Quine-McCluskey Method

K-Map Pros and Cons

- K-Map is systemic
- Require the ability to identify and visualize the prime implicants in order to cover all minterms
- But effective only up to 5-6 input variables!

Quine-McCluskey Algorithm

- Tabular Method
  - Compute all prime implicants
  - Find a minimum expression for Boolean functions
- No visualization of prime implicants
- Can be programmed and implemented in a computer

QM Method Example

$$F(W,X,Y,Z) = \sum m(0,3,5,6,7,10,12,13) + \sum d(2,9,15)$$

- Step 1 : Divide all the minterms (and don't cares) of a function into groups

For Minterms:

Minterm ID	W	X	Y	Z
0	0	0	0	0
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
10	1	0	1	0
12	1	1	0	0
13	1	1	0	1

For don't cares:

Minterm ID	W	X	Y	Z
2	0	0	1	0
9	1	0	0	1
15	1	1	1	1

QM Method Example

- Step 1 : Divide all the minterms (and don't cares) of a function into groups

Groups	Minterm ID	W	X	Y	Z	Merge Mark
G0	0	0	0	0	0	
G1	2	0	0	1	0	
G2	3	0	0	1	1	
	5	0	1	0	1	
	6	0	1	1	0	
	9	1	0	0	1	
	10	1	0	1	0	
	12	1	1	0	0	
G3	7	0	1	1	1	
	13	1	1	0	1	
G4	15	1	1	1	1	

QM Method Example

- Step 2: Merge minterms from adjacent groups to form a new implicant table

Groups	Minterm ID	W	X	Y	Z	Merge Mark
G0	0	0	0	0	0	✓
G1	2	0	0	1	0	✓
G2	3	0	0	1	1	✓
	5	0	1	0	1	✓
	6	0	1	1	0	✓
	9	1	0	0	1	✓
	10	1	0	1	0	✓
	12	1	1	0	0	✓
G3	7	0	1	1	1	✓
	13	1	1	0	1	✓
G4	15	1	1	1	1	✓

Groups	Minterm ID	W	X	Y	Z
G0'	0, 2	0	0	d	0
G1'	2, 3	0	0	1	d
	2, 6	0	d	1	0
	2, 10	d	0	1	0
G2'	3, 7	0	d	1	1
	5, 7	0	1	d	1
	6, 7	0	1	1	d
	5, 13	d	1	0	1
	9, 13	1	d	0	1
	12, 13	1	1	0	d
G3'	7, 15	d	1	1	1
	13, 15	1	1	d	1

QM Method Example

- Step 3: Repeat step 2 until no more merging is possible

Groups	Minterm ID	W	X	Y	Z	Merge Mark
G0'	0, 2	0	0	d	0	
G1'	2, 3	0	0	1	d	✓
	2, 6	0	d	1	0	✓
	2, 10	d	0	1	0	
G2'	3, 7	0	d	1	1	✓
	5, 7	0	1	d	1	✓
	6, 7	0	1	1	d	✓
	5, 13	d	1	0	1	✓
	9, 13	1	d	0	1	
	12, 13	1	1	0	d	
G3'	7, 15	d	1	1	1	✓
	13, 15	1	1	d	1	✓

Groups	Minterm ID	W	X	Y	Z
G1''	2, 3, 6, 7	0	d	1	d
	2, 6, 3, 7	0	d	1	d
G2''	5, 7, 13, 15	d	1	d	1
	5, 7, 13, 15	d	1	d	1

QM Method Example

- Step 3: Repeat step 2 until no more merging is possible

Groups	Minterm ID	W	X	Y	Z	Merge Mark
G0''	0, 2	0	0	d	0	
G1''	2, 3, 6, 7	0	d	1	d	
	2, 10	d	0	1	0	
G2''	5, 7, 13, 15	d	1	d	1	
	9, 13	1	d	0	1	
	12, 13	1	1	0	d	

- **No more merging possible!**

QM Method Example

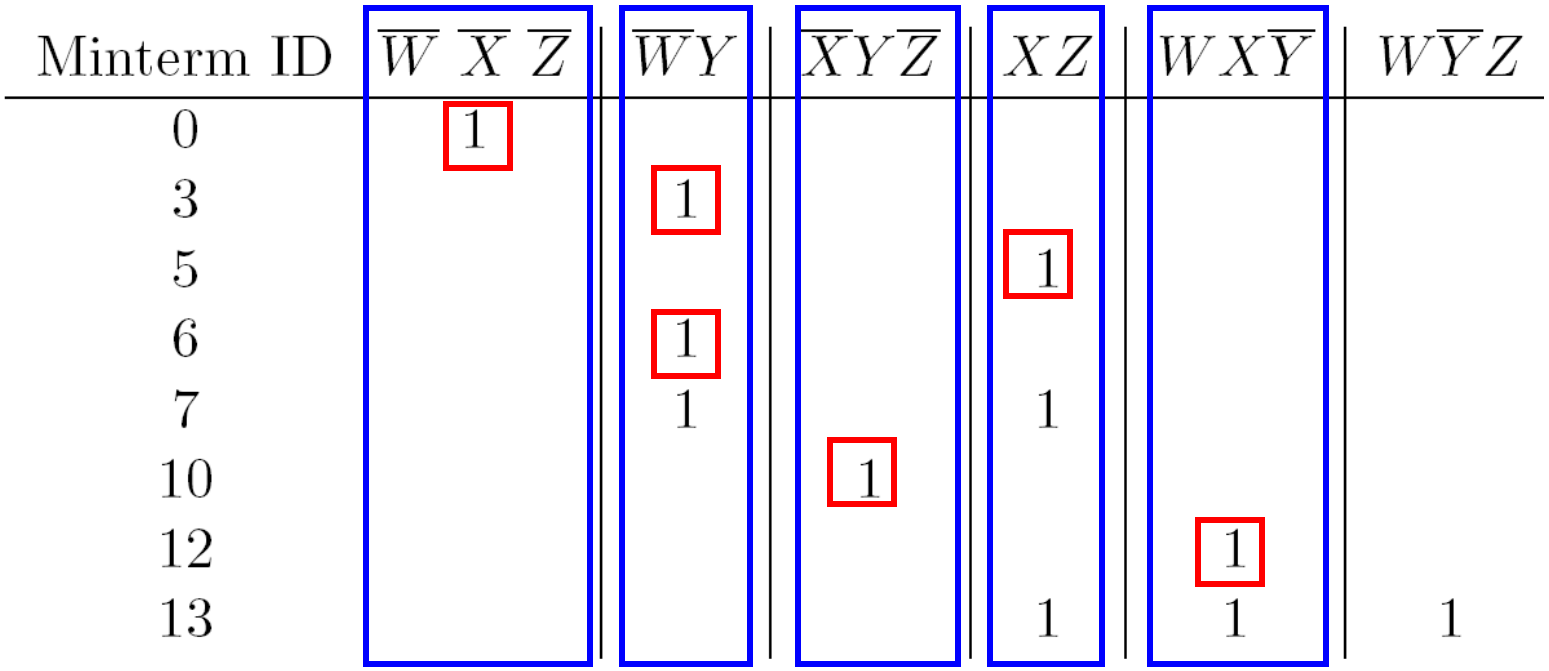
- Step 4: Put all prime implicants in a cover table (don't cares excluded)

Minterm ID	$\overline{W}\ \overline{X}\ \overline{Z}$	$\overline{W}Y$	$\overline{X}Y\overline{Z}$	XZ	$WX\overline{Y}$	$W\overline{Y}Z$
0	1					
3		1				
5				1		
6		1				
7		1		1		
10			1			
12					1	
13				1	1	1

Need not include don't cares

QM Method Example

- Step 5: Identify essential minterms, and hence essential prime implicants

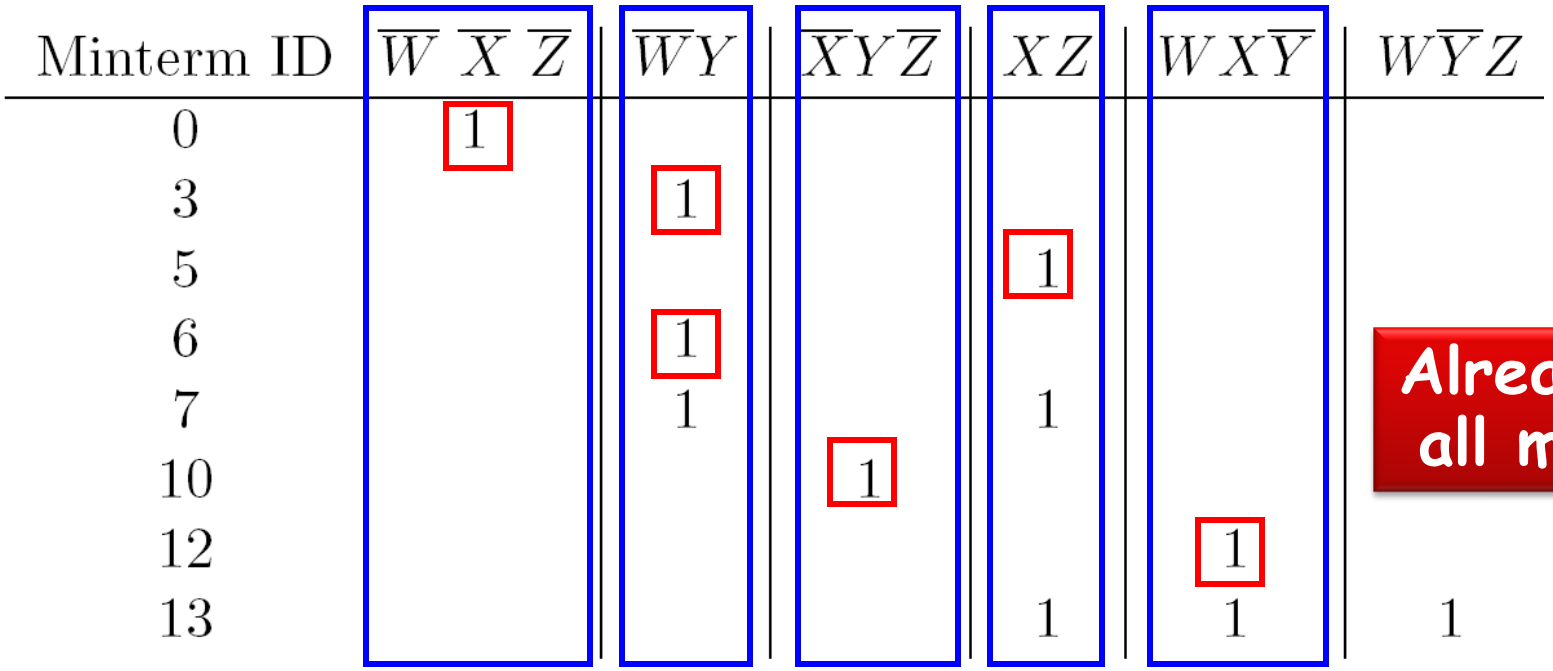


Minterm ID	$\overline{W}\ \overline{X}\ \overline{Z}$	$\overline{W}Y$	$\overline{X}Y\overline{Z}$	XZ	$WX\overline{Y}$	$W\overline{Y}Z$
0	1					
3		1				
5				1		
6		1				
7		1		1		
10			1			
12					1	
13				1	1	1

E.M.T E.P.I

QM Method Example

- Step 6: Add prime implicants to the minimum expression of F until all minterms of F are covered



QM Method Example

$$F(W,X,Y,Z)=\sum m(0,3,5,6,7,10,12,13)+\sum d(2,9,15)$$

- So after simplification through QM method, a minimum expression for F(W, X, Y, Z) is:

$$F(W,X,Y,Z)=\overline{W}\,\overline{X}\,\overline{Z}+\overline{W}Y+X\overline{Y}\,\overline{Z}+XZ+WX\,\overline{Y}$$

Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

	Step 1			Step 2			Step 3		
2		5							
		9							
3		7							
		11							
		13							
4		15							

List minterms by the number of 1s it contains.

Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
	5	0101						
	9	1001						
	7	0111						
	11	1011						
	13	1101						
	15	1111						

Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
	5	0101		5,7				
	9	1001	2	5,13				
				9,11				
	7	0111		9,13				
	11	1011						
	13	1101		7,15				
			3	11,15				
	15	1111		13,15				

Enter combinations of minterms by the number of 1s it contains.

Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
⊠	5	0101		5,7	01-1			
⊠	9	1001		5,13	-101			
				9,11	10-1			
⊠	7	0111		9,13	1-01			
⊠	11	1011						
⊠	13	1101		7,15	-111			
				11,15	1-11			
⊠	15	1111		13,15	11-1			

Check off elements used from Step 1.

Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
⊠	5	0101		5,7	01-1		5,7,13,15	-1-1
⊠	9	1001		5,13	-101		5,13,7,15	-1-1
				9,11	10-1		9,11,13,15	1- -1
⊠	7	0111		9,13	1-01		9,13,11,15	1- -1
⊠	11	1011						
⊠	13	1101		7,15	-111			
				11,15	1-11			
⊠	15	1111		13,15	11-1			

Enter combinations of minterms by the number of 1s it contains.

Finding Prime Implicants (PIs)

$$F(W,X,Y,Z) = \Sigma(5,7,9,11,13,15)$$

Step 1			Step 2			Step 3		
⊠	5	0101	⊠	5,7	01-1		5,7,13,15	-1-1
⊠	9	1001	⊠	5,13	-101		5,13,7,15	-1-1
			⊠	9,11	10-1		9,11,13,15	1- -1
⊠	7	0111	⊠	9,13	1-01		9,13,11,15	1- -1
⊠	11	1011						
⊠	13	1101	⊠	7,15	-111			
			⊠	11,15	1-11			
⊠	15	1111	⊠	13,15	11-1			

The entries left unchecked are Prime Implicants.

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms 5	7	9	11	13	15
	- 1 - 1	5,7,13,15						
	1 - - 1	9,13,11,15						

Enter the Prime Implicants and their minterms.

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms					
			5	7	9	11	13	15
	- 1 - 1	5,7,13,15	X	X			X	X
	1 - - 1	9,13,11,15			X	X	X	X

Enter Xs for the minterms covered.

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms					
			5	7	9	11	13	15
	- 1 - 1	5,7,13,15	X	X			X	X
	1 - - 1	9,13,11,15			X	X	X	X

Circle Xs that are in a column singularly.

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms 5	7	9	11	13	15
⊠	- 1 - 1	5,7,13,15	(X)	(X)			X	X
⊠	1 - - 1	9,13,11,15			(X)	(X)	X	X

The circled Xs are the Essential Prime Implicants, so we check them off.

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms 5	7	9	11	13	15
⊠	- 1 - 1	5,7,13,15	X	X			X	X
⊠	1 - - 1	9,13,11,15			X	X	X	X
			⊠	⊠	⊠	⊠	⊠	⊠

We check off the minterms covered by each of the EPIs.

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms 5	7	9	11	13	15
⊠	- 1 - 1	5,7,13,15	X	X			X	X
⊠	1 - - 1	9,13,11,15			X	X	X	X
			⊠	⊠	⊠	⊠	⊠	⊠

EPIs:

W	X	y	Z
-	1	-	1
1	-	-	1

F = (X .Z) +(W.Z)
= (X +W).Z

Finding Prime Implicants (PIs)

$F(W,X,Y,Z) = \Sigma(2,3,6,7,8,10,11,12,14,15)$

Step 1			Step 2			Step 3			Step 4		
	2	0010									
	8	1000									
	3	0011									
	6	0110									
	10	1010									
	12	1100									
	7	0111									
	11	1011									
	14	1110									
	15	1111									

Finding Prime Implicants (PIs)

$F(W,X,Y,Z) = \Sigma(2,3,6,7,8,10,11,12,14,15)$

Step 1			Step 2			Step 3			Step 4		
⊠	2	0010		2,3	001-						
⊠	8	1000		2,6	0-10						
				2,10	-010						
⊠	3	0011		8,10	10-0						
⊠	6	0110		8,12	1-00						
⊠	10	1010									
⊠	12	1100		3,7	0-11						
				3,11	-011						
⊠	7	0111		6,7	011-						
⊠	11	1011		6,14	-110						
⊠	14	1110		10,14	1-10						
				10,11	101-						
⊠	15	1111		12,14	11-0						
				7,15	-111						
				11,15	1-11						
				14,15	111-						

Finding Prime Implicants (PIs)

F(W,X,Y,Z) = Σ(2,3,6,7,8,10,11,12,14,15)

Step 1			Step 2			Step 3			Step 4		
⊠	2	0010	⊠	2,3	001-		2,3,6,7	0-1-			
⊠	8	1000	⊠	2,6	0-10		2,6,3,7	0-1-			
			⊠	2,10	-010		2,3,10,11	-01-			
⊠	3	0011	⊠	8,10	10-0		2,6,10,14	- - 10			
⊠	6	0110	⊠	8,12	1-00		2,10,3,11	- 01-			
⊠	10	1010					2,10,6,14	- - 10			
⊠	12	1100	⊠	3,7	0-11		8,10,12,14	1 - - 0			
			⊠	3,11	-011		8,12,10,14	1 - - 0			
⊠	7	0111	⊠	6,7	011-						
⊠	11	1011	⊠	6,14	-110		3,7,11,15	- - 11			
⊠	14	1110	⊠	10,14	1-10		3,11,7,15	- - 11			
			⊠	10,11	101-		6,7,14,15	- 11 -			
⊠	15	1111	⊠	12,14	11-0		6,14,7,15	- 11 -			
							10,14,11,15	1 - 1 -			
			⊠	7,15	-111		10,11,14,15	1 - 1 -			
			⊠	11,15	1-11						
			⊠	14,15	111-						

Finding Prime Implicants (PIs)

$F(W,X,Y,Z) = \Sigma(2,3,6,7,8,10,11,12,14,15)$

Step 1			Step 2			Step 3			Step 4		
⊠	2	0010	⊠	2,3	001-	⊠	2,3,6,7	0-1-		2,3,6,7,10,14,11,15	- - 1 -
⊠	8	1000	⊠	2,6	0-10	⊠	2,6,3,7	0-1-		2,3,10,11,6,14,7,15	- - 1 -
			⊠	2,10	-010	⊠	2,3,10,11	-01-		2,6,3,7,10,11,14,15	- - 1 -
⊠	3	0011	⊠	8,10	10-0	⊠	2,6,10,14	- - 10		2,6,10,14,3,7,11,15	- - 1 -
⊠	6	0110	⊠	8,12	1-00	⊠	2,10,3,11	- 01-		2,10,3,11,6,7,14,15	- - 1 -
⊠	10	1010				⊠	2,10,6,14	- - 10		2,10,6,14,3,11,7,15	- - 1 -
⊠	12	1100	⊠	3,7	0-11		8,10,12,14	1 - - 0			
			⊠	3,11	-011		8,12,10,14	1 - - 0			
⊠	7	0111	⊠	6,7	011-						
⊠	11	1011	⊠	6,14	-110	⊠	3,7,11,15	- - 11			
⊠	14	1110	⊠	10,14	1-10	⊠	3,11,7,15	- - 11			
			⊠	10,11	101-	⊠	6,7,14,15	- 11 -			
⊠	15	1111	⊠	12,14	11-0	⊠	6,14,7,15	- 11 -			
						⊠	10,14,11,15	1 - 1 -			
			⊠	7,15	-111	⊠	10,11,14,15	1 - 1 -			
			⊠	11,15	1-11						
			⊠	14,15	111-						

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms									
			2	3	6	7	8	10	11	12	14	15
	1 - - 0	8,12,10,14										
	- - 1 -	2,3,6,7,10,11,14,15										

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms									
			2	3	6	7	8	10	11	12	14	15
	1 - - 0	8,12,10,14					X	X		X	X	
	- - 1 -	2,3,6,7,10,11,14,15	X	X	X	X		X	X		X	X

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms									
			2	3	6	7	8	10	11	12	14	15
	1 - - 0	8,12,10,14					X	X		X	X	
	- - 1 -	2,3,6,7,10,11,14,15	X	X	X	X		X	X		X	X

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms									
			2	3	6	7	8	10	11	12	14	15
⊠	1 - - 0	8,12,10,14					X	X		X	X	
⊠	- - 1 -	2,3,6,7,10,11,14,15	X	X	X	X		X	X		X	X

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms									
			2	3	6	7	8	10	11	12	14	15
⌦	1 - - 0	8,12,10,14					X	X		X	X	
⌦	- - 1 -	2,3,6,7,10,11,14,15	X	X	X	X		X	X		X	X
			⌦	⌦	⌦	⌦	⌦	⌦	⌦	⌦	⌦	⌦

Finding Essential Prime Implicants (EPIs)

	Prime Implicants	Covered Minterms	Minterms									
			2	3	6	7	8	10	11	12	14	15
⌦	1 - - 0	8,12,10,14					X	X		X	X	
⌦	- - 1 -	2,3,6,7,10,11,14,15	X	X	X	X		X	X		X	X
			⌦	⌦	⌦	⌦	⌦	⌦	⌦	⌦	⌦	⌦

EPIs:

W	X	Y	Z
1	-	-	0
-	-	1	-

$F = (W.Z') + Y$