Instituto Tecnológico Autónomo de México

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Dynamic Macroeconomics 1: Final Project

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Contents

1. Setting

Considering the overlapping generations model. Whereas, in the first period, agents work and are subject to random and idiosyncratic productivity shock for which there are no complete markets, but they can accumulate a risk-free asset. We assume further that there are no credit restrictions. In the second period, agents are retired; thus, their consumption depends only on accumulated assets. Agent's problem is given by:

$$\begin{aligned} max \quad & u(c_t^1) + \beta u(c_{t+1}^2) \\ \text{s.t.} \quad & c_t^1 + a_{t+2} = e^{z_t} w_t \\ & c_{t+1}^2 = R_{t+1} a_{t+1}^2 \end{aligned}$$

Let $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ and $F(H_t) = Y_t = K_t^\alpha$. Furthermore, we suppose that z_t is a random variable following an AR(1):

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \quad where \;\; \varepsilon \sim \mathcal{N}(0,1)$$

Assuming that: $\alpha = 0{,}33$, $\beta = 0{,}96$, $\delta = 0{,}08$, $\sigma = 1{,}5$, $\rho = 0{,}95$, $\sigma_\varepsilon = 0{,}004$

(i)

Using Tauchen's method, we obtain the discretized approximation fo z_t's AR(1) process using a 5 point grid.

Listing 1.1: Tauchen Method

```
% Question (i)
zeta =

   -0.0384
   -0.0192
         0
    0.0192
    0.0384

% Transition Matrix
P =

```

```
    0.9727    0.0273    0.0000         0         0
    0.0041    0.9806    0.0153    0.0000         0
    0.0000    0.0082    0.9837    0.0082    0.0000
    0.0000    0.0000    0.0153    0.9806    0.0041
    0.0000    0.0000    0.0000    0.0273    0.9727

%Asymptotic Distribution
P_Asym =

    0.0361    0.2392    0.4494    0.2392    0.0361
    0.0361    0.2392    0.4494    0.2392    0.0361
    0.0361    0.2392    0.4494    0.2392    0.0361
    0.0361    0.2392    0.4494    0.2392    0.0361
    0.0361    0.2392    0.4494    0.2392    0.0361

    % Verification
xFix =

    0.0361    0.2392    0.4494    0.2392    0.0361
```

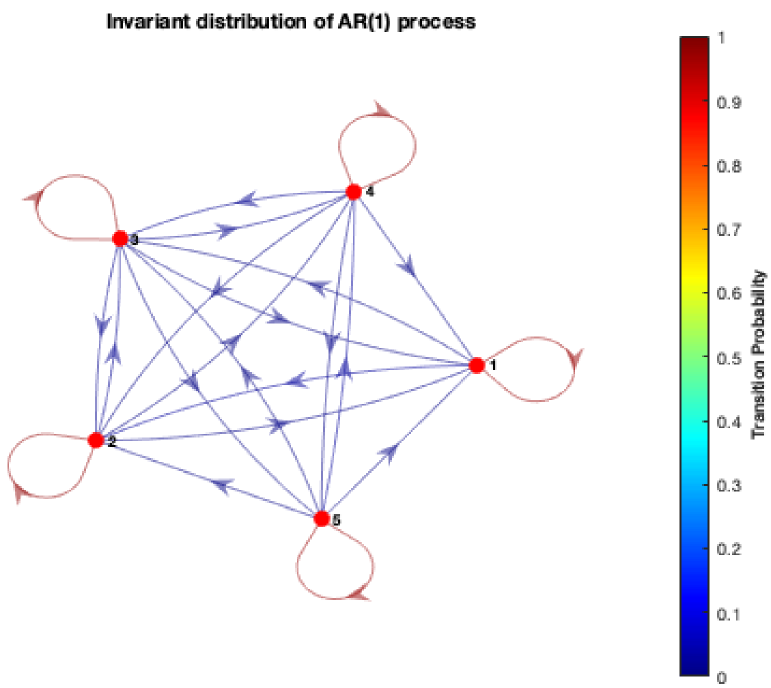

Figura 1.1: Invariant Distribution of AR(1) process

(ii)

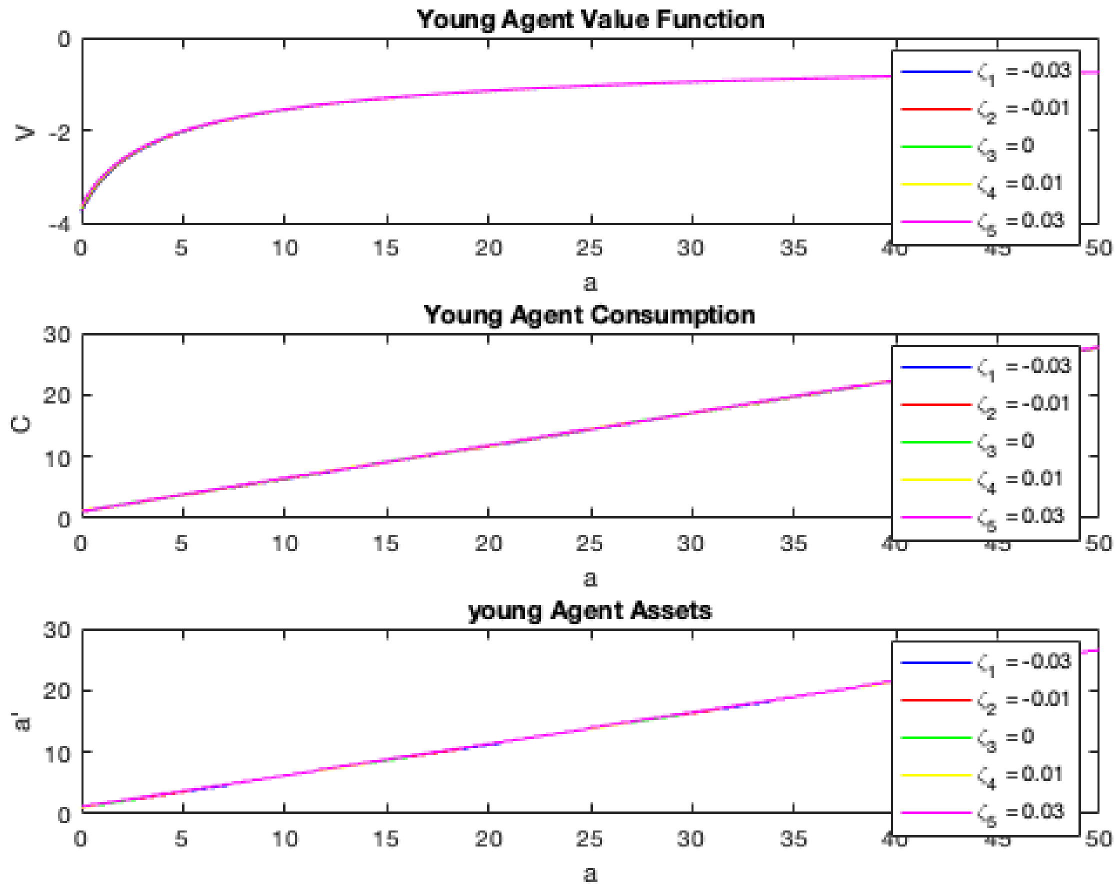

Figura 1.2: Young Agent's Value Function, Consumption and Assets

Figura 1.3: Retired Agent's Value Function, Consumption and Assets

(iii)

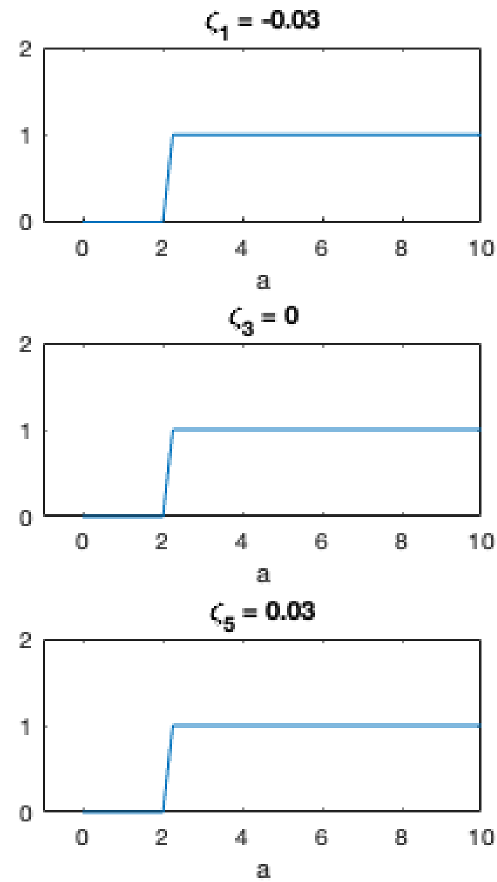

Figura 1.4: Invariant Distribution of Assets for each shock value

(iv)

Now, assuming agents are really **risk-averse** i.e $\sigma = 3{,}7$ (recall: the higher the ratio of risk-aversion then the more risk-averse the agent is). Let us take $\sigma_\varepsilon = 0{,}094$ so we can see the difference in AR(1) process realizations more clearly, as the standard deviation between them grows larger.

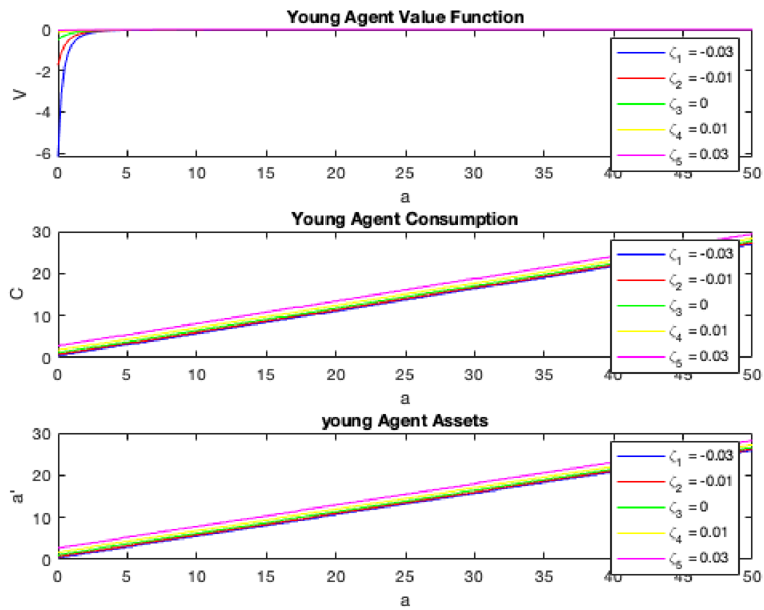

Figura 1.5: Young Agent with $\sigma = 3{,}7$

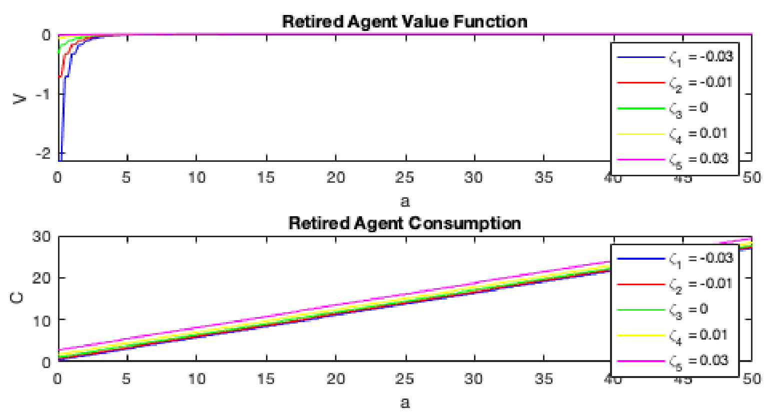

Figura 1.6: Retired Agent with $\sigma = 3{,}7$

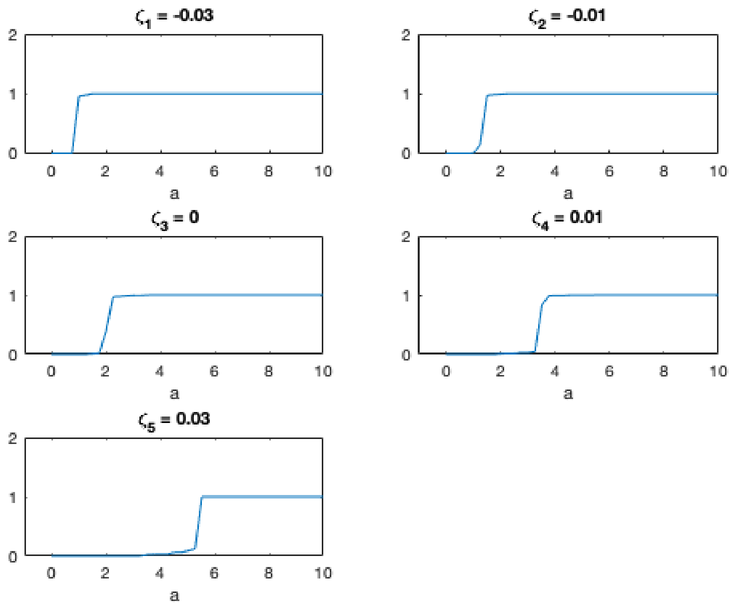

Figura 1.7: Invariant schocks distributions

We can see the more risk-averse our agent is, the more quickle the value function grows and converges to zero. Seems that consumption and asset accumulation follow the same pattern. However, further analysis is required: As risk-aversion grows, premiums on capital accumulation should get higher; thus, increasing non-laboral income by capital returns. This implies that, our agent will save more by accumulating more assets and will be better-off in retirement.
Wages should not be directly affected by this risk-aversion ratio increase; however there can be a substitution effect internalized by them. Following this line of though, consumption should increase in the short term and smooth over future periods. That is, as agents get more risk-averse, they will prefer the risk-free asset over consumption; implying that savings should increase.
To sum up, agents will prefer to over-accumulate as they value having more assets as retirees than having them as employees. The income they gaing, they will either save it or invest it in the risk-free asset (given that capital returns should increase).