Digital Design

A Datapath and Control Approach

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List of Processes

Digital Design Body Of Knowledge

The focus of this text is very much on the datapath and control approach. To achieve this end in a single semester, sacrafices in coverage must be made.

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Chapter 1

Sequential Building Blocks

In order to design complex systems, a suitable abstraction is required for building them. This requirement stems from the limitations of the human mind to only manage about a dozen items at once. A complex system containing hundreds of components simply cannot be organized in one pass. Instead, the components are organized into larger units which compose the system. Thus, the number of components need to be considered at a development stage is reduced by an order of complexity. These intermediate units should have a high utility and be modular. In other words, they should be useful and applicable in a wide variety of design situations.

As an example, consider the design problem of writing a technical document describing the operation of a pacemaker for a human heart. This process does not begin by thinking about the spelling of the individual words, instead it makes more sense to first draft an outline. This outline is an abstraction of the written document, it is a simplified representation of the final written document. In somewhat the same way as an author ignores spelling issues when constructing the outline, a designer is not concerned with the operation of AND and OR gates when designing a digital-signal processing chip.

Clearly, choosing components, or as they are called "basic building blocks," which are reusable and have non-overlapping functionality, result in a small number of highly useful components. The set of available building blocks has largely been determined by the electronics industry which provides basic blocks as off-the-shelf prepackaged components. These time-tested components have established themselves over the years as the accepted language of hardware design. Several sequential logic building blocks are examined, next. The word "sequential" in their name implies that these building blocks are different from those presented in Chapter 4 because they have memory.

Like the combinational building blocks shown in Figure ?? each of the sequential basic building blocks have control and data inputs, and status and data outputs. In addition, being sequential devices, most also have an edge-sensitive clock input. First to be examined is the most basic sequential basic building block, the register.

1.1   The Register

Nomenclature:	N-bit register					
Data Input:	N-bits vector $D = d_{N-1} \ldots d_1 d_0$.					
Data Output:	N-bit vector $Q = q_{N-1} \ldots q_1 q_0$					
Control:	1-bit C					
Status:	none					
Others:	1-bit edge-sensitive clock. 1-bit asynchronous active low reset.					
Behavior:	$reset$	clk	C	D	Q^+	comment
	0	x	x	x	0	reset
	1	0,1,falling	x	x	Q	hold
	1	rising	0	x	Q	hold
	1	rising	1	D	D	load

An N-bit register is very much like a wide D flip flop. It samples its N data inputs, denoted D on the rising edge of the clock input. Depending on the control input, C, the register either holds its current value when $C = 0$ or loads the new value when $C = 1$. The stored value of the register is asserted on its output, denoted Q. The columns in the register's state table are organized from left to right, from highest priority to lowest priority. Holding the asynchronous active low reset line to 0 causes the stored value and the outputs to remain at 0 regardless of the value on any other input; the reset input has priority over all other inputs.

A timing diagram for a 4-bit register is shown in Figure 1.1. The initial value of the register is arbitrarily set to $A_{16} = 0001_2$. Since the value of Q is represented using four bits, its value on the timing diagram is shown as a wide trace. This reflects the fact that Q is composed of many bits. At time=10, a positive edge of the clock arrives with $C = 1$, hence the register loads $D = 5_{16} = 0101_2$, as its new value. The fact that the Q outputs changes slightly after time=10 is an acknowledgment that the circuit elements inside the register have propagation delay. The goofy behavior of the C input around time=20 has no effect on the Q outputs of the register because the clock is not rising. The rising clock edge at time=30 does not change the stored value of the register because $C = 0$, hence the register holds its stored value. The change in the Q output at $time 50$ results from the rising clock edge and $C = 1$.

An N-bit register is constructed using N, D flip flops. A common error committed by beginning students, and even some text books, is to AND the clk and C signals together, sending the AND gate output to the clock input of a D flip flop. This technique is incorrect because it causes the D flip flops to sample their input when the $clk = 1$ and C rises, contrary to the behavior described in the register's state table. As a general rule, avoid modifying the clock signal unless it is absolutely necessary.

The correct construction of an N-bit register is shown in Figure 1.2. Two modes are present for this circuit, corresponding to $C = 0$ and $C = 1$. When $C = 1$, the four multiplexers shown in Figure 1.2, all route the data input D_i to the input of the D flip flop. When a clock edge arrives, each D_i is loaded into its respective flip flop and soon thereafter appears on the Q output.

When $C = 0$, the four multiplexers shown in Figure 1.2, all route their data output Q_i back to the input of the D flip flop. When a clock edge arrives, each Q_i is loaded into its respective flip flop and soon thereafter appears on the Q output. Thus, the Q outputs appear to have held their output value even though the internal D flip flops have loaded a value.
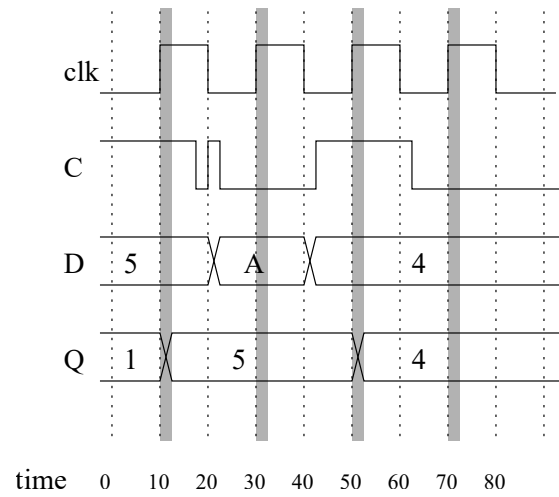
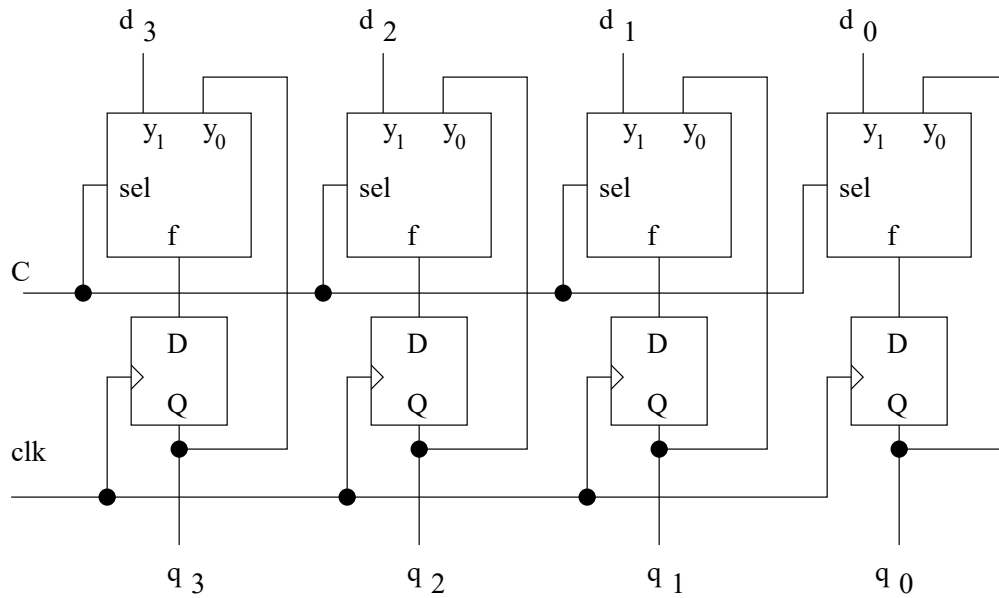Figure 1.1: A timing diagram for a 4-bit register.

Figure 1.2: The internal organization of a 4-bit register.

1.2 The Shift Register

A shift register is a register with the additional capability of shifting its stored bits to the left or to the right. The input, output, and behavior of a shift register are shown in the following table.

Nomenclature:	N-bit shift register with parallel load					
Data Input:	N-bits vector $D = d_{N-1} \ldots d_1 d_0$.					
Data Output:	N-bit vector $Q = q_{N-1} \ldots q_1 q_0$					
Control:	2-bits $C = c_1 c_0$					
Status:	none					
Others:	1-bit edge-sensitive clock. 1-bit asynchronous active low reset.					
Behavior:	$reset$	clk	C	D	Q^+	comment
	0	x	xx	x	0	reset
	1	0,1,falling	xx	x	Q	hold
	1	rising	00	x	Q	hold
	1	rising	01	x	$Q >> 1$	shift right
	1	rising	10	x	$Q << 1$	shift left
	1	rising	11	x	D	load

If $Q = 0110$ then shifting Q to the left, denoted $Q << 1$, yields 1100. The symbol "$<<$" denotes a shift left and the "1" describes how many bits to shift. Shifting the original value of Q to the right by one bit, denoted $Q >> 1$, yields 0011. The "$>>$" symbol denotes a shift right and the "1" describes how many bits.

Shifting is used to examine bits one at a time and in the multiplication and division of binary numbers. The bits could be examined by looking at the LSB of a shift register as it shifted its bits successively to the right. Multiplication and division involve a bit more explanation.

For instance, consider multiplying a 4-bit binary number $X = x_3 x_2 x_1 x_0$ by 2. This task is accomplished by shifting X to the left one bit, yielding $X << 1 = x_3 x_2 x_1 x_0 0$. That is a "0" is place in the LSB. In order to verify this, write down the decimal equivalent of the shifted value of $X << 1$, $x_3 * 2^4 + x_2 * 2^3 + x_1 * 2^2 + x_0 * 2^1$. Now, factor a 2 from each component of the sum, yielding $2 * (x_3 * 2^3 + x_2 * 2^2 + x_1 * 2^1 + x_0 * 2^0)$. But this is $2 * X$.

For each shift left by one bit, each of the exponents in the decimal representation of X increases by 1, adding a factor of 2 to every term of X which can be factored out. Hence, every shift left increases X by a factor of 2. These factors accumulate for each shift, so that shifting X left three bits increases X by a factor of $2^3 = 8$. Shifting can be used to multiply by constants which are not powers of two by rewriting the constant as the sum of powers of two. For example, to multiple a binary number X by 10, rewrite 10 as $8 + 2$ yielding $10X = (8 + 2)X = 8X + 2X$. So $10 * X$ is computed by adding together X shifted left by three bits to X shift left by one bit.

Shifting left may create a result which cannot fit in the prescribed word-size. For example, if $X = 12_{10} = 1100_2$ is shifted left one bit in a 4-bit shift register, the result $1000_2 = 8_{10}$ does not represent 24_{10} because this value cannot fit into four bits. It is easy to see a shift left results in overflow whenever the MSB equals 1.

Dividing binary numbers by powers of two is accomplished by shifting the bits to the right. Since it is possible that division by two results in a fraction and combined with the fact that binary numbers represent integers, some form of rounding must occur. For example, if $X = 5_{10} = 0101$ is shifted right by one bit, the result is $010_2 = 2_{10}$. The 1 in the LSB of X is lost. Hence, shifting to the right divides a number by 2 and rounds down when there is a fractional result.

In order to accommodate, the diverse set of situations when shifting can be used, three types of shifts are available: arithmetic, logical, and circular. Since each of these can occur to the left or right, then six possible effects are to be considered due to shifting a 4-bit string $x_3 x_2 x_1 x_0$. These are enumerated in the following table.

	Left	Right
Arithmetic	$x_2x_1x_0 0$	$x_3x_3x_2x_1$
Circular	$x_2x_1x_0x_3$	$x_0x_3x_2x_1$
Logical	$x_2x_1x_0 0$	$0x_3x_2x_1$

All these shifts are characterized by how they "fill in" the void created by the shift. Logical shifts always fill in the void with a 0 and are used mainly for multiplication and division of binary numbers.

Circular shifts fill in the void with the bit that "falls off" from the other end of the shift. For example, circularly shifting 0101 to the right yields 1010 because the 1 that falls off the LSB is inserted into the void created at the MSB. Circular shifts are useful when each bit of a register needs to be inspected without destroying the registers contents.

Arithmetic shifts are used mainly to manipulate 2's-complement numbers. An arithmetic shift right fills the void with a duplicate of the MSB, maintaining the "sign" of the 4-bit 2's-complement number. This process is the same as the one governing the sign-extending of 2's-complement numbers as discussed on page **??**. An arithmetic shift left fills in the void with a 0 because this multiplies both positive and negative quantities by 2.

To better understand its organization the design of a 4-bit circular shift register that holds its value, circularly shifts its contents to the right, circularly shifts its contents to the left, or loads an external 4-bit input is examined. Since this circular shift register has four functions, it requires two bits of control, denoted c_1c_0. The assignment of bit values to the various functions is arbitrary and defined in the table below. The external 4-bit data input will be denoted D. A clock signal, clk, indicates when the circular shift register should perform its function. Finally, the 4-bit output from the circular shift register is denoted Q.

clk	C	D	Q^+	comments
0,1,falling	x	x	Q	
rising	00	x	Q	hold
rising	01	x	$Q(0)\|Q >> 1$	CSR
rising	10	x	$Q << 1\|Q(3)$	CSL
rising	11	D	D	load

The notation in Q^+ column of the state table needs some further explanation. The $Q(0)$ symbol refers to the LSB of Q, the | symbol denotes concatenation, the merging of the bits to the left and right of the | symbol. Thus, the expression $Q(0)|Q >> 1$ means that the LSB of Q should be "glued" to the most significant three bits of Q.

The circuit for the circular shift register is shown in Figure 1.3 and consists of two major components, D flip flops and 4:1 muxes.

The organization of the shift register is similar to the register, the difference being the larger mux. When the 2-bit control signal, denoted by the slash with a 2 through it, is 00, Q_i is routed to the input of each D flip flop. The rising edge of the clock causes each D flip flop to latch its previous output, causing no change in the outputs. When $C = 01$, the data input to each D flip flop is Q, circularly shifted to the right. This movement is denoted by writing the name of each Q bit on the mux input instead of drawing the lines because otherwise the diagram quickly becomes messy and confusing. Hence, when on the rising edge of the clock, the D flip flops latch the shifted value of the outputs, making all the outputs appear to circularly shift to the right, one bit. The $C = 10$ input cause the inputs to circularly shift to the left. The $C = 11$ input, sometimes called a parallel load because all four bits are loaded simultaneously, loads each bit of the external 4-bit input, D, to its respective flip flop.
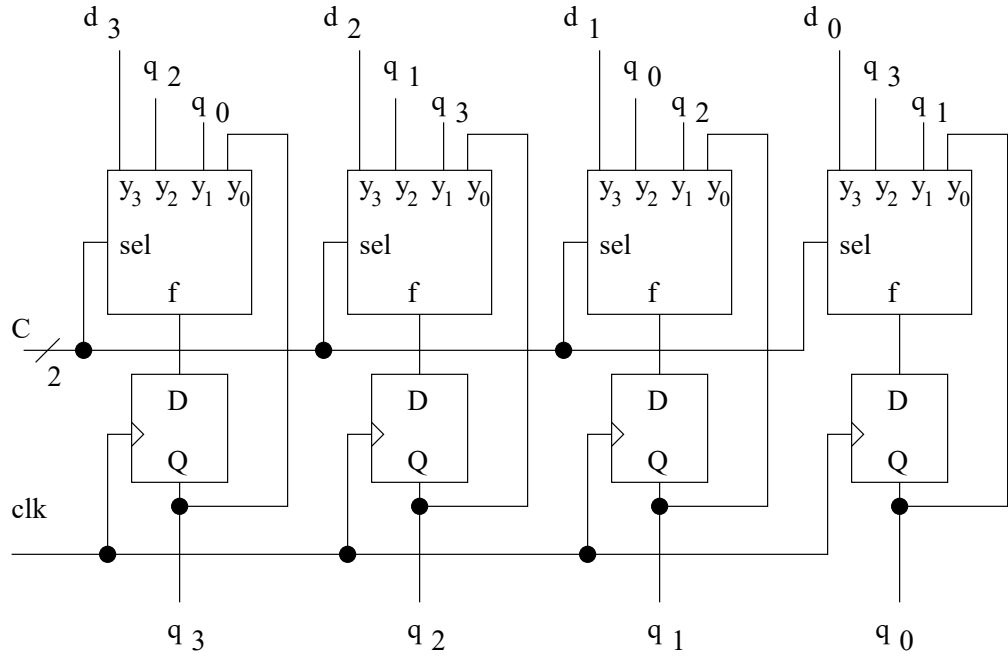
Figure 1.3: The internal organization of a 4-bit circular shift register.

1.3   The Counter

A counter is a simple but surprisingly versatile piece of hardware. Its behavior is obvious from its name; it counts up when instructed to do so.

Nomenclature:	N-bit counter with parallel load					
Data Input:	N-bits vector $D = d_{N-1} \ldots d_1 d_0$.					
Data Output:	N-bit vector $Q = q_{N-1} \ldots q_1 q_0$					
Control:	2-bits $C = c_1 c_0$					
Status:	none					
Others:	1-bit edge-sensitive clock. 1-bit asynchronous active low reset.					
	$reset$	clk	C	D	Q^+	comment
	0	x	xx	x	0	reset
	1	0,1,falling	xx	x	Q	hold
Behavior:	1	rising	00	x	Q	hold
	1	rising	01	x	D	load
	1	rising	10	D	$Q+1$	count up
	1	rising	11	x	x	

When the 2-bit control input equals 10 and a clock edge arrives, the counter counts up. Since number of bits are limited, the counting will at some point overflow. When this happens, the count value rolls-over back to 0 and begins counting up again. For example, a 4-bit counter rolls-over to 0 when it tries to count up at 15. This behavior is similar to the behavior of the digits of a car's odometer rolling over from 9 to 0.

A timing diagram for a 4-bit counter is shown in Figure 1.4. The initial value of the counter was arbitrarily set to $E_{16} = 1110_2$. At time=10, a positive edge of the clock arrives. Since the

C input is equal to 10_2 at time=10, then the counter counts up to $F_{16} = 1111_2$. The goofy behavior of the C input between time=10 and time=20 has no effect on the Q outputs of the counter because the clock is not rising. At time=30, the C input is equal to 00_2 so the counter holds the current count value. At time=50, the C input is equal 10_2 so the counter counts up rolling over to 0_{16}. At time=70 the counter counts up to $1_{16} = 0001_2$. At time=90, the counter loads $7_{16} = 0111_2$. Notice, as in Figure 1.1, the counter timing diagram shows a small propagation delay for the Q output.
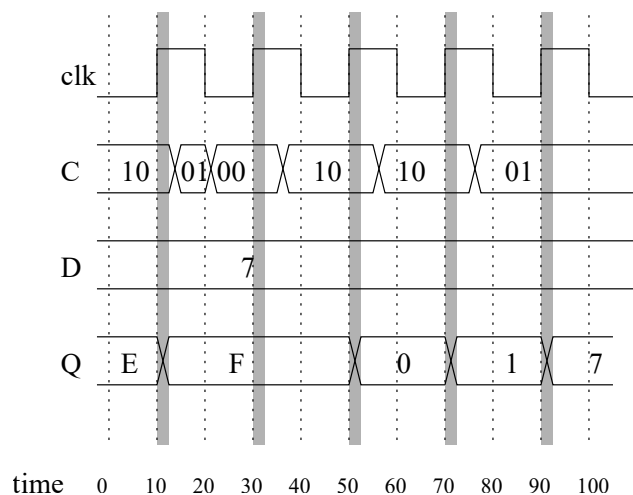
Figure 1.4: A timing diagram for a 4-bit counter.

The internal organization of a counter is very similar to the structure of the register and shift register. A set of D flip flops holds the current count value. A mux decides which input is presented to the D flip flops to be latched up when the positive clock edge arrives. An adder is used to add 1 to the outputs of the flip flops (current count value). If the overflow output of the adder is ignored, then the adder has the advantage of rolling over to 0 when the count value is at the maximum. It is easy to verify that $111...1 + 1 = 000...0$. The entire circuit is shown in Figure 1.5.

Notice, the data inputs to the mux shown in Figure 1.5 all have a slash through them with the number 4. This notation indicates each of the data inputs is a four bit wide vector, and consequently, this device is a multibit mux, as discussed on page **??**. Additionally, the y_3 input is unused. This inefficiency is the cost of using a generalized building block.

1.4 The Static RAM

Random Access Memory (RAM) is an unfortunate name for a digital circuit that has nothing random about it. The name "RAM" originated in the early days of computing when engineers used cassette tapes as an inexpensive way to store "lots" of data. One of the downfalls of a cassette tape is the sequential nature of data access. That is, the time to access a piece of data depends on its position on the tape and the current position of the tape. RAMs do not suffer from the limits of a sequential access devices; the amount of time to access any *random location* is the same.

In order to understand how RAMs work, you will need to understand a few concepts first, so let's start.
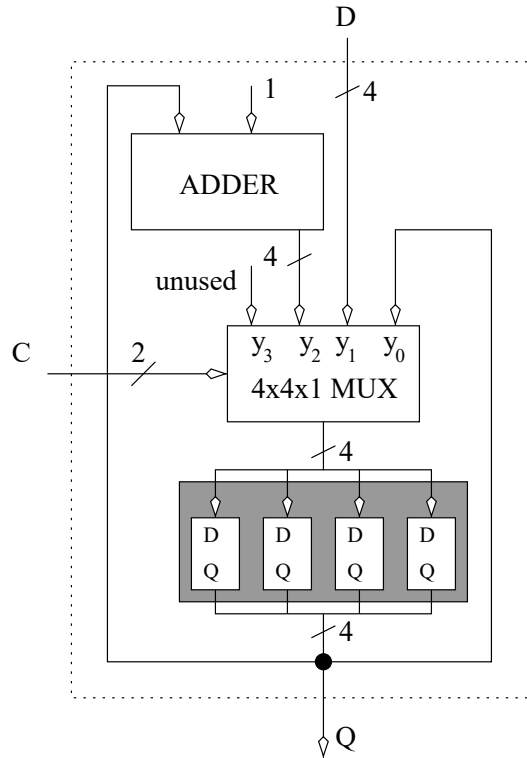
Figure 1.5: The internal organization of a 4-bit counter.

A RAM is a device that stores and retrieves bundles of bits, called a word, from an address. You can envision a RAM as tower of binary numbers like that shown in Figure 1.6. The width of a RAM is called the **word size**. In Figure 1.6 the word size is 4-bits. Each set of 4-bits is called a word. In Figure 1.6, the words are organized in rows. Each word in the RAM has an address. By convention addressing starts at location 0. In Figure 1.6, the word at address 5 is 0011. Retrieving information from a RAM is called a **read** operation. Storing information into a RAM is called a **write** operation. The terms read/write should invoke the idea that the RAM is a book or ledger that you are reading with your eyes or writing into it with a pen.

0	0110
1	1010
2	1101
3	0010
4	1000
5	0001
6	1101
7	1111

Figure 1.6: A 8x4 RAM has eight words each containing four bits. The addresses are shown to the left.

While there is no relationship between the word size and the number of words stored in the

RAM, there is a relationship between the number of words stored in the RAM and the number of address bits: The number of bits in the address must be sufficient to assign each word a unique binary address. In Figure 1.6, the addresses range from 0 to 7, hence the address is a 3-bit value. In general, if a RAM has 2^N words, it requires a N bit address so that each word is assigned a unique/distinct address.

The number of words in a RAM is often described using the metric system notation.

1k	2^{10}	kilo
1M	2^{20}	mega
1G	2^{30}	giga
1T	2^{40}	tera

The metric names are only close approximations to the actual, binary values they describe. For example, 1 kilometer is a 1,000 meters, but 1k bytes of memory is 2^{10} bytes, or 1024 bytes. The number of address bits can be determined quickly from the metric abbreviations. For example, a 256k RAM has $256k = 2^8 * 2^{10} = 2^{18}$ words, or 18 bits of address.

A wide variety of random access memories are available to meet the wide variety of applications in which users need to store data. For situations where data needs to be retained even though power is removed, non-volatile memories are employed. These memories typically trade-off access and storage speed for the convenience of non-volatility. Volatile memories, memories which lose their contents when power is removed, can be either static/dynamic, or synchronous/asynchronous.

Dynamic memories store data on tiny capacitors and consequently require periodic refreshing in order to retain their values. These versions are typically the highest density memories available, but the refresh circuitry adds to their complexity. Static memories store data in an arrangement of transistors which does not need refreshing. Static memories generally faster, more expensive, and consume lesas power than their dynamic counterparts.

Access to a synchronous memory requires a clock and is reminiscent of a flip flop. Asynchronous memories require the control and data signals be applied for certain minimum duration in order for the operations to take effect.

In order to convey the behavior and utilization of a RAM we consider a popular type of RAM used in many FPGAs, synchronous static RAM. The input, output, and behavior of a synchronous static RAM is defined by the following table.

Nomenclature:	NxM RAM (random access memory)
Data Input:	M-bit vector $D = d_{M-1}\ldots d_1 d_0$ $log_2(N)$-bit address $A = a_{log_2(N)-1}\ldots a_1 a_0$
Data Output:	M-bit vector $D = d_{M-1}\ldots d_1 d_0$
Control:	1-bit enb (read enable), 1-bit wen (write enable),
Status:	none
Others:	1-bit edge sensitive clock
Behavior:	

clk	enb	wen	A	D_{in}	D_{out}^+	Note
0, 1 falling	x	x	x	x	D_{out}	no clock edge
rising	0	0	x	x	D_{out}	hold
rising	1	0	A	x	RAM[A]	Read
rising	0	1	A	D_{in}	D_{out}	RAM[A] $\Leftarrow D_{in}$
rising	1	1	A	D_{in}	D_{in}	RAM[A] $\Leftarrow D_{in}$

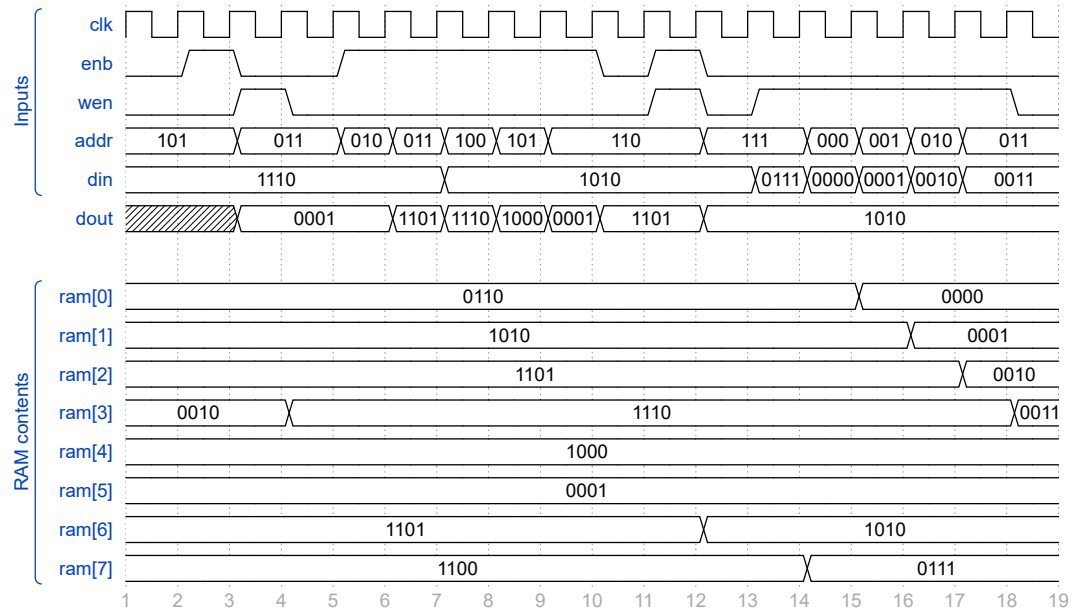Let's examine how the truth table for a RAM is applied to the timing diagram shown in Figure 1.7.

Figure 1.7: A simulation of the behavior of a 8x4 RAM whose initial values are given in Figure 1.6 .

time = 300ns Read

time = 400ns Write

time = 600ns...1,000ns Consectutive reads.

time = 1,200ns Simultanous read/write

time = 1,400ns...1,800ns Consectutive writes.

Cases occur when it is necessary to build a larger RAM from several smaller RAMs. RAMs have two dimensions which can be increased: (1) the word size or (2) the number of words.

Increasing the word size of a RAM is fairly straightforward because each RAM chip gets all the address lines and handles some portion of the data lines. Structurally, this transformation is accomplished by placing several RAM chips side-by-side as shown. For example, in order to construct a 256kx32 RAM from 256kx8 RAM chips, then four, 256kx8 RAM are placed side-by-side as shown in Figure 1.6.

Each of the four chips have the same address lines and control lines R/W', and CS. Thus, all the RAM chips behave in exactly the same fashion, each handling its own set of eight bits. Their actions are coordinated by the common address and control signals.

Combining RAM chips to increase the number of words involves some manipulation of addresses. For example, consider the problem of using 64kx8 RAM chips to construct a circuit which behaves like a 256kx8 RAM. Stacking four, 64kx8 RAM chips above one another would create the required depth of 256k words. However, each of the 64k RAMs would have 16 address lines, but the 256k RAM being constructed requires 18 address lines. How is this discrepancy of the extra two address bits resolved? The 18 bits of address are split into two components; the lower 16 bits are sent to all four of the 64kx8 RAMs and the upper two bits
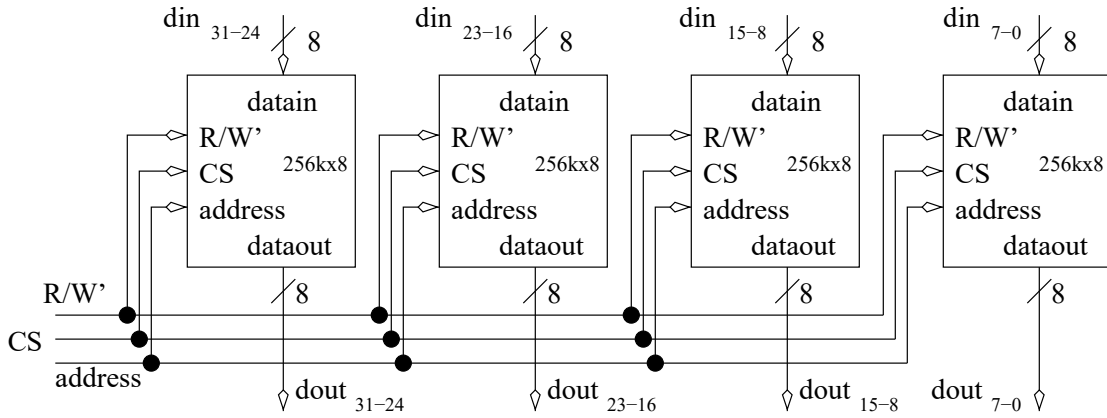
Figure 1.8: The construction of a 256kx32 RAM from 256kx8 RAM chips.

are used to decide which of the four 64k RAM chips is activated. A decoder uses the two bits
as select, and routes a chip select signal to one of its four outputs which are attached to the
chip select lines of each 64k RAM. Since a decoder routes the chip select to only one of the
RAMs, there is no potential conflicts on the data lines and they can safely be tied together.
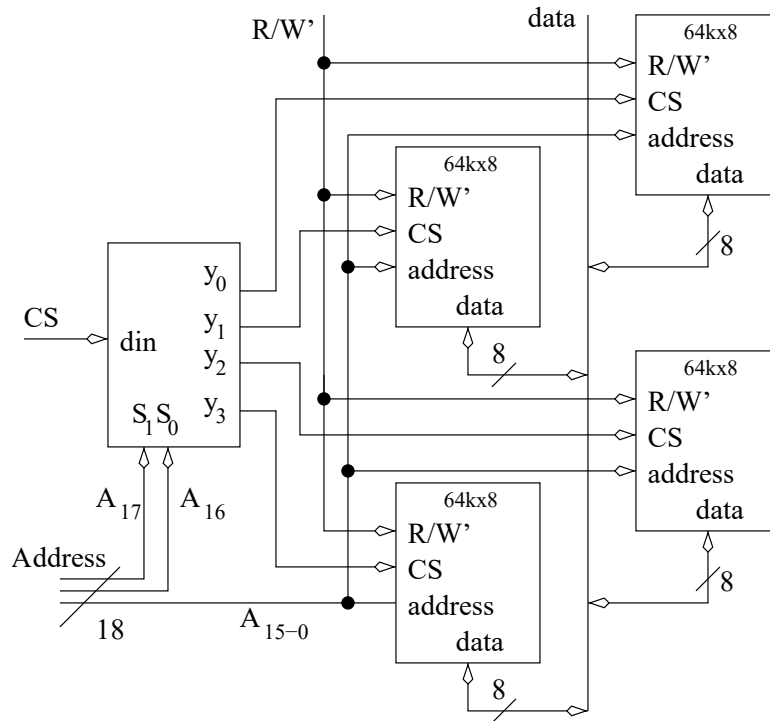Figure 1.9 shows the complete circuit diagram.

Figure 1.9: The construction of a 256kx4 RAM from 64kx4 RAM chips.

1.5   The Dynamic RAM

1.6 Register Transfer

A digital system designed using the datapath and control approach transforms data into some predetermined sequence. For now, focus on the task of transforming the data performed by the datapath. The datapath is composed of the basic building blocks discussed in Chapters 4 and 6; their input, output and behavior is summarized on page **??**. Although a gross simplification, a datapath can be considered as some combinational logic "sandwiched" between registers. The clock signal to the datapath governs how data moves in the datapath. After the rising edge of the clock, the register outputs become valid. The output data from the registers flows into the combinational logic which transforms this input into an output. The outputs of the combinational logic flow into the register inputs. The next rising edge of the clock causes the registers to latch these new values. This process then proceeds into the next clock cycle. In order to understand this discussion, consider the simplified datapath shown in Figure 1.10. In this datapath, an adder is sandwiched between three registers. Here, the control inputs on all three registers are assumed to be hardwired to 1, causing them to load on every positive edge. The inputs A and B to the two registers are provided by some external agent.
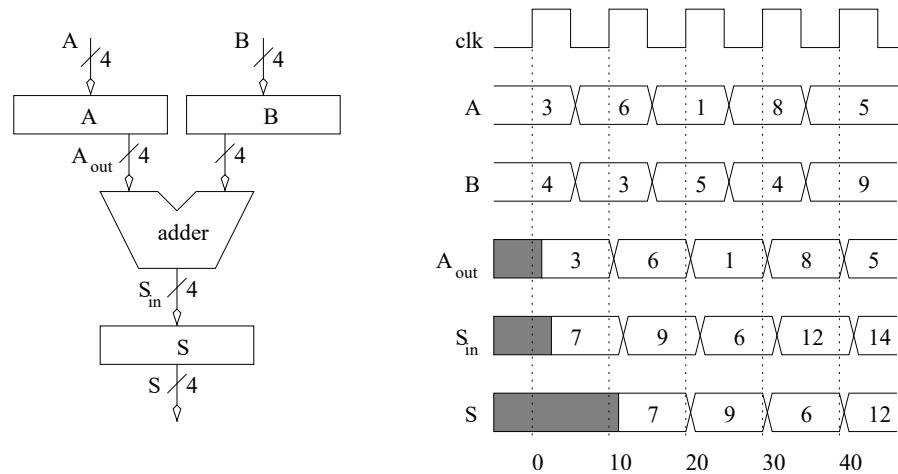

Figure 1.10: A simple datapath and the timing diagram describing its behavior.

When a signal has an unknown value, it is given a shaded regions. Prior to time=0, none of the registers contains a known value, therefore all their outputs are shaded. Since $A = 3$ and $B = 4$ prior to the positive edge at time=0, the outputs of register A/B equals 3/4 after the positive edge at time=0, respectively. The slight delay in the A_{out} signal becoming valid, emphasizes how the real outputs of a register exhibit propagation delay. Note, the output of the B register is not shown because its behavior is similar to A_{out} and would clutter up the timing diagram. Since the outputs of the A and B registers are unknown prior to time=0, causing the inputs to the adders to be unknown, causing the outputs of the adder to be unknown, causing the input to the S register to be unknown. Since the inputs to the S register are unknown when the clock edge arrives at time=0, the outputs of the S register remain unknown after that clock edge.

It might seem as if the new outputs of registers A/B available just after the clock edge at time=0 would be able to propagate to the S-register's input, allowing it to latch a valid value on the time=0 clock edge. This is incorrect because all the registers in the datapath latch their values at exactly the same instant in time and then output their new values. This assertion is

simply a restatement of the observation made on page **??**: the propagation delay of a flip flop should be larger than the hold time in order to allow flip flops to be daisy-chained together.

The A and B signals are changed at time=5, on a negative edge of the clock, in order to keep the changes on the register inputs as far away from a positive edge of the clock as possible.

After the rising edge of the clock at time=0, the outputs of the A and B registers become valid, causing the inputs to the adder to become valid, causing the output to become valid, causing the input to the S register to become valid. Thus, the S_{in} signal becomes valid after the positive clock edge at time=0.

When the positive clock edge at time=10 arrives at the circuit in Figure 1.10, registers $A/B/S$ latch the values on their inputs, 6/3/7, respectively. The new outputs of A and B are sent to the adder causing $S_{in} = 9$. This value cannot be latched into the S register until the next rising edge of the clock at time=20, because the rising edge at time=10 is long gone.

The positive clock edge at time=20 causes registers $A/B/S$ to latch 1/5/9, respectively. Once the analysis is started, it is a matter of waiting for a positive clock edge, latch all the register inputs simultaneously, propagating outputs through the combinational logic, and waiting for the next positive edge.

1.7   Combinations

The basic building blocks in Chapters 4 and 6 can be combined in interesting ways to produce complex behavior. The behavior of these digital systems can be described using a programming-language-like syntax, initially presented in Chapter 4. First, examine the counter, counting over a subinterval of its possible range.

```
while(1) {
    if (count<10) count += 1;
    else count = 3;
}
```

The `while(1)` statement means that the statements contained in `while`-brackets should be executed forever. In other words, it is a never-ending loop. The `if` statement checks the count value. If it is less than 10, `count` is incremented, otherwise the value is reset back to 3. The circuit is implemented with a counter to perform the count-up function and a comparator to perform the magnitude comparison between `count` and 9. Why 9? Because if the value is compared against 10, then the `count` value would have to reach 10 in order for the L output of the comparator to change, by which time it would be too late to stop the `count` value from reaching 10. The completed circuit and a timing diagram is shown in Figure 1.11.

The L output of the comparator is used because the condition checked is `count < 10`. This single-bit output cannot be directly connected to the counter's 2-bit control input because there just are not enough bits. The "glue" box shown in Figure 1.11 contains glue-logic, a combinational logic circuit which interfaces or glues together two pieces of logic. When the counter's output is less than 10, $L = 1$, and the counter is supposed to count up by 1. According to page 6, this requires the control to be $c_1c_0 = 10$.

When the counter's output is greater than or equal to than 10, $L = 0$, the counter should load 3. According to page 6, a load is elicited by asserting $c_1c_0 = 01$ on the counter's control input. The resulting truth table for the glue logic is shown in the margins. From this table, it is easy to ascertain that $c_1 = L$ and $c_0 = L'$.

The timing diagram shows the counter starting at 6. Since 6 is less than 10, then $L = 1$. When $L = 1$ is present on the input of the glue logic it will output $C = c_1c_0 = 10$, causing

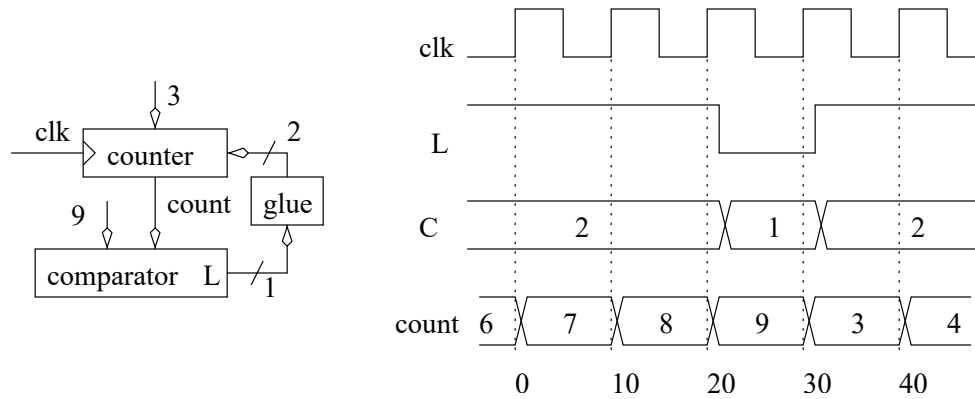| L	c_1	c_0
0	0	1
1	1	0

Figure 1.11: A circuit to count between 3 and 9 and its associated timing diagram.

the counter to count up when the clock input arrives at time=0. Successive clock edges see the count value increment to 9 at time=20. At this moment, the L output of the comparator changes to 0 causing the glue logic box to output $C = c_1 c_0 = 01$ telling the counter to load 3 on the next positive clock edge at time=30. When the value of 3 is loaded into the counter at time=30, the L output of the comparator changes to 1, causing the glue logic box to tell the counter to count up on the next positive clock edge at time=40. Without a finite state machine, it is difficult to get a combination of basic building blocks to perform a sequence of actions. Difficult, but not impossible as the following circuit shows.

Design a circuit which searches the first 100 memory locations of an eight bit wide random access memory for the smallest value is examined. Assume the RAM is preloaded with data values, so only the memory needs to be read. The approach reads each value and checks if it is smaller than the smallest 8-bit value found thus far. This smallest value is stored in a register, `min`, which will be assumed as initialized to the largest possible value, 0xFF. The "0x" in front of "0xFF" is used in many program languages to signify that "FF" is a hexadecimal number.

```
// assume that min is initialized to 0xFF
for(i=0; i<100; i++) {
    MBR = RAM[i];
    if (MBR < min) MBR = min;
}
```

The variable `MBR`, standing for memory buffer register, is a generic term applied to a register used to buffer data operations between a RAM and a datapath. The line
`for(i=0; i<100; i++)` will be implemented with a counter and a comparator. The comparator examines the counter's output and stops the counter when the count value reaches 99. The line `MBR = RAM[i];` is implemented with a RAM and a register. The address of the RAM `[i]` comes from the counter, and the data output of the RAM is sent to the data input of a register called MBR. The line `if (MBR < min) MBR = min;` is realized with a comparator and a register. The comparator compares the output of the MBR and the min registers and asserts its L output when MBR is less than min. This L output runs to the control input of the min register. The data input of the min register comes from the data output of the MBR register. The control of the MBR register should also be controlled by the counter comparator so that the MBR register stops loading when the count value is greater or equal to 99. The circuit diagram is shown in Figure
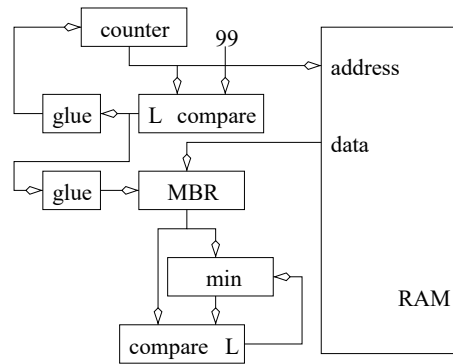
Figure 1.12: A circuit to find the smallest value in a RAM. The min register is initialized to 0xFF.

S2	S1	S0	Operation
0	0	0	Hold
0	0	1	Load
0	1	0	ASR
0	1	1	ASL
1	0	0	LSR
1	0	1	LSL
1	1	0	CSR
1	1	1	CSL

Table 1.1: The truth table for a universal shift register.

1.8 Exercises

1. **(8 points)** Build a 4-bit universal shift register in Table 1.1 using D flip-flops and 8:1 multiplexers.

2. **(8 pts.)** Use a counter and a comparator to implement the following circuits.

   a) Show how to modify the counter (by adding some external logic) to implement a mod-10 counter. A mod-10 counter counts from 0 to 9 and then goes back to 0. It spends one full clock cycle on each of these count values.

   b) Use four mod-10 counters to build a 4-digit decimal counter which counts up from 0 to 9999. Draw a schematic for the 4-digit decimal counter.

3. **(8 pts.)** Design a circuit which contains three 8-bit registers X,Y,Z. The behavior of the circuit is determined by the statement:

   ```
   if (X > Y) then Z = X+X else Z = Y+Y
   ```

   The registers are preloaded with values in them. Submit a circuit diagram showing the building blocks uses, their interconnections and any miscellaneous logic required to make them operate together.

4. **(8 pts.)** Design a circuit which contains three registers X,Y,Z. The behavior of the circuit is determined by the statements:

   ```
   1. while (X > 0) {
   2.      Z = Z+Y;
   3.      X = X-1;
   4. }
   ```

   The registers are preloaded with values in them. Submit a circuit diagram showing the building blocks used, their interconnections and any miscellaneous logic required to make them operate together. The design should use an adder and an adder subtractor plus some other building blocks. Hint, use the enable inputs of the registers to control when they latch information.

5. **(8 pts.)** Given three 32-bit registers A,B,PC, design a circuit which adds PC and A (putting the result back into PC) when A is equal to B. Otherwise, add 1 to PC. The contents of A and B are to remain unchanged.

6. **(8 pts.)** Build a circuit that performs the following:

   ```
   for(i=0; i<100; i++)
       total = total + i;
   ```

   Use the counter described in this chapter for the i variable; assume that the counter is initialized to 0. `total` is stored in a register and its initialized to 0. Use a comparator to shut down the counter and put the register in hold when the count value reaches a critical value. Until this critical value is reached the comparator should allow the counter to count and the register to load.

7. **(8 pts.)** Build a circuit that performs the following:

   ```
   for(i=0; i<100; i++)
       total = total + 1;
   ```

8. **(8 pts.)** Design a circuit that can shift (circular to the right) the contents of register X by an amount given in register Y. X is stored in the circular shift register described in this chapter. The solution will require a comparator and a counter.

9. **(8 pts.)** Assume a 32kx8 RAM is full of data. Show the hardware required to realize the following algorithm.

   ```
   for(i=0; i<32767; i++)
       total = total + M[i];
   ```

   Where `M[i]` is the 8-bit word stored at address `i`. Assume the total register is initialized to 0. The i variable should be the output of a counter. Use a comparator to shut down the counter and to put the register in hold when the count value reaches a critical value.

10. **(8 pts.)** Show how to initialize a 32kx8 RAM in the following manner.

    ```
    for(i=0; i<32767; i++)
        M[i] = i mod 256;
    ```

    Where the "i mod 256" statement means store the least significant eight bits of the i variable into the RAM.

11. **(5 pts.)** Complete the timing diagram in Figure 1.13 for a 4-bit arithmetic shift register. Use the control setting from the truth table on page 4.

12. **(5 pts.)** Complete the timing diagram in Figure 1.14 for a 4-bit counter. Use the control setting from the truth table on page 6.

13. **(5 pts.)** Complete the timing waveforms for A_1, A_0, Q_1, Q_0 based on the circuit diagram shown in Figure 1.15. Use the truth table on page 2 for the register. Put the decimal representation of the signals in the timing diagram (like the timing diagram in Figure **??**).

14. **(4 pts.)** The circuit shown in Figure 1.16 generates a Fibonacci sequence, a sequence starting with 1,1,2,3... The next number in the sequence is the sum of the preceding two numbers. Complete the timing diagram, assuming the circuit starts with the values shown. Identify the signal which generates a complete Fibonacci sequence.
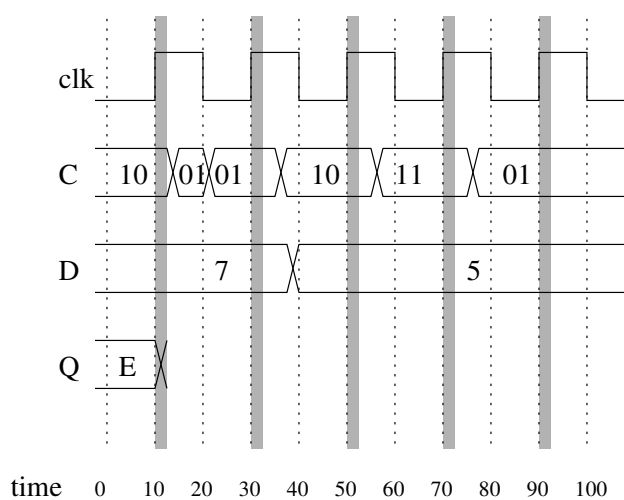
Figure 1.13: The timing diagram for a 4-bit arithmetic shift register in Problem 11.
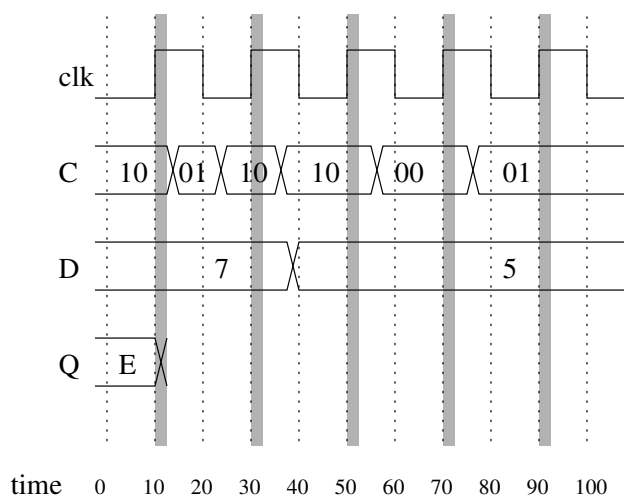

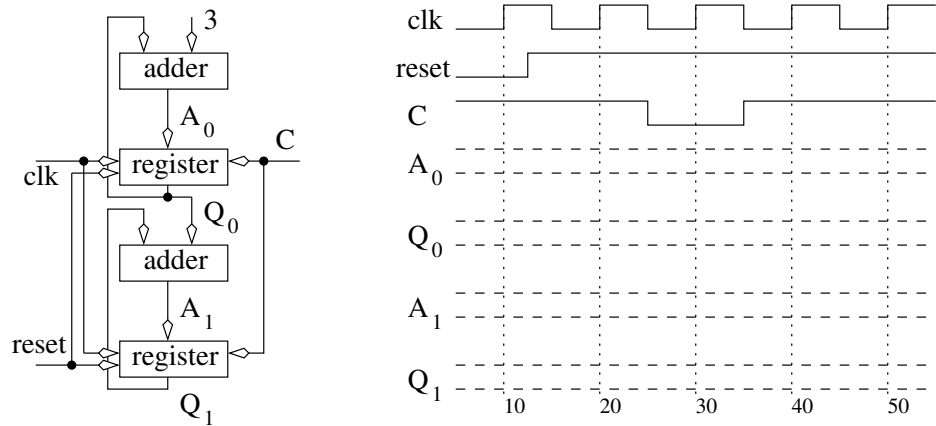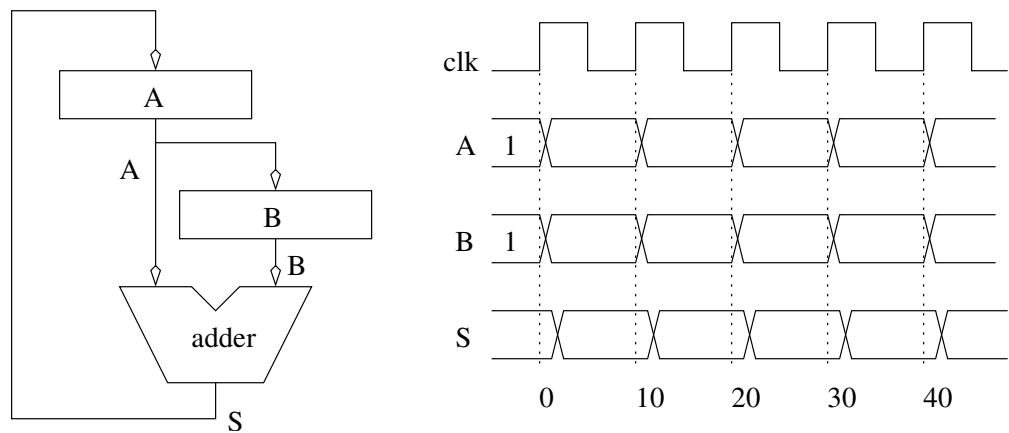Figure 1.14: The timing diagram for a 4-bit counter in Problem 12.

Figure 1.15: The circuit diagram and incomplete timing diagram for Problem 13.

Figure 1.16: A circuit which generates a Fibonacci sequence.

Appendix A

74LS00 Data Sheets

Since the device documentation for the TI chips is covered by TI's copyright it was decided to leave these pages out of this text. The documents discussed in the text can be found onilne at:

http://focus.ti.com/lit/ds/symlink/sn74ls00.pdf

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