Empirical Comparison of Three Adaptive Moment Optimization Methods

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Abstract

Stochastic gradient-based optimization led to the relatively recent development of adaptive moment estimation methods, which are known to outperform traditional stochastic gradient-based techniques. This paper presents the comparative analysis of three of such methods (Adam, AdamW, and AMSGrad) when applied to industry-standard, real-world datasets in the context of a multi-class classification problem.

1 Introduction

First proposed in 1951, the idea of stochastic approximation revolved around the minimization of an objective or risk function with the adjunctive use of noise as part of the optimization process (Robbins & Monro, 1951):

$$\underset{x}{minimize} \quad F(x) = \mathbb{E}(f(x;\zeta))$$

Given ζ a random seed, and $f(.)$ the composite of a loss function l and a prediction function h (Bottou et al., 2018). Here the noise represents a random pick from the dataset, over which the gradient descent will be performed:

$$\theta = \theta - \eta.\nabla_\theta F(\theta, x^{(i)}, y^{(i)})$$

Given $\theta \in \mathbb{R}^d$, the model parameters, $\nabla_\theta F$, the gradient of the objective function, η, the learning rate determining the size of a step taken towards a minimum, and $x^{(i)}$ and $y^{(i)}$, a data point and its label respectively (in the case of a classification example).

Overall, stochastic gradient descent (SGD) makes use of practical data in a more efficient way than batch methods. Indeed, compared to batch gradient descent, SGD does not perform superfluous gradient computations on a dataset: a growing concern as dataset size increase (Ruder, 2017). Furthermore, the intrinsic randomness allows the descent to shift towards potentially better minimas. The use of random picks from the available data thus enable a more efficient gradient update than when all data is simultaneously iterated over. It has been shown that SGD is an efficient method that achieves fast initial improvement with low cost (Nemirovski et al., 2009).

However, standard SGD suffers from the consequence of a static learning rate and its inherent noisy fluctuation, impeding the convergence of the descent to the exact, achievable minimum (Bottou & Bousquet, 2007). Dealing with this overshooting is a long-standing area of research, which led to the recent development of adaptive moment estimation methods.

2 Adaptive Moment Estimation Methods

To answer the overshooting problem of standard SGD, development in optimization methods led to a foray in how stepsizes are selected at each parameter update during a gradient descent optimization process. This adaptive approach found its early expression in the first-order Adaptive Gradient method or AdaGrad (Duchi et al., 2011), its variant AdaDelta (Zeiler, 2012), or the unpublished Root Mean Square Propagation or RMSProp (Tieleman & Hinton, 2012). While the two former accumulate gradients (all of them for AdaGrad, and a window selection for AdaDelta), the latter divides the gradient by a running average over its recent magnitude.

2.1 Adam

More recently, an expansion over this concept led to the development of a new kind of optimization algorithm: the Adaptive Moment Estimation method or Adam (Kingma & Ba, 2017). Where the standard SGD method keeps a fixed stepsized for all updates, the Adam methods takes inspiration from AdaGrad and RMSProp. With Adam, the parameter learning rate is updated based on the gradient's first and second moments and the process stores an exponentially decaying average of past gradients such that:

$$\begin{aligned} m_t &= \beta_2 m_{t-1} + (1-\beta_1) g_t \\ v_t &= \beta_1 2 v_{t-1} + (1-\beta_1) g_t^2 \\ \hat{m}_t &= \frac{m_t}{1-\beta_1^t} \\ \hat{v}_t &= \frac{v_t}{1-\beta_2^t} \\ \theta_{t+1} &= \theta_t - \frac{\eta}{\sqrt{\hat{v}}+\epsilon}\hat{m}_t \end{aligned}$$

Given m_t, the estimate of the first moment (mean), v_t, the estimate of the second moment (uncentered variance), $\hat{m}_t$ and $\hat{v}_t$ their biased-corrected counterparts, and ϵ, very small value to avoid dividing by zero.

With the early success of this method, having shown that it is effective in practice with large datasets, multiple variants emerged.

2.2 AdamW

One such a variant takes advantage of adding a weight decay, resulting in a new algorithm: the Adam with decoupled weight decay or AdamW (Loshchilov & Hutter, 2019). The idea that spurred the algorithm's development was the highlight that, in opposition to the SGD method, L_2 regularization and weight decay were not identical in the case of Adam (For the case of Standard SGD, see the proof reproduced in Appendix A). As L_2 regularization was found not to be as effective with Adam, the modification over Adam focused on the update step, inspired by the weight decay described by Hanson & Pratt (1988):

$$\theta_{t+1} = \theta_t - \alpha_t\left(\frac{\eta\, m_t}{\sqrt{\hat{v}}+\epsilon} + \lambda\theta_t\right)$$

Given λ, the rate of the weight decay at each step, and α_t, a $SetScheduleMultiplier(t)$ allowing to reset the learning rate during optimization (Loshchilov & Hutter, 2019).

2.3 AMSGrad

The final Adaptive Moment Estimation method we are interested in began with the observation of a flaw in the Adam method: Adam can fail at converging towards an optimal solution in a convex setting (Reddi et al., 2019). Indeed, there is an underlying risk that positive definiteness could be violated for methods other than Standard SGD.

Given the quantity Γ_t measuring the change in the inverse of the learning rate for an adaptive method with regards to time such that:

$$\begin{aligned} \Gamma_{t+1} &= \frac{\sqrt{V_{t+1}}}{\eta_{t+1}} - \frac{\sqrt{V_t}}{\eta_t} \\ V_t &= diag(v_t) \end{aligned}$$

It happens that for Standard SGD, $\forall t \in [T] \quad \Gamma_t \geq 0$ with T, the number of rounds. However, this property is not always true for exponential moving average methods such as Adam. Indeed, a convex optimization problem like Adam was shown to have a potentially non-zero average regret

(Shalev-Shwartz, 2012), i.e, the delta between the loss of a possible action and the action taken given an hypothesis class $h^\star$ would not converge to zero (Reddi et al., 2019):

$$\frac{R_T}{T} \underset{T\to\infty}{\not\to} 0$$

$$R_T(h^\star) = \sum_{t=1}^{T} l(p_t, y_t) - \sum_{t=1}^{T} l(h^\star(x_t), y_t)$$

To resolve this issue (that the quantity Γ_t can be negative for Adam), a new exponential moving average method was developed: AMSGrad (Reddi et al., 2019). The algorithm is meant to guarantee convergence while conserving the advantages of the Adam method. The modification over Adam corresponds to the computation of $\hat{v}_t$ and of the update step (note the use of m_t rather than $\hat{m}_t$) such that:

$$\hat{v}_t = max(\hat{v}_{t-1}, v_t)$$

$$\theta_{t+1} = \theta_t - \frac{\eta}{\sqrt{\hat{v}_t} + \epsilon} m_t$$

The main difference with Adam happens with the preserving of the maximum of all past values v_t, which results in never-increasing stepsizes, which was the underlying risk of the Adam method.

3 Comparison Methodology

In this section, we present an empirical comparison procedure for the three Adaptive Moment Estimation methods we previously covered.. Our experiments will focus on the problem of multi-class classification using industry-standard and real-world datasets with various kind of neural networks.

3.1 Datasets

Our comparative analysis will rely on four industry-standard, fixed-size image datasets: MNIST (Lecun et al., 1998), Fashion-MNIST (Xiao et al., 2017), German Traffic Sign Recognition Benchmark GTSRB (Stallkamp et al., 2012), and CIFAR-10 (Krizhevsky, 2012).

The MNIST dataset corresponds to grayscale, normalized and centered fixed-size images (28×28 pixels) of handwritten digits split between 60,000 training and 10,000 testing examples, and 10 classes. The more recent Fashion-MNIST follows the same format: grayscale, normalized and centered fixed-size images (28×28 pixels) of fashion and clothing articles also split between 60,000 training and 10,000 testing examples, and 10 classes.

The GTSRB dataset corresponds to color, fixed-size images (32×32 pixels with 3 channels) split between 34,799 training and 4,410 testing examples (the provided validation set was not included), and 43 classes. Similarly, the CIFAR-10 dataset corresponds to color, fixed-size images (32×32 pixels with 3 channels) split between 50,000 training and 10,000 testing examples, and 10 classes.

Each of those datasets is organized into classes, providing us a standard multi-class classification problem. The problem compatibility between the four datasets allows us to compare Adam, AdamW, and AMSGrad using a variety of interchangeable neural network models.

3.2 Models

We compare the three Adaptive Moment Estimation methods using four distinct model types: a shallow neural network, a deep (fully-connected) neural network, a convolutional neural network, and a residual neural networks. The graphs for MNIST and Fashion-MNIST can be found in Appendix B, those for the GTSRB and CIFAR-10 in Appendix C – the only difference affects the number of input channels. The Python code can be found in Appendix D.

Each model will use the same suite of hyper-parameters, notably being trained for 100 epochs with batch sizes of 32 elements with the categorical crossentropy loss function. Each optimization algorithm will be initialized with default parameters using the TensorFlow library (Abadi et al., 2015). The used parameters for AdamW and AMSGrad are reproduced in Table 1.

Table 1: AdamW and AMSGrad Parameters

Parameters	**Value**
learning rate	$1e^{-2}$
β_1	0.9
β_2	0.999
weight decay	$1e^{-4}$
ϵ	$1e^{-8}$
decay	$0.$

3.3 Comparison approach

Our comparison approach will be two-fold. We will first take note of the comparative speed of convergence between the three different optimizers' losses (for each training session and model). Afterwards, we will be to derive possible ranges of applicability for the optimization algorithms based on the models and datasets where they performed the best.

4 Experiment Results

The first observation we make relates to how Adam performs with regards to AdamW and AMSGrad. Based on our visual assessment, Adam performs differently than the latter two, who display distinct, similar patterns (For the results, see Appendix E for MNIST, Appendix F for Fashion-MNIST, Appendix G for GTSRB, and Appendix H for CIFAR-10).

Our second observation relates to how both AdamW and AMSGrad algorithms show a faster loss convergence than Adam, for all the datasets used in this exercise. We can say that, visually, AdamW and AMSGrad will generally converge around a mean value in less than 40 epochs. This translates into a faster convergence on the testing set, except in the case of the shallow neural network on GTRSB and CIFAR-10 where the loss is too noisy to draw conclusions. As such, we observe a slower convergence for Adam, if not at all. For instance, using a deep neural network on GTRSB with Adam as an optimizer will not translate into any loss descent. Using the other datasets, we also find that Adam might result in an increasing loss (e.g. for our deep neural network implementation, and our shallow neural network implementation in the case of the MNIST dataset). This overfitting validates the observation made by Loshchilov & Hutter (2019) that Adam will not always converge.

A third observation relates to performance itself, here based on model accuracy. We find that, although AdamW and AMSGrad converge quickly, they do not always achieve a better accuracy that Adam in the same amount of epochs. Indeed, with most models, Adam ends up achieving a better accuracy value on the testing set (For the results, see Appendix I for MNIST, Appendix J for Fashion-MNIST, Appendix K for GTSRB, and Appendix L for CIFAR-10).

5 Conclusion & Discussion

We saw that the optimizers AdamW and AMSGrad perform better in terms of convergence than Adam when applied to four industry-standard, real-world datasets, adding to the literature empirical proof of their soundness. A visible caveat, however, is that their faster convergence will not always translate into a better model performance.

Beyond this short comparative analysis, further explorations could be done in two regards. First, as we saw that better convergence does not always imply better accuracy, a thorough exploration of initialization parameters for AdamW and AMSGrad should be explored. The goal would be to find the best mix of parameters to Adam in terms of accuracy performance. Another area of exploration relates to the development of newer Adaptive Moment Optimization methods, which could further be covered: Adabound (Luo et al., 2019), Radam (Liu et al., 2020), LAMB (You et al., 2020), and MAS (Landro et al., 2020).

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A FORMAL ANALYSIS OF WEIGHT DECAY VS. L_2 REGULARIZATION FOR STANDARD SGD

Proposition (Weight decay is equivalent to L_2 regularization for standard SGD). *Standard SGD coupled with a base learning rate η executes identical steps between loss functions $f_t(\theta)$ with a weight decay λ and on regularized loss functions without weight decay $f_t^{reg}(\theta) = f_t(\theta) + \frac{\lambda'}{2}||\theta||_2^2$ with $\lambda' = \frac{\lambda}{\eta}$.*

Proof of Proposition

The weights update for SGD with regularization and SGD with weight decay are represented respectively by the following iterates:

$$\theta_{t+1} \leftarrow \theta_t - \eta \nabla f_t^{reg}(\theta_t) = \theta_t - \alpha \nabla f_t(\theta_t) - \eta \lambda' \theta_t$$
$$\theta_{t+1} \leftarrow (1 - \lambda)\theta_t - \alpha \nabla f_t(\theta_t)$$

Resulting in the following observation:

$$\lambda' = \frac{\lambda}{\eta}$$

B Graphs of Neural Networks used for Empirical Comparison For 1-channel images (MNIST and Fashion-MNIST)

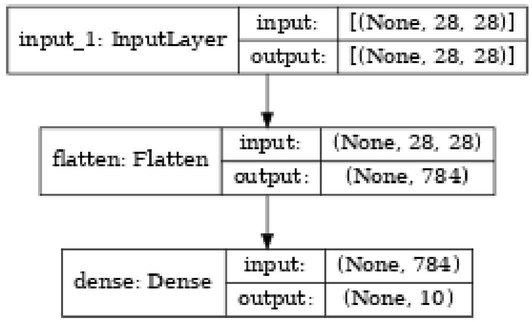

Figure 1: Shallow Neural Network

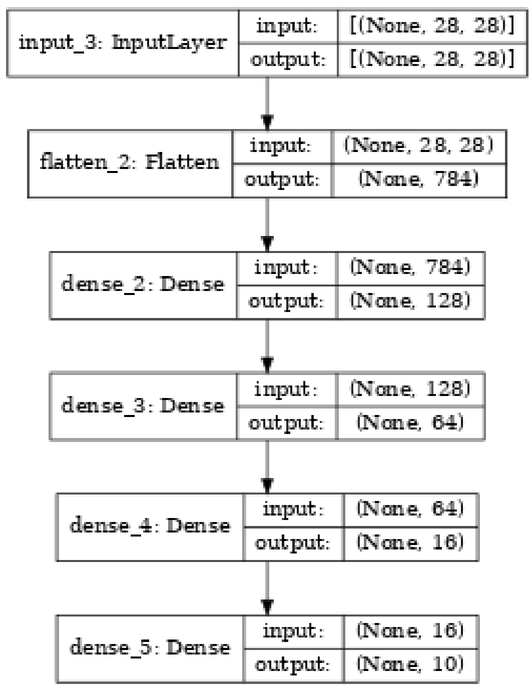

Figure 2: Deep (Fully-Connected) Neural Network

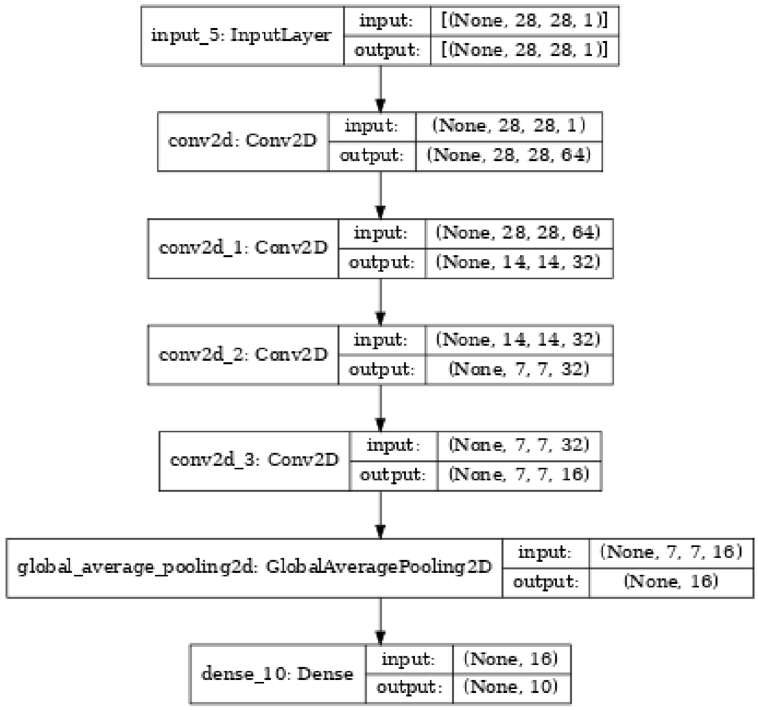

Figure 3: Convolutional Neural Network

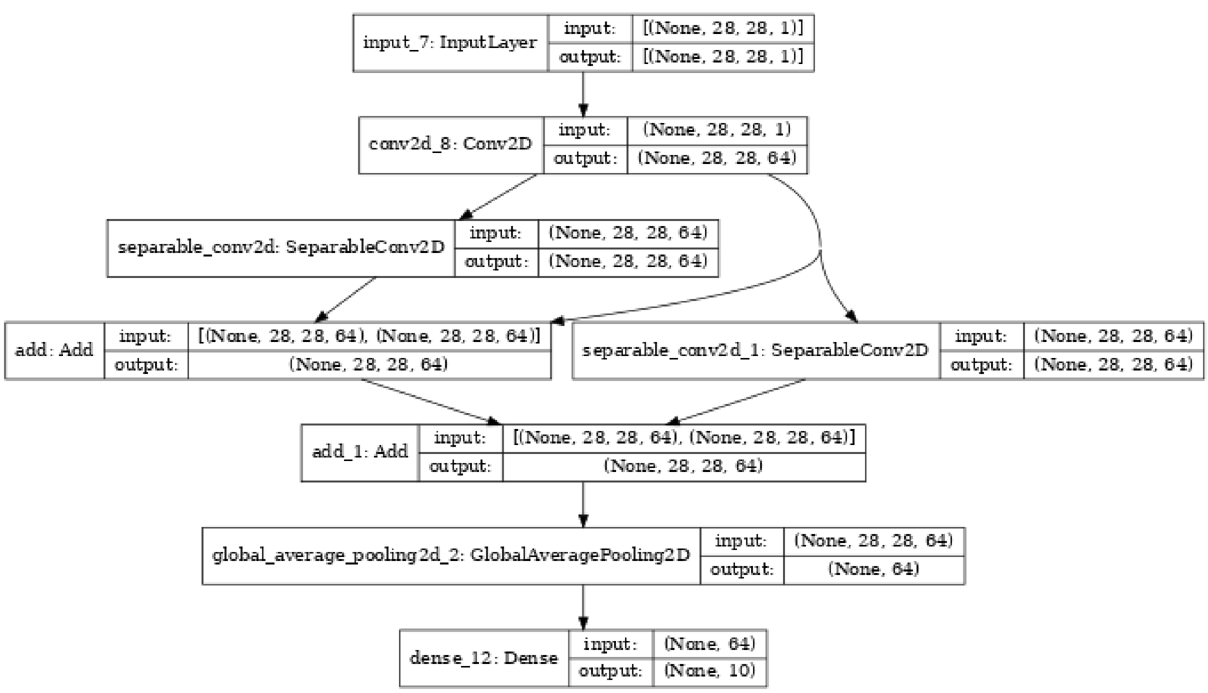

Figure 4: Residual Neural Network

C Graphs of Neural Networks used for Empirical Comparison For 3-channel images (GTSRB and CIFAR-10)

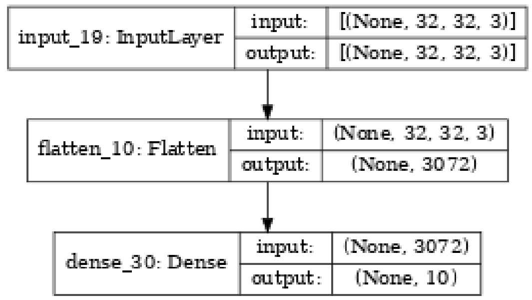

Figure 5: Shallow Neural Network

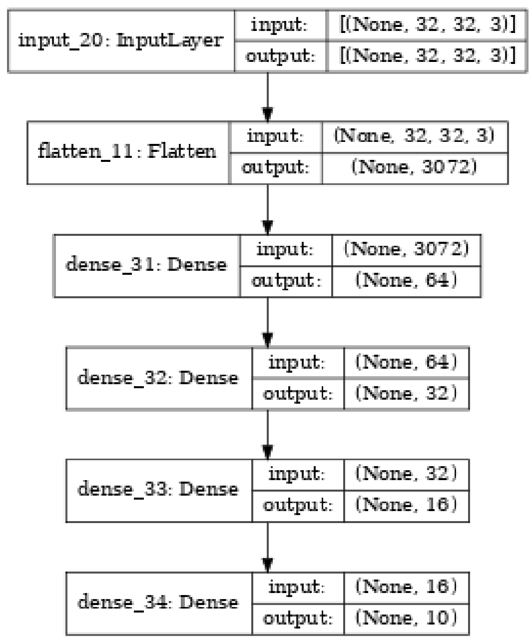

Figure 6: Deep (Fully-Connected) Neural Network

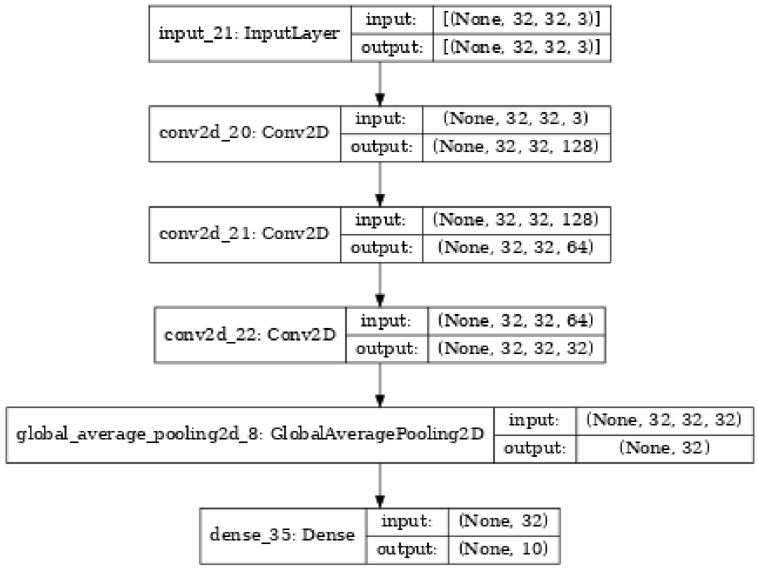

Figure 7: Convolutional Neural Network

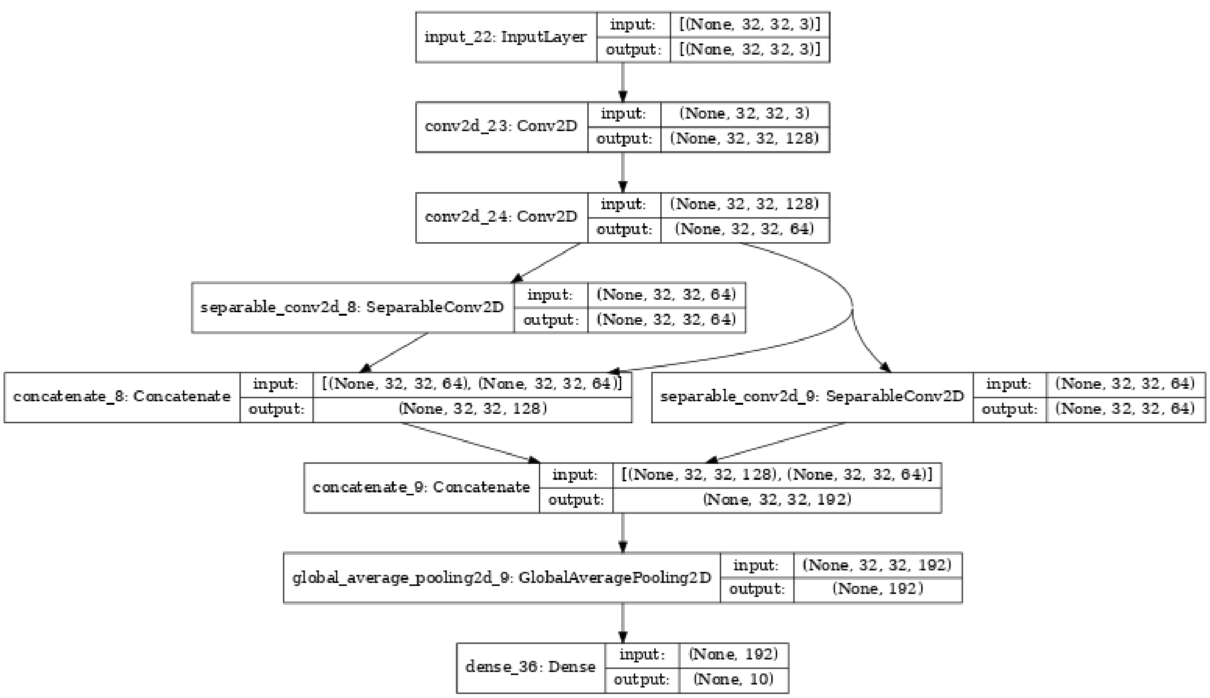

Figure 8: Residual Neural Network

D CODE IMPLEMENTATION USING PYTHON AND TENSORFLOW

Listing 1: Library Imports

```
import matplotlib.pyplot as plt
import numpy as np
import pickle
import random
import tensorflow as tf

from plot_keras_history import plot_history
from tensorflow.keras import Input, Model
from tensorflow.keras.datasets import mnist, fashion_mnist, cifar10
from tensorflow.keras.layers import Flatten, Dense, Concatenate
from tensorflow.keras.layers import Conv2D, SeparableConv2D
from tensorflow.keras.layers import BatchNormalization, Activation
from tensorflow.keras.layers import GlobalAveragePooling2D
from tensorflow.keras.optimizers import Adam
from tensorflow_addons.optimizers import AdamW
from tensorflow.keras.utils import plot_model
from keras.utils import np_utils
```

Listing 2: Custom Function Declarations

```
def normalize(dataset):
    """
    Normalizes the pixel values of an image stored as an array
    """
    return dataset/255.

def one_hot_labels(labels):
    """
    One-hot encodes the imported labels
    """
    return tf.one_hot(labels, depth=10)

def shallow_neural_network(input_shape, num_classes):
    """
    Creates a shallow neural network
    """
    inputs = Input(shape=input_shape)
    flatten = Flatten()(inputs)
    outputs = Dense(num_classes, activation="softmax")(flatten)
    return Model(inputs, outputs)

def deep_neural_network(input_shape, num_classes):
    """
    Creates a Deep (Fully-Connected) Neural Network
    """
    inputs = Input(shape=input_shape)
    flatten = Flatten()(inputs)
    layer_1 = Dense(64, activation="relu")(flatten)
    layer_2 = Dense(32, activation="relu")(layer_1)
    layer_3 = Dense(16, activation="relu")(layer_2)
    outputs = Dense(num_classes, activation="softmax")(layer_3)
    return Model(inputs, outputs)

def convolutional_neural_network(input_shape, num_classes):
    """
    Creates a convolutional neural network
    """
    inputs = Input(shape=input_shape)
    x = Conv2D(128, 3, padding="same", activation="relu")(inputs)
    x = Conv2D(64, 3, padding="same", activation="relu")(x)
    x = Conv2D(32, 3, padding="same", activation="relu")(x)
```

```
    x = GlobalAveragePooling2D()(x)
    outputs = Dense(num_classes, activation="softmax")(x)
    return Model(inputs, outputs)

def residual_neural_network(input_shape, num_classes):
    """
    Creates a residual neural network
    """
    inputs = Input(shape=input_shape)
    x = Conv2D(128, 3, padding="same", activation="relu")(inputs)
    x = Conv2D(64, 3, padding="same", activation="relu")(x)
    previous_block_activation_1 = x
    previous_block_activation_2 = x
    residual = SeparableConv2D(64, 3,
                               padding="same",
                               activation="relu")(previous_block_activation_2)
    x = Concatenate()([x, residual])
    previous_block_activation = x
    residual = SeparableConv2D(64, 3,
                               padding="same",
                               activation="relu")(previous_block_activation_1)
    x = Concatenate()([x, residual])
    x = GlobalAveragePooling2D()(x)
    outputs = Dense(num_classes, activation="softmax")(x)
    return Model(inputs, outputs)

def compile_fit(model, X, y, X_val, y_val,
                optimizer, loss, metrics,
                epochs, batch_size):
    """
    Compiles and fits a model given a set of hyperparameters
    """
    model.compile(
        optimizer=optimizer,
        loss=loss,
        metrics=metrics
    )
    history = model.fit(
        x=X, y=y,
        epochs=epochs, batch_size=batch_size,
        validation_data = (X_val, y_val),
        verbose=0
    )
    return history

def plot_loss(model, dataset):
    """
    Plots a comparative representation of obtained losses for
    a specific model for all three selected optimizers
    """
    if dataset == "MNIST":
        res = results_mnist
    elif dataset == "Fashion-MNIST":
        res = results_fmnist
    elif dataset == "Road":
    else:
        res = results_cifar
    a = res[f"{model}_adam"].history["loss"]
    b = res[f"{model}_adam"].history["val_loss"]
    c = res[f"{model}_adamw"].history["loss"]
    d = res[f"{model}_adamw"].history["val_loss"]
    e = res[f"{model}_amsgrad"].history["loss"]
    f = res[f"{model}_amsgrad"].history["val_loss"]

    plt.figure(figsize=(12,10))
```

```
    plt.plot(a, '--', linewidth=1, color="maroon")
    plt.plot(b, linewidth=3, alpha=0.5, color="firebrick")
    plt.plot(c, '--', linewidth=1, color="lightseagreen")
    plt.plot(d, linewidth=3, alpha=0.5, color="teal")
    plt.plot(e, '--', linewidth=1, color="purple")
    plt.plot(f, linewidth=3, alpha=0.5, color="mediumvioletred")
    plt.xlabel("EPOCHS", fontsize=13)
    plt.ylabel("LOSS VALUE", fontsize=13)
    plt.title("Observed loss values per epoch given " + \
              f"a specific optimizer on the {dataset} dataset",
              fontsize=15)
    plt.legend(["Adam Training Loss", "Adam Testing Loss",
                "AdamW Training Loss", "Adamw Testing Loss",
                "AMSGrad Training Loss", "AMSGrad Testing Loss"],
               fontsize=13)
    plt.show()
```

Listing 3: Dataset Imports

```
# 1. Import dataset
# 2. Normalize X arrays (features)
# 3. One-hot encode Y arrays (labels)

# MNIST import
(mx_train, my_train), (mx_test, my_test) = mnist.load_data()
mx_train, mx_test = normalize(mx_train), normalize(mx_test)
my_train, my_test = one_hot_labels(my_train), one_hot_labels(my_test)

# Fashion-MNIST import
(fmx_train, fmy_train), (fmx_test, fmy_test) = fashion_mnist.load_data()
fmx_train, fmx_test = normalize(fmx_train), normalize(fmx_test)
fmy_train, fmy_test = one_hot_labels(fmy_train), one_hot_labels(fmy_test)

# GTSRB import
!git clone https://bitbucket.org/jadslim/german-traffic-signs
!ls german-traffic-signs
with open('german-traffic-signs/train.p', 'rb') as f:
    train_data = pickle.load(f)
with open('german-traffic-signs/valid.p', 'rb') as f:
    val_data = pickle.load(f)
sign_x_train, sign_y_train = train_data['features'], train_data['labels']
sign_x_test, sign_y_test = val_data['features'], val_data['labels']
sign_x_train, sign_x_test = normalize(sign_x_train), normalize(sign_x_test)
sign_y_train = one_hot_labels(sign_y_train, 43)
sign_y_test = one_hot_labels(sign_y_test, 43)

# CIFAR-10 import
(cifar_x_train, cifar_y_train), (cifar_x_test, cifar_y_test) = cifar10.load_data()
cifar_x_train = normalize(cifar_x_train)
cifar_x_test = normalize(cifar_x_test)
cifar_y_train = one_hot_labels(cifar_y_train)
cifar_y_test = one_hot_labels(cifar_y_test)
cifar_y_train = tf.reshape(cifar_y_train, [50000, 10])
cifar_y_test = tf.reshape(cifar_y_test, [10000, 10])
```

Listing 4: Model Declarations

```
# Global variable declarations
input_shape = (28, 28)
input_shape_cnn = (28, 28, 1)
input_shape_color = (32, 32, 3)
num_classes = 10 #or 43 for GTSRB

# Declaring Shallow Models
```

```
shallow = shallow_neural_network(input_shape=input_shape,
                                 num_classes=num_classes)

# Declaring Deep (Fully-Connected) Models
dnn = deep_neural_network(input_shape=input_shape,
                          num_classes=num_classes)

# Declaring Convolutional Models
cnn = convolutional_neural_network(input_shape=input_shape_cnn,
                                   num_classes=num_classes)

# Declaring ResNet Models
resnet = residual_neural_network(input_shape=input_shape_cnn,
                                 num_classes=num_classes)
```

Listing 5: Experiments/Computing Loss Results

```
# Global variable declarations
epochs = 100
batch_size = 32
loss = "categorical_crossentropy"
metrics = ["accuracy"]

# Generic learning rate and weight decay for AdamW and AMSgrad
lr=0.001
beta_1=0.9
beta_2=0.999
weight_decay=1e-4
epsilon=1e-8
decay=0.

optimizers = [("adam", Adam()),
              ("adamw", AdamW(lr=lr, beta_1=beta_1, beta_2=beta_2,
                              weight_decay=weight_decay, epsilon=epsilon,
                              decay=decay)),
              ("amsgrad", AdamW(lr=lr, beta_1=beta_1, beta_2=beta_2,
                                weight_decay=weight_decay, epsilon=epsilon,
                                decay=decay,
                                amsgrad=True))]

results = {}

# Computes loss results for a kind of model
for name, optimizer in optimizers:
    res = compile_fit(model,
                      x_train, y_train,
                      x_test, y_test,
                      optimizer, loss, metrics,
                      epochs, batch_size)
    results[f"{name}"]=res
```

Listing 6: Plotting Results/Data Visualization example with the Shallow model and MNIST dataset

```
plot_history(results["f{model}_adam"].history)
plot_history(results["f{model}_adamw"].history)
plot_history(results["f{model}_amsgrad"].history)
plot_loss("f{model}", "f{dataset}")
```

E OBSERVED LOSS VALUES FOR EACH OPTIMIZERS AND MODELS TRAINED ON THE MNIST DATASET

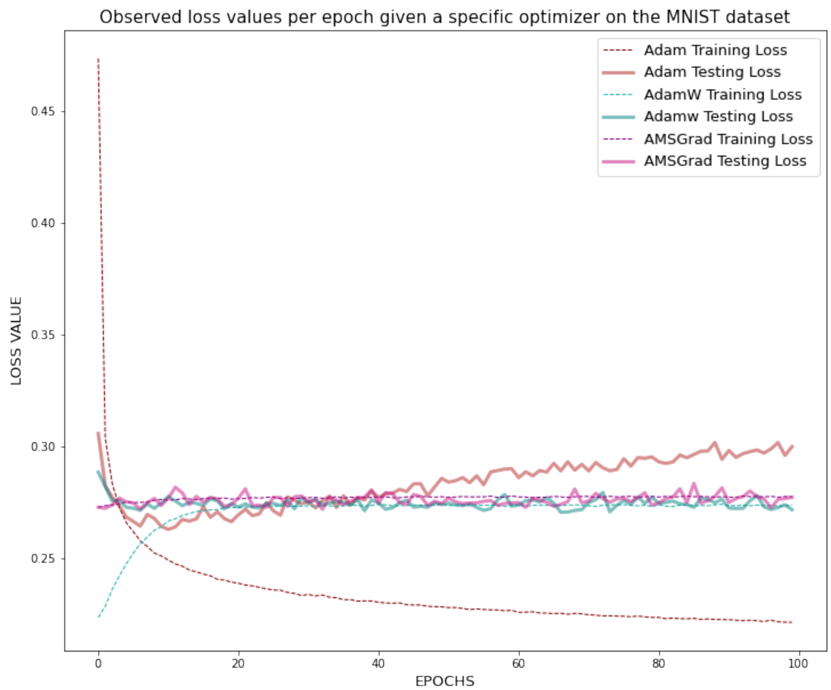

Figure 9: Results for the shallow neural network trained on MNIST

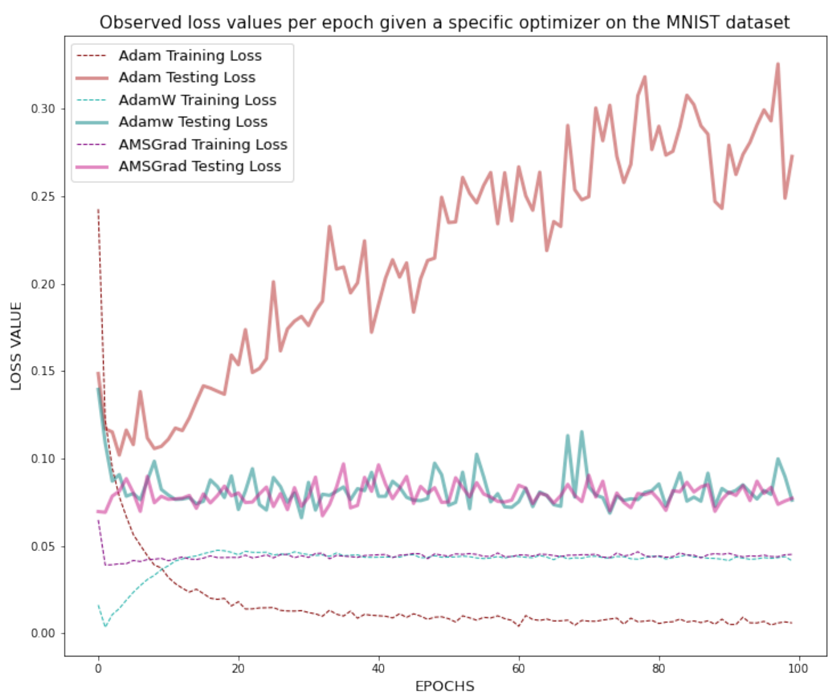

Figure 10: Results for the deep neural network trained on MNIST

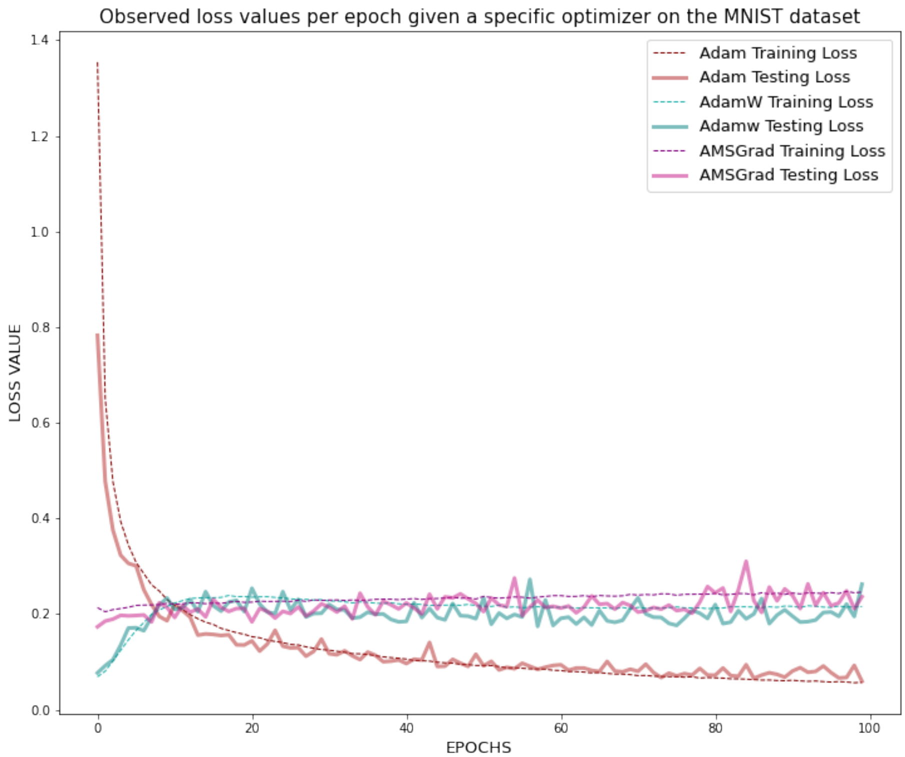

Figure 11: Results for the convolutional neural network trained on MNIST

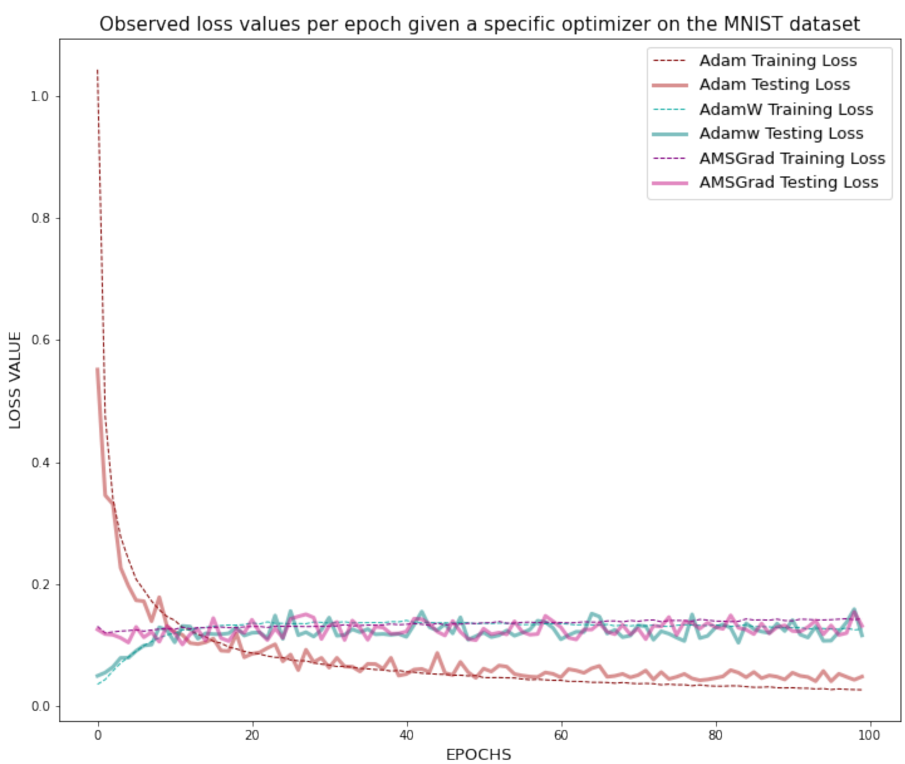

Figure 12: Results for the residual neural network trained on MNIST

F OBSERVED LOSS VALUES FOR EACH OPTIMIZERS AND MODELS TRAINED ON THE FASHION-MNIST DATASET

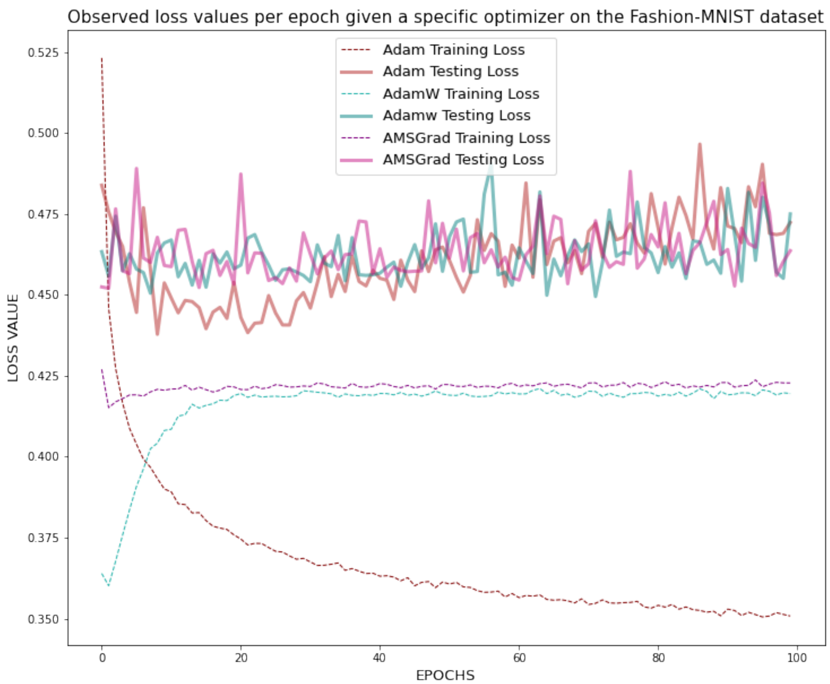

Figure 13: Results for the shallow neural network trained on Fashion-MNIST

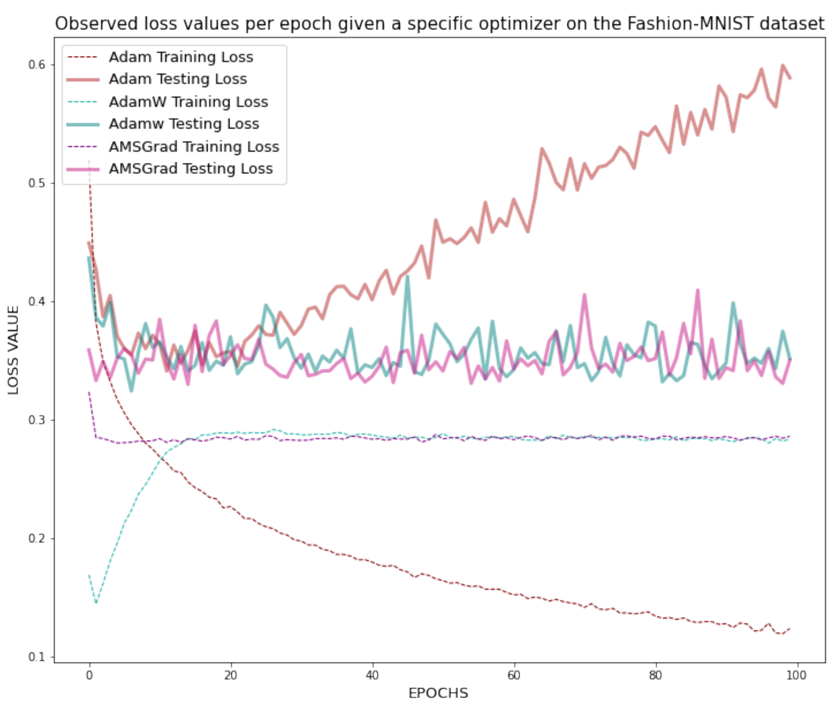

Figure 14: Results for the deep neural network trained on Fashion-MNIST

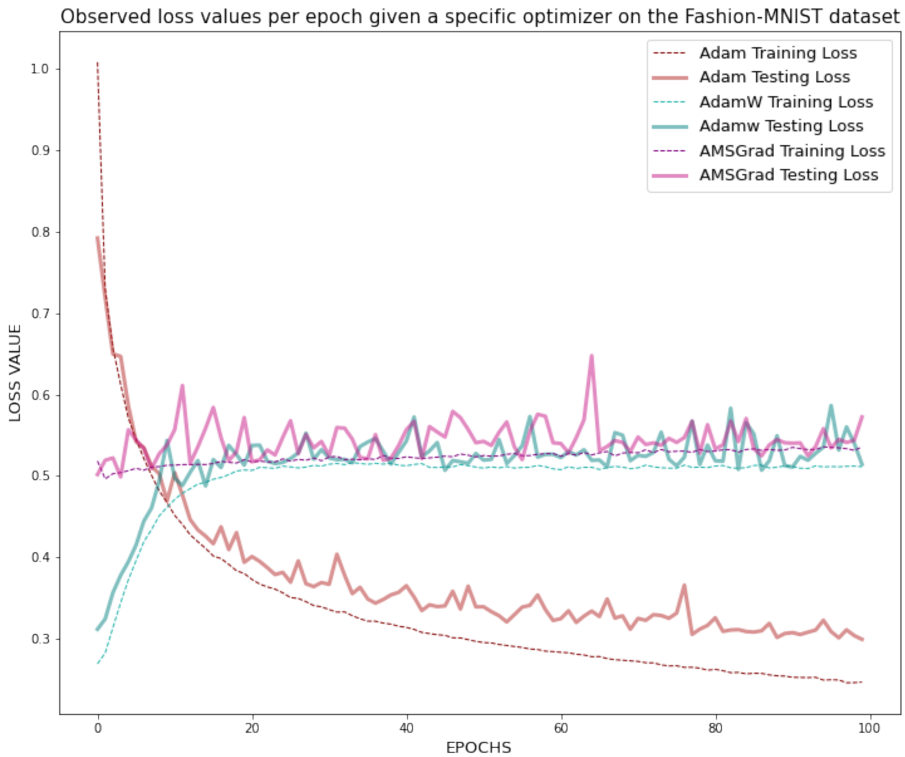

Figure 15: Results for the convolutional neural network trained on Fashion-MNIST

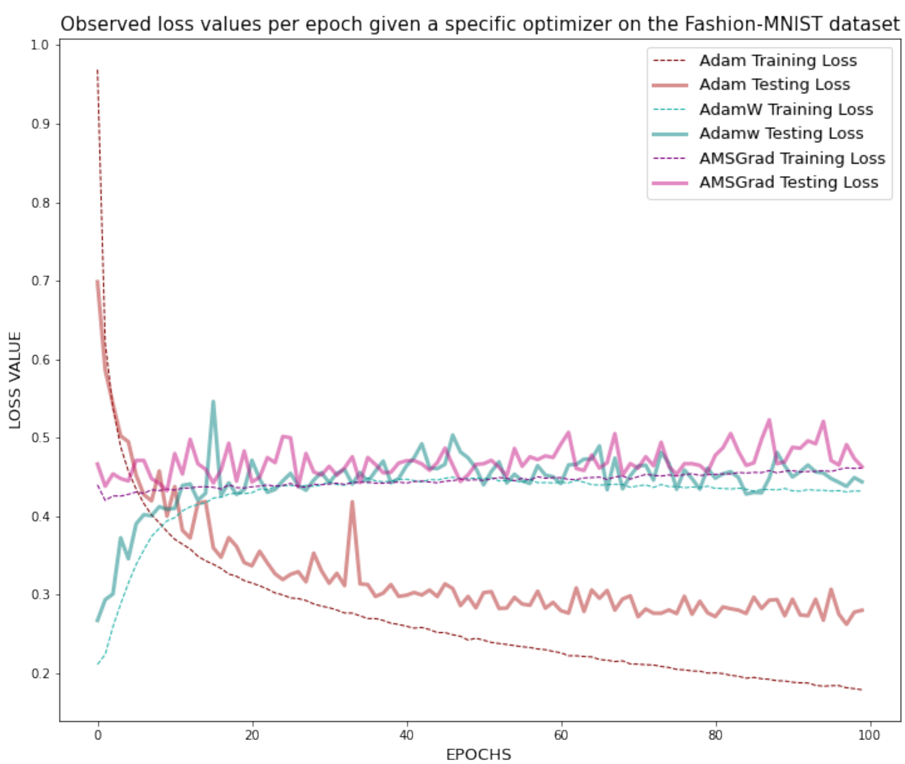

Figure 16: Results for the residual neural network trained on Fashion-MNIST

G OBSERVED LOSS VALUES FOR EACH OPTIMIZERS AND MODELS TRAINED ON THE GERMAN TRAFFIC SIGN RECOGNITION BENCHMARK DATASET

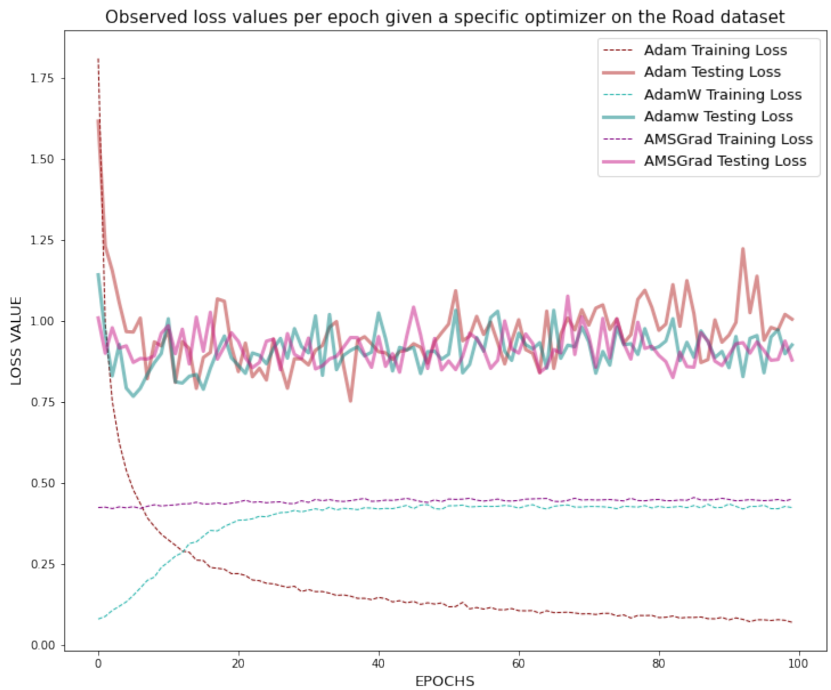

Figure 17: Results for the shallow neural network trained on GTSRB

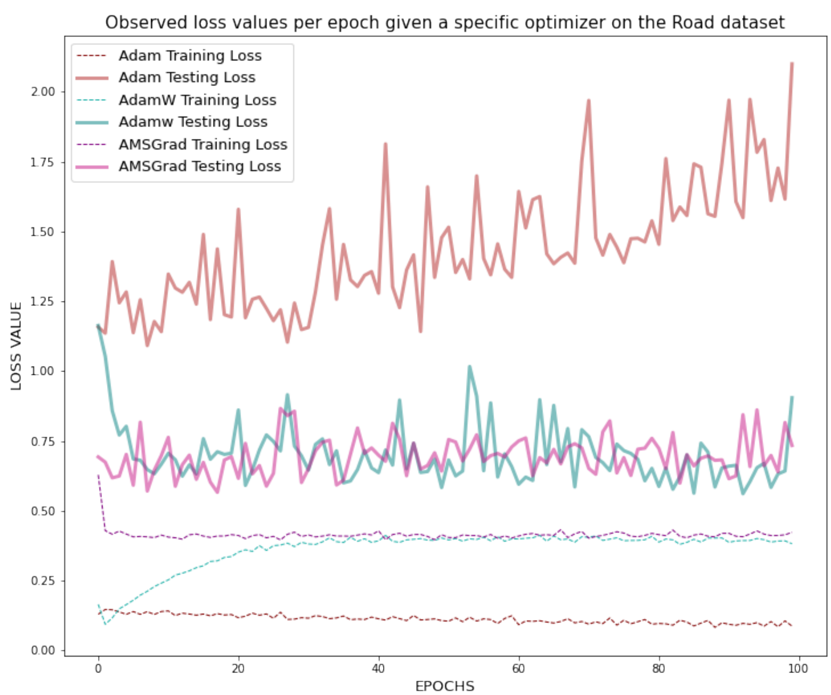

Figure 18: Results for the deep neural network trained on GTSRB

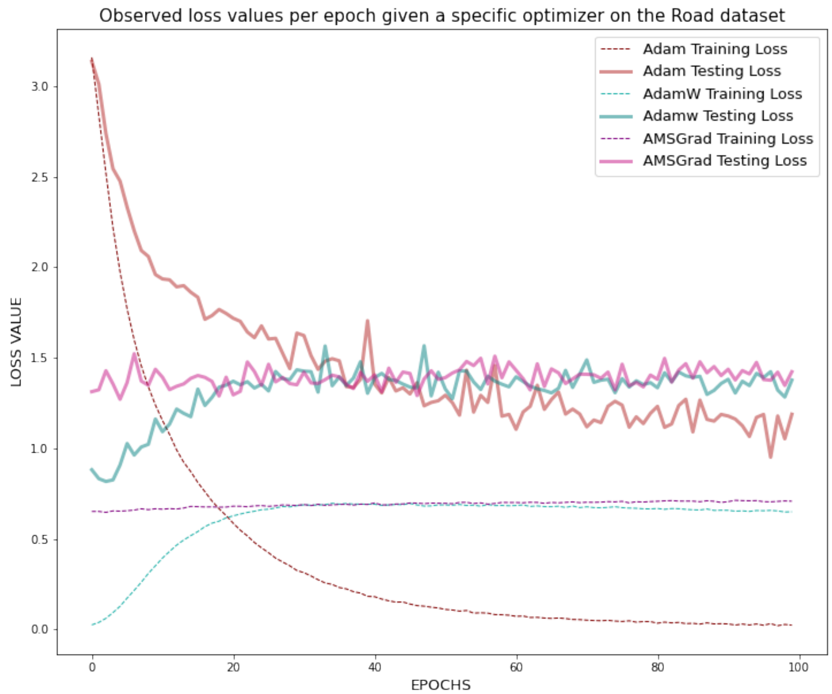

Figure 19: Results for the convolutional neural network trained on GTSRB

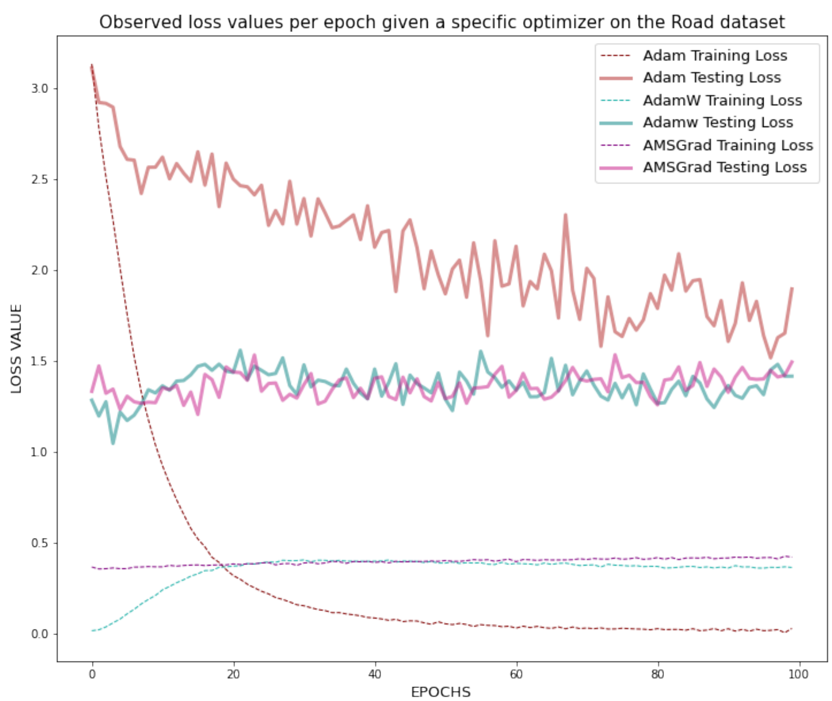

Figure 20: Results for the residual neural network trained on GTSRB

H Observed loss values for each optimizers and models trained on the CIFAR-10 dataset

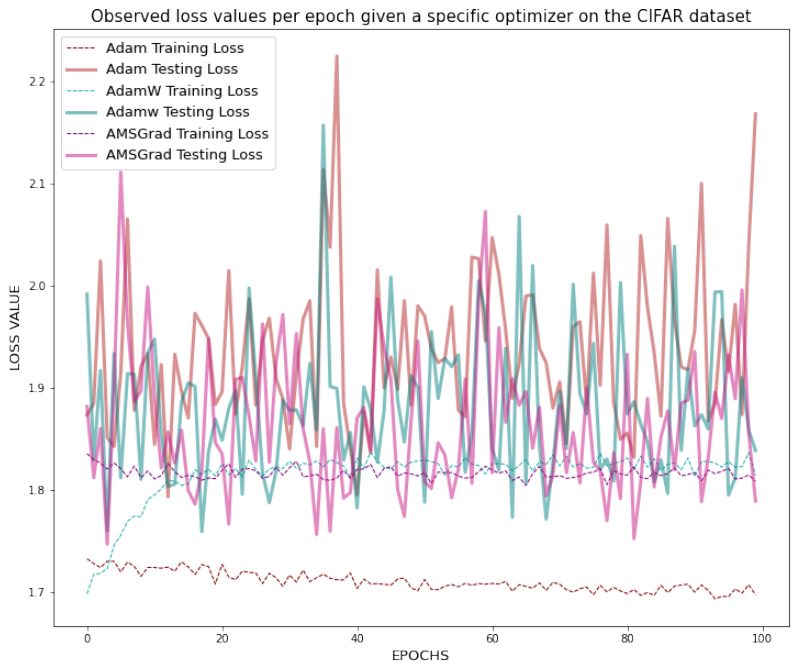

Figure 21: Results for the shallow neural network trained on CIFAR-10

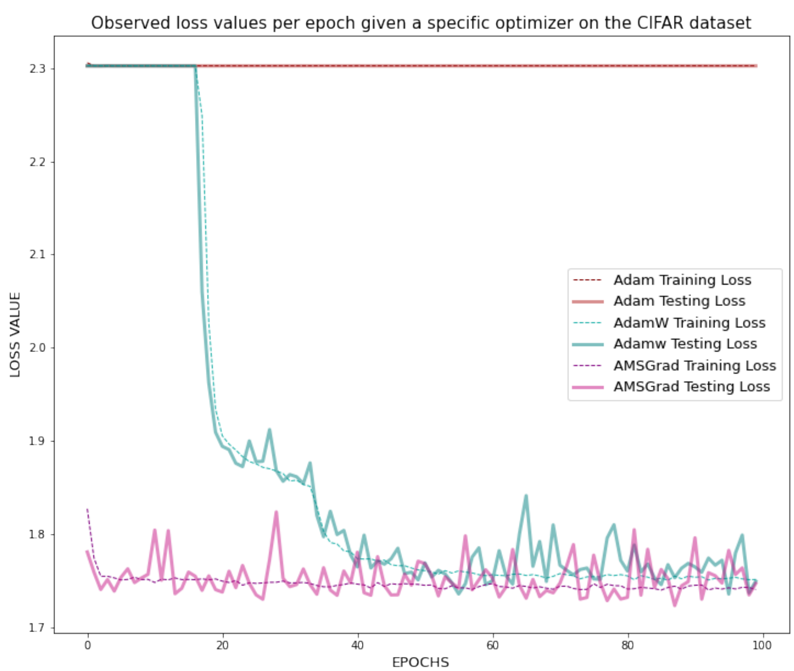

Figure 22: Results for the deep neural network trained on CIFAR-10

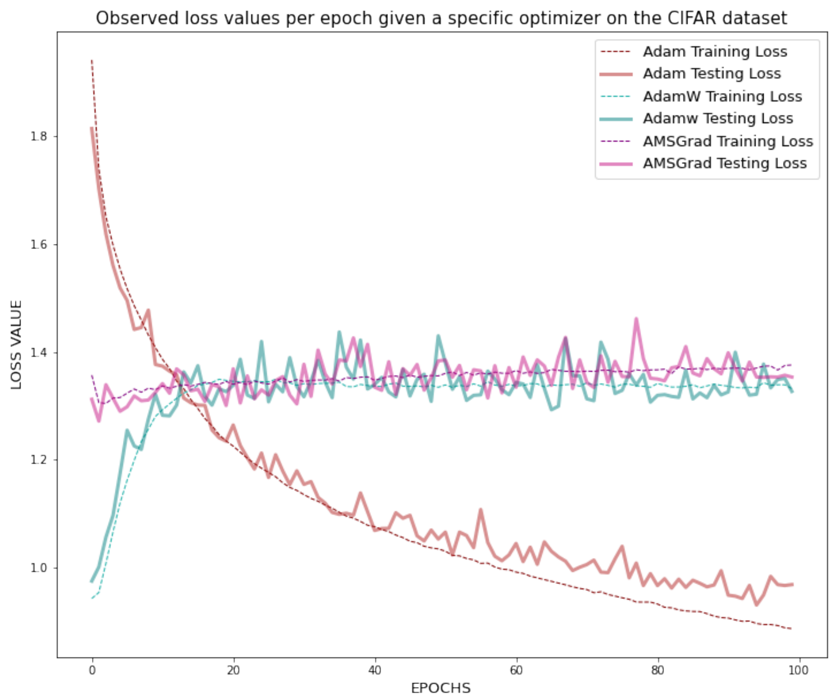

Figure 23: Results for the convolutional neural network trained on CIFAR-10

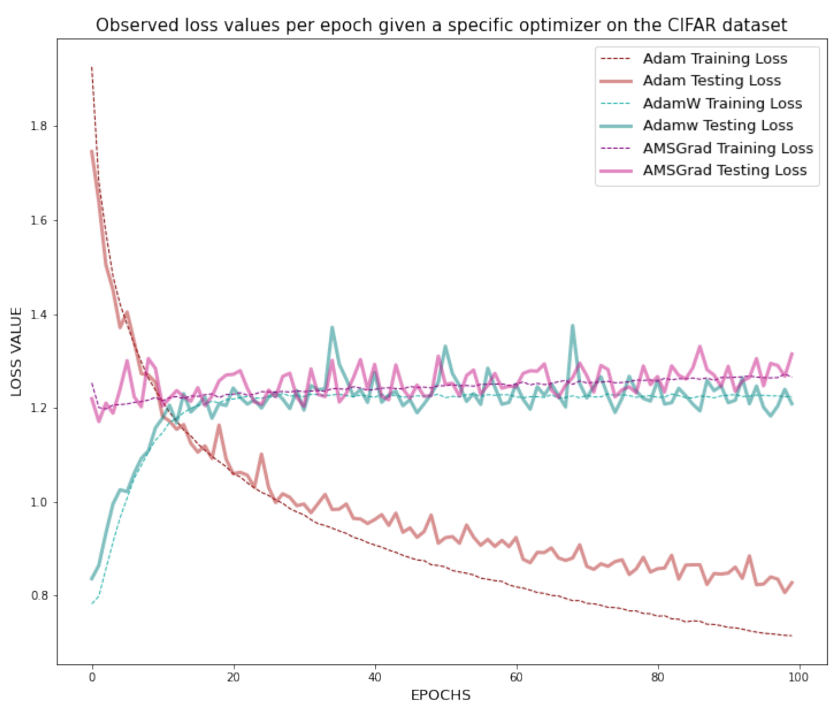

Figure 24: Results for the residual neural network trained on CIFAR-10

I Observations of loss and accuracy per model and optimizer for the MNIST dataset

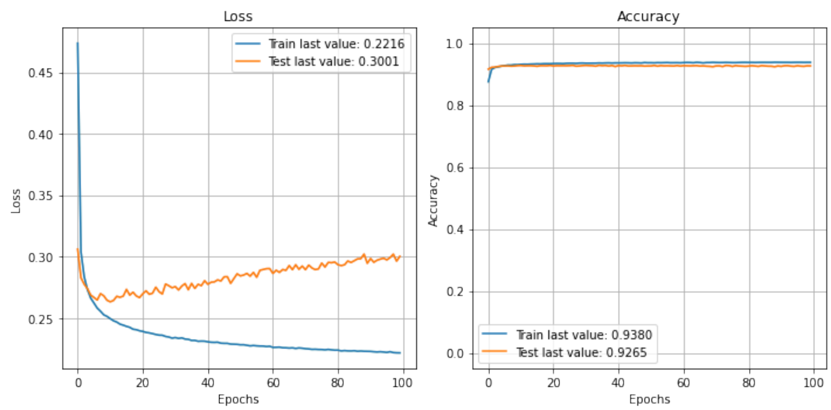

Figure 25: Loss and accuracy for the shallow neural network with Adam optimizer on MNIST

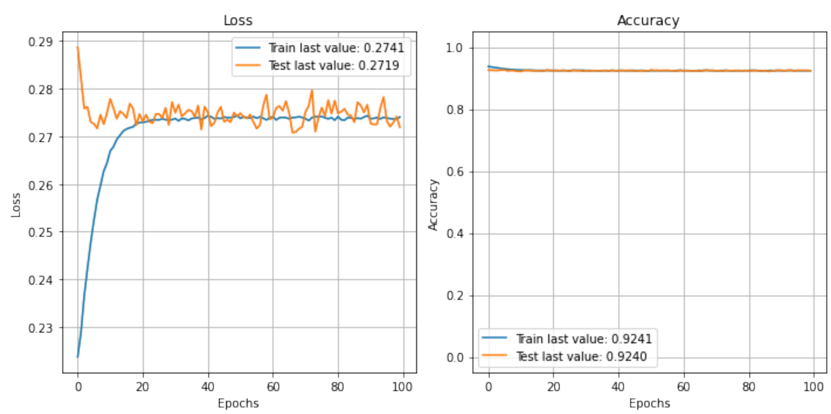

Figure 26: Loss and accuracy for the shallow neural network with AdamW optimizer on MNIST

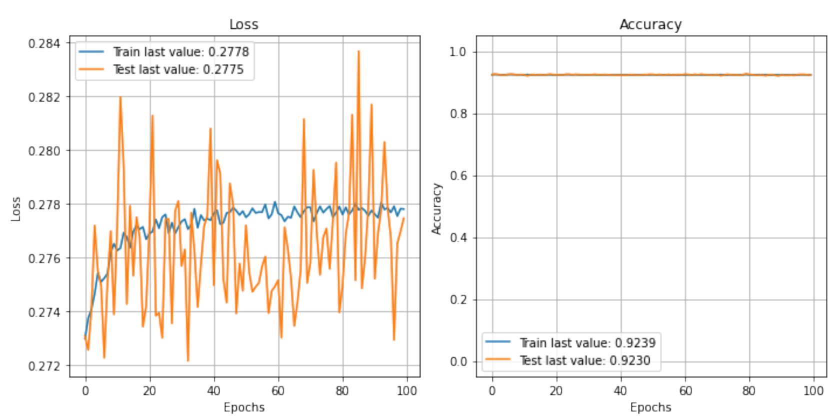

Figure 27: Loss and accuracy for the shallow neural network with AMSGrad optimizer on MNIST

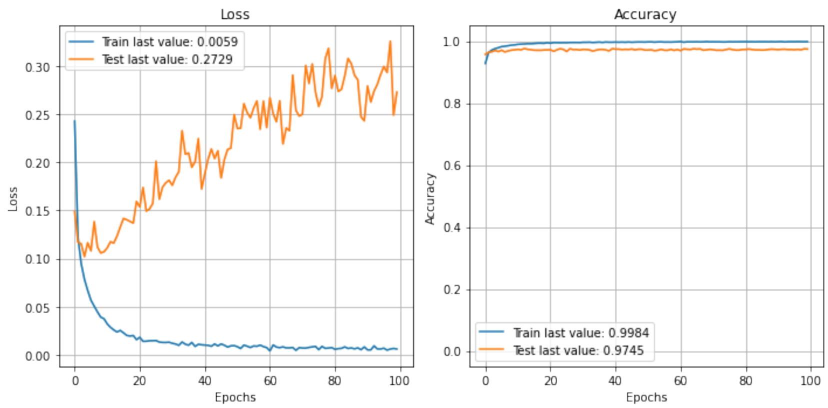

Figure 28: Loss and accuracy for the deep neural network Adam optimizer on MNIST

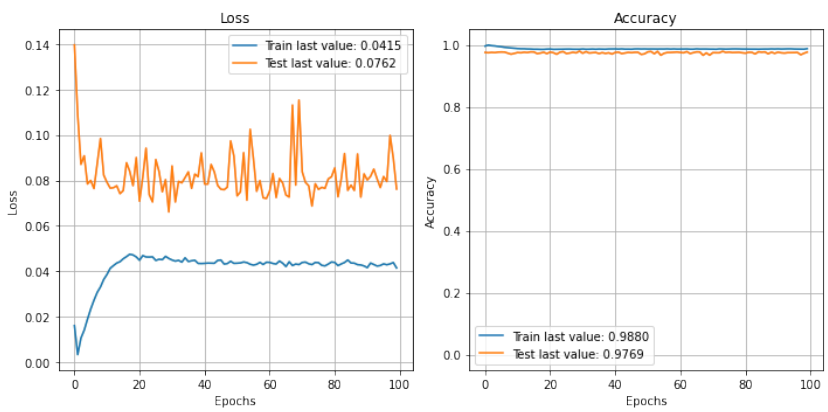

Figure 29: Loss and accuracy for the deep neural network with AdamW optimizer on MNIST

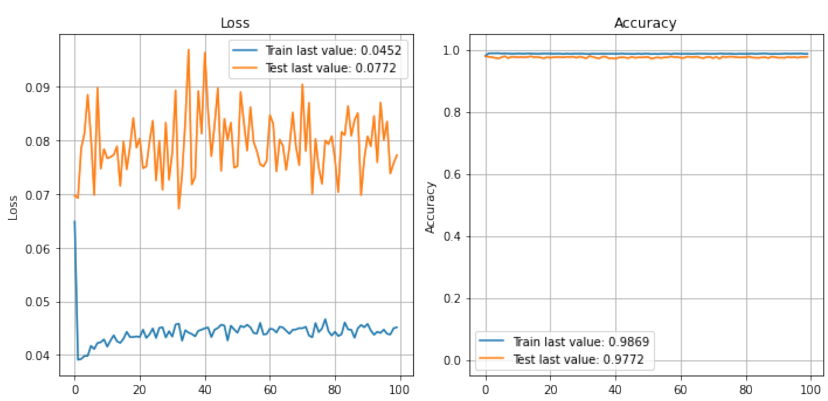

Figure 30: Loss and accuracy for the deep neural network with AMSGrad optimizer on MNIST

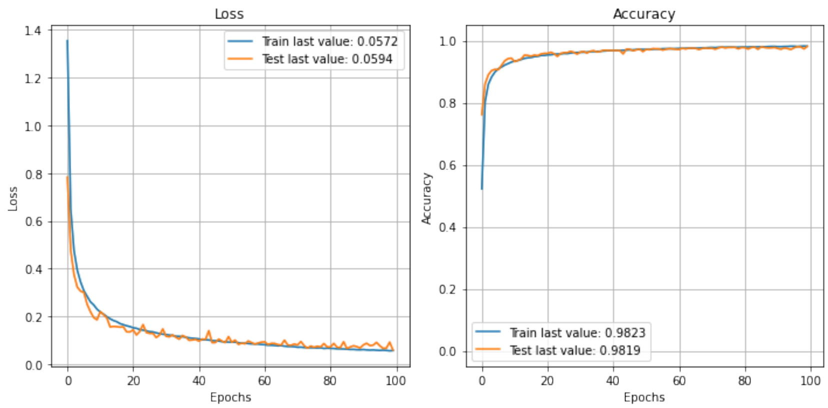

Figure 31: Loss and accuracy for the convolutional neural network with Adam optimizer on MNIST

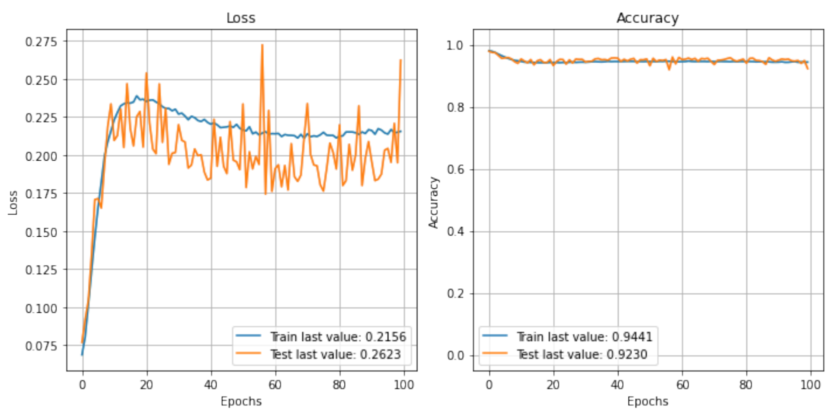

Figure 32: Loss and accuracy for the convolutional neural network with AdamW optimizer on MNIST

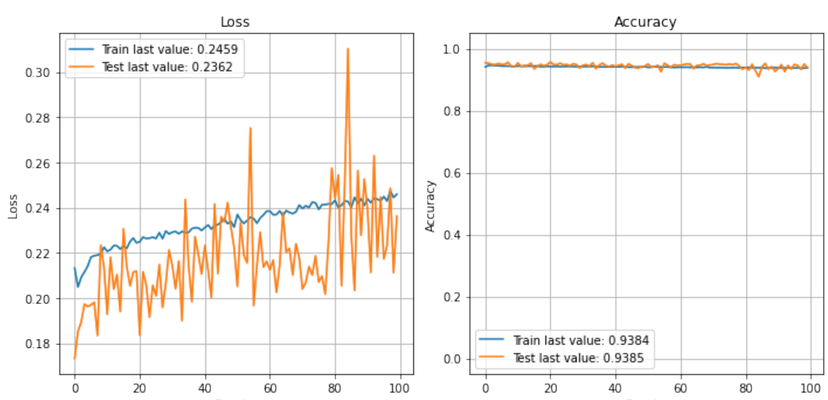

Figure 33: Loss and accuracy for the convolutional neural network with AMSGrad optimizer on MNIST

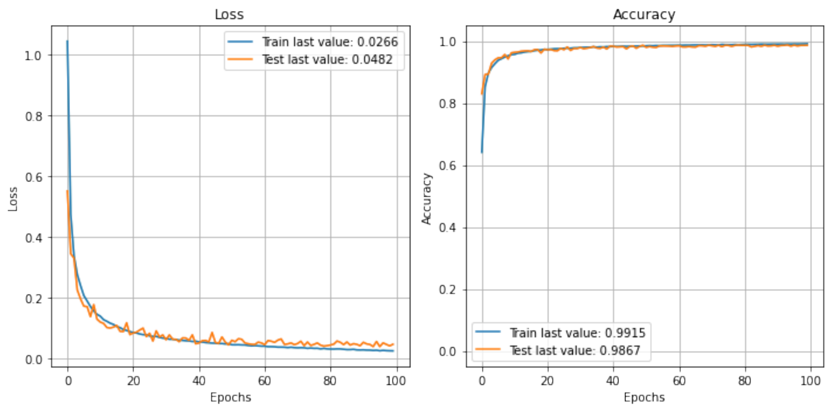

Figure 34: Loss and accuracy for the residual neural network with Adam optimizer on MNIST

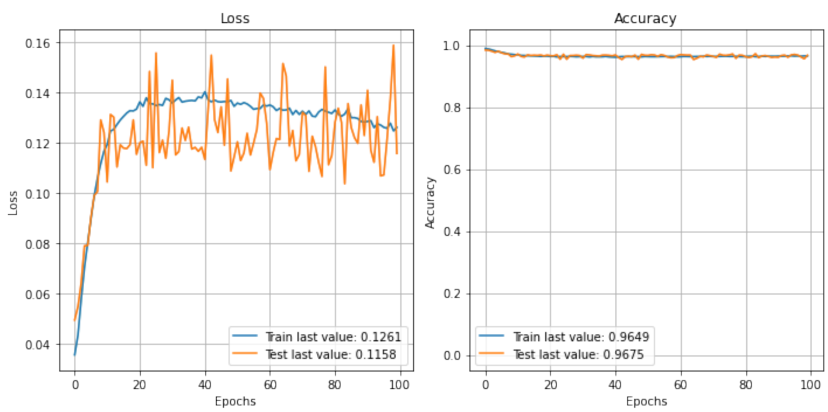

Figure 35: Loss and accuracy for the residual neural network with AdamW optimizer on MNIST

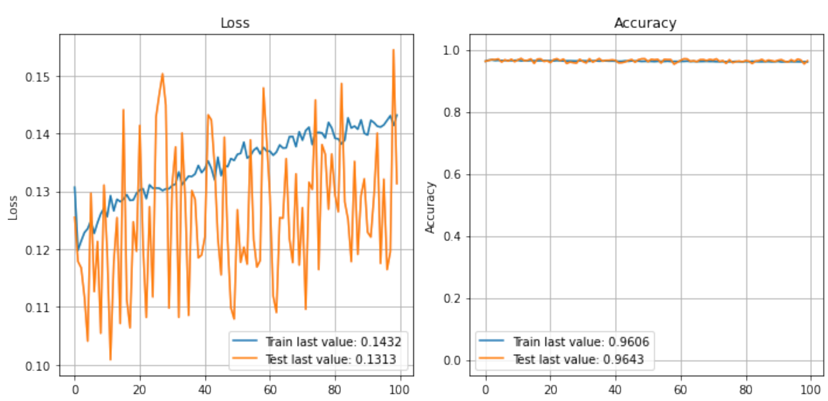

Figure 36: Loss and accuracy for the residual neural network with AMSGrad optimizer on MNIST

J OBSERVATIONS OF LOSS AND ACCURACY PER MODEL AND OPTIMIZER FOR THE FASHION-MNIST DATASET

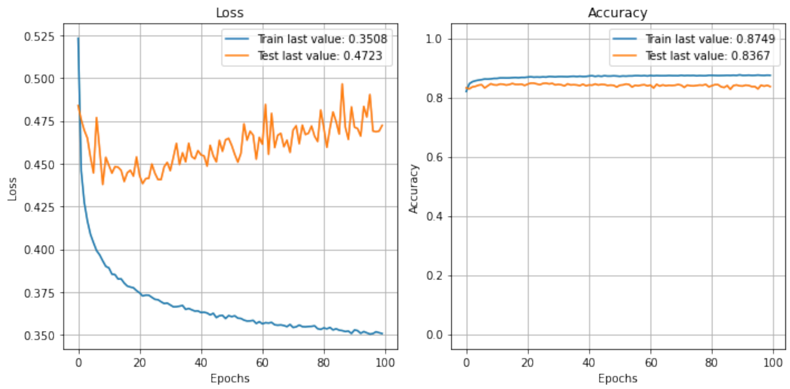

Figure 37: Loss and accuracy for the shallow neural network with Adam optimizer on Fashion-MNIST

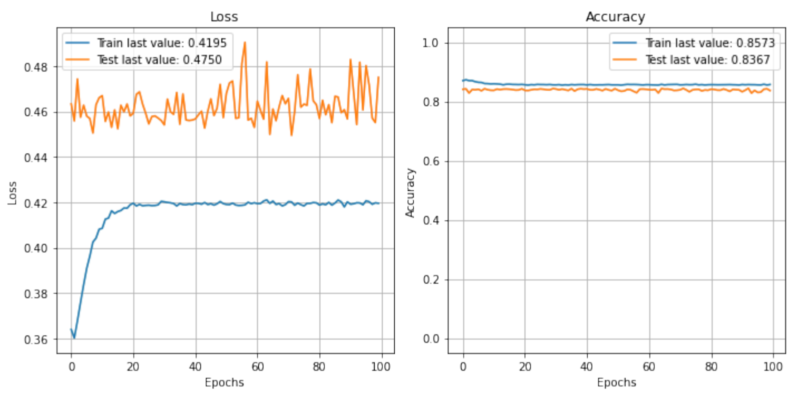

Figure 38: Loss and accuracy for the shallow neural network with AdamW optimizer on Fashion-MNIST

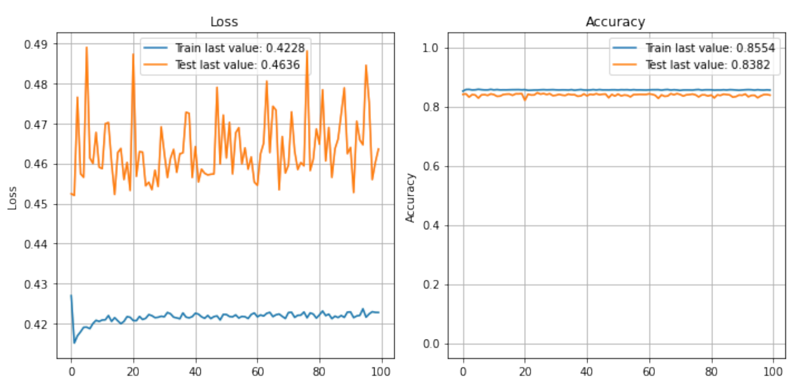

Figure 39: Loss and accuracy for the shallow neural network with AMSGrad optimizer on Fashion-MNIST

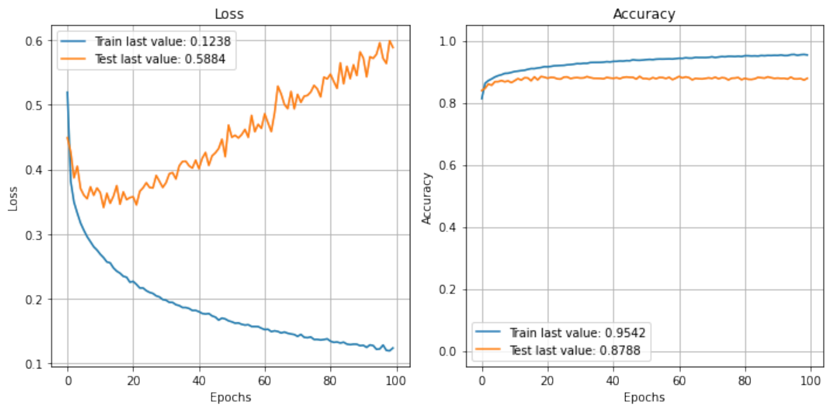

Figure 40: Loss and accuracy for the deep neural network Adam optimizer on Fashion-MNIST

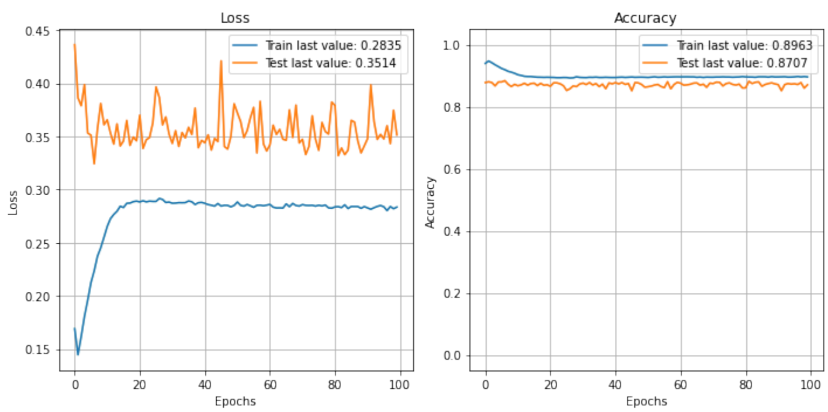

Figure 41: Loss and accuracy for the deep neural network with AdamW optimizer on Fashion-MNIST

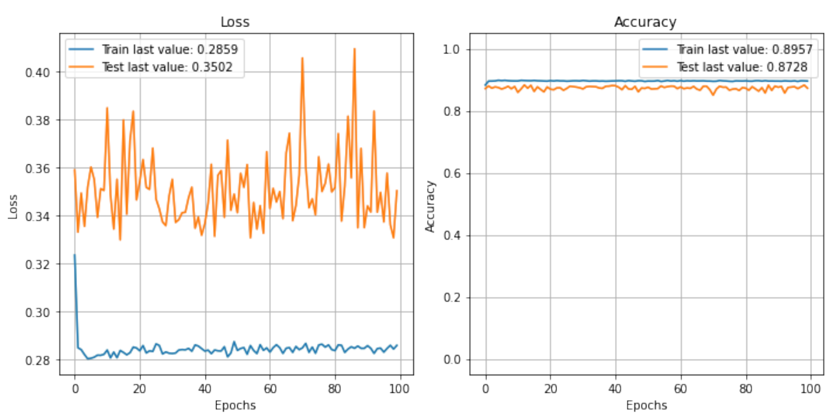

Figure 42: Loss and accuracy for the deep neural network with AMSGrad optimizer on Fashion-MNIST

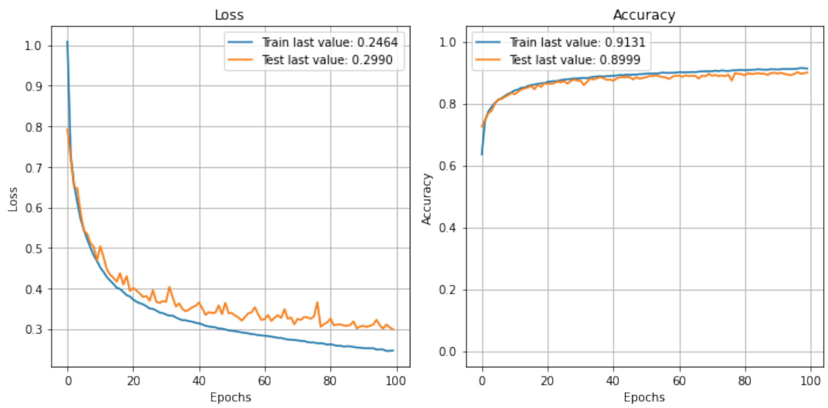

Figure 43: Loss and accuracy for the convolutional neural network with Adam optimizer on Fashion-MNIST

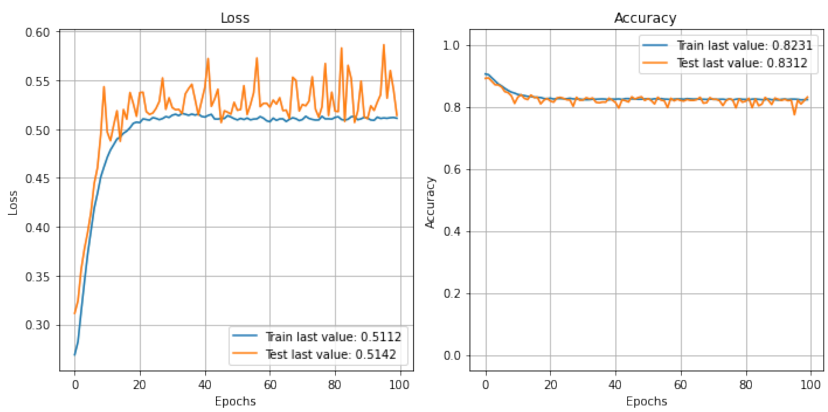

Figure 44: Loss and accuracy for the convolutional neural network with AdamW optimizer on Fashion-MNIST

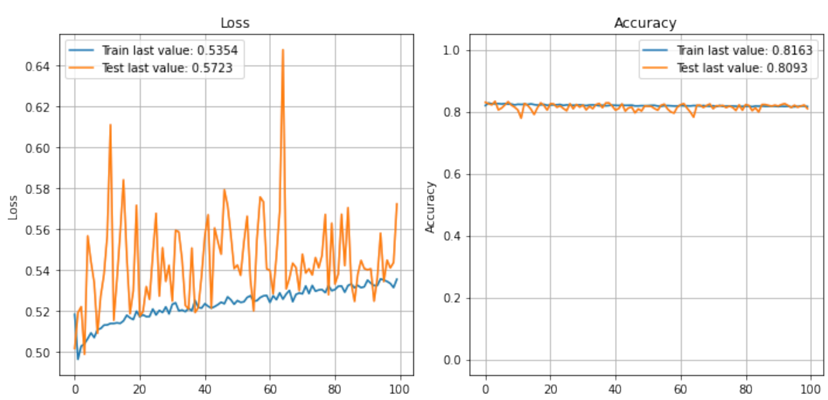

Figure 45: Loss and accuracy for the convolutional neural network with AMSGrad optimizer on Fashion-MNIST

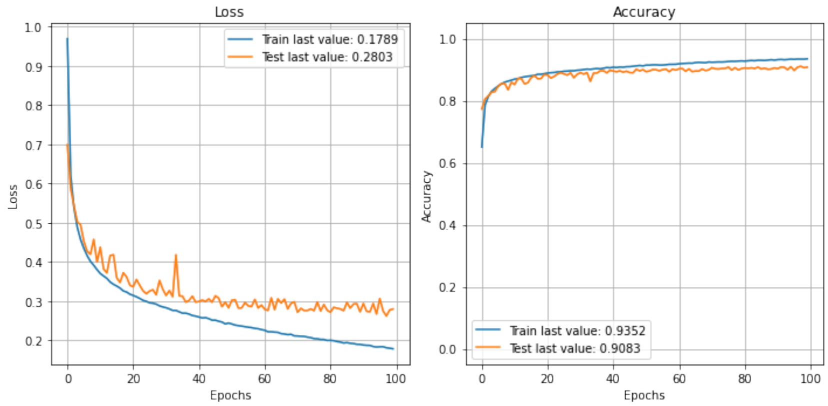

Figure 46: Loss and accuracy for the residual neural network with Adam optimizer on Fashion-MNIST

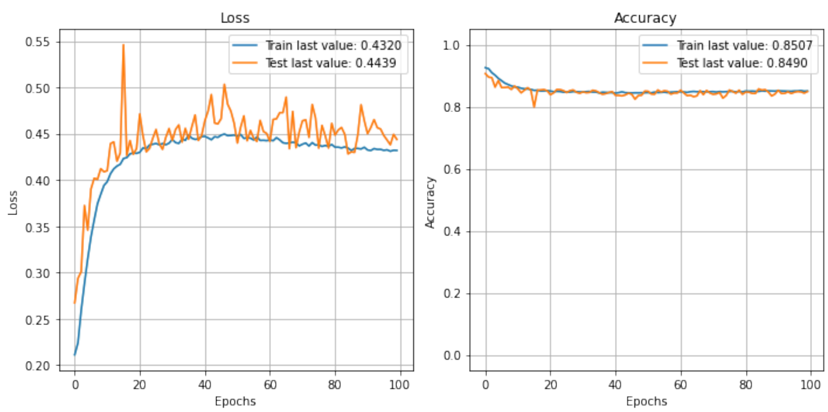

Figure 47: Loss and accuracy for the residual neural network with AdamW optimizer on Fashion-MNIST

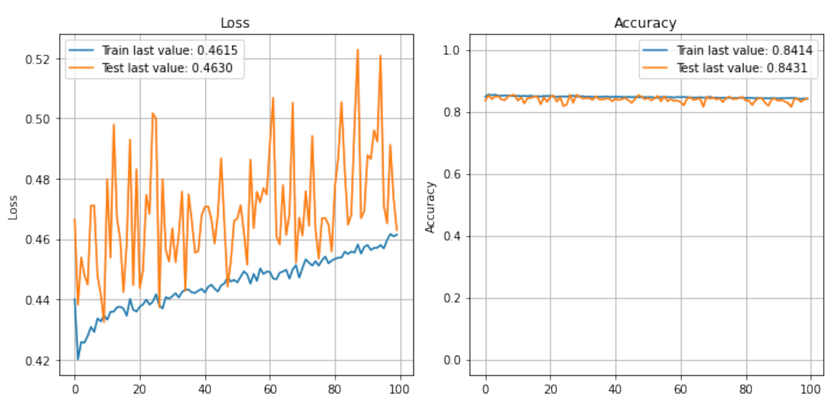

Figure 48: Loss and accuracy for the residual neural network with AMSGrad optimizer on Fashion-MNIST

K Observations of loss and accuracy per model and optimizer for the German Traffic Sign Recognition Benchmark dataset

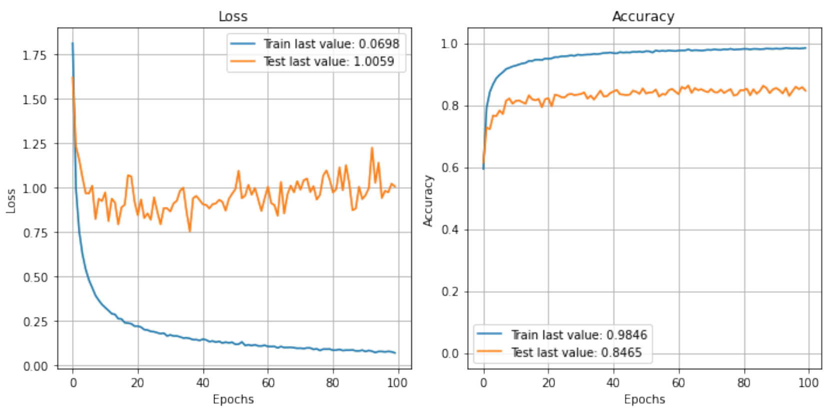

Figure 49: Loss and accuracy for the shallow neural network with Adam optimizer on GTSRB

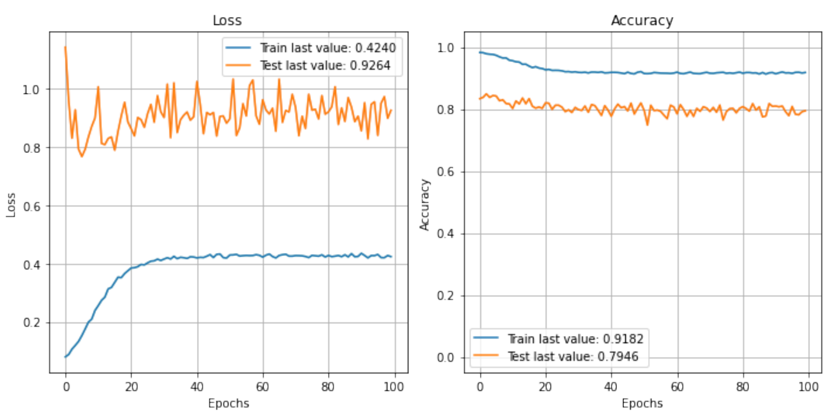

Figure 50: Loss and accuracy for the shallow neural network with AdamW optimizer on GTSRB

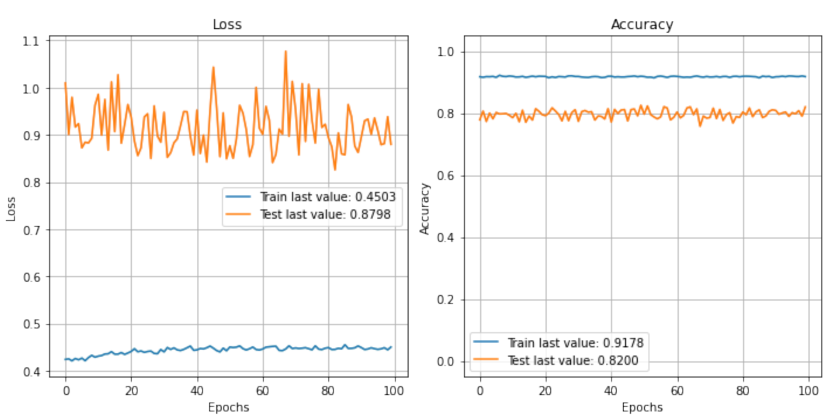

Figure 51: Loss and accuracy for the shallow neural network with AMSGrad optimizer on GTSRB

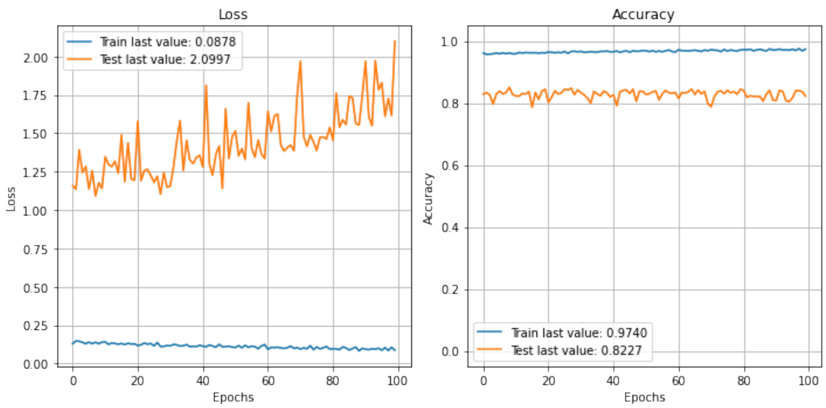

Figure 52: Loss and accuracy for the deep neural network Adam optimizer on GTSRB

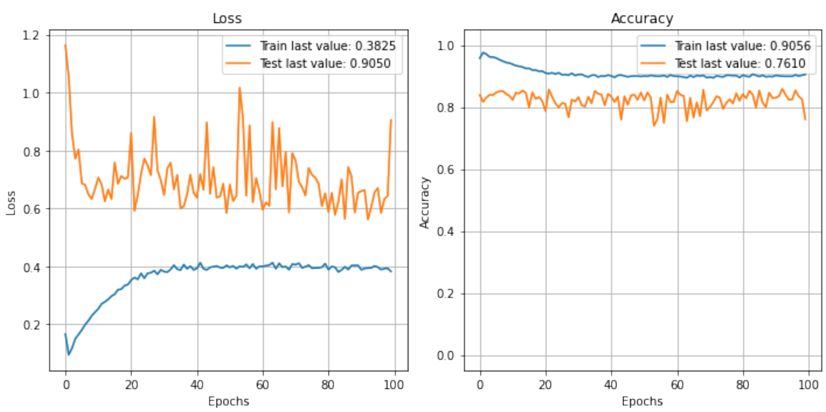

Figure 53: Loss and accuracy for the deep neural network with AdamW optimizer on GTSRB

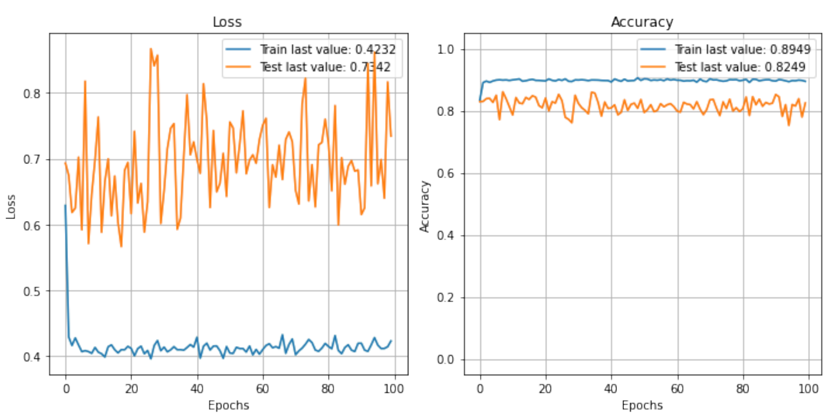

Figure 54: Loss and accuracy for the deep neural network with AMSGrad optimizer on GTSRB

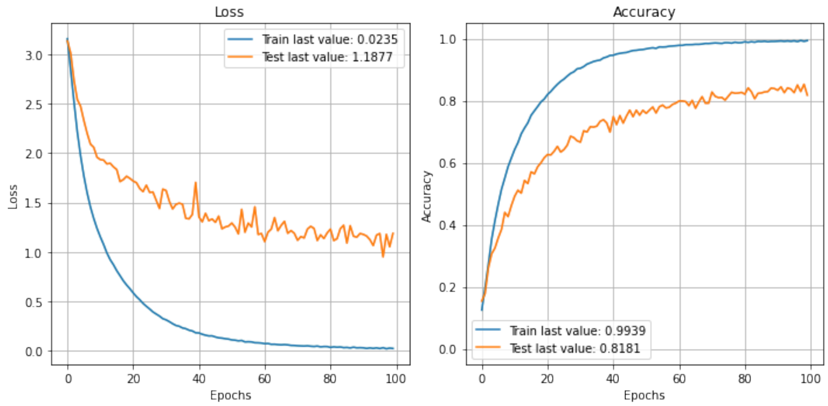

Figure 55: Loss and accuracy for the convolutional neural network with Adam optimizer on GTSRB

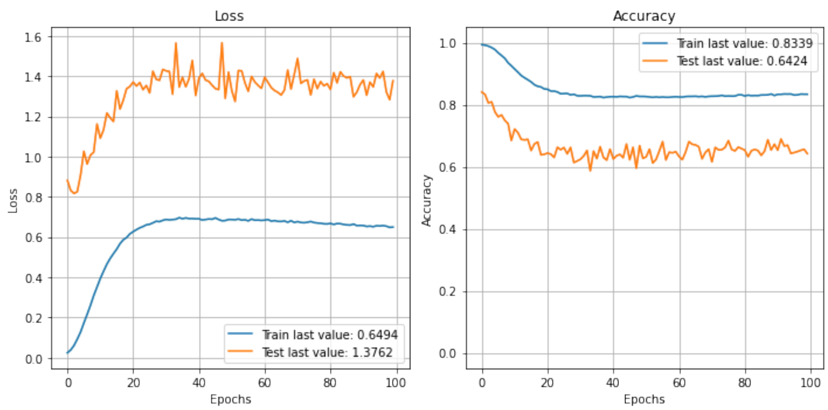

Figure 56: Loss and accuracy for the convolutional neural network with AdamW optimizer on GTSRB

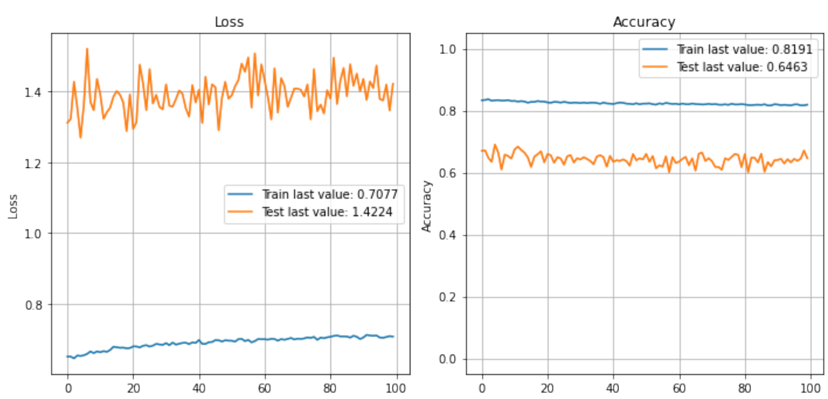

Figure 57: Loss and accuracy for the convolutional neural network with AMSGrad optimizer on GTSRB

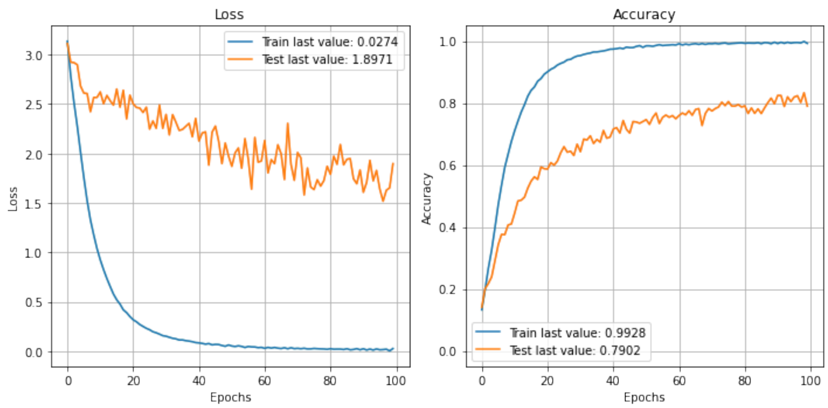

Figure 58: Loss and accuracy for the residual neural network with Adam optimizer on GTSRB

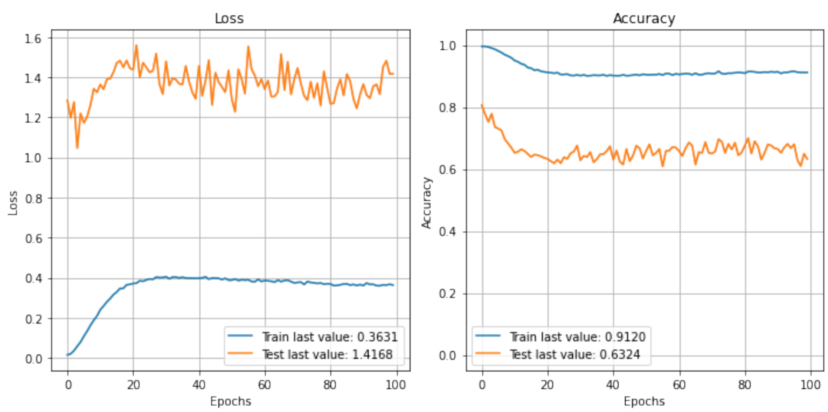

Figure 59: Loss and accuracy for the residual neural network with AdamW optimizer on GTSRB

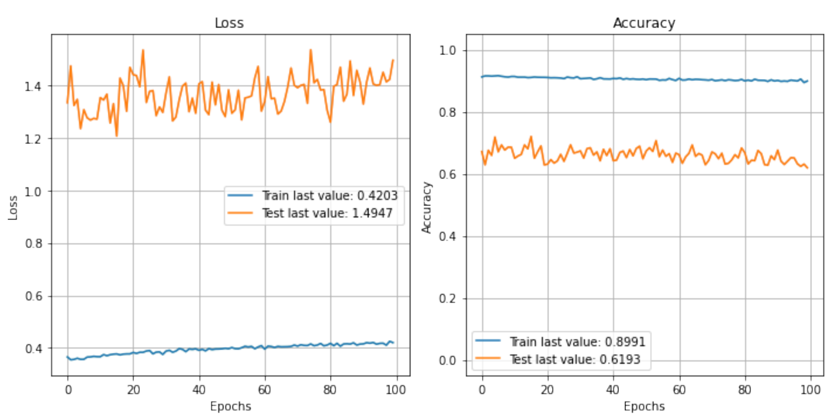

Figure 60: Loss and accuracy for the residual neural network with AMSGrad optimizer on GTSRB

L Observations of loss and accuracy per model and optimizer for the CIFAR-10 dataset

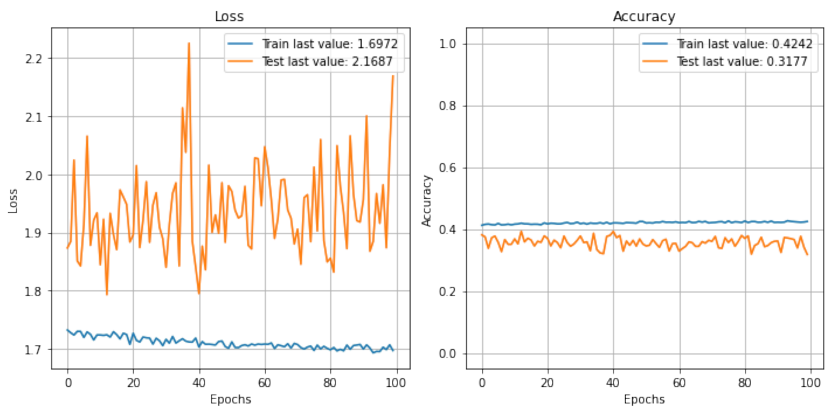

Figure 61: Loss and accuracy for the shallow neural network with Adam optimizer on CIFAR-10

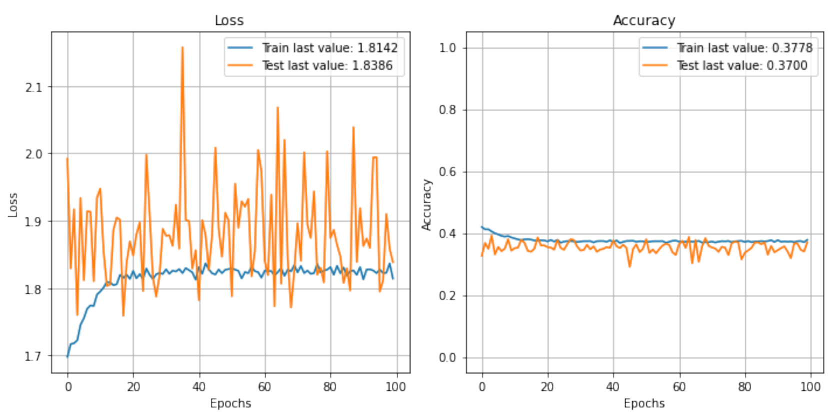

Figure 62: Loss and accuracy for the shallow neural network with AdamW optimizer on CIFAR-10

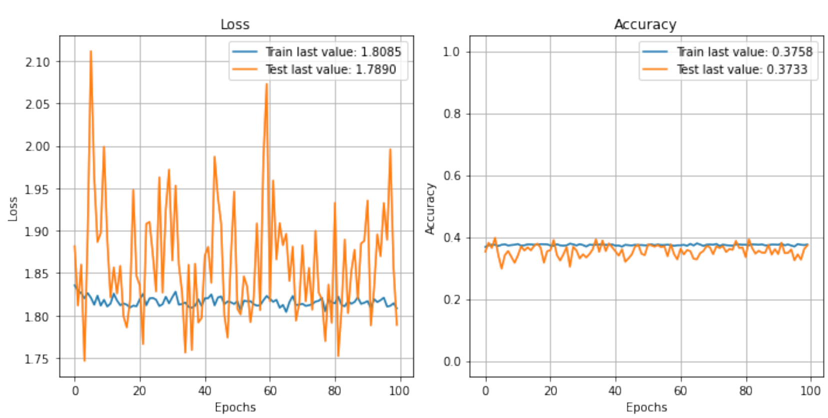

Figure 63: Loss and accuracy for the shallow neural network with AMSGrad optimizer on CIFAR-10

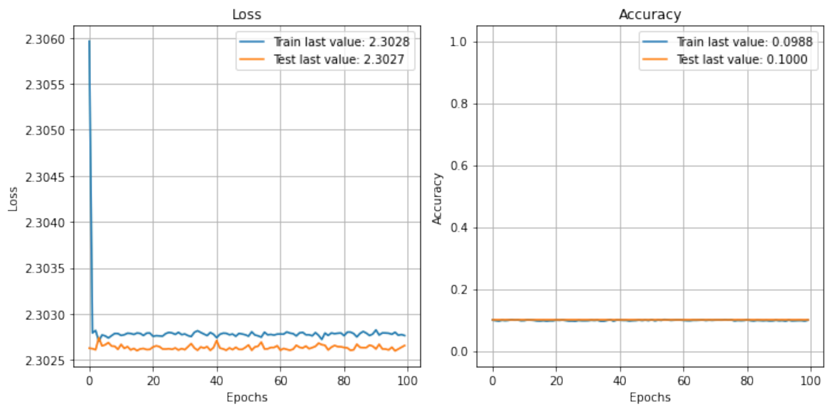

Figure 64: Loss and accuracy for the deep neural network Adam optimizer on CIFAR-10

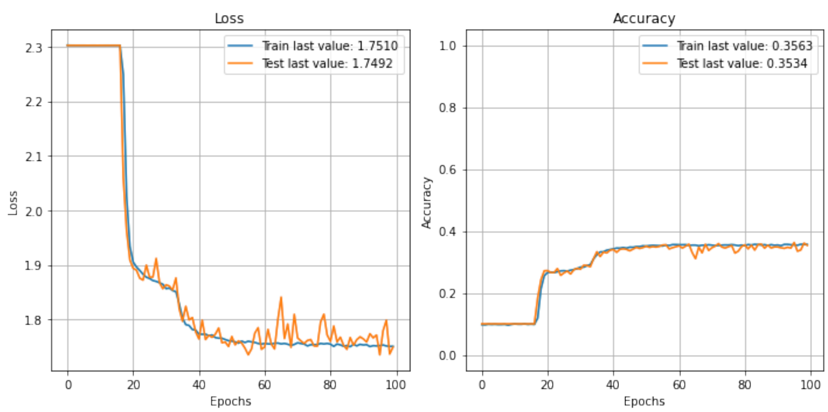

Figure 65: Loss and accuracy for the deep neural network with AdamW optimizer on CIFAR-10

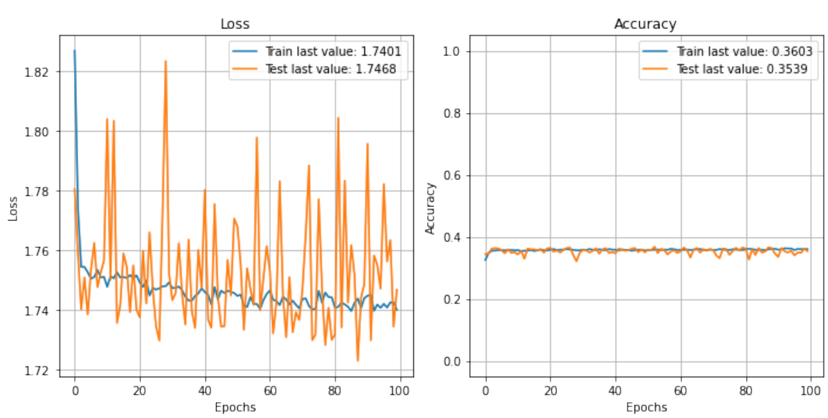

Figure 66: Loss and accuracy for the deep neural network with AMSGrad optimizer on CIFAR-10

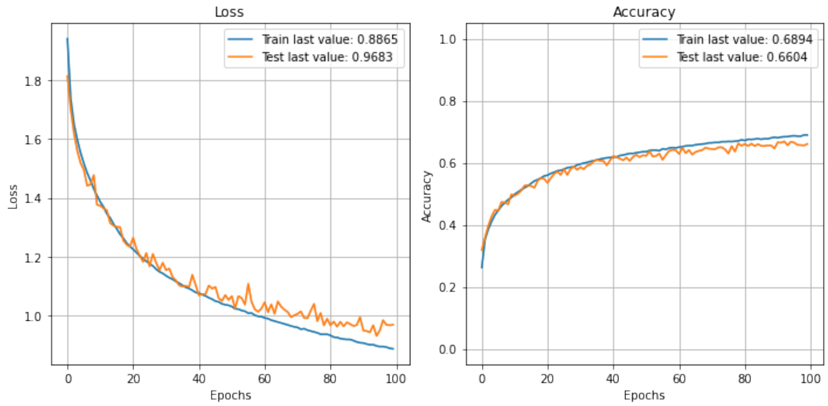

Figure 67: Loss and accuracy for the convolutional neural network with Adam optimizer on CIFAR-10

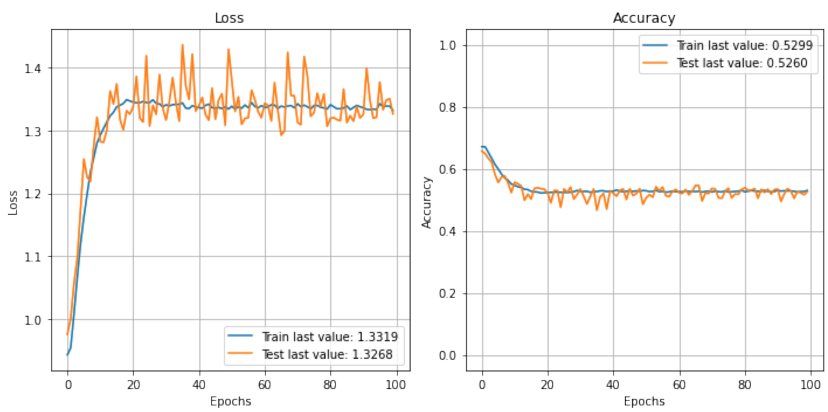

Figure 68: Loss and accuracy for the convolutional neural network with AdamW optimizer on CIFAR-10

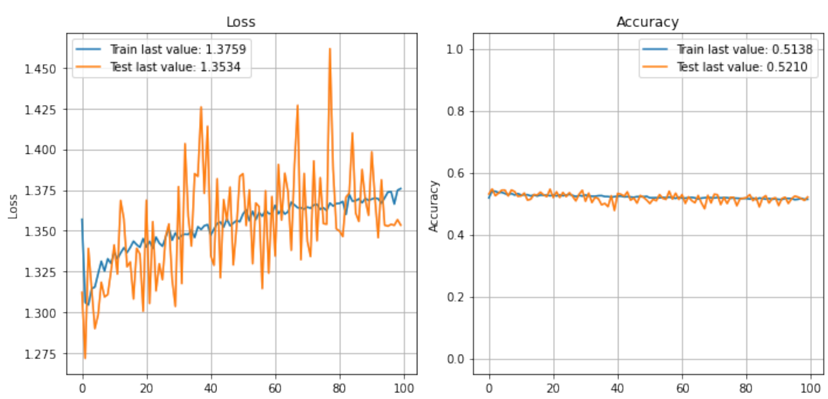

Figure 69: Loss and accuracy for the convolutional neural network with AMSGrad optimizer on CIFAR-10

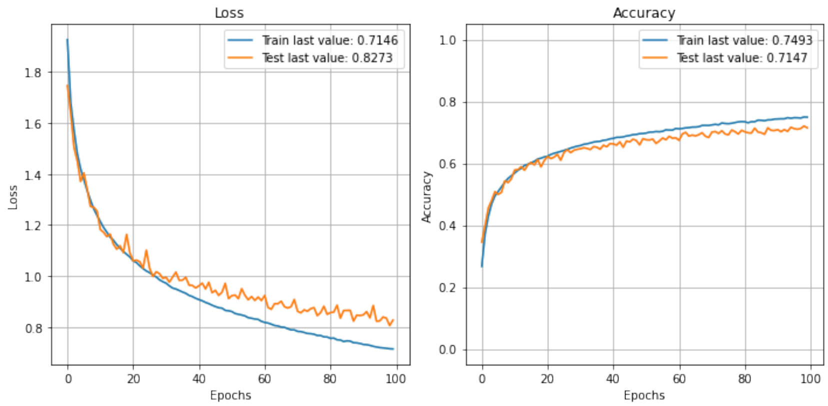

Figure 70: Loss and accuracy for the residual neural network with Adam optimizer on CIFAR-10

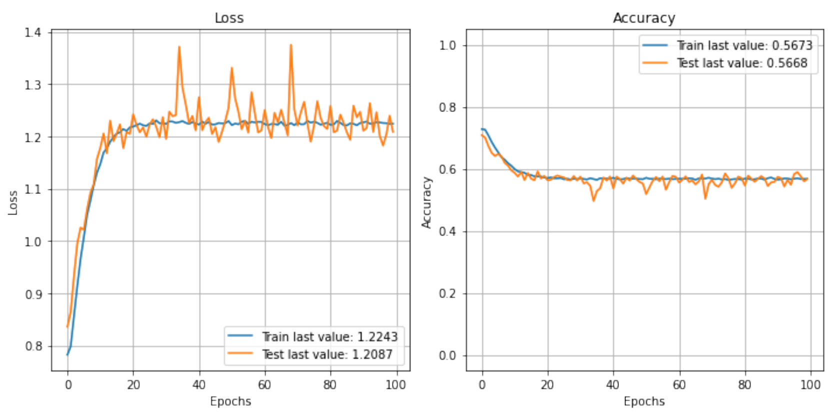

Figure 71: Loss and accuracy for the residual neural network with AdamW optimizer on CIFAR-10

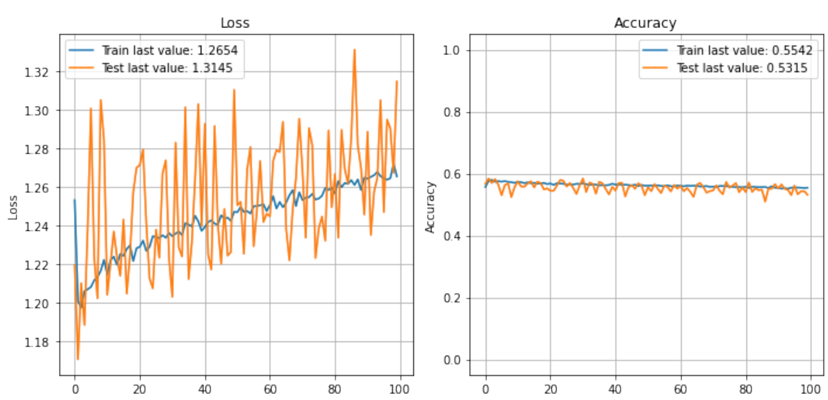

Figure 72: Loss and accuracy for the residual neural network with AMSGrad optimizer on CIFAR-10