(6) Symbolic to Symbolic

Boolean Algebra is a system of rules to manipulate expressions consisting of Boolean Variables, AND, OR and NOT into equivalent forms.

Algebra Ex: $x^2+3x+2 = (x+2)(x+1)$

Law	Primary	Dual
1	$x+0=x$	$x*1=x$
2	$x+1=1$	$x*0=0$
3	$x+x=x$	$x*x=x$
4	$x''=x$	
5	$x+x'=1$	$x*x'=0$
6	$x+y=y+x$	$xy=yx$
7	$x+(y+z)=(x+y)+z$	$x(yz)=(xy)z$
8	$x*(y+z)=xy+xz$	$x+yz=(x+y)(x+z)$
9	$(x+y)'=x'y'$	$(xy)'=x'+y'$

Dual is formed by swapping: AND $\leftrightarrow$ OR, 0 $\leftrightarrow$ 1

Each Law is identified by its number followed by "D" if its a dual.

We will use these Laws to:

- Show 2 expressions are equal
- Simply an expression

Ex: Prove $A + A'B' = A + B'$

$A + A'B'$	=	Law 8D
$(A + A')(A + B')$	=	Law 5
$1\ (A + B')$	=	Law 1D
$A + B'$		QED

Ex: Prove $A(A + B) = A$

$A(A + B)$	=	Law 8
$AA + AB$	=	Law 3
$A + AB$	=	Law 8D
$(A + A)(A + B)$	=	Law 3
$A(A + B)$		

$A(A + B)$	=	Law 8
$AA + AB$	=	Law 3
$A + AB$	=	Law 1D
$A \cdot 1 + AB$	=	Law 8
$A(1 + B)$	=	Law 2
$A \cdot 1$	=	Law 1
A		

"The Bull Dozer" a method for proving or disproving any equivalence btw Boolean expressions.

* The expansion trick = convert a product into the sum of fully populated product terms.
* The sum of two terms is their OR.
* A product term is the AND of variables (or their negation)
  e.g. $A'BC$ is $A'(B+C)$ is not
* The set of variables that appear in an expression is called the population
* A fully populated product term is a product term that contains all the variables (or their negation) in the population.

Ex: Prove $A'B'C' + B'C + ABC = A'B' + AC$ Population = {A, B, C}

$A'B'C' + 1B'C + ABC = A'B'1 + A1C$

$A'B'C' + (A+A')B'C + ABC = A'B'(C+C') + A(B+B')C$

$A'B'C' + AB'C + A'B'C + ABC = A'B'C + A'B'C' + ABC + AB'C$

(3) (1) (4) (2)

$A'B'C + AB'C + A'B'C + ABC$

① Expand to SOP
② Add missing
Law 1D
Law 5
make fully populated product term
③ Eliminate duplicate products
Law 3
④ Rewrite order
Law 6

Proof now flows down then up