Tuto_1

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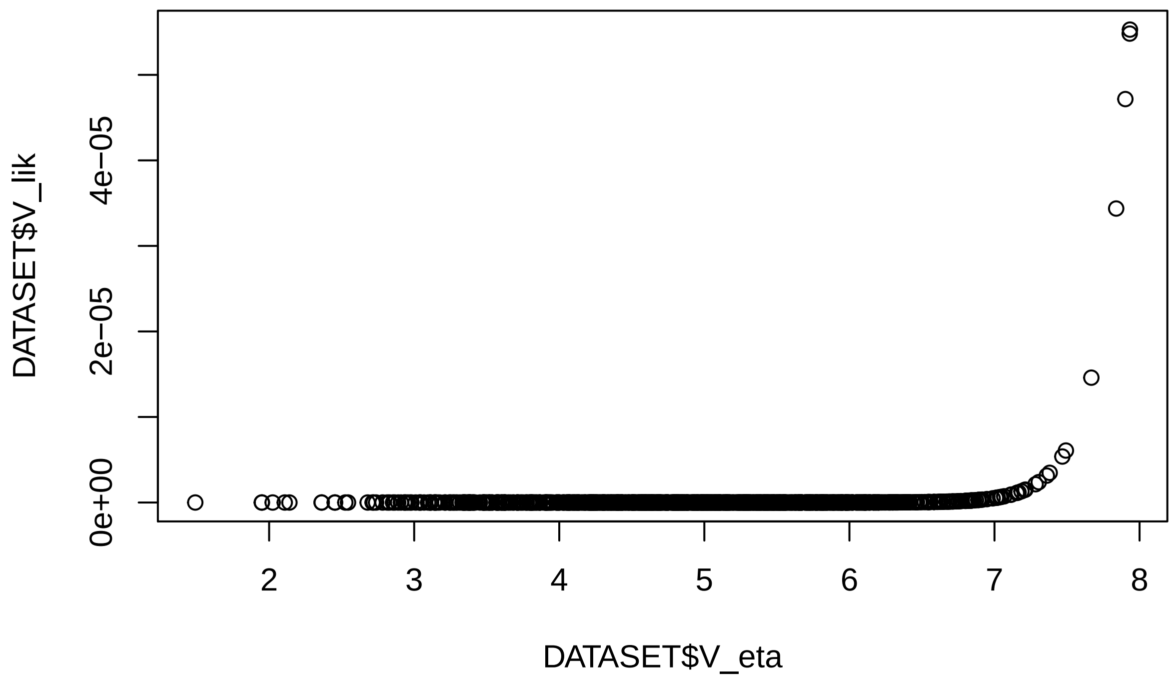
1.a

1.b

```r
#genereting 10 IID samples
set.seed(50) # Setting a seed

x <- rnorm(10, mean = 10, sd = 1)


lik <- function(f_eta){
  theta<- f_eta[1] #theta
  eta<- f_eta[2]
  out <-theta*exp(-theta*(sum(x)-length(x)*eta))
  return(out)
}


#plot(f_eta, lik(f_eta), type="l", lwd=3, main="logliklihood_Weibull, n=1000")


theta=0.5
#genereting eta
V_eta <- rnorm(1000, mean = 5, sd = 1)

V_lik<- c()
for (el in V_eta) {

  f_eta<- c(theta,el) #theta    eta
  V_lik<-append(V_lik, lik(f_eta))
}
```

```r
#creating dataset with eta column and  liklihood column
DATASET<- data.frame(V_eta, V_lik)
#plot
plot(DATASET$V_eta,DATASET$V_lik)
```




```r
#maxium value

DATASET[c(which.max(DATASET$V_lik)),]
```

```
##        V_eta        V_lik
## 416 7.933991 5.528858e-05
```

1.c

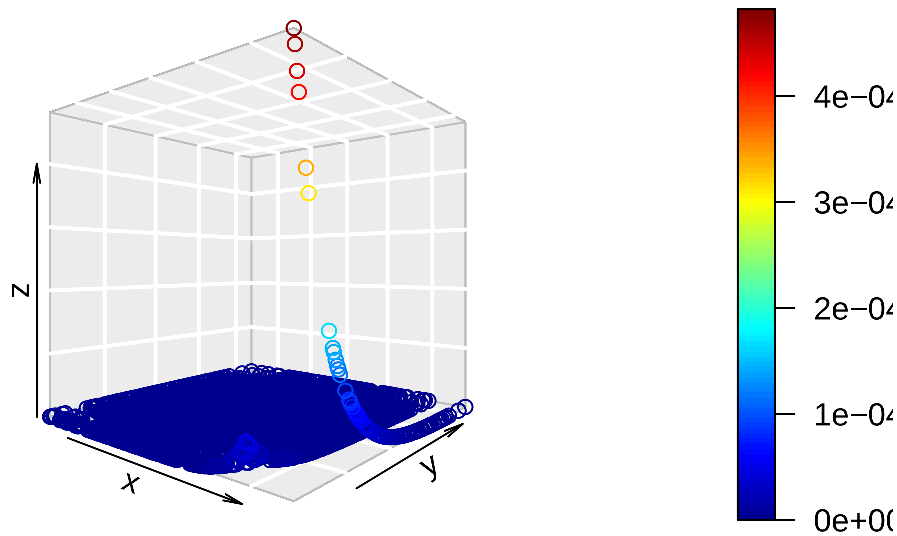
1.d

```r
#genereting 10 IID samples
set.seed(50) # Setting a seed

x <- rnorm(10, mean = 10, sd = 1)
```

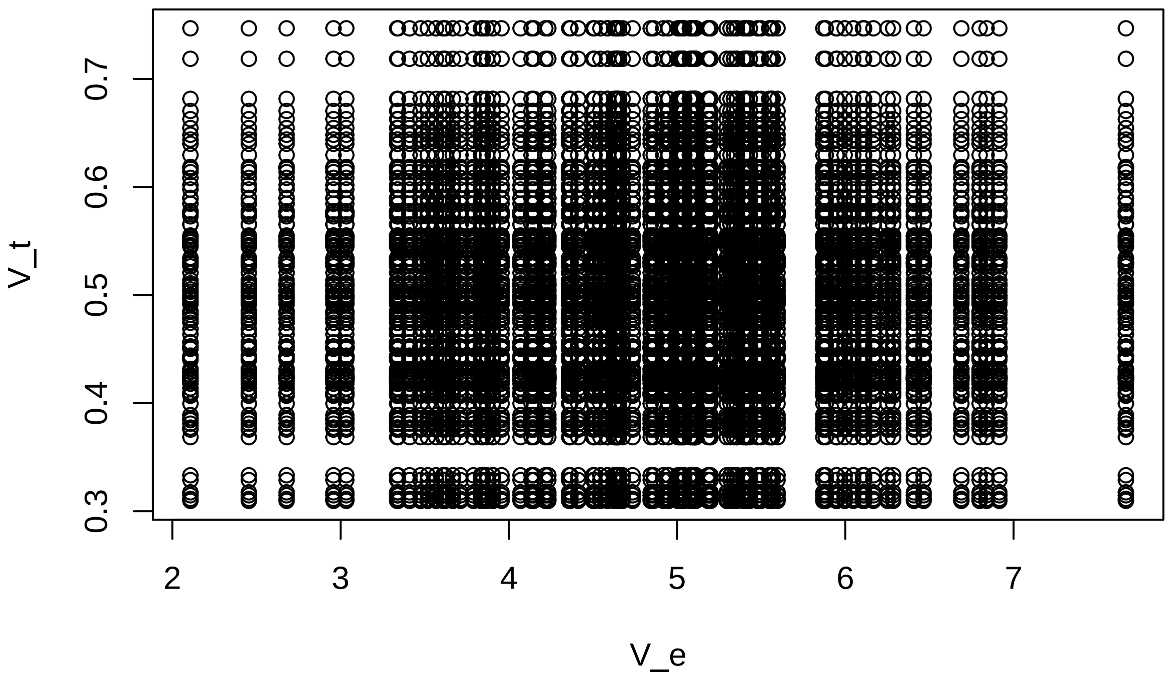
```
lik <- function(f_eta){
  theta<- f_eta[1] #theta
  eta<- f_eta[2]
  out <-theta*exp(-theta*(sum(x)-length(x)*eta))
  return(out)
}


#plot(f_eta, lik(f_eta), type="l", lwd=3, main="logliklihood_Weibull, n=1000")



#genereting eta
V_eta <- rnorm(100, mean = 5, sd = 1)
V_THETA <- rnorm(100, mean = 0.5, sd = 0.1)

V_lik<- c()
V_t<- c()
V_e<- c()

for (theta in V_THETA) {


        for (el in V_eta) {

          f_eta<- c(theta,el) #theta    eta
          V_lik<-append(V_lik, lik(f_eta))
          V_t<-append(V_t, theta)
          V_e<-append(V_e, el)
        }

}


#creating dataset with eta column and  liklihood column
DATASET<- data.frame(V_e, V_t, V_lik)
#plot
require(plot3D)
```

```
## Loading required package: plot3D
```

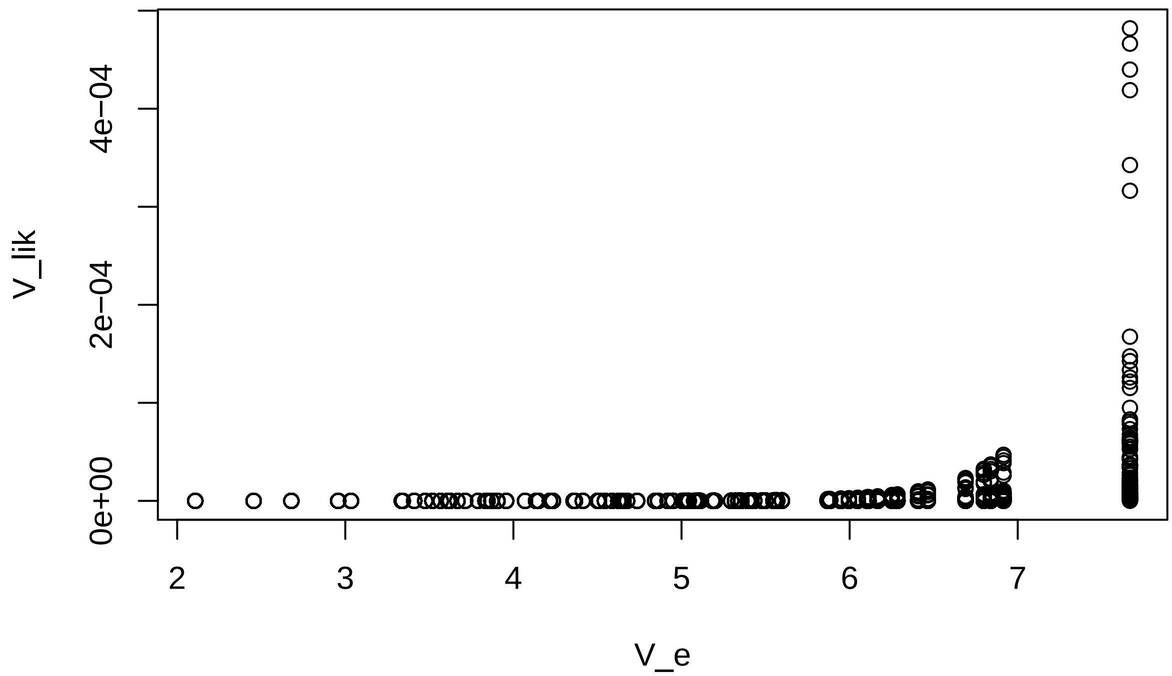
```
scatter3D(V_e, V_t, V_lik, phi=0, bty="g")
```



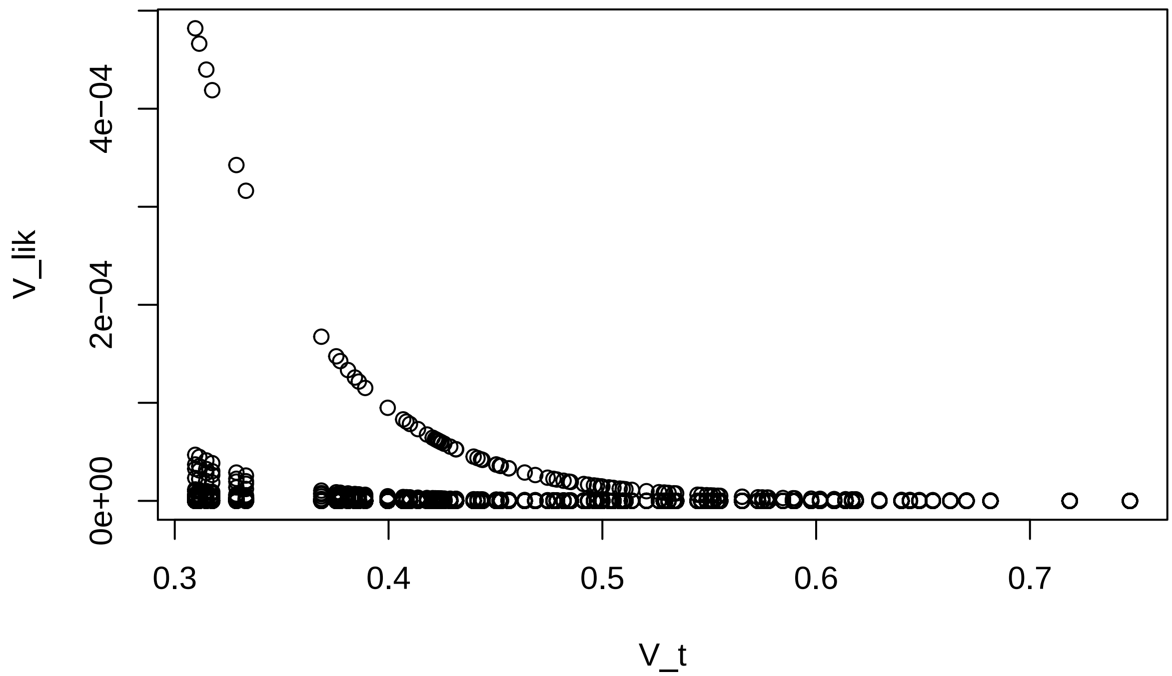

```
plot(V_e, V_t)
```




```
plot(V_e, V_lik)
```



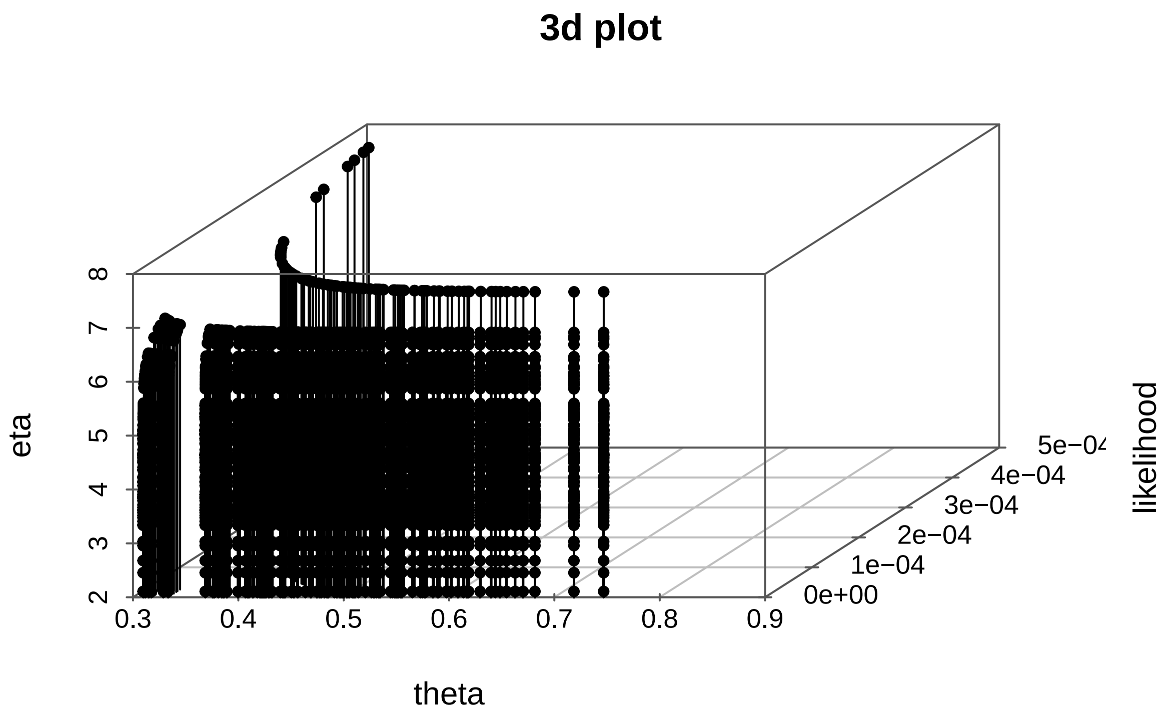

```
plot(V_t, V_lik)
```




```r
require(scatterplot3d)
```

```
## Loading required package: scatterplot3d
```

```r
ex1 <- scatterplot3d(DATASET[,2],
                     DATASET[,3],
                     DATASET[,1],
                     type="h",
                     xlab = "theta",
                     ylab = "likelihood",
                     zlab = "eta",
                     highlight.3d=FALSE,
                     col.axis="gray34",
                     col.grid="gray",
                     angle=50,
                     main="3d plot",
                     pch = 20)
```




```r
#maxium value
DATASET[c(which.max(DATASET$V_lik)),]
```

```
##           V_e       V_t        V_lik
## 5016 7.667633 0.3095818 0.0004819928
```

1.e

```r
require(STAR)
```

```
## Loading required package: STAR
```

```
## Loading required package: survival
```

```
## Loading required package: mgcv
```

```
## Loading required package: nlme
```

```
## This is mgcv 1.8-34. For overview type 'help("mgcv-package")'.
```

```
## Loading required package: R2HTML
```

```
## Loading required package: gss

## Loading required package: codetools
```

```r
?STAR
```

```
## starting httpd help server ...

##  done
```

```r
data(e070528citronellal)
x1<-as.vector(e070528citronellal[["neuron 1"]][[1]])
x2<-as.vector(e070528citronellal[["neuron 2"]][[1]])
x3<-as.vector(e070528citronellal[["neuron 3"]][[1]])
x4<-as.vector(e070528citronellal[["neuron 4"]][[1]])

lik <- function(f_eta){
  theta<- f_eta[1] #theta
  eta<- f_eta[2]
  out <-theta*exp(-theta*(sum(f_eta[3])-length(f_eta[3])*eta))
  return(out)
}

f_eta<- c(0.3095818,7.667633,x1)
lik(f_eta)
```

```
## [1] 3.24782
```

```r
f_eta<- c(0.3095818,7.667633,x2)
lik(f_eta)
```

```
## [1] 2.97167
```

```r
f_eta<- c(0.3095818,7.667633,x3)
lik(f_eta)
```

```
## [1] 3.249548
```

```r
f_eta<- c(0.3095818,7.667633,x4)
lik(f_eta)
```

```
## [1] 3.231053
```

Tuto 1

Yassin RAISSOUNI and Eslam MAYHOB

1- a. $f(x_1, x_2, \dots, x_n \mid \theta)$

$= f(x_1 \mid \theta) \times f(x_2 \mid \theta) \times \dots \times f(x_n \mid \theta)$ (Model of i.i.d interspike intervals)

$= \prod_{i=1}^{n} f(x_i \mid \theta)$

$= \prod_{i=1}^{n} \theta e^{-\theta(x_i - \eta)} 1_{x \geq \eta}$

$= \theta \prod_{i=1}^{n} e^{-\theta(x_i - \eta)} 1_{x \geq \eta}$

$= \theta e^{\sum_{i=1}^{n}(-\theta(x_i - \eta))} 1_{x \geq \eta}$

$= \theta e^{-\theta \sum_{i=1}^{n}(x_i - \eta)} 1_{x \geq \eta}$

$= \boxed{\theta e^{-\theta\left(\sum_{i=1}^{n} x_i - n\eta\right)} 1_{x \geq \eta}}$ $(\theta, \eta \geq 0)$

$\Rightarrow$ Likelihood : $\theta \longmapsto f_\theta(x) = \theta e^{-\theta\left(\sum_{i=1}^{n} x_i - n\eta\right)} 1_{x \geq \eta}$

1- b. "R"

1- c. After seeing what happened on the plot, and by looking at the formula : $\hat{\eta} \simeq 7{,}9 \simeq 8$.

$$\ell(\theta) = \log f_\theta(x) = \left(\log(\theta) - \theta\left(\sum_{i=1}^{n} x_i - 8n\right)\right) 1_{x \geq 8}$$

$$\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmax}}\, \ell(\theta)$$

$$\frac{d\,\ell(\theta)}{d\theta} = 0 \iff \left(\frac{1}{\theta} - \sum_{i=1}^{n} x_i + 8n\right) 1_{x \geq 8} = 0$$

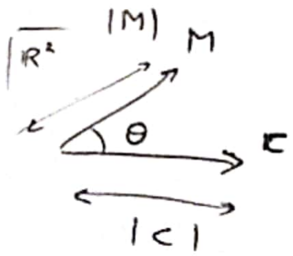
$$\iff \frac{1}{\theta} = \sum_{i=1}^{n} x_i - 8n$$

$$\iff \boxed{\hat{\theta} = \frac{1}{\sum_{i=1}^{n} x_i - 8n}}$$

1.d - "R": We can see that η converges to 8 and θ to 0,3, when we choose several values of η and θ & n tends to infinity.

1.e - "R".

2 - a -



We know that : $\langle M, C\rangle = |M|\,|C| \cos(\theta)$

As M and C are unit vectors of the plan $\mathbb{R}^2$, then : $|M| = |C| = 1$

So : $\boxed{\cos(\theta) = \langle M, C\rangle}$