

"The Bull Dozer" a method for proving or disproving any equivalence btw Boolean expressions.

\* The expansion trick = convert a product into the sum of fully populated product terms.

\* The sum of two terms is their OR.

\* A product term is the AND of variables (or their negation)  
e.g.  $A'BC$  is  $A'(B+C)$  is not

\* The set of variables that appear in an expression is called the population.

\* A fully populated product term is a product term that contains all the variables (or their negation) in the population.

Ex: Prove  $A'B'C' + B'C + ABC$  =  $A'B' + AC$       Population =  $\{A, B, C\}$

$$A'B'C' + 1B'C + ABC = A'B'1 + A1C$$

$$A'B'C' + (A+A')B'C + ABC = A'B'(C+C') + A(B+B')C$$

$$A'B'C' + AB'C + A'B'C + ABC = \underbrace{A'B'C'}_{(3)} + \underbrace{AB'C}_{(1)} + \underbrace{ABC}_{(4)} + \underbrace{A'B'C}_{(2)}$$

$$A'B'C + AB'C + A'B'C + ABC$$

Proof now flows down then up

① Expand to SOP

② Add missing

Law 10

Law 5

make fully populated product term

③ Eliminate duplicate products  
Law 3

④ Rewrite order  
Law 6

Exception when multiplying out is a big pain  
 Rule

$$(A+B)(A'+B)(B+C) = B$$

$$((A+B)(A'+B)(B+C))'' = (B')'$$

$$((A+B)' + (A'+B) + (B+C)')' = \left( \underbrace{AB'C}_{(3)} + \underbrace{AB'C'}_{(4)} + \underbrace{A'B'C}_{(1)} + \underbrace{A'B'C'}_{(2)} \right)'$$

$$(A'B' + AB' + B'C')'$$

$$(A'B'C + \underline{A'B'C'} + \underline{ABC} + \underline{AB'C'} + \cancel{AB'C'} + \cancel{A'B'C'})'$$