ITAM
Departamento de Estadística

Inferencia Estadística– Laboratorio #5
Estimación por Intervalos de Confianza

Otoño 2021

1. (Verdadero o Falso)Suponcamos que tenemos un intervalo de confianza al 95 % para el parámetro θ, por lo que se puede asegurar que el parámetro se encuentra dentro del intervalo con una probabilidad del 95 %.

   (a) V

   (b) F

   En caso de ser Falso, comentar por qué.

Interpretación del intervalo Uno podría irse con la finta de que el intervalo lo que significa es que con 80% de probabilidad la σ^2 está ahí. ESO NO ES CIERTO. El 80% cuantifica el mecanismo para construir intervalos. Éste quiere decir que si construimos intervalos (para distintos experimentos) con este mecanismo el 80% de esos (infinitos) intervalos contendrán el verdadero valor. Esto **no** significa que **este** intervalo contenga el valor con probabilidad de 80%. Este intervalo O LO CONTIENE O NO PERO COMO σ^2 es un real y el intervalo $[0.003, 0.012]$ son también reales AQUÍ YA NO HAY NADA DE ALEATORIO Y NO SE PUEDE HABLAR DE PROBA. El intervalo contiene el valor o no pero ya no hay nada probabilístico; lo probabilístico está en que sobre múltiples (infinitas) muestras la forma de construir los intervalos funcionará para contener el valor verdadero 80% de las veces.

2. Si $L(x)$ y $U(x)$ satisfacen $\mathbb{P}_\theta\{L(X) \leq \theta\} = 1 - \alpha_1$ y $\mathbb{P}_\theta\{U(X) \geq \theta\} = 1 - \alpha_2$, sabemos que $L(x) \leq U(x)$ para toda x, demuestre que $\mathbb{P}_\theta\{L(X) \leq \theta \leq U(X)\} = 1 - \alpha_1 - \alpha_2$

Definimos $A = \{x \mid L(x) \leq \theta\}$; $B = \{x \mid U(x) \geq \theta\}$

$\therefore \quad A \cap B = \{x \mid L(x) \leq \theta \leq U(x)\}$

y se tiene que: $\qquad\qquad\qquad\qquad \because L(x) \leq U(x) \; \forall x$

$$1 \geq \mathbb{P}\{A \cup B\} = \mathbb{P}\{L(x) \leq \theta \vee \theta \leq U(x)\} \geq \mathbb{P}\{\overbrace{L(x) \leq \theta \vee \theta \leq L(x)}\} = 1$$

$$\therefore \quad \mathbb{P}\{A \cap B\} = \mathbb{P}\{A\} + \mathbb{P}\{B\} - \mathbb{P}\{A \cup B\}$$

$$= \mathbb{P}\{L(x) \leq \theta\} + \mathbb{P}\{\theta \leq U(x)\} - \mathbb{P}\{A \cup B\}$$

$$= 1 - \alpha_1 + 1 - \alpha_2 - 1 = 1 - \alpha_1 - \alpha_2 \quad \blacksquare$$

3. Sea $X_1, X_2, ..., X_n \overset{v.a.i.i.d}{\sim} N(\theta, 1)$. Prueba que $\bar{X} \pm \frac{1{,}96}{\sqrt{n}}$ es un intervalo de con 95 % para θ. Sea **p** la probabilidad de que una v.a.i.i.d independiente X_{n+1} caiga dentro del intervalo. ¿**p** es menor, igual o mayor a .95? Prueba tu respuesta.

Sabemos que $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ en este caso $\bar{X} \sim N\left(\theta, \frac{1}{n}\right)$

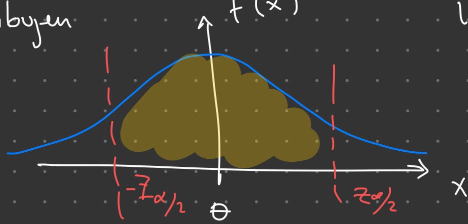
$\Rightarrow \dfrac{\bar{X} - \theta}{\frac{1}{\sqrt{n}}} \sim N(0,1)$ (Luego aprenderán qué es una cantidad pivote)

$$\Rightarrow \mathbb{P}\left\{ L(x) \leq \frac{\bar{X} - \theta}{\frac{1}{\sqrt{n}}} \leq U(x) \right\} = \mathbb{P}\left\{ -Z_{\alpha/2} \leq \frac{\bar{X} - \theta}{\frac{1}{\sqrt{n}}} \leq Z_{\alpha/2} \right\} = .95$$

Ahora, vamos a calcular el valor de α (Esto es súper fácil)

queremos que $1 - \alpha = .95 \iff 1 - .95 = \alpha \iff \alpha = .05 \therefore \frac{\alpha}{2} = .025$

Dibujen



luego.

$$\mathbb{P}\left\{ -Z_{.025} \leq \frac{\bar{X} - \theta}{1/\sqrt{n}} \leq Z_{.025} \right\}$$

$$= \mathbb{P}\left\{ -Z_{.025}\left(1/\sqrt{n}\right) \leq \bar{X} - \theta \leq Z_{.025}\left(\tfrac{1}{\sqrt{n}}\right) \right\}$$

$$= \mathbb{P}\left\{ \bar{X} - Z_{.025}\left(\tfrac{1}{\sqrt{n}}\right) \leq \theta \leq \bar{X} + Z_{.025}\left(\tfrac{1}{\sqrt{n}}\right) \right\}$$

$$= \mathbb{P}\left\{ \bar{X} - \frac{1.96}{\sqrt{n}} \leq \theta \leq \bar{X} + \frac{1.96}{\sqrt{n}} \right\} \therefore \text{IC al 95\%} \quad \bar{X} \pm \frac{1.96}{\sqrt{n}}$$

Evidentemente $\bar{X} \perp\!\!\!\perp X_{n+1} \Rightarrow X_1, \dots, X_{n+1} \overset{v.a.iid}{\sim} N(\theta, 1)$

Ahora, $T = X_{n+1} - \bar{X} \sim ?$ Recuerde $\bar{X}$ es INSESGADO!

$$E(T) = E(X_{n+1} - \bar{X}) = E(X_{n+1}) - E(\bar{X}) = \theta - \theta = 0 \checkmark$$

$$V(T) = V(X_{n+1} - \bar{X}) = V(X_{n+1}) + V(\bar{X}) - 2\,\underbrace{\cancel{Cov(X_{n+1}, \bar{X})}}_{0}$$

$$= 1 + \frac{1}{n} \quad \because \quad V(\bar{X}) = V\left(\sum_1^n \frac{X_i}{n}\right) = \frac{\sum_1^n V(X_i)}{n^2} + \sum_{i \neq j}^n \sum^n \underbrace{\cancel{Cov(X_i, X_j)}}_{0}$$

$$= \frac{n+1}{n} \qquad = \frac{1}{n^2}(n) = \frac{1}{n}$$

$$\therefore T \sim \left(0, \frac{n+1}{n}\right) \quad \therefore \quad \frac{T - 0}{\sqrt{\frac{n+1}{n}}} \sim N(0,1)$$

$$\Rightarrow p = \mathbb{P}\left\{ \bar{X} - \frac{1.96}{\sqrt{n}} \leq X_{n+1} \leq \bar{X} + \frac{1.96}{\sqrt{n}} \right\} = \mathbb{P}\left\{ \frac{-1.96}{\sqrt{n}} \leq X_{n+1} - \bar{X} \leq \bar{X} + \frac{1.96}{\sqrt{n}} \right\}$$

$$= \mathbb{P}\left\{ -1.96 \leq \sqrt{n}(X_{n+1} - \bar{X}) \leq \bar{X} + 1.96 \right\} = \mathbb{P}\left\{ -\frac{1.96}{\sqrt{n+1}} \leq \underbrace{\frac{X_{n+1} - \bar{X}}{\sqrt{\frac{n+1}{n}}}}_{Z} \leq \frac{1.96}{\sqrt{n+1}} \right\} < .95$$

$$\therefore \left| \frac{1.96}{\sqrt{n+1}} \right| < 1.96 \quad \therefore p < .95$$

4. Sea $X_1, X_2, ..., X_n \overset{v.a.i.i.d}{\sim} U(0,\theta)$. Encuentra un IC al $(1-\alpha)*100\%$ para θ.

$$\left[\frac{\quad}{2n^2 \bar{X}} , \frac{\quad}{2n^2 \bar{X}} \right]$$

(2) $X_1, \dots, X_n \overset{iid}{\sim} U(0,\theta)$ CI for θ?

$X_{(n)}$ is a suff and comp stat. → I only care about this property.

- $X_i \sim U(0,\theta)$

$\underset{(4)}{\Rightarrow} \frac{X_i}{\theta} \sim U(0,1) \underset{(3)}{\Rightarrow} \frac{X_{(n)}}{\theta} \sim Beta(n,1)$

$\underset{(5)}{\Rightarrow} -2n \ln\left(\frac{X_{(n)}}{\theta}\right) \sim \chi^2_2 \perp \theta$ → It's dist. is free from θ.

$\Rightarrow Q = -2n \ln\left(\frac{X_{(n)}}{\theta}\right)$

$\Rightarrow \mathbb{P}\left(\chi^2_{(2,1-\alpha/2)} < -2n\ln\left(\frac{X_n}{\theta}\right) < \chi^2_{(2,\alpha/2)}\right) = 1-\alpha$

$\Rightarrow \mathbb{P}\left(\frac{\chi^2_{(2,\alpha/2)}}{-2n} < \ln\left(\frac{X_{(n)}}{\theta}\right) < \frac{\chi^2_{(2,1-\alpha/2)}}{-2n}\right) = 1-\alpha$

Take exp

$\Rightarrow \mathbb{P}\left(\exp\left[\frac{\chi^2_{(2,1-\alpha/2)}}{-2n}\right] < \frac{X_{(n)}}{\theta} < \exp\left[\frac{\chi^2_{(2,\alpha/2)}}{-2n}\right]\right) = 1-\alpha$

$\Rightarrow \mathbb{P}\left(\frac{X_{(n)}}{\exp\left[\frac{\chi^2_{(2,1-\alpha/2)}}{-2n}\right]} < \theta < \frac{X_{(n)}}{\exp\left[\frac{\chi^2_{(2,\alpha/2)}}{-2n}\right]}\right) = 1-\alpha$

$\Rightarrow \left[X_{(n)} \exp\left[\frac{\chi^2_{(2,1-\alpha/2)}}{2n}\right], X_{(n)} \exp\left[\frac{\chi^2_{(2,\alpha/2)}}{2n}\right] \right]$ CI for θ.

Remark
(3) $X_i \sim U(0,1)$
$\Rightarrow X_{(n)} \sim Beta(n,1)$
(4) $X_i \sim U(a,b)$
$\Rightarrow \frac{X-a}{b-a} \sim U(0,1)$
(5) $X \sim Beta(\alpha, 1)$
$\Rightarrow -\ln(X) \sim Exp(\alpha)$
$\Rightarrow -2\alpha \ln(X) \sim \chi^2_2$

Lem. If X_i's $\overset{iid}{\sim} F(x;\theta)$ → cdf. Then $-2n\sum \ln[F(x_i;\theta)] \sim \chi^2_{2n}$ is a pivotal quantity.

Dem

→ random variable

$U = F(X;\theta) \sim U(0,1)$

$-2\ln[F(X;\theta)] \sim Exp(\frac{1}{2}) \equiv Gamma(1, \frac{1}{2}) \equiv \chi^2_2$

$\Rightarrow -2\ln[F(X;\theta)] \sim \chi^2_2$

$Q = -2\ln[F(\underset{\sim}{X};\theta)] = -2\ln\left[\prod_1^n F(x_i;\theta)\right] = -2\sum_1^n \ln[F(x_i;\theta)] \sim \chi^2_{2n}$

Remark
$U \sim U(0,1)$
$-2\ln(U) \sim Exp(1/2)$
$-\alpha\ln(U) \sim Exp(1/\alpha)$

(3) $X_1, \dots, X_n \overset{iid}{\sim} Beta(\theta, 1)$. Find CI for θ.

$f(x,\theta) = \theta x^{\theta-1} \quad 0<x<1, \quad \theta>0$

$F(x,\theta) = x^\theta$

$\Rightarrow F(X;\theta) = X^\theta \sim U(0,1)$

Remark: $Beta(a,b)$
$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$

5. Sea $X_1, X_2, ..., X_n \overset{v.a.i.i.d}{\sim} \Gamma(r, \lambda)$ donde r es conocido. Encuentra un IC al $(1-\alpha) * 100\%$ para λ.

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ink 1.5

I T TASK

Point Est. $\left\{ \begin{array}{l} 1.\ \text{MME} \\ 2\ \text{MLE} \\ 3\ \text{UMVUE} \end{array} \right.$

Interval Est.

$\Downarrow$

Pivotal Quant

$$\left.\begin{array}{l} z \\ t \\ \chi^2 \\ F \end{array}\right\} \leftarrow Q(\underset{\sim}{x}, \theta) \sim \perp\!\!\!\perp \theta$$

Ind.

equal Tails

Min length.

Ex 1 $X_1, \ldots, X_n \overset{iid}{\sim} \text{Gamma}(r, \lambda)$

$$f(x; r, \lambda) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x} \qquad \begin{array}{l} x > 0 \\ r, \lambda > 0 \end{array}$$

Suppose r is known.

Find $100(1-\alpha)\%$ CI for λ

- $\sum X_i$ is suff & complete $\sim \text{Gamma}(r, n\lambda)$

Use Rework (1)

$$\underbrace{2n\lambda \sum X_i}_{Q} \sim \text{Gamma}\left(r, \frac{n\lambda}{2n\lambda}\right) = \text{Gamma}\left(r, \frac{1}{2}\right) \equiv \chi^2_{2r} \perp\!\!\!\perp \theta$$

Rework

(1) $X \sim \text{Gamma}(r, \lambda)$

$kX \sim \text{Gamma}(r, \lambda/k)$

(2) X_i's $\overset{iid}{\sim}$ Gamma(r, λ)

$\Rightarrow \sum X_i \sim \text{Gamma}(r, n\lambda)$

$$\mathbb{P}\left(\chi^2_{(2r, 1-\alpha/2)} < 2n\lambda \sum X_i < \chi^2_{(2r, \alpha/2)}\right) = 1-\alpha$$

$$\Rightarrow \mathbb{P}\left(\frac{\chi^2_{(2r, 1-\alpha/2)}}{2n\sum X_i} < \lambda < \frac{\chi^2_{(2r, \alpha/2)}}{2n\sum X_i}\right) = 1-\alpha$$

$$n^2 \underbrace{\frac{1}{n}\sum X_i}_{\bar{X}} = n \sum X_i$$

So mi $100(1-\alpha)\%$ CI is $\left[\dfrac{\chi^2_{(2r, 1-\alpha/2)}}{2n^2 \bar{X}}, \dfrac{\chi^2_{(2r, \alpha/2)}}{2n^2 \bar{X}}\right]$ for λ

6. Sea $X_1, X_2, ..., X_n \overset{v.a.i.i.d}{\sim} Exp(\theta)$. Encuentra un IC de 90 % para θ si $n = 9$. (Simula en R este ejercicio)

Remember if
$X \sim Gamma(\alpha, \beta)$
$kX \sim Gamma(\alpha, \frac{\beta}{k})$

Q.1 Let $X_1, \ldots, X_n$, be iid from Exp(θ).

(a) Find a 90% confidence interval (CI) for θ, when $n = 9$.

$\underline{n=9}$ We know $\sum X_i$ is a suff stat for θ

$\Rightarrow \sum X_i \sim Gamma(n, \theta) \Rightarrow 2\theta \sum X_i \sim Gamma(n, \theta/2\theta)$
$\sim Gamma(n, \frac{1}{2})$
$\sim \chi^2_{2n}$

Now
$.90 = 1-\alpha \Rightarrow \alpha = \underline{.10}$

$\Rightarrow \mathbb{P}\{\chi^2_{(2n, \alpha/2)} \leq 2\theta \sum X_i \leq \chi^2_{(2n, 1-\alpha/2)}\} = 1-\alpha$

$\Rightarrow \mathbb{P}\left\{\frac{\chi^2_{(2n, \alpha/2)}}{2\sum X_i} \leq \theta \leq \frac{\chi^2_{(2n, 1-\alpha/2)}}{2\sum X_i}\right\} = \mathbb{P}\left\{\frac{\chi^2_{(2n, \alpha/2)}}{2n\bar{X}} \leq \theta \leq \frac{\chi^2_{(2n, 1-\alpha/2)}}{2n\bar{X}}\right\}$

Given then $\bar{X} = \frac{1}{n}\sum X_i \Rightarrow n\bar{X} = \sum X_i$

$= 1-\alpha$

$\therefore$ my 90% CI for θ is $\left[\frac{\chi^2_{(2n, \alpha/2)}}{2n\bar{X}}, \frac{\chi^2_{(2n, 1-\alpha/2)}}{2n\bar{X}}\right]$

$n=9$, $\alpha = .1$

$= \left[\frac{\chi^2_{(18, .05)}}{18\bar{X}}, \frac{\chi^2_{(18, .95)}}{18\bar{X}}\right]$

7. Sean $X_1, X_2, ..., X_n \overset{v.a.i.i.d}{\sim} N(\mu_1, \sigma_1^2)$ y $Y_1, Y_2, ..., Y_m \overset{v.a.i.i.d}{\sim} N(\mu_2, \sigma_2^2)$ independientes. Demuestra que:

   (a) $\bar{X} - \bar{Y} \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m})$

   (b) Calclua el IC al $(1-\alpha) * 100\,\%$ para $\mu_1 - \mu_2$ si las varianzas son conocidas.

   (c) Calclua el IC al $(1-\alpha) * 100\,\%$ para $\mu_1 - \mu_2$ si las varianzas son desconocidas.

$X_1, \dots, X_n \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$

$Y_1, \dots, Y_m \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$

We know that $\bar{x} \sim N(\mu_1, \sigma^2/n)$, $\bar{y} \sim N(\mu_2, \sigma^2/m)$

$V(\bar{x}-\bar{y}) = V(\bar{x}) + V(\bar{y}) - 2\underbrace{Cov(\bar{x},\bar{y})}_{0} = \sigma_1^2/n + \sigma_2^2/m$

$\Rightarrow \bar{x} - \bar{y} \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m})$

$\Rightarrow \underbrace{\dfrac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}}}_{Q(\underline{x}, \mu_1-\mu_2)} \sim N(0,1)$

(a) We assume σ_1^2, σ_2^2 are known

$P\left(-z_{\alpha/2} \le \dfrac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}} \le z_{\alpha/2}\right) =$

$= P\left((\bar{x}-\bar{y}) - \sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}\, z_{\alpha/2} \le \mu_1-\mu_2 \le (\bar{x}-\bar{y}) + \sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}\, z_{\alpha/2}\right) = 1-\alpha$

$\Rightarrow$ CI $100(1-\alpha)\%$ for $\mu_1-\mu_2$ $(\bar{x}-\bar{y}) \pm \sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}\, z_{\alpha/2}$

(b) Assume par's σ_1^2, σ_2^2 are unknown. First, suppose n, m are suff. large

$S_1^2 \longleftrightarrow \sigma_1^2$

Now I have $Q(\underline{x}, \mu_1-\mu_2) = \dfrac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{S_1/\sqrt{n} + S_2/\sqrt{m}} \sim N(0,1)$

Then, my CI is $(\bar{x}-\bar{y}) \pm \sqrt{\frac{S_1^2}{n}+\frac{S_2^2}{m}}\, z_{\alpha/2}$

Now, suppose $\sigma_1^2 = \sigma_2^2$

S_1 is not necessarily equal to S_2

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$\Rightarrow Q = \dfrac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{\sigma\sqrt{\frac{1}{n}+\frac{1}{m}}} \sim N(0,1)$

we have to replace it $\Rightarrow$ Pooled Variance (Mixed) $\to$ (*)

$S_p^2 = \dfrac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}$

Using (*)

$\Rightarrow Q^* = \dfrac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{S_p\sqrt{\frac{1}{n}+\frac{1}{m}}} = (\circledast) \sim t_{n+m-2}$

Remember $\dfrac{(n-1)S_1^2}{\sigma_1^2} \sim \chi^2_{n-1}$, $\dfrac{(m-1)S_2^2}{\sigma_2^2} \sim \chi^2_{m-1}$ it's evident this 2 are independent

* Remark $U \perp\!\!\!\perp V$ and $U \sim \chi^2_u$, $V \sim \chi^2_v$ $\Rightarrow U+V \sim \chi^2_{u+v}$

From (#) we know $\dfrac{(n-1)S_1^2 + (m-1)S_2^2}{\sigma^2} \sim \chi^2_{n+m-2} \longrightarrow (**)$

Assuming $\sigma_1^2 = \sigma_2^2$

So, $\bar{x}-\bar{y}-(\mu_1-\mu_2)$

$= P\left((\bar{x}-\bar{y}) - \sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}\, z_{\alpha/2} \le \mu_1-\mu_2 \le (\bar{x}-\bar{y}) + \sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}\, z_{\alpha/2}\right) = 1-\alpha$

$\Rightarrow$ CI $100(1-\alpha)\%$ for $\mu_1-\mu_2$ $(\bar{x}-\bar{y}) \pm \sqrt{\frac{\sigma_1^2}{n}+\frac{\sigma_2^2}{m}}\, z_{\alpha/2}$

(b) Assume par's σ_1^2, σ_2^2 are unknown. First, suppose n, m are suff. large

$S_1^2 \longleftrightarrow \sigma_1^2$

Now I have $Q(\underline{x}, \mu_1-\mu_2) = \dfrac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{S_1/\sqrt{n} + S_2/\sqrt{m}} \sim N(0,1)$

Then, my CI is $(\bar{x}-\bar{y}) \pm \sqrt{\frac{S_1^2}{n}+\frac{S_2^2}{m}}\, z_{\alpha/2}$

Now, suppose $\sigma_1^2 = \sigma_2^2$

S_1 is not necessarily equal to S_2

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$\Rightarrow Q = \dfrac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{\sigma\sqrt{\frac{1}{n}+\frac{1}{m}}} \sim N(0,1)$

we have to replace it $\Rightarrow$ Pooled Variance (Mixed) $\to$ (*)

$S_p^2 = \dfrac{(n-1)S_1^2 + (m-1)S_2^2}{n+m-2}$

Using (*)

$\Rightarrow Q^* = \dfrac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{S_p\sqrt{\frac{1}{n}+\frac{1}{m}}} = (\circledast) \sim t_{n+m-2}$

Remember $\dfrac{(n-1)S_1^2}{\sigma_1^2} \sim \chi^2_{n-1}$, $\dfrac{(m-1)S_2^2}{\sigma_2^2} \sim \chi^2_{m-1}$ it's evident this 2 are independent

* Remark $U \perp\!\!\!\perp V$ and $U \sim \chi^2_u$, $V \sim \chi^2_v$ $\Rightarrow U+V \sim \chi^2_{u+v}$

From (#) we know $\dfrac{(n-1)S_1^2 + (m-1)S_2^2}{\sigma^2} \sim \chi^2_{n+m-2} \longrightarrow (**)$

Assuming $\sigma_1^2 = \sigma_2^2$

So, $\dfrac{\dfrac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{\sigma\sqrt{\frac{1}{n}+\frac{1}{m}}}}{\sqrt{\dfrac{(n-1)S_1^2+(m+1)S_2^2}{(n+m-2)\sigma^2}}} = (\circledast) \sim t_{n+m-2}$

$P(-t(n+m-2, \alpha/2) < Q < t(n+m-2, \alpha/2))$

$= P\left((\bar{x}-\bar{y}) - S_p\sqrt{\frac{1}{n}+\frac{1}{m}}\; t(n+m-2, \alpha/2) < \mu_1-\mu_2 < (\bar{x}-\bar{y}) + S_p\sqrt{\frac{1}{n}+\frac{1}{m}}\; t(n+m-2, \alpha/2)\right)$

$= 1-\alpha$

$\Rightarrow$ CI for $\mu_1-\mu_2$: $(\bar{x}-\bar{y}) \pm S_p\sqrt{\frac{1}{n}+\frac{1}{m}}\; t(n+m-2, \alpha/2)$

Numerical Ex. 95% ...

(8) Comentar y empezar a resolver el ejercicio de los elefantes de Rod con las herramientas vistas en este lab.