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Advanced Microeconometrics: Exam 2

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Contents

1. A Model of School Choice and Competition Under Social Interactions

This exam is inspired by Allende (2019). Dubin and McFadden (1984) should also be consulted. We will analyze a model of school choice and competition under social interactions, which will be modeled as externalities in the demand side of the market.

1.1 Demand

We start by modeling families' demand for schools. Each family must choose a school where to enroll a child. Preferences for schools can be represented by the following indirect utility function,

$$U_{ijt} = \beta_{1i} p_{jt} + \beta_2 q_{jt} + \beta_{3i}' X_{jt} + \beta_{4i} d_{ijt} + \beta_5 \hat{z}^x_{jt} + \xi_{jt} + \varepsilon_{ijt},$$

where the subscript ijt refers to student (family) i in school j in market t. The variable p_{jt} denotes the tuition (price) charged by the school. The variable q_{jt} is school's quality in the form of a value added measure for test scores. The vector X_{jt} groups other K observable characteristics of the school. The variable d_{ijt} denotes distance from home to school j. The variable $\hat{z}^x_{jt}$ is student i's expectation of school j's average body composition in terms of the demographic variable x (e.g. family income, parents' level of education). As is usual, ξ_{jt} captures school's unobserved (to the econometrician) characteristics that are relevant for the family's decision. Finally, ε_{ijt} is an idiosyncratic Type I Extreme Value error term.

Important to note is that $\hat{z}^x_{jt}$ is an equilibrium object in this model. Furthermore, we assume that students' beliefs are consistent, in the sense that students' expectations lead to choices that result in schools' equilibrium average body compositions that are consistent with students' expectations.

Note that preferences include heterogeneous coefficients. Specifically, let $\gamma_i = (\beta_{1i}, \beta_{3i}, \beta_{4i})$, and $\gamma = (\beta_1, \beta_3, \beta_4)$. We assume,

$$\gamma_i = \gamma + \Gamma W_i$$

where W_i is a $w \times 1$ vector of demographic variables, and Γ is a $(K+2) \times w$ matrix of coefficients that measure how taste varies with demographics.

While each family must choose one and only one school, the family is allowed to choose a school outside the market. For the "outside school", we normalize the indirect utility to be

$$U_{i0t} = \varepsilon_{i0t}.$$

We will assume that we are working with individual level data.

1. Write down the probability that a particular student i chooses school j, with j not the outside school.
   Answer:
   The student (family) will choose school j to maximize U_{ijt} i ntheir respective choice set Ω_{it}. Each market is characterized by a set of nodes (N_m) and schools (N_s), where w^m_{nk} is the distribution of students type k across nodes and π^m_k is the proportion of the students in the market who are of type k. Now, if we assume that ε_{ijt} has the standard extreme.value distribution we can calculate the probability that a familiy of observable type k, who lives at node n and has an unobservable type v_i will select school j as follows:

$$\mathcal{S}^{nk}_{ijt}(\mu, p, z^y, z^e) = \left(\frac{exp(U_{ijt}(\mu, p, z^y, z^e)}{\sum_{l\in\Omega_{it}} exp(U_{ilt}(\mu, p, z^y, z^e))} \right) \tag{1.1}$$

   Notice, that this choice probability is taking school j as not the ouside school. Furthermore, we can compute for the shares of each school at the **market level**:

$$s^k_{jt}(\mu, p, z^y, z^e) = \sum_{n}^{N_m} s^{nk}_{jt} \cdot \omega^m_{nk} \tag{1.2}$$

$$s_{jt}(\mu, p, z^y, z^e) = \sum_{k}^{K} \sum_{n}^{N_m} s^{nk}_{jt} \cdot \omega^m_{nk} \cdot \pi^m_k \tag{1.3}$$

   Where (1.2) is the probability of family type k chooses school j (market share of school j), and (1.3) is the **total** market share

2. Write down an expression for $\hat{z}^x_{jt}$. Note that $\hat{z}^x_{jt}$ is function of all other students' probability of choosing school j. In fact, the equilibrium realization of $\hat{z}^x_{jt}$ is a fixed point of all students' probability of choosing school j.
   Answer:
   Let us characterize now, the student body composition un terms of the proportion of families with high income (z^y_{jt}) and high human capital (z^e_{jt}). Assuming that families observe the characteristics of schools in their choice set (Ω_{it}) and individual preferences shocks one can obtain:

$$\hat{z}^x_{jt} = \mathcal{Z}^x_{jt}(\mu, p, z^y, z^e) = \frac{N \times \pi^m_{k=x} \times \sum_n^{N_m} \sum_i^{N_v} \mathcal{S}^{n,k=x}_{ijt}(\mu, p, z^y, z^e) \times \omega^v \times \omega^m_{n,k=x}}{N \times \sum_k^K \sum_n^{N_m} \sum_i^{N_v} \mathcal{S}^{nk}_{ijt}(\mu, p, z^y, z^e) \times \omega^v \times \omega^m_{nk} \times \pi_k} \tag{1.4}$$

   for $x = \{y, e\}$ and $j = \{1, ..., N^m_j\}$ and ω^v are the corresponding weights associated with both observable and unobservable types N_v.

3. Rewrite the indirect utility as $U_{ijt} = \delta_{jt} + \mu_{ijt} + \varepsilon_{ijt}$. Be explicit about the components of the indirect utility that enter δ_{jt} and those that enter μ_{ijt}.
   Answer:
   Now, rewriting the indirect utility[1] as required, we have,

$$\delta_{jt} = -\alpha_i p_{jt} + \beta^{z^y}_i \hat{z}^y_{jt} + \beta^{z^e}_i \hat{z}^e_{jt} + \beta^d_i D_{ij} + \beta^{net}_i net_{jt} + r'_{jt}\beta^r + \xi_{jt}$$
$$\mu_{ijt} = \beta^\mu_i \mu_{jt}$$

   Therefore, $U_{ijt} = \delta_{jt} + \mu_{ijt} + \varepsilon_{ijt}$

[1] Taking from Allende (2019)[1], our indirect utility function is given by $U_{ijt} = \beta^\mu_i \mu_{jt} - \alpha_i p_{jt} + \beta^{z^y}_i \hat{z}^y_{jt} + \beta^{z^e}_i \hat{z}^e_{jt} + \beta^d_i D_{ij} + \beta^{net}_i net_{jt} + r'_{jt}\beta^r + \xi_{jt} + \varepsilon_{ijt}$

4. Rewrite down the probability that a particular student i chooses school j, with j not the outside school, but now in terms of the δ_{jt} and μ_{ijt} terms.
Answer:
Answer to question 3 is incredibly similar to a **Random Coefficients Multinomial Logit Model**, following the line of though for this result we have that the probability of a particular student i of choosing the not outside school j is given by

$$s_{ijt} = \frac{exp(\delta_{jt} + \mu_{ijt})}{1 + \sum_k^K exp(\delta_{kt} + \mu_{ikt})}$$

5. We want to estimate the "nonlinear" parameters (those that enter μ_{ijt}) and the δ_{jt} terms by maximum likelihood. Write down the corresponding likelihood function.
Answer:
In the context of Moon, et al. (2018) [4] and Maximum Likelihood and Classical Non-Linear Models we can write the associated likelihood function as

$$ln(\mathcal{L}) = \sum_i^I \sum_j^{N_j} \mathbb{I}_{ij} s_{ijt} = \sum_i^I \sum_j^{N_j} \frac{exp(\delta_{jt} + \mu_{ijt})}{1 + \sum_j^{N_j} exp(\delta_{jt} + \mu_{ijt})} \tag{1.5}$$

Where $\mathbb{I}_{ij}$ is the indicator function that takes value 1 whenever school j is selected.

1.2 Student Achievement

An additional piece of this model is students' counterfactual outcomes, i.e. test scores. Each student i has a counterfactual test score for every school she may attend. Specifically,

$$Y_{ijt} = A_{it}\omega + q_{jt} + \epsilon_{ijt},$$

where A_{it} is a vector of student i's demographic characteristics that determine test scores, q_{jt} is school j's value added, and ϵ_{ijt} is an idiosyncratic mean zero and finite variance error term orthogonal to A_{it} and q_{jt}. The realized outcome for student i is $Y_{it} = \sum_{j=1}^J 1\{S_{it} = j\} Y_{ijt}$, where $S_{it} \in \{1, \ldots, J\}$ denotes student i's school choice.

1. Suppose we want to estimate the parameters in the test scores equation imposing a "selection on observables" assumption, following Allende (2019). Explain how you would perform such estimation.
Answer:
Following Allende (2019) and Neilson (2013) [5] and taking the fact that students' counterfactual outcomes (test scores) take the form of a linear regression we can estimate first using the assumption of *Suitable Unit Treatment Value* or SUTVA[2] we can first estimate school j's value-added by a simple OLS [3]. Then in a second stage (take 2SLS if you will) run a regression to estimate the remaining parameter. A key feture in this estimation is the orthogonality of the error terms and the parameters we wish to estimate. As a matter of fact, both papers emphasize in the importance of this feature.

[2] Alende, 2019. p.36
[3] Following Neilson (2013)

2. Now, suppose we are unsure about the validity of the "selection on observables" assumption, and want to correct the estimates for school selection. Using the control function strategy in Dubin and McFadden (1984)[3], that applies to a joint logistic choice and continuous outcome model, explain how you would perform a selection-corrected consistent estimation of the parameters in the test scores equation.
   Answer:
   Following Dubind and McFadden (1984) one can use non-parametric or parametric methods for determining a discrete/continuous demand for a particular student i for school j using the indirect utility function [4]. Once we have this discrete/continuous demand one can make use of **Conditional Expectation Correction Method**[5] which applies OLS to the demand equation conditional on school's observables and student characteristics (in our case). Considering the unobserved characteristics of student i one can compute the conditional expectation of the unobserved (to the econometrician) characteristics of i over i i.e $\mathbb{E}[\nu|i]$ and the unobserved characteristics of school j conditioned on j i.e $\mathbb{E}[\eta_{jt}|j]$[6]. The terms involving the estimated probabilities of choice permit us to hvae a consistent estimate of these two conditional expectations.
   Now, we can now do a 2SLS over student i's counterfactual outcome to perform a selection-corrected consisten estimation of the parameters in test scores equation.

1.3 Back to Demand Estimation

With estimates for δ_{jt} and q_{jt} in hand, you should now be able to finalize the estimation of the "linear" parameters of the demand model, i.e. those that enter the δ_{jt} terms.

1. Suppose we do not worry about endogeneity in any of the variables. Explain how you would estimate the "linear" parameters of the demand model.
   Answer:
   Taking from Allende (2019) we can estimate parameters for utility equation by using a method of moments estimator (GMM) combining aggregate, micro and IV for demand and supply moments[7]. By computing the method of moments for each of the previsly mentioned requirements we can estimate the "linear" parameters for the demand model.

2. Now, we worry that p_{jt} and q_{jt} are endogenous. Propose a set of instruments for these variables.
   Answer:
   We will propose a set of instruments that sift the marginal cost and the cost of increasing quality – which should not be correlated with scholl j's observables.Furthermore, adding two sets of instruments controlling for variations in local demographics and in market structure; this instruments should shift markups and markdowns. Instruments are:

   - Public school teacher vacancies (matching control by distance weighted)
   - Public school teacher wage index
   - Stock of graduates with education degrees

[4] See. Dubin and McFadden (2013) p.346-350
[5] See Dubin and McFadden (1984) p. 355-356
[6] See page 1's definition of η_{jt}
[7] Author follows Berry et. al. line of thought closely

- Test scores for tachers
- New graduates with education degrees in the market

3. We now additionally worry that $\hat{z}^x_{jt}$ is endogenous. Which set of instruments are good candidates for this variable?
   Answer:
   In the case we have a many endogeneous characteristics on student i's expectation of school j's student body composition ($\hat{z}^x_{jt}$) a natural proposal is to use BLP/BLP-style instruments. Then we can propose exogeneous geographical instruments that i) provide independent and differential exposure for different schools, and ii) make use of each family's type and shift behavior. Instruments are:
   - Strike Exposure[8] (weighted by distance)
   - Local Market Exposure
   - Local demographics.

   The last two items should be categorized by distance-relevance data.

4. Explain how you would perform a 2SLS regression that estimates all "linear" parameters, taking into account the endogeneity of the p_{jt}, q_{jt}, and $\hat{z}^x_{jt}$ variables.
   Answer:
   I propose the following steps:
   (a) I would specify a model (sub-model if you will) for each parameter ($p_{jt}, q_{jt}, \hat{z}^x_{jt}$) according to the set of instruments proposed in the previous two questions.
   (b) Run OLS on each *sub-model* and obtain the estimated parameters, in order to generate the predicted price, quality and student body composition.
   (c) Plug them into the demand function.
   (d) Finally, run OLS on the modified demand equation and estimate linear parameters.

1.4 Supply

Schools are profit-maximizers, and compete choosing the tuition (p_{jt}) they charge to families in a static complete information game. We assume that schools' marginal cost is constant, and that it can be modeled as a linear function of quality, q_{jt}, other observable characteristics, B_{jt}, and an unobserved (to the econometrician) shock, η_{jt}.

1. In this context, write down school j's profit function.
   Answer:
   Following Allende (2019)[9] and taking $s_{jt}(\mu, p, z^y(p), z^e(p)) = s_{jt}$, we can obtain the profit

[8] Allende (2019) uses Lagged strike exposure index

[9]
$$MC_j(\mu_{jt}) = \sum_l \gamma_l \omega^l_{jt} + \gamma_\mu \mu_{jt} + \omega_{jt} = \gamma \mu_{jt} + \sum_l \gamma_l B_{jt} + \eta_{jt} = MC_{jt}$$
$$FC_j(\mu_{jt}) = \sum_l \lambda_l \omega^l_{jt} + \phi_{jt}\mu_{jt},\ where\ \ \phi_{jt} = \bar{\phi}_{jt} + \Delta\phi_{jt} = FC_{jt}$$

function for a school j in a market with N students:

$$\pi_{jt}(\mu, p, z^y(p), z^e(p)) = \Big(p_{jt} - MC_{jt}\Big) \times N \times s_{jt} - FC_{jt} \tag{1.6}$$

2. Write down school j's first order condition. Assume away corner solutions. Do not forget to take into account the externalities present in the demand. Hint: you should follow Allende (2019).
Answer:
Let $\pi_{jt(\mu,p,z^y(p),z^e(p))} = \pi_{jt}$ and $s_{jt}(\mu, p, z^y(p), z^e(p)) = s_{jt}$, then the first order condition for price for school j is:
$$\frac{d\pi_{jt}}{dp_{ij}} = s_{jt} + \frac{ds_{jt}}{dp_{jt}} \times (p_{jt} - MC_{jt}) = 0 \tag{1.7}$$

3. How different are schools' first order conditions with demand externalities relative to those without demand externalities? Can you predict whether externalities bring up or down prices relative to prices without demand externalities?
Answer:
One can rearrange (1.7) as the follows:

$$p^*_{jt} = MC_{jt} + s_{jt} \times \left(\frac{ds_{jt}}{dp_{jt}}\right)^{-1} \tag{1.8}$$

The term given by $s_{jt} \times \left(\frac{ds_{jt}}{dp_{jt}}\right)^{-1}$ is defined by the **price markup with social interctions** as it takes into account the effects the social interactions have on the school's pricing strategy. It is easy to see that this particular derivative differs from the version without social interactions. Why? It considers both the direct and strategic effects of inclusing the terms $z^y(p), z^e(p)$ in the demand.
One can apply further analysis by decomposing the derivative $\frac{ds_{jt}}{dp_{jt}}$. Two main effects arise:

(a) Direct effect of prices on demand: Reflects a combination betwen market power (baseline) and the direct social effect. The one related to market power emerges from product differentiation[10]. The case of direct social effecs results from an expansion in product differentiation due to social interactions, which changes how the distribution of demand responses to prices. We can see that this particular term is negativ; thus, as it tends to zero, the less sensitive demand is to prices, and the larger market power. Then, in a setting **without** externalities, schools can charge **higher** tuitions.

(b) Strategic social effects of prices on demand: This term depends mainly on how sensitive is demand is to peers on one hand; and how much do peers respond to prices. Now, if prices and peers direct and strategic **complements** in demand then the markup increases for schools that are better at attracting high-qualified (SES) peers. Thus, the strategic social effect will make those schools **gain** market power.

[10]In the case without social interactions schools increase their market power; thus increasing their markup; therefore increasing their price (tuition).

4. Define a Nash-Bertrand equilibrium in this game.
Answer:
A Nash-Bertrand equilibrium in our case is a tuple $\left\{\{z^y, z^x_{j\in J}\}, \{p\}_{j\in J}, \{\mu\}_{j\in J}\right\}$ that satisfies:

   (a) $\hat{z}^y_i = z^y$ and $\hat{z}^e_i = z^e$, $\forall\ i \in I$
   (b) $p^*_j(p^*_{-j}) = argmax_{\{p\}}\ \pi_j(\mu, p, p^*_{-j}, z^y(p, p^*_{-j}), z^e(p, p^*_{-j}))$, $\forall\ j \in J$
   (c) $\mu^*_j(\mu^*_{-j}) = argmax_{\{\mu\}}\ \pi_j(\mu, \mu^*_{-j}, p^*(\mu, \mu^*_{-j}), z^y(\mu, \mu^*_{-j}), z^e(\mu, \mu^*_{-j}))$, $\forall\ j \in J$

5. Explain how you would perform estimation of the marginal cost parameters.
Answer:
Marginal costs can be infered from markups also, they can be easily recovered as a function of shares, market size and matrix of partial derivatives for price and quality (functions of demand parameters). I propose two methos to estimate the MC_{jt} parameters:

   (a) Following the line of thought from Allende (2019) and Berry et al. (2004) [2] I would estimate the parameters for the marginal cost function using a GMM. Combining aggregate, micro and IV(demand) moments in this setting with moments that exploit the discontinuous variation in price generated by the scholarship assignment and IV(supply) moments. Assuming orthogonality between unobserved costs and instruments one should estimate the parameters rather easily.

   (b) A bit more straight-foreward but not that precise method –given the set up in this particular exam– one can make use of theleariy of the marginal cost function and approximate the parameters by maximum likelihood. Solving the endogeinity problems that may arise from the model, one could apply a 2SLS for better estimation.

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