$

$$= \left[\frac{1}{4^2}x^4\right]_2^4 - \left[\frac{1}{6}x^3\right]_2^4 - \left[\frac{1}{4^2}x^4\right]_4^6 + \left[\frac{1}{2}x^3\right]_4^6$$

$$= \frac{4^4}{4^2} - \frac{2^4}{4^2} - \frac{4^3}{6} + \frac{2^3}{6} - \frac{6^4}{4^2} + \frac{4^4}{4^2} + \frac{6^3}{2} - \frac{4^3}{2}$$

$$= 4^2 - 1 - \frac{32}{3} + \frac{4}{3} - 81 + 4^2 + 108 - 32$$

$= 50/3$ meanwhile $V[X] = E[X^2] - (E[X])^2$

$$= 50/3 - 4^2$$

$$\boxed{V[X] = \frac{2}{3}}$$

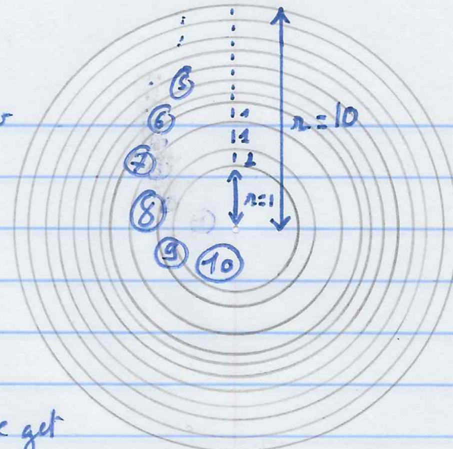
3. $P(3 \leq X \leq 6) = \int_3^4 f_x(x)\,dx + 0.5 = 0.5 + \left[\frac{1}{8}x^2\right]_3^4 - \left[\frac{1}{2}x\right]_3^4$

method 1 method 2 $= 0.5 + 7/8 - 1/2 \Rightarrow \boxed{\frac{7}{8}}$

$P(1 \leq X \leq 4)$ = area of triangle of width $(4-2)$ and height $(\frac{1}{4} \times 4 - \frac{1}{2})$

$= (2 \times 0.5)/2 = \boxed{1/2}$

Let's consider the following object: Throwing an arrow at this dart target yields a r.v. X, the number associated to the location of the area it fell on
P(X = Location) is proportional to the area of it.



1) determine the distribution of X
2) we assume that if we obtain the values 6 to 10, we get the respective amount in € (gain). In all other cases we give away €2 (loss)
Is the game interesting for the player?

1 area of circle = πr^2 $\Rightarrow$ total area of target $= \pi . 10^2$
and $= 100.\pi$

$P(X = Loc) =$ area of Loc / total area
$X(\Omega) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

by the property of the area of a circle, we can ~~recursively~~ define
$P(X = Loc)$ as

$P(X = Loc) = [\pi . (21 - 2Loc)] / 100\pi$

$\forall Loc \in \{1; ...; 9\}$, $P(X = Loc) = (\pi[(11 - Loc)^2 - (10 - Loc)^2]) \times \frac{1}{100\pi}$
$P(X = 10) = \pi . 1^2 / (100\pi) = 1/100$
Can be abstracted out

The area of each ring is determined by $r^2\pi - (r-1)^2\pi$
$= 2(r-1)\pi$

by cross multiplication the probability is $\frac{(2r-1)}{100}$

$f(x) = (21 - 2x)/100$

$X(\Omega) = \{1, 2, 3 ... 10\}$
$f_X(x) = (21 - 2x)/100$
$\forall x \in X(\Omega)$

2 We can define the gain or loss as the following sum:

+ given $PnL_{Loc} \in \{-2, 6, 7, 8, 9, 10\}$ w/ $\begin{cases} PnL_{Loc} = -2 \text{ if } loc \in \{1, 2, 3, 4, 5\} \\ PnL_{Loc} = x. \end{cases}$

$g(x) = \begin{cases} x & \forall x \in \{6, 7, 8, 9, 10\} \\ -2 & \forall x \in \{1, 2, 3, 4, 5\} \end{cases}$

total $PnL = \sum_{Loc=1}^{10} PnL_{Loc} \times P(X = Loc)$

$= 0.3$ It is interesting as PnL total ≥ 0

we compute $E(g(x))$ as g is the distribution of a discrete r.v.

x	area	P	gain
10	100π	$19\pi/100\pi$	-2
9	81π	$17\pi/100\pi$	-2
8	64π	$15\pi/100\pi$	-2
7	49π	$13\pi/100\pi$	-2
6	36π	$11\pi/100\pi$	-2
5	25π	$9\pi/100\pi$	6
4	16π	$7\pi/100\pi$	7
3	9π	$5\pi/100\pi$	8
2	4π	$3\pi/100\pi$	9
1	π	$\pi/100\pi$	-10

$\sum \Pi = 0.3$